AUTUMN MEETING 2026
BRAZILIAN PHYSICAL SOCIETY

May 17 - 21
Cuiabá, MT - Brazil

# Nonlinear data analysis tools for complex systems research

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AGAUR

GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
UNITED STATES AIR FORCE

## Presentation

- Bachelor and Master in Physics in Universidad de la República, Uruguay (1986-1991).
- PhD in Laser Physics (1999, Bryn Mawr College, PA, USA).
- Universitat Politecnica de Catalunya (2004 –to date; Prof. Catedratica since 2018).

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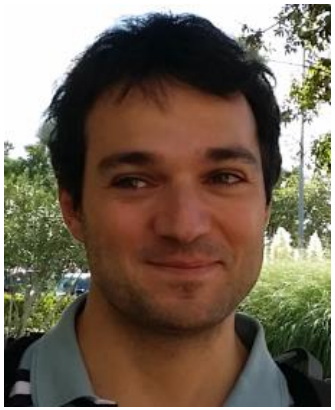
# Doers, Mentors and Adviser

Maria Masoliver



Dario Zappala

Maria Duque-Gijon

Juan Gancio

Bruno Boaretto
*Universidade Federal de Sao Paulo, Brazil*

Jordi Tiana
*Universitat de Barcelona*

Giulio Tirabassi
*Universitat de Girona*

Marcelo Barreiro
*Universidad de la República, Uruguay*



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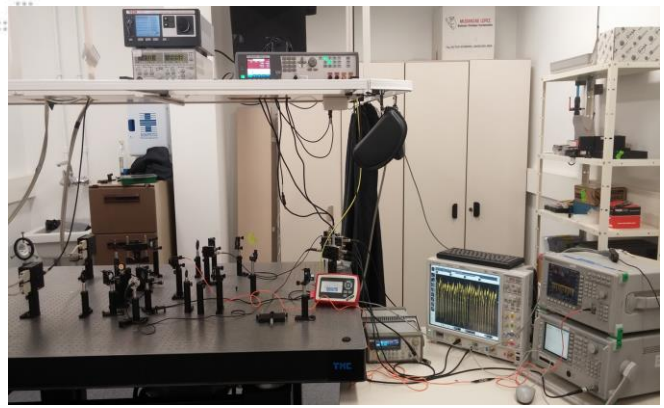
# Where are we? Universitat Politècnica de Catalunya Campus Terrassa





Semiconductor laser lab



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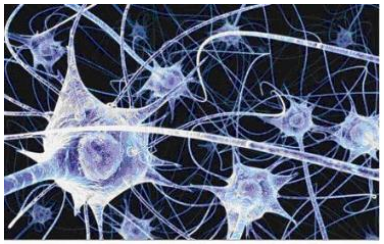
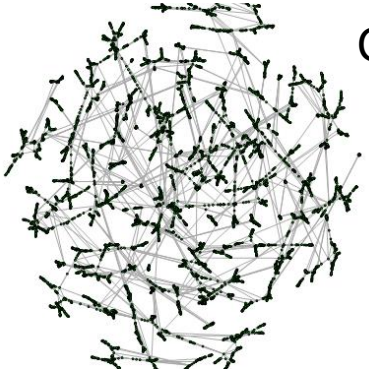
# Research lines

Dynamics of semiconductor lasers

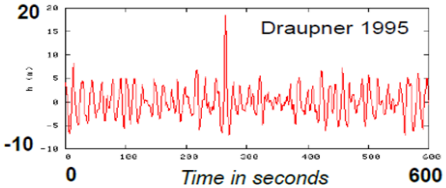


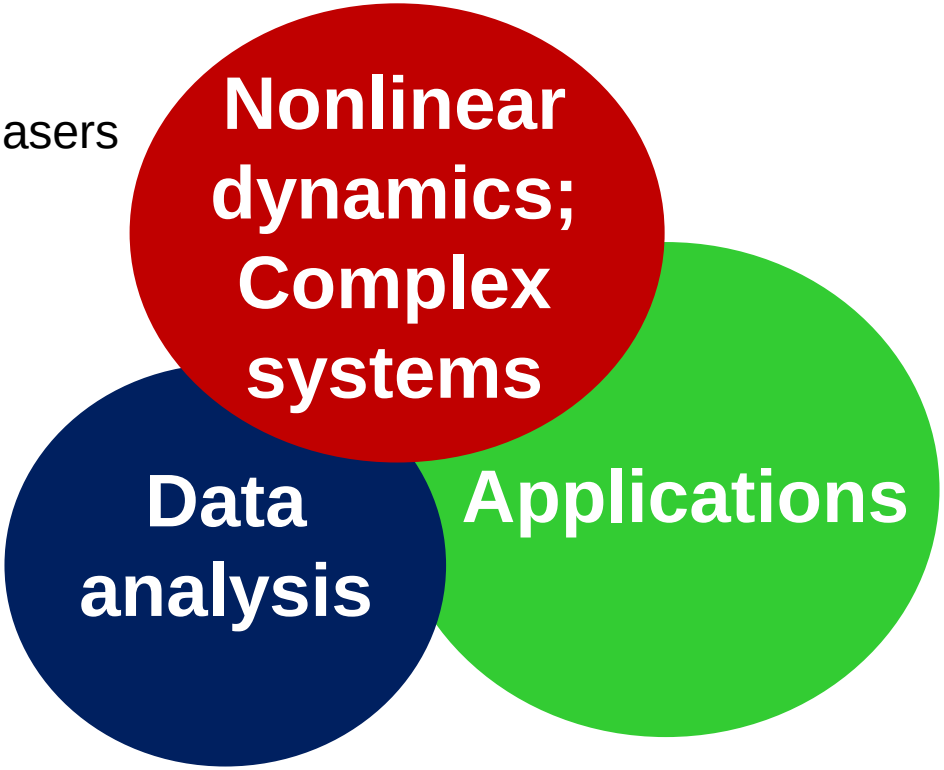
Neuronal dynamics



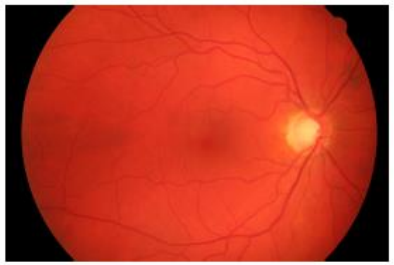
Complex networks

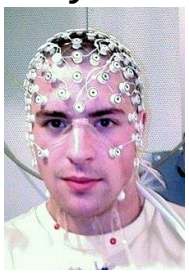
Extreme events

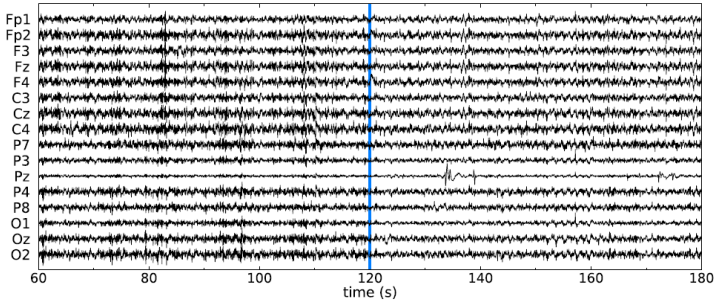


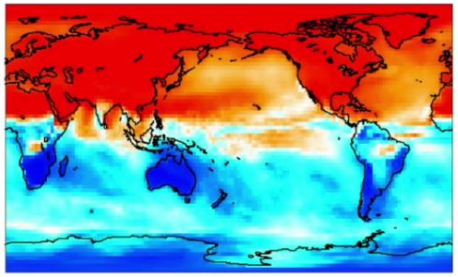

Biomedical data analysis








Climate data analysis





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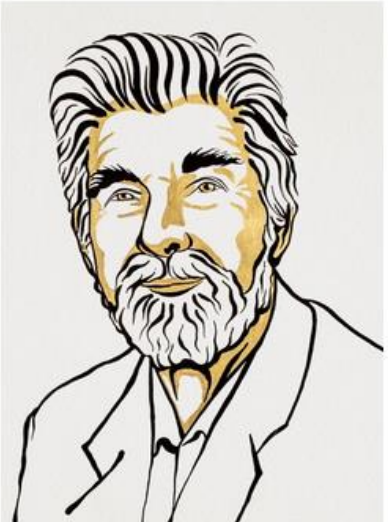
# The Nobel Prize in Physics 2021


for groundbreaking contributions to our understanding of **complex systems**

½ Syukuro Manabe and Klaus Hasselmann "*for the physical modelling of Earth's climate, quantifying variability and reliably predicting global warming*"

½ Giorgio Parisi "*for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales*."

# What is a complex system?



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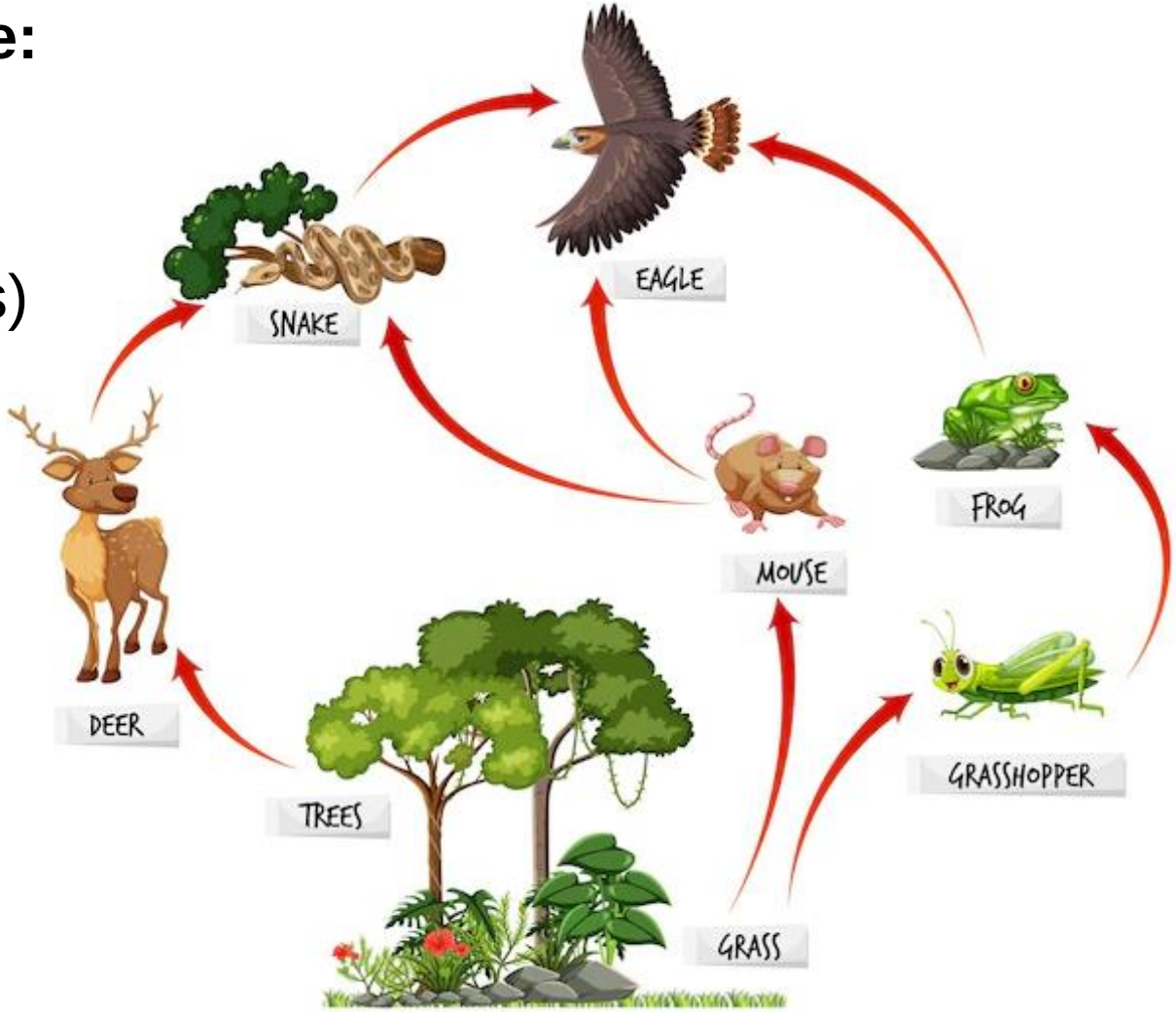
## Complex systems are:

- High-dimensional (huge number of variables or elements)
- Nonlinear (the elements and/or the interactions are nonlinear)
- Heterogeneous, multiscale, have memory and adapt
- Often modeled as networks / graphs



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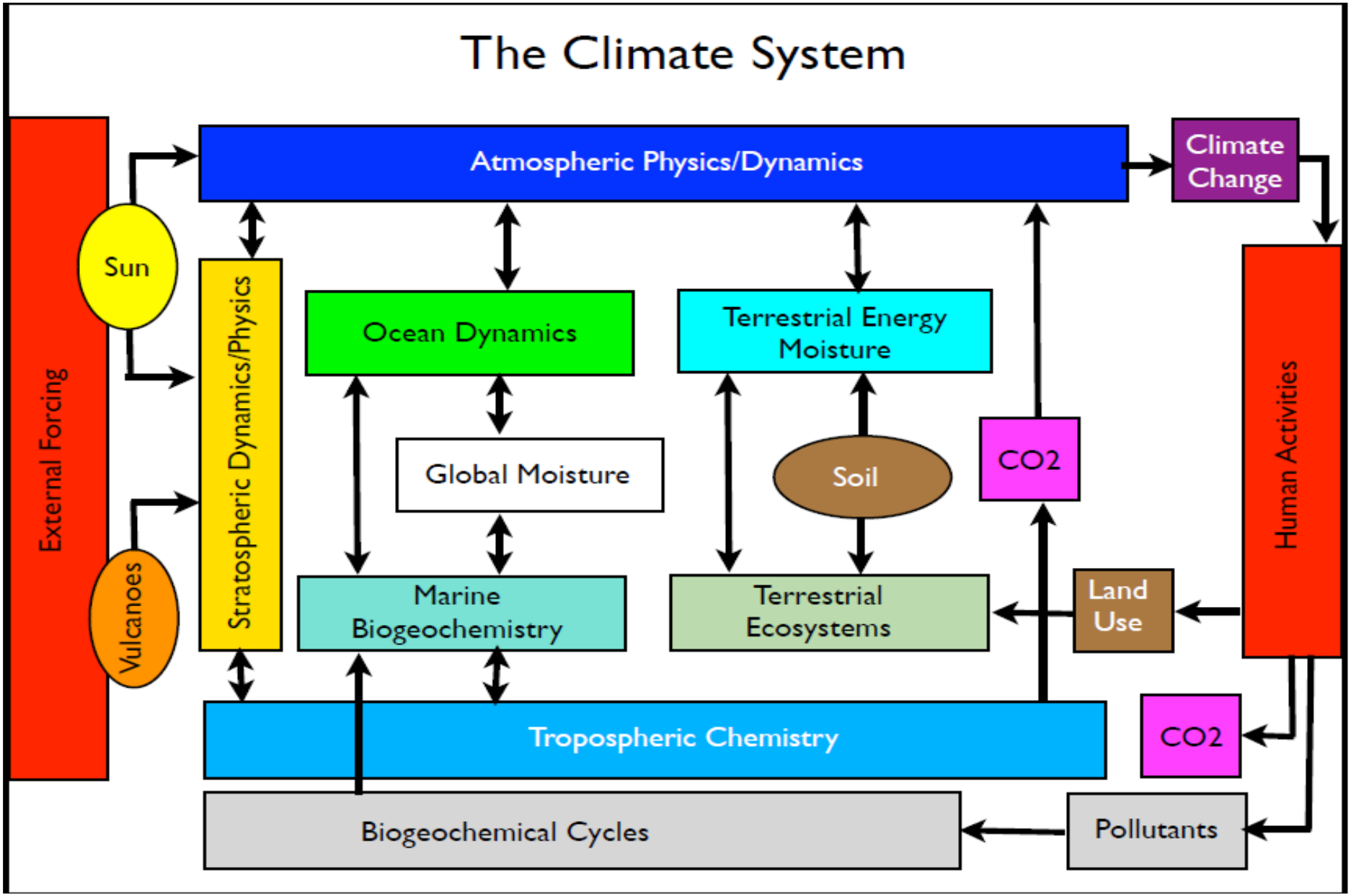


*Courtesy of Henk Dijkstra (Universidad de Ultrech)*



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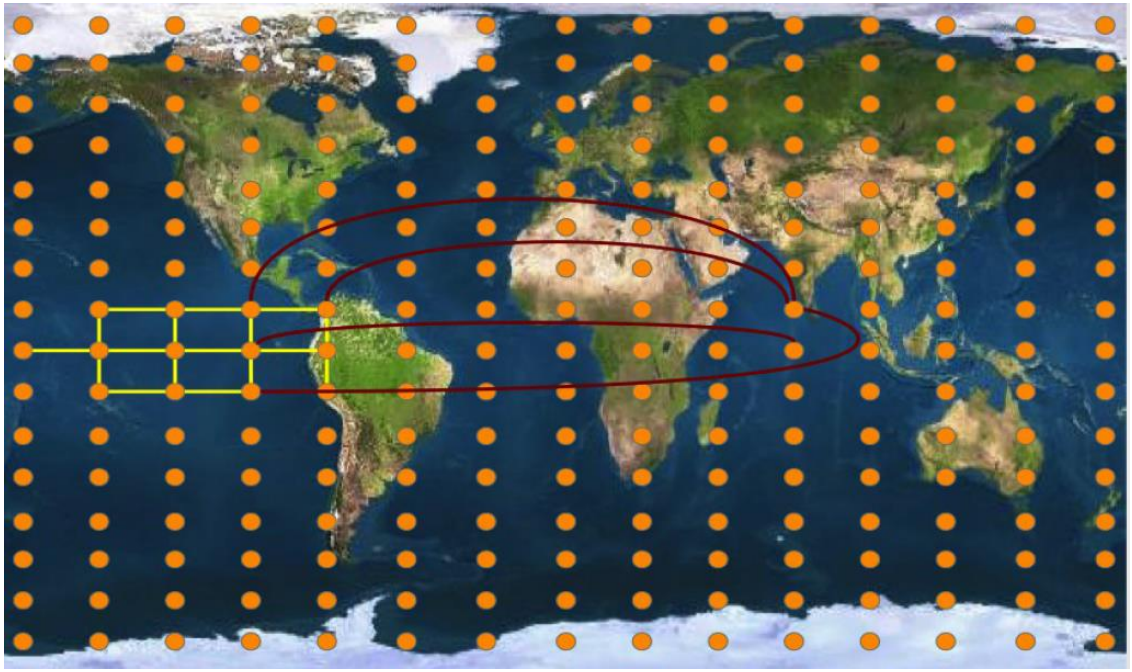
# The analysis of interdependencies and causal interactions uncovers long-range tele-connections



H. A. Dijkstra, E. Hernandez-Garcia, C. Masoller and M. Barreiro, "*Networks in Climate*", Cambridge University Press 2019



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# Complex systems usually display:

## 1. Emergent synchronized behavior

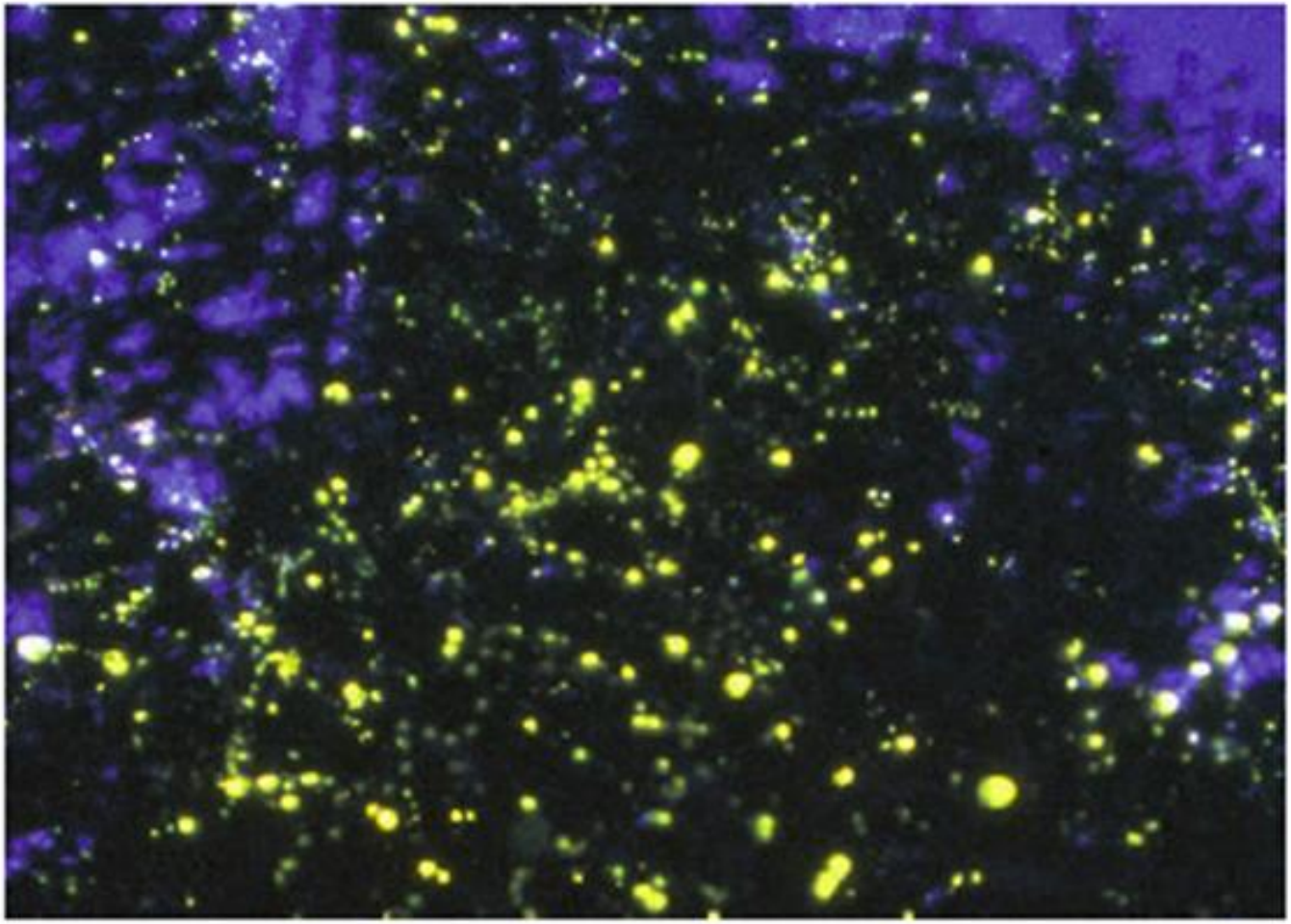

**Figure 1 | Fireflies, fireflies burning bright.** In the forests of the night,



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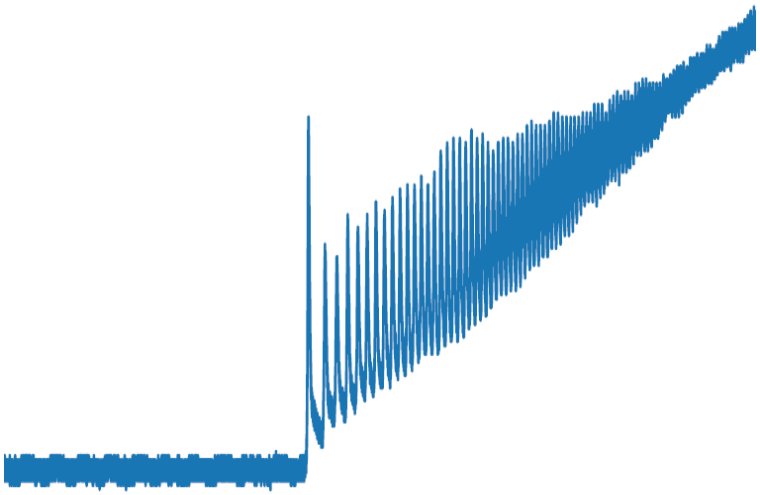
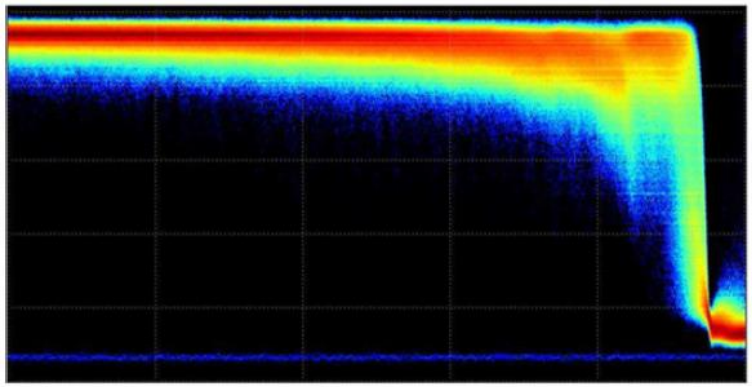
# Complex systems display:

## 2. Transitions to different regimes



Time



Time



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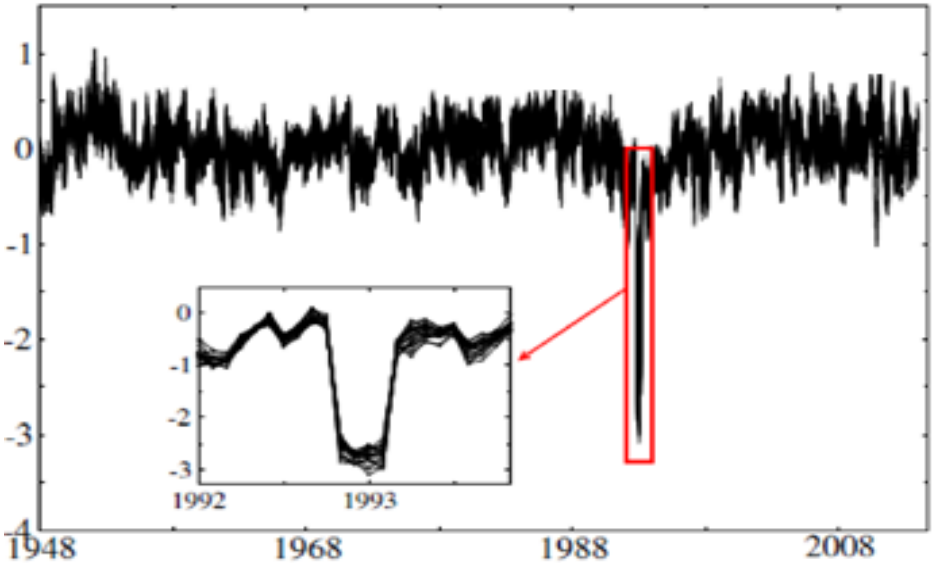
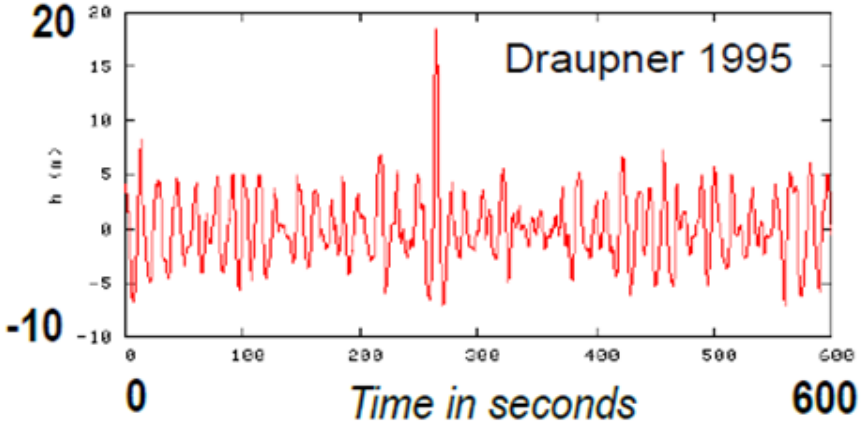
# Complex systems typically display:

## 3. Extreme Events




What is NOT, in my opinion, a complex system:

A linear system (no matter how big).

A low dimensional system.



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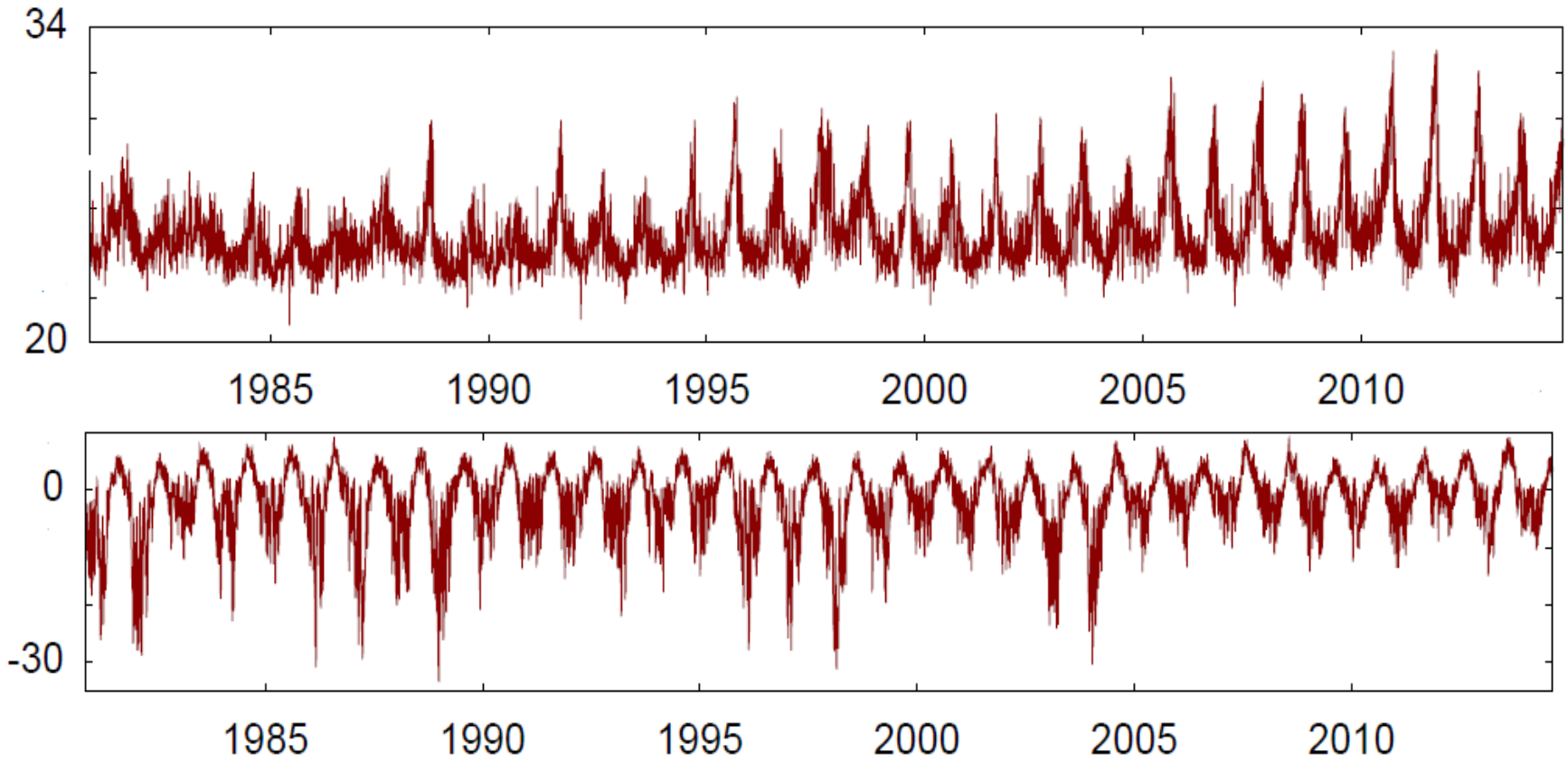
# Sometimes changes occur gradually

Surface Air Temperature (SAT) in two geographical regions




**How to quantify the differences?**

D. A. Zappala, M. Barreiro and C. Masoller, Earth Syst. Dynam. 9, 383 (2018)



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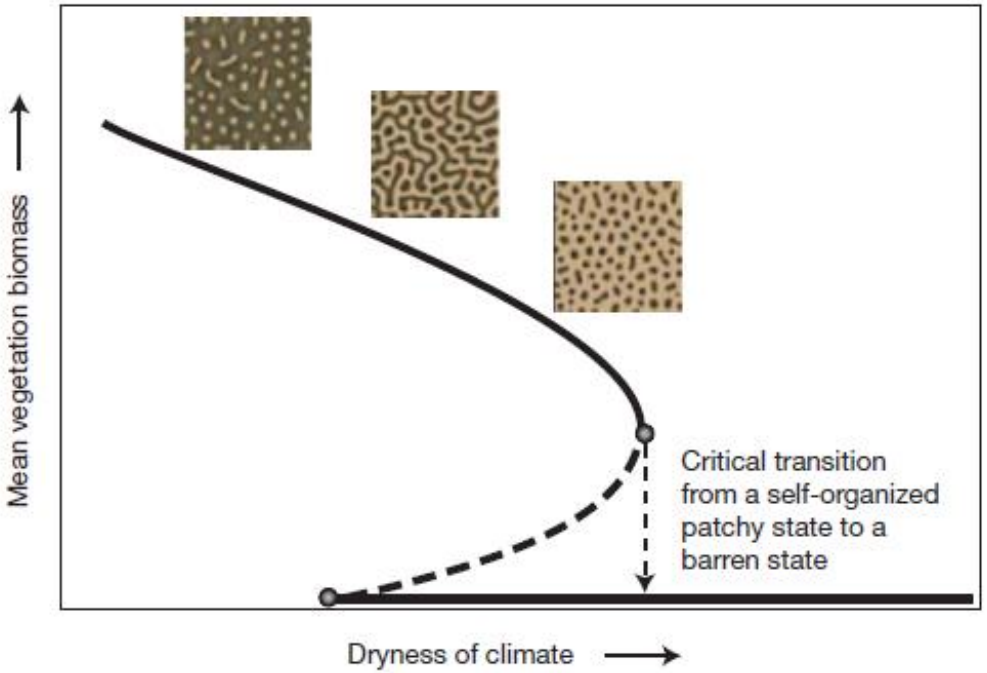
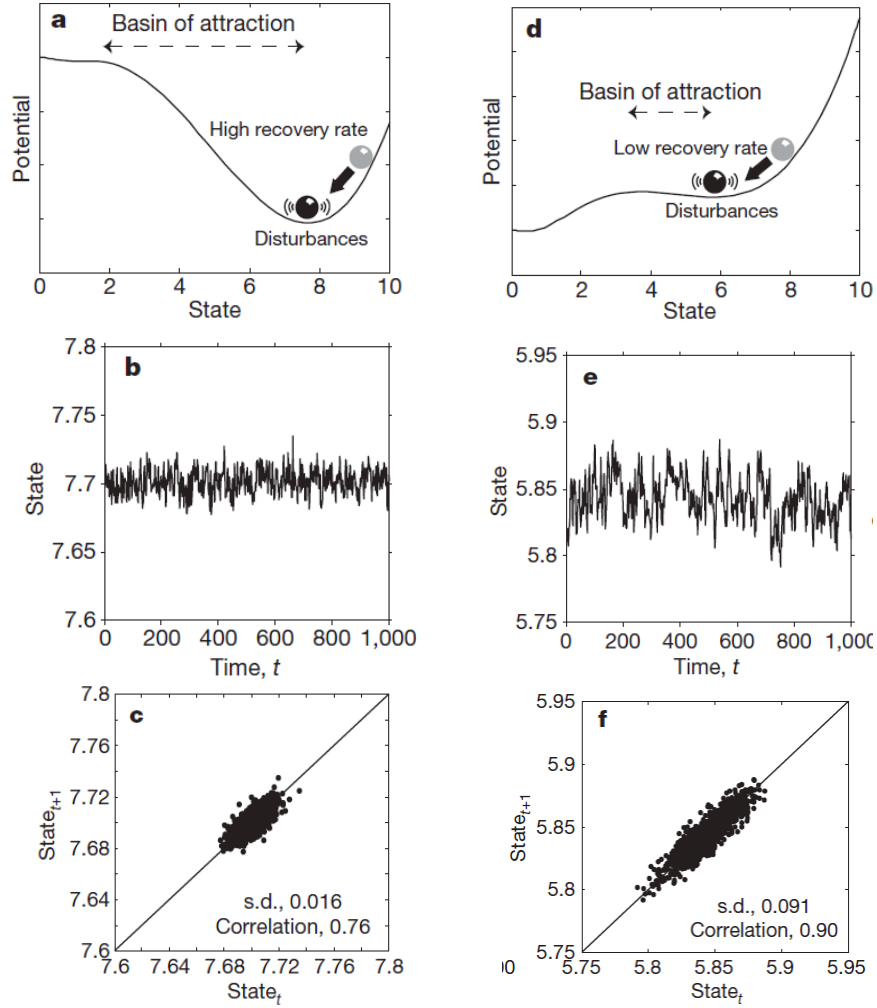
# But some critical transitions can occur abruptly




*M. Scheffer et al., Nature 461, 53 (2009)*

Far from bifurcation | Close to bif.




$\Rightarrow$ increase of variance and autocorrelation (*critical slowing down*)



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**Oceans**

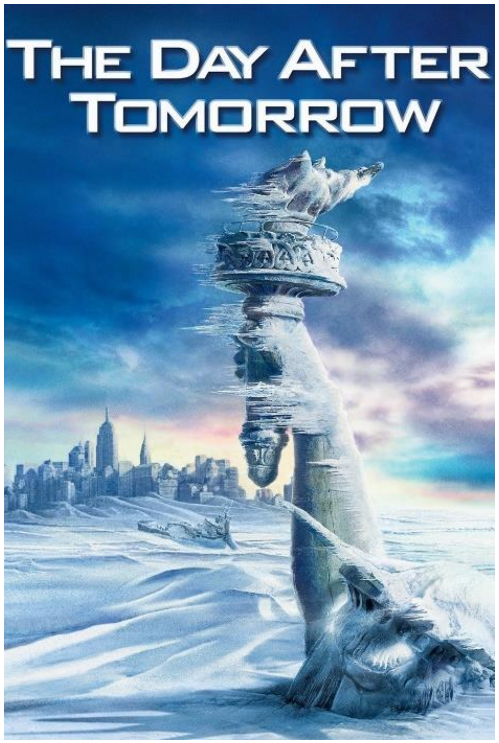
The Guardian

# Atlantic Ocean circulation nearing 'devastating' tipping point, study finds

**Collapse in system of currents that helps regulate global climate would be at such speed that adaptation would be impossible**

**Jonathan Watts**

@jonathanwatts
Fri 9 Feb 2024 20.00 CET




Sea levels in the Atlantic would rise by a metre in some regions and temperatures around the world would fluctuate far more erratically. Photograph: Henrik Egede-Lassen/Zoomedia/PA

The circulation of the Atlantic Ocean is heading towards a tipping point that is "bad news for the climate system and humanity", a study has found.



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# Data-based “early warning indicators” are crucial for successful adaptation/mitigation strategies

Global Climate Risks

ISSUES RESOURCES SOLUTIONS INSIGHTS LOCATIONS HUBS

→ Insights → Blog → Iceland Flags AMOC a National Security Threat

GLOBAL RISKS POLAR CHANGE

## Iceland Flags AMOC a National Security Threat

NOVEMBER 28, 2025 EUROPE ▲ ICELAND

3 min read

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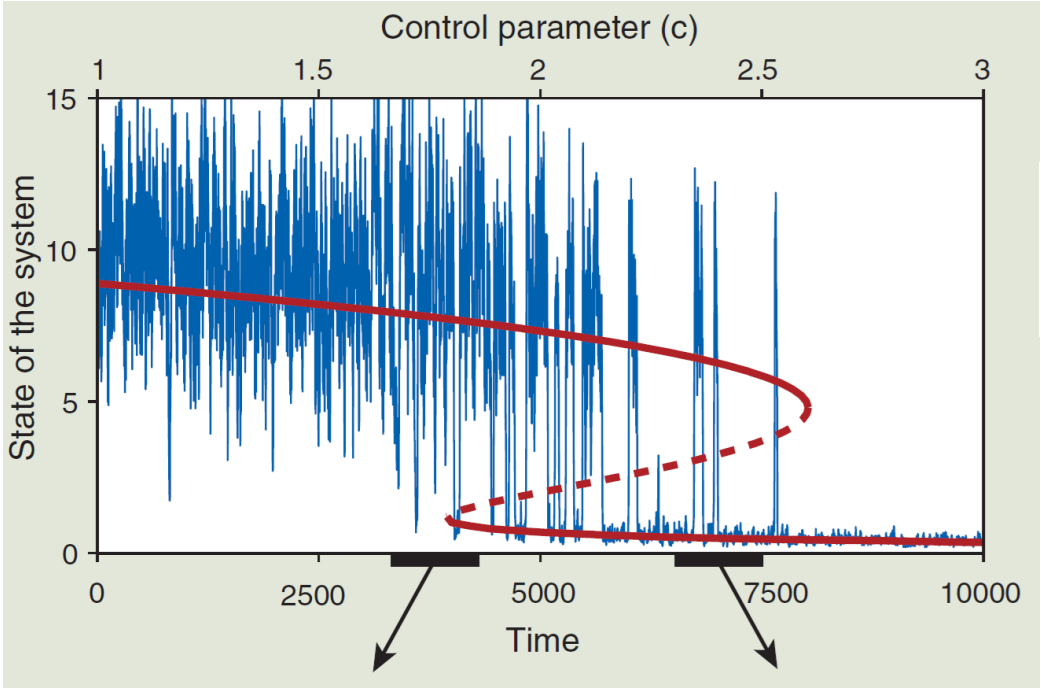
# Are climatic extremes becoming more extreme? More rare?




*M. Scheffer et al., Science 338, 344 (2012)*

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**Extreme events in the ocean: Rogue Waves**

Huge "*waves that appear from nowhere & disappear without a trace".*
Risk for oil platforms, offshore wind farms and traffic.

*N. Akhmediev et al., Phys. Letts. A 2009*

There are scary videos on internet.



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Science & Environment

# Rogue waves occurring less but 'becoming more extreme'

By Rebecca Morelle
Science Correspondent, BBC News

21 March 2019

Share

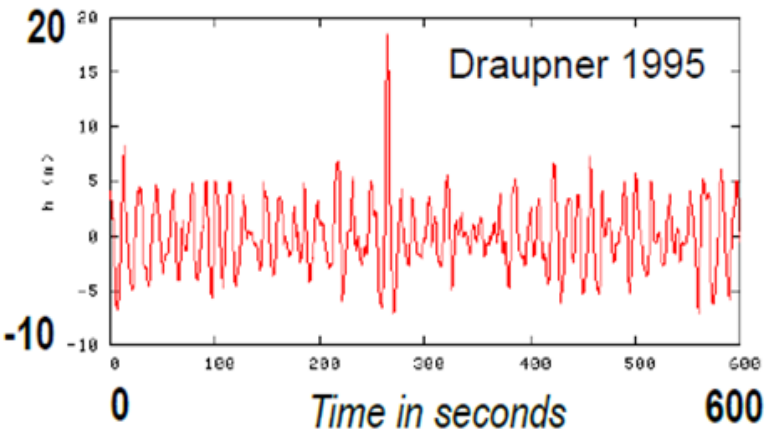

Rogue waves are a growing threat for the global shipping industry

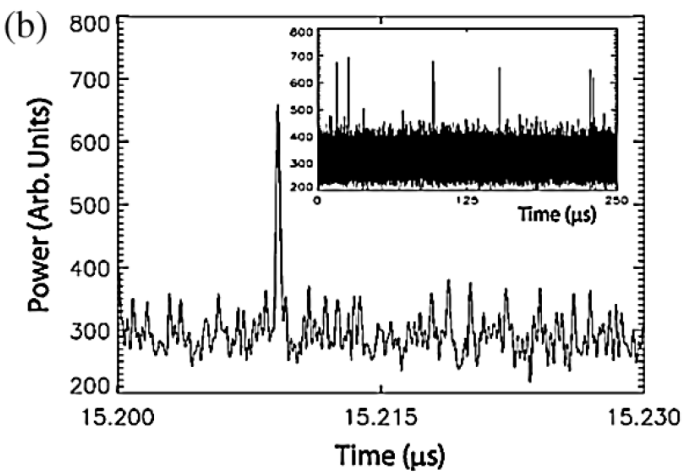
# Controlled experiments with complex systems are rare

**The power of analogies** NATURE PHOTONICS | VOL 8 | JANUARY 2014 |

Black holes, gravitational lenses, turbulence, chaotic flow and rogue waves are just a few examples of complex physical phenomena that can be conveniently modelled using photonics.

Time scales of photonic systems allow recording long data sets in short times.




*C. Bonatto, M. Feyereisen, S. Barland, M. Giudici, C. Masoller, J. R. Rios Leite, J. R. Tredicce, Phys. Rev. Lett. 107, 053901 (2011).*



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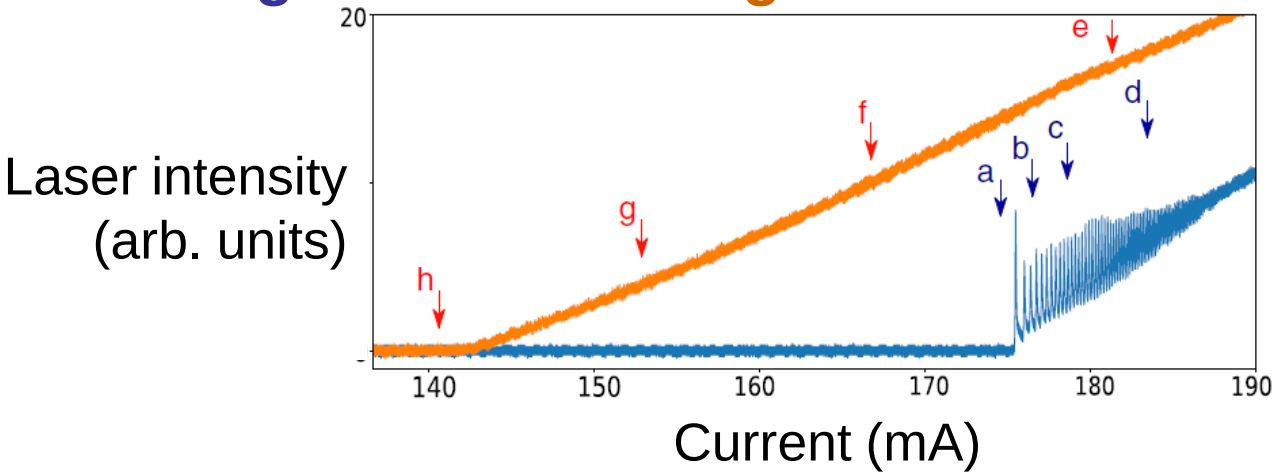
# *Critical slowing down* (a classical early warning indicator) can occur **after** the bifurcation

PHYSICAL REVIEW LETTERS **125,** 134102 (2020)

**Testing Critical Slowing Down as a Bifurcation Indicator in a Low-Dissipation Dynamical System**

M. Marconi,[1] C. Métayer,[2] A. Acquaviva,[1] J. M. Boyer,[2] A. Gomel,[3]
T. Quiniou,[2] C. Masoller,[4,*] M. Giudici,[1] and J. R. Tredicce[2,3]
[1]*Université Côte d' Azur, Institut de Physique de Nice, CNRS-UMR 7010, Sophia Antipolis, France*
[2]*Université de la Nouvelle Calédonie, ISEA, BP R4-98851 Nouméa Cedex, Nouvelle Calédonie*
[3]*Universidad de Buenos Aires, Departamento de Física, Intendente Guiraldes 2160, CABA, Buenos Aires, Argentina*
[4]*Departamento de Física, Universitat Politecnica de Catalunya, St Nebridi 22, Barcelona 08222, Spain*

**Increasing** and **Decreasing** the laser current



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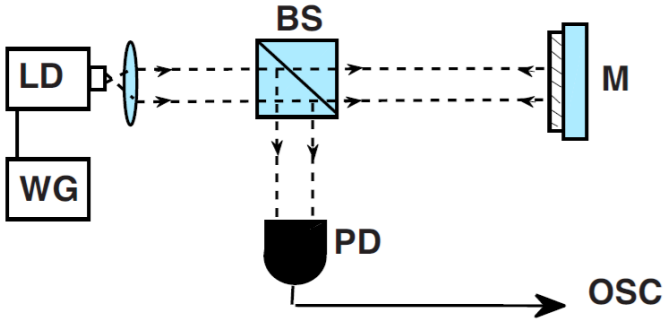
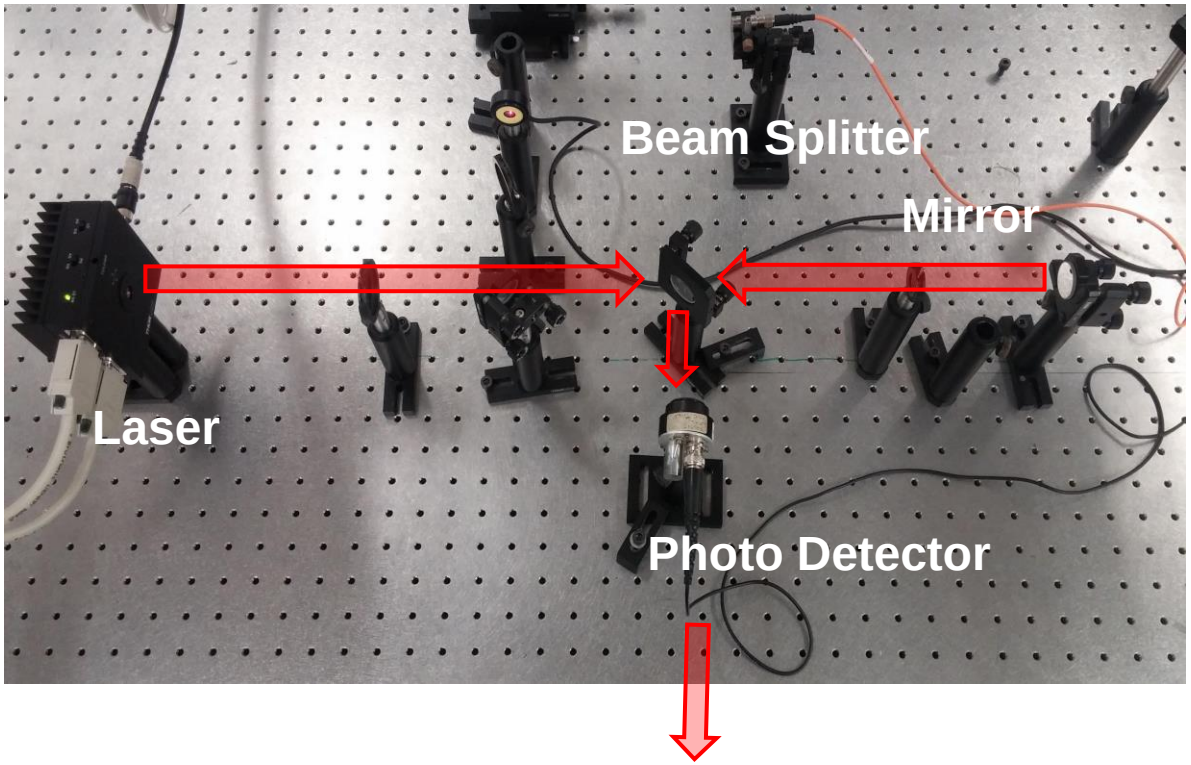
# In our lab: experiments with diode lasers with optical feedback




WG: wave generator used to modulate the laser current.




Oscilloscope

**What do we see in the oscilloscope?**
**(with 1 GHz resolution)**



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# How optical noise (quantum spontaneous emission) gradually turns into a complex signal?

## Quantitative identification of dynamical transitions in a semiconductor laser with optical feedback

Carlos Quintero, Jordi Tiana-Alsina, Jordi Roma,
M. Carme Torrent, and Cristina Masoller.

Grup de Recerca en Dinàmica No Lineal, Òptica No Lineal i Làsers
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Dinàmica i Òptica No Lineal i Làsers (DONLL)
Dept. Física, Terrassa, Barcelona, Spain



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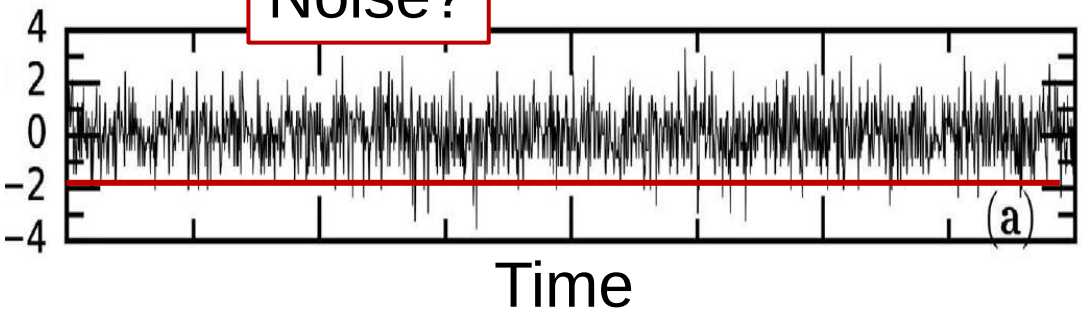
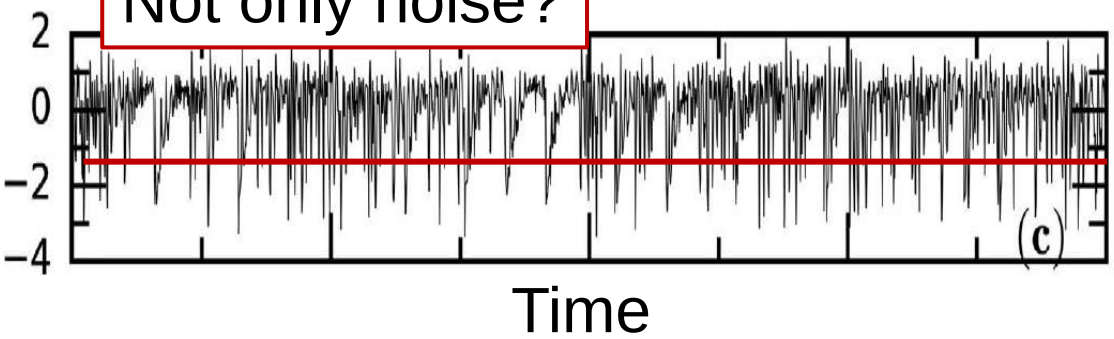
# By counting the number of "threshold crossings", we can distinguish dynamical regimes

$$Z = \frac{X - \mu}{\sigma}$$

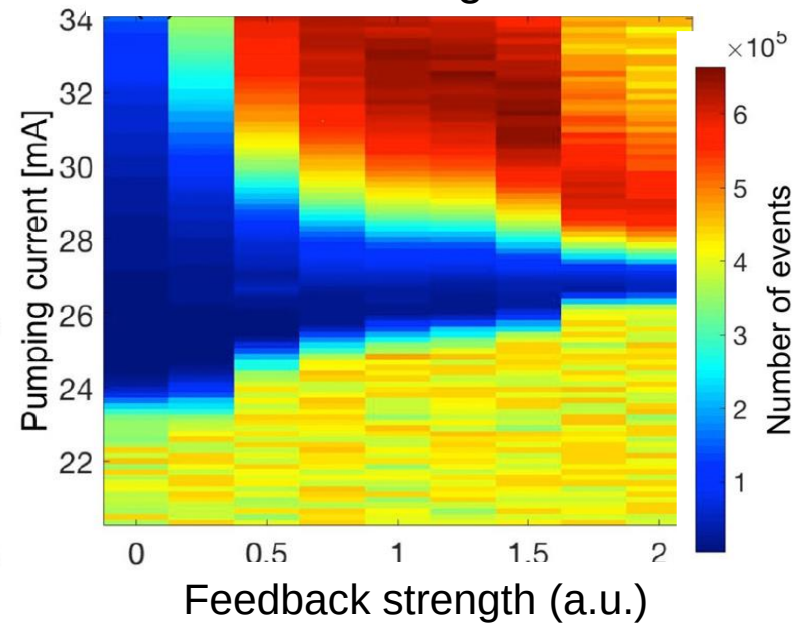
Noise?



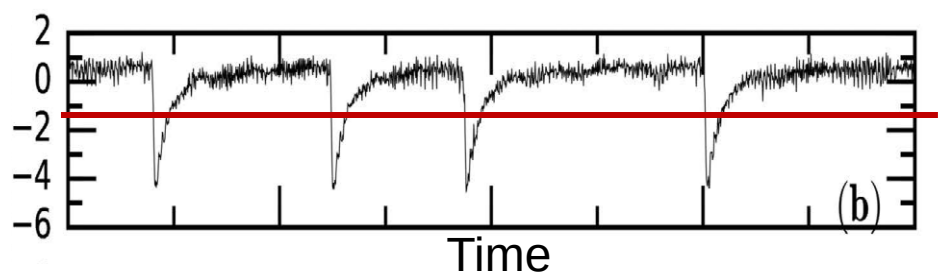

Not only noise?




Number of crossings below -1.5σ in log scale




Blue region? Optical Spikes




M. Panozzo, C. Quintero, J. Tiana-Alsina, M.C. Torrent, C. Masoller, Chaos 27, 114315 (2017).



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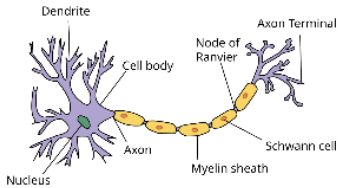
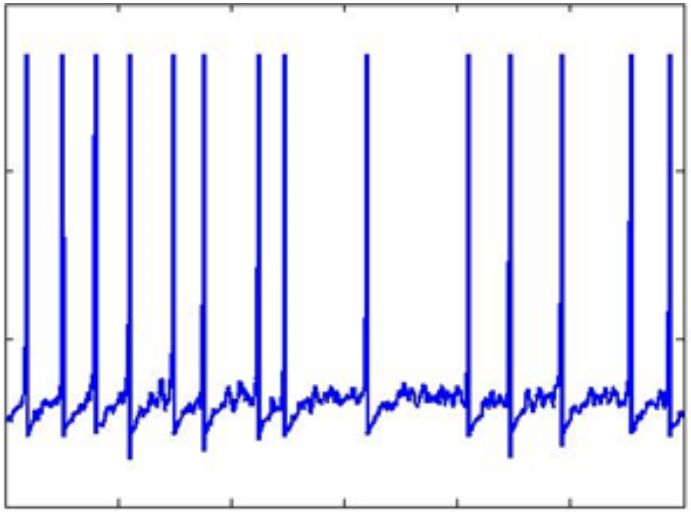
# Intensity emitted by a diode laser and simulated neuronal spikes





Time $10^{-9}$ s



Time $10^{-3}$ s

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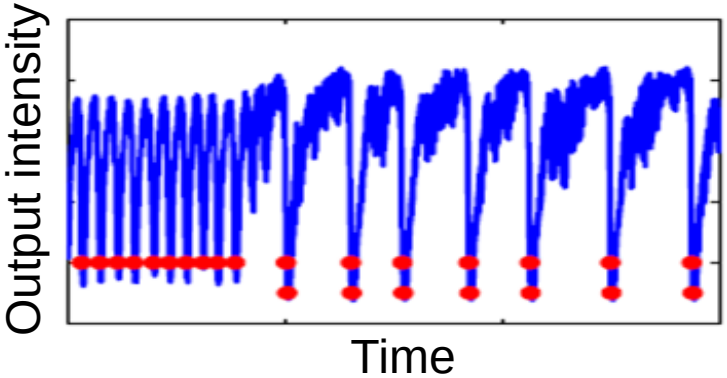
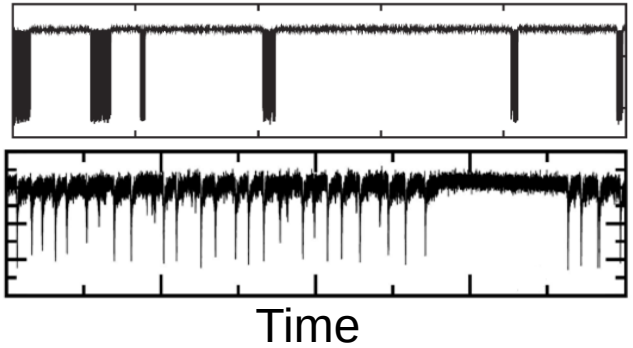
# Uncovering similarities between neuron and lasers… Interesting but relevant?

- Data centers, AI and HPC systems consume a lot of power.
- Big concern in the context of climate change.
- The human brain works with only 20 Watts.
- **Diode laser-based neurons** should work as neurons, but
  - Much faster
  - With much less energy consumption

Some time series recorded in our lab resemble neuronal activity




**Statistically similar to neurons?**

A. Aragoneses, S. Perrone, T. Sorrentino, M. C. Torrent, C. Masoller, Sci. Rep. **4**, 4696 (2014).
C. Quintero-Quiroz, J. Tiana-Alsina, J. Roma, M. C. Torrent, C. Masoller, Sci. Rep. **6** 37510 (2016).



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# Why using diode lasers for implementing photonic neurons?

- Widely used in datacom and telecom systems.
- Inexpensive, compact & energy-efficient.
- Micro and Nano lasers: ultra low power & large arrays can be integrated on chips (photonics integrated chips =PICs).
- There are several configurations in which diode lasers emit well-defined pulses of light ("optical spikes").



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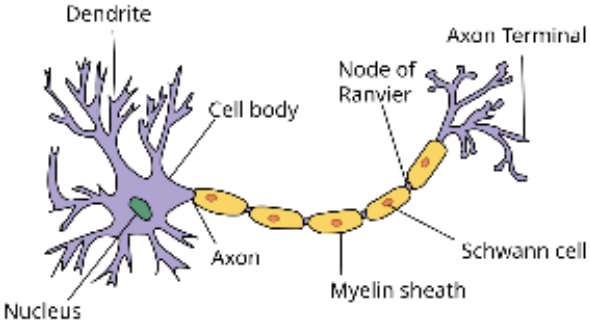
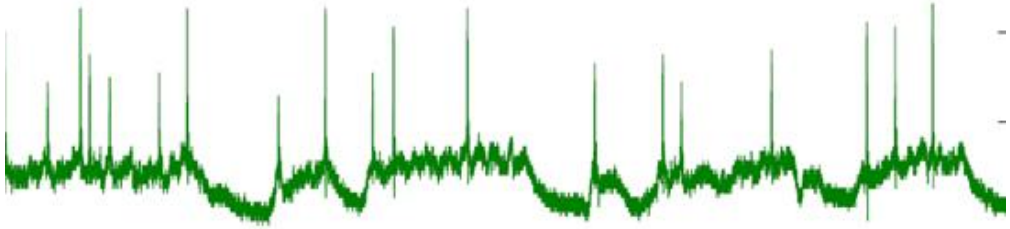
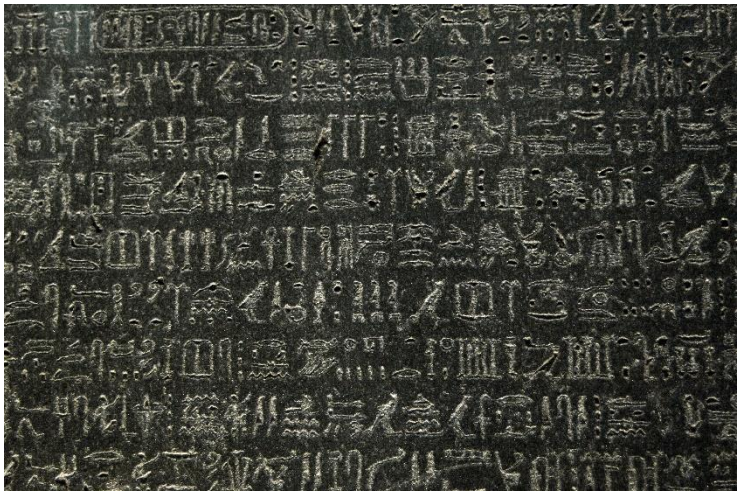
# How neurons encode information?




Intracellular recording



We need to decipher the neural coding mechanisms.



And use them in spiking diode lasers, for processing information.

Goal: **photonic spiking neural networks** that use neural coding mechanisms.

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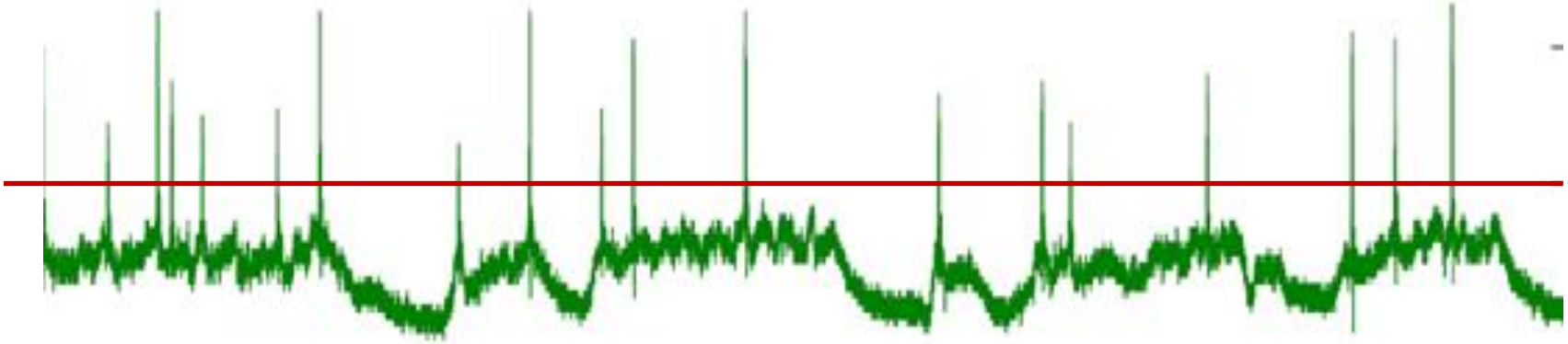
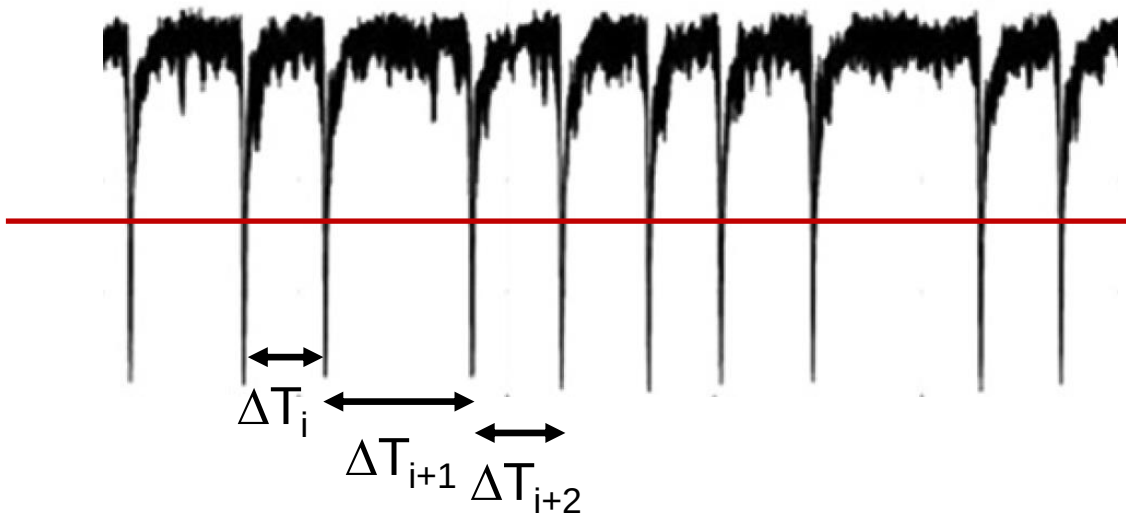
# Thresholding detects the spike times ⇒ Point Process ⇒ sequence of inter-spike-intervals (ISIs)

Intracellular recording






Laser output seen in 1 GHz oscilloscope

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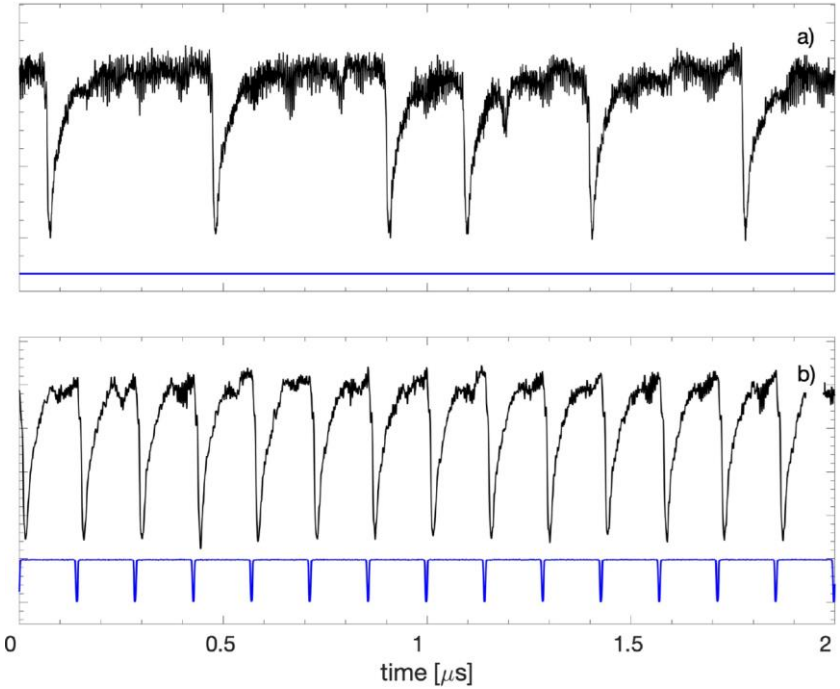
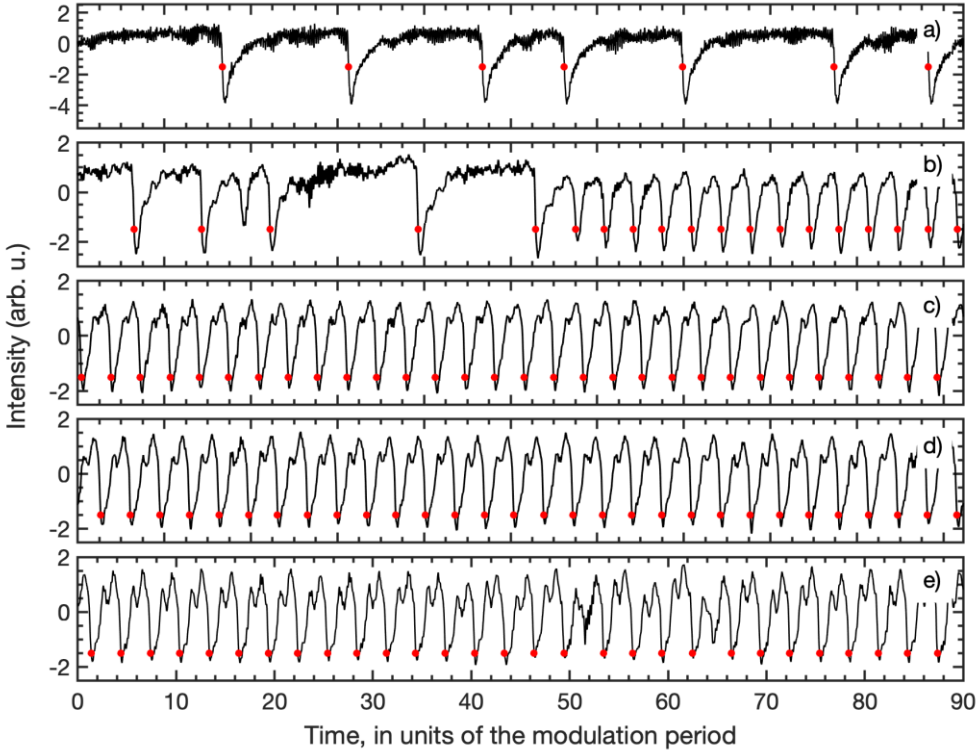
# What happens when a small-amplitude modulation ("external input") is added to the laser current?

**Input: pump current**

Output: laser intensity




Sinusoidal modulation, increasing amplitude




J. Tiana-Alsina, C. Masoller, Sci. Rep. 12, 4914 (2022); Optics Express 30, 9441 (2022).



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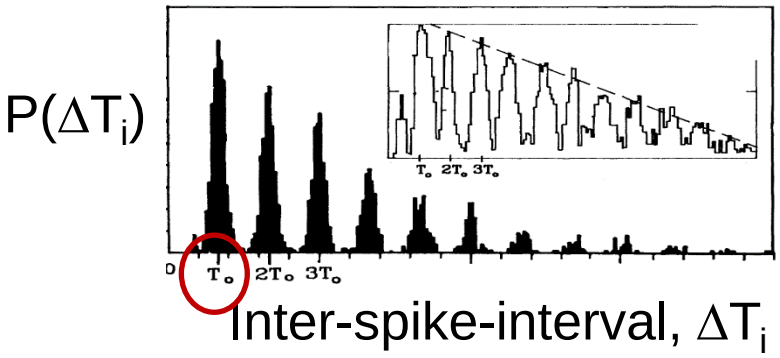
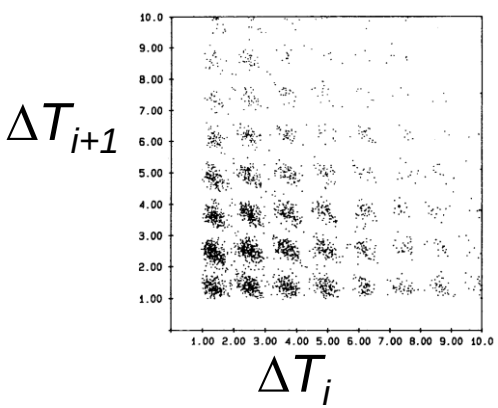
# Neuronal inter-spike-intervals (ISI) distribution

Spikes in a monkey's auditory nerve when the monkey hears a pure tone sound.



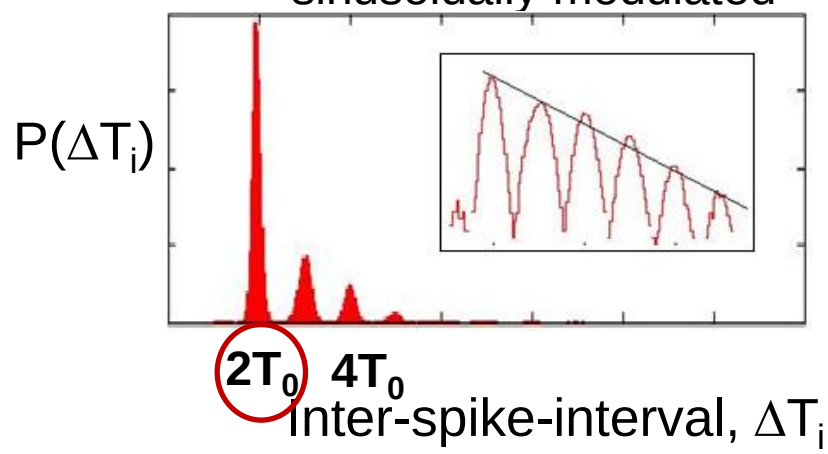

Spikes in a cat's auditory nerve when the cat hears a pure tone sound.

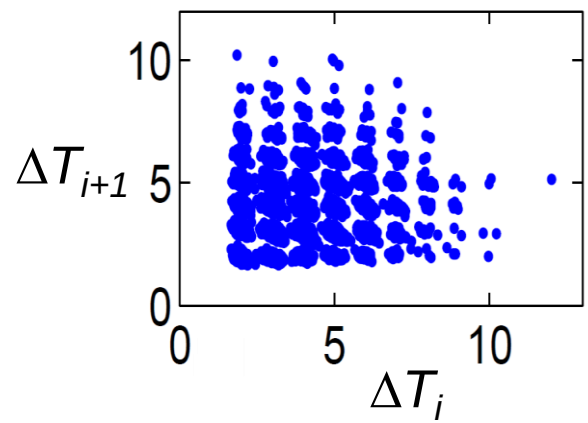



A. Longtin et al. PRL (1991), IJBC (1993).

# Diode laser ISI distribution

when the laser current is sinusoidally modulated




A. Aragoneses, T. Sorrentino, S. Perrone, D.J. Gauthier, M.C. Torrent, C. Masoller Opt. Express (2014).

**Similar ISI sequences? How to quantify?**



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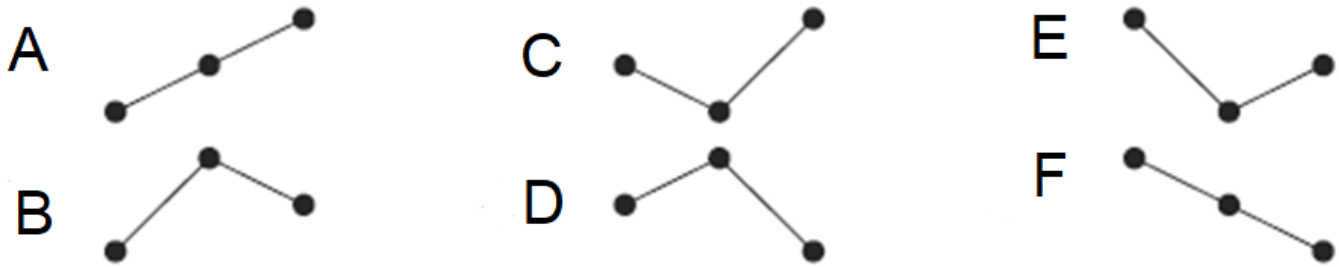
# First time series analysis method: ordinal analysis

$$\{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$$

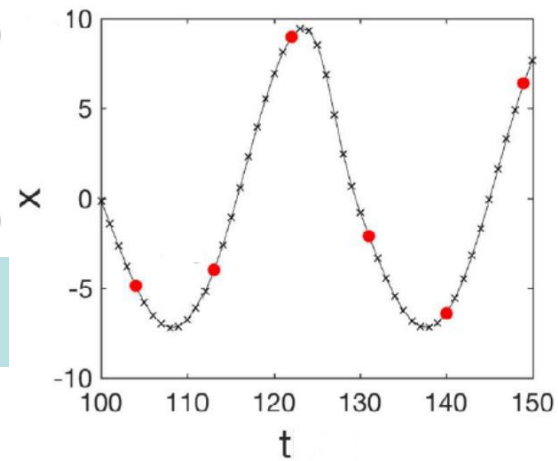
Possible order relations among three numbers (e.g., 2, 5, 7)?

| | | |
|---|---|---|
| {…2, 5, 7…} | {…5, 2, 7…} | {…7, 2, 5 …} |

**Which is the code?**




| | | |
|---|---|---|
| {…2, 7, 5…} | {…5, 7, 2…} | {…7, 5, 2… |




A B F C

C. Bandt and B. Pompe, Phys. Rev. Lett. 88, 174102 (2002).



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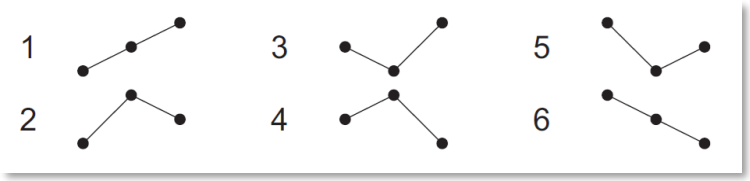
# Drawbacks of ordinal analysis:

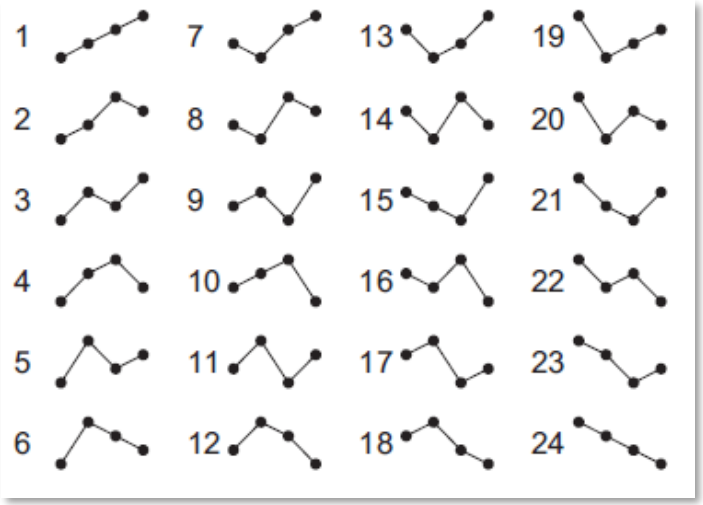
1. Information about the data values is lost.
(5,7,2) and (5,70,2) are both represented by the same symbol.

2. The number of possible patterns grows with the length (D) of the pattern as D!

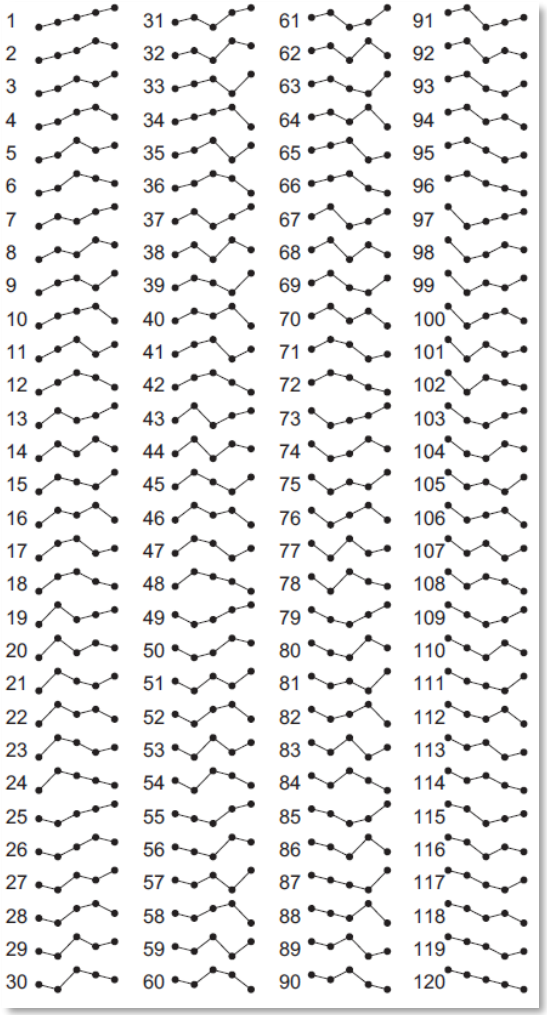
D=3




D=4




D=5




*U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327*



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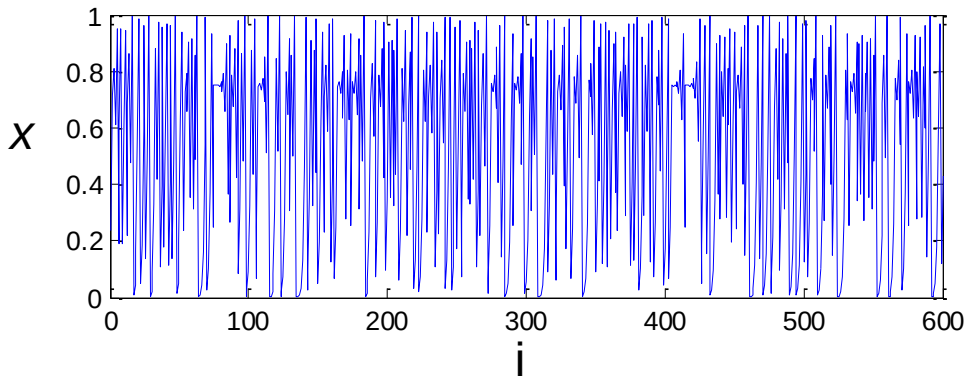
# Example: chaotic time series generated with the Logistic map

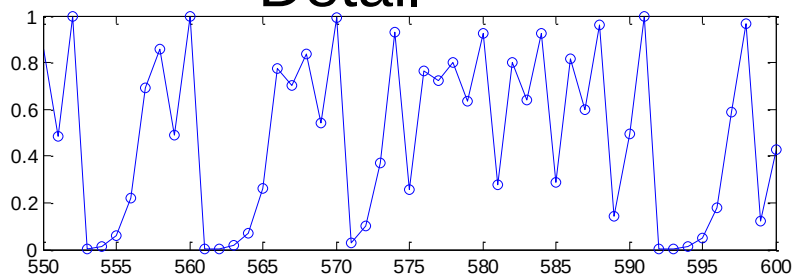
$$x(i+1) = r\, x(i)[1 - x(i)]$$

$x(1) \in (0,1)$

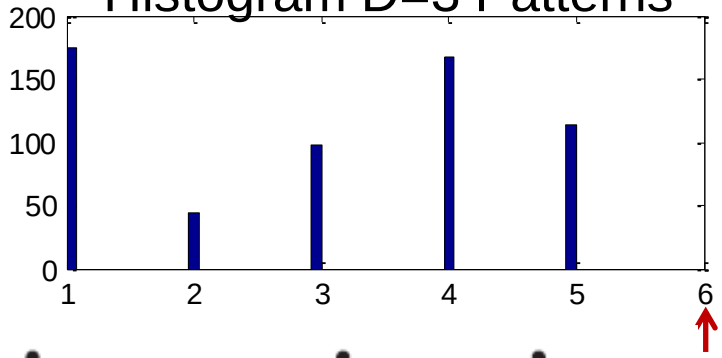
r=3.99



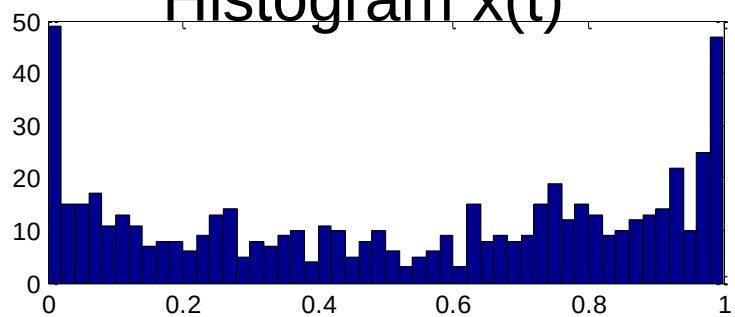

Detail

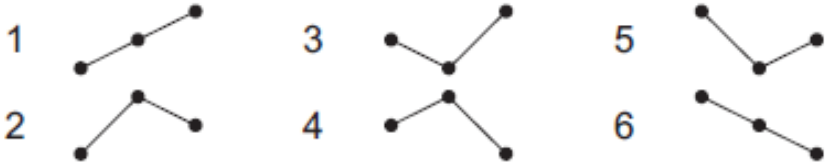



Histogram D=3 Patterns




Histogram x(t)




**forbidden**



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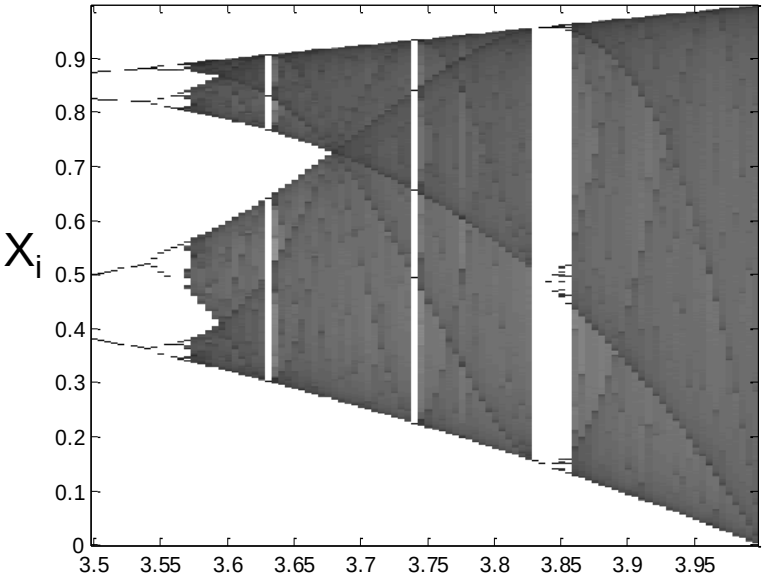
# “Normal” and “Ordinal” bifurcation diagrams of the Logistic map

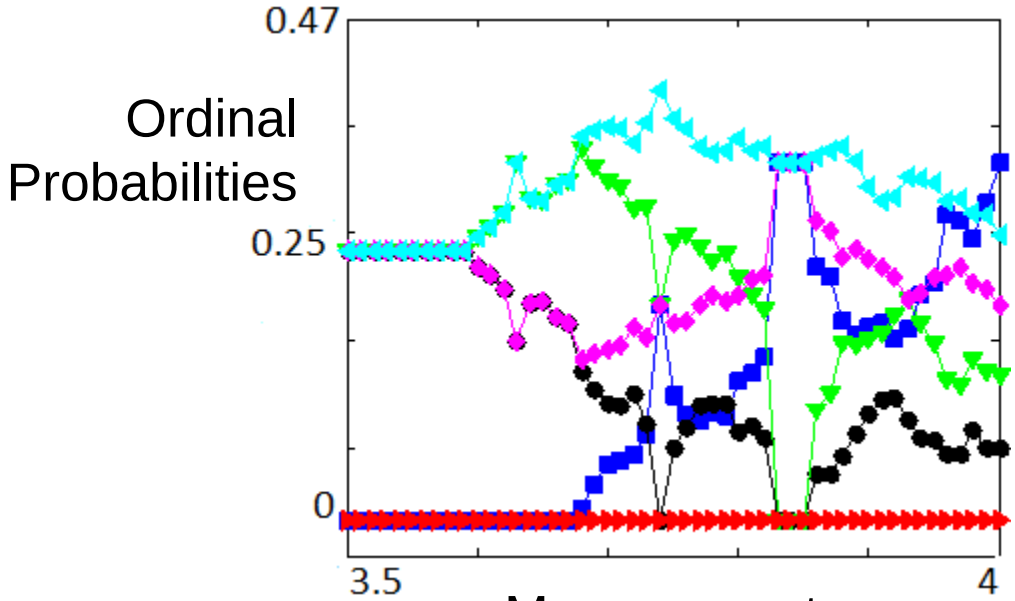
$$x(i+1) = r\, x(i)[1 - x(i)]$$

## Normal bifurcation diagram




Map parameter, r

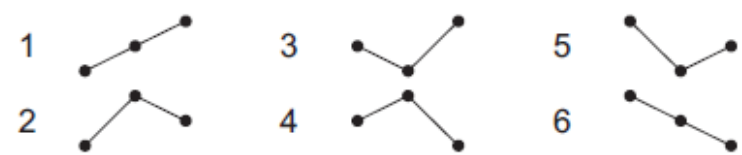
## Ordinal diagram with D=3




Map parameter, r

Pattern **210** is always forbidden; pattern **012** becomes more probable as r increases

**012 021 102 120 201 210**



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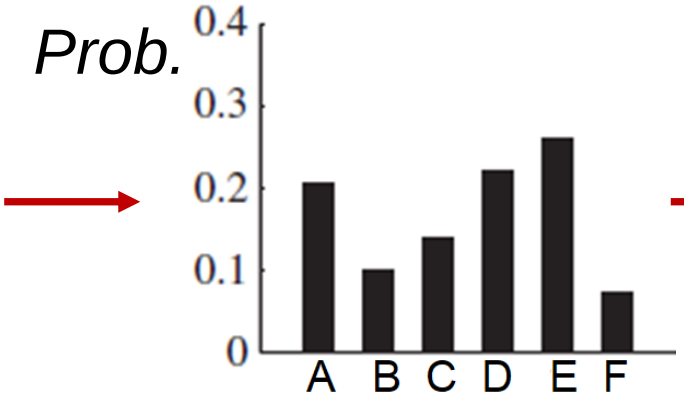
**From a sequence of data points, counting the occurrence of the patterns we estimate the "ordinal probabilities"**




?

**What to do next?**

1. **Permutation Entropy**:

$p_i = p_j \; \forall i, j \Rightarrow H=1$

$p_i = 1, \; p_j = 0 \; \forall \; j \neq i \Rightarrow H=0$

$$H = -\frac{1}{\ln N!} \sum_{i=1}^{N} p_i \ln p_i$$

(Nonlinear dimensionality reduction)

2. Analyze the **Ordinal Probabilities** (e.g., use them as features for machine learning algorithms)



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# Quick review of Shannon entropy

$$\boxed{H = -\sum_{i=1}^{N} p_i \ln p_i} \qquad \sum_{i=1}^{N} p_i = 1$$

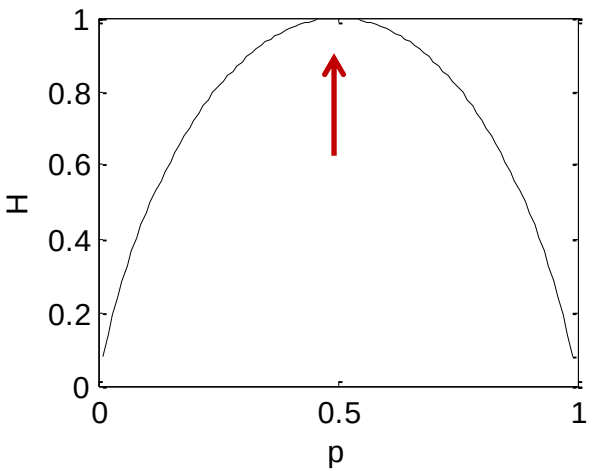
- Interpretation: "*quantity of* ***surprise*** *one should feel upon reading the result of a measurement* ".

- Example: a random variable takes values 0 or 1 with probabilities:

  $p(0) = p, \qquad p(1) = 1 - p.$

  $H = -p \ln(p) - (1 - p) \ln(1 - p).$

  $\Rightarrow$ p=0.5: Maximum **unpredictability.**




*C. Shannon, "A Mathematical Theory of Communication", Bell System Technical Journal. 27 (3): 379–423 (1948). Bell System Technical Journal. 27 (4): 623–656 (1948).*



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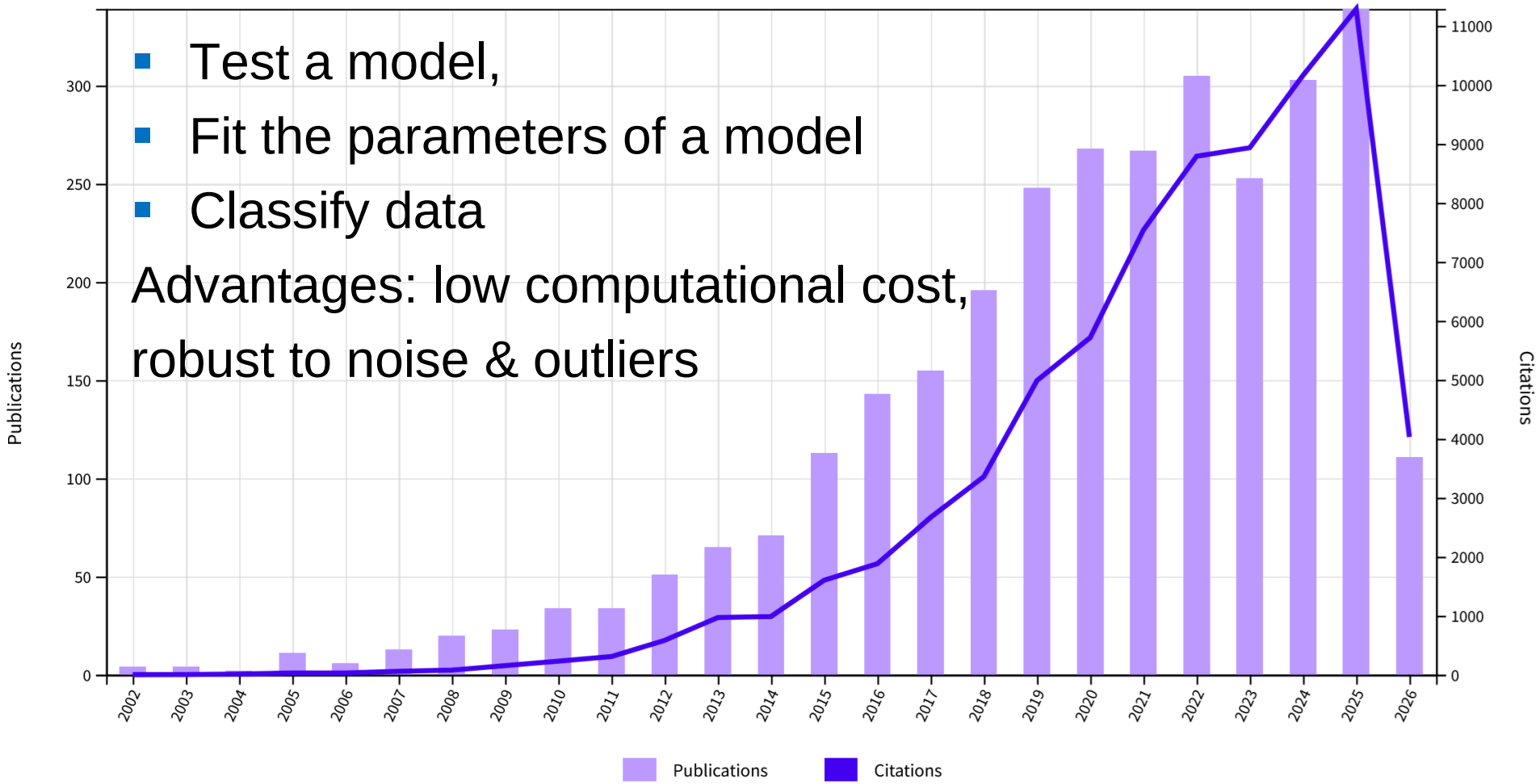
# Ordinal analysis and Permutation Entropy are popular to:




- Test a model,
- Fit the parameters of a model
- Classify data

Advantages: low computational cost, robust to noise & outliers

I. Leyva, J. M. Martinez, C. Masoller, O. A. Rosso, M. Zanin, *"20 Years of Ordinal Patterns: Perspectives and Challenges"*, EPL 138, 31001 (2022).



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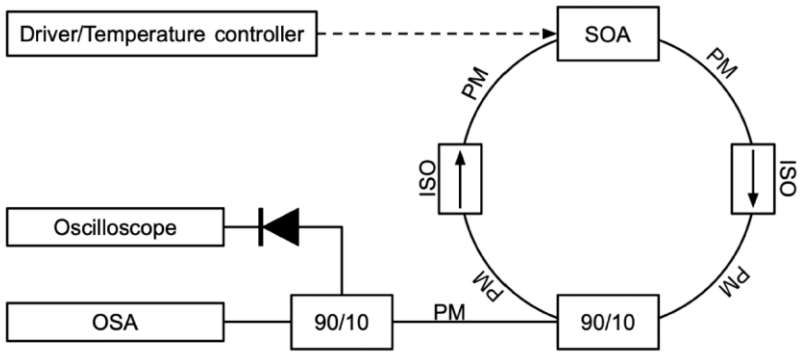
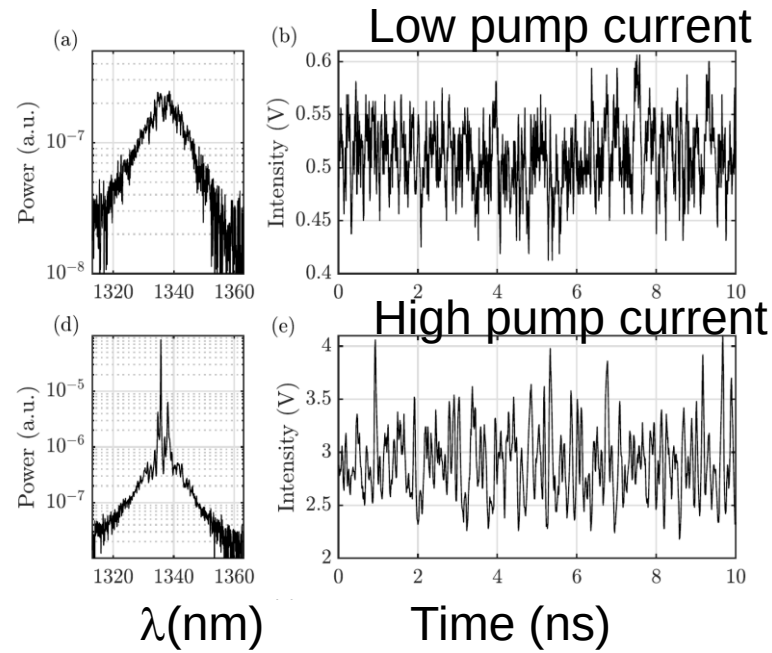
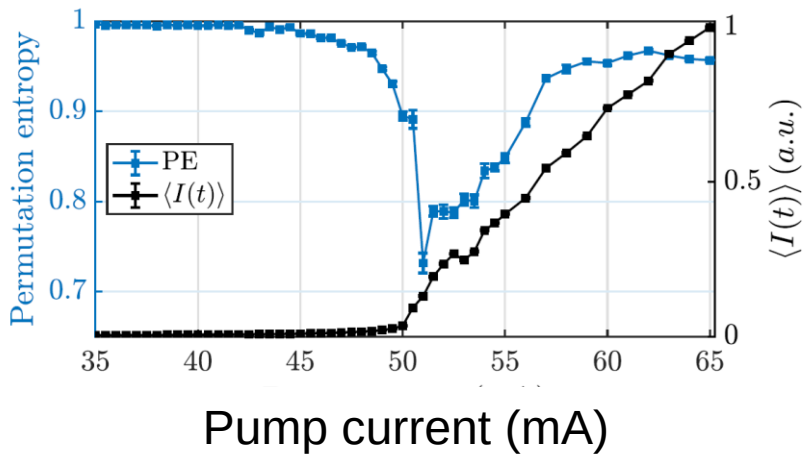
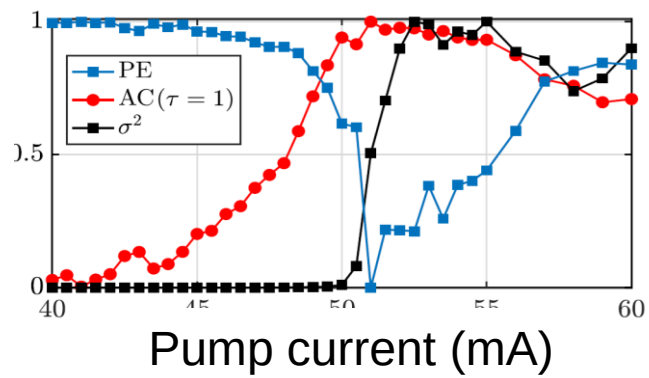
# Application: turn-on transition of a complex multimode laser




$$\text{PE} = -\frac{1}{\ln N!}\sum_{i=1}^{N} p_i \ln p_i$$




J. Gancio, C. Masoller, M. Marconi, "*Identifying and anticipating the threshold bifurcation of a complex laser with permutation entropy*", Phys. Rev. Lett. 135, 093802 (2025).



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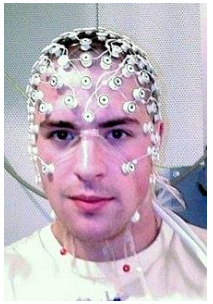
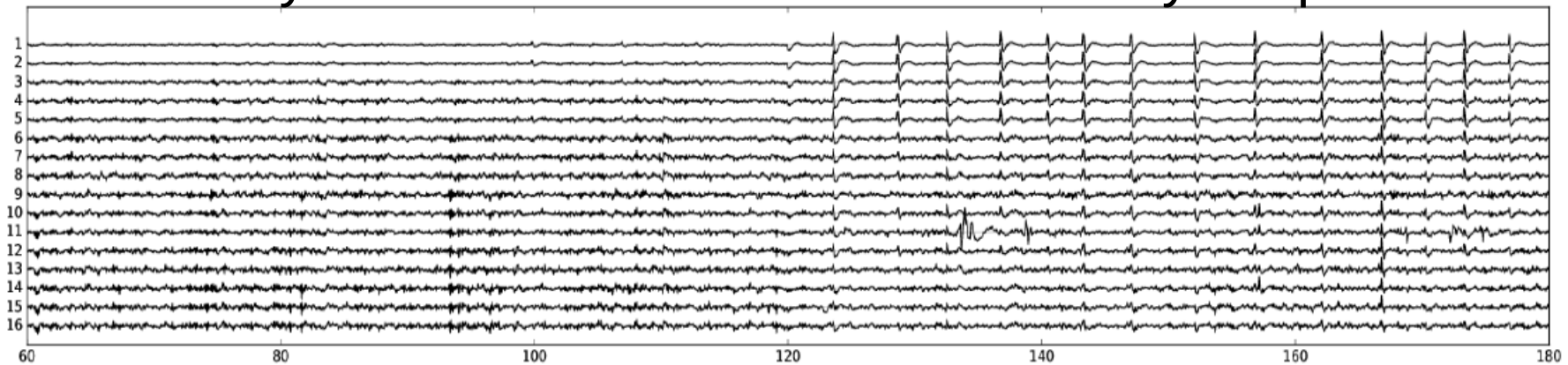
# A classification example: can Permutation Entropy distinguish eyes-closed and eyes-open states?



EEG Eyes closed EEG Eyes open




Time (seconds)

TABLE I. Description of the datasets used.

| | DTS1 | DTS2 |
|---|---|---|
| Sampling rate (Hz) | 256 | 160 |
| Time task (seg) | 120 | 60 |
| Total points | 30 720 | 9600 |
| Number of electrodes | 16 | 64 |
| Number of subjects (healthy) | 71 | 109 |

DTS1: Bitbrain (Zaragoza)
DTS2: Physionet



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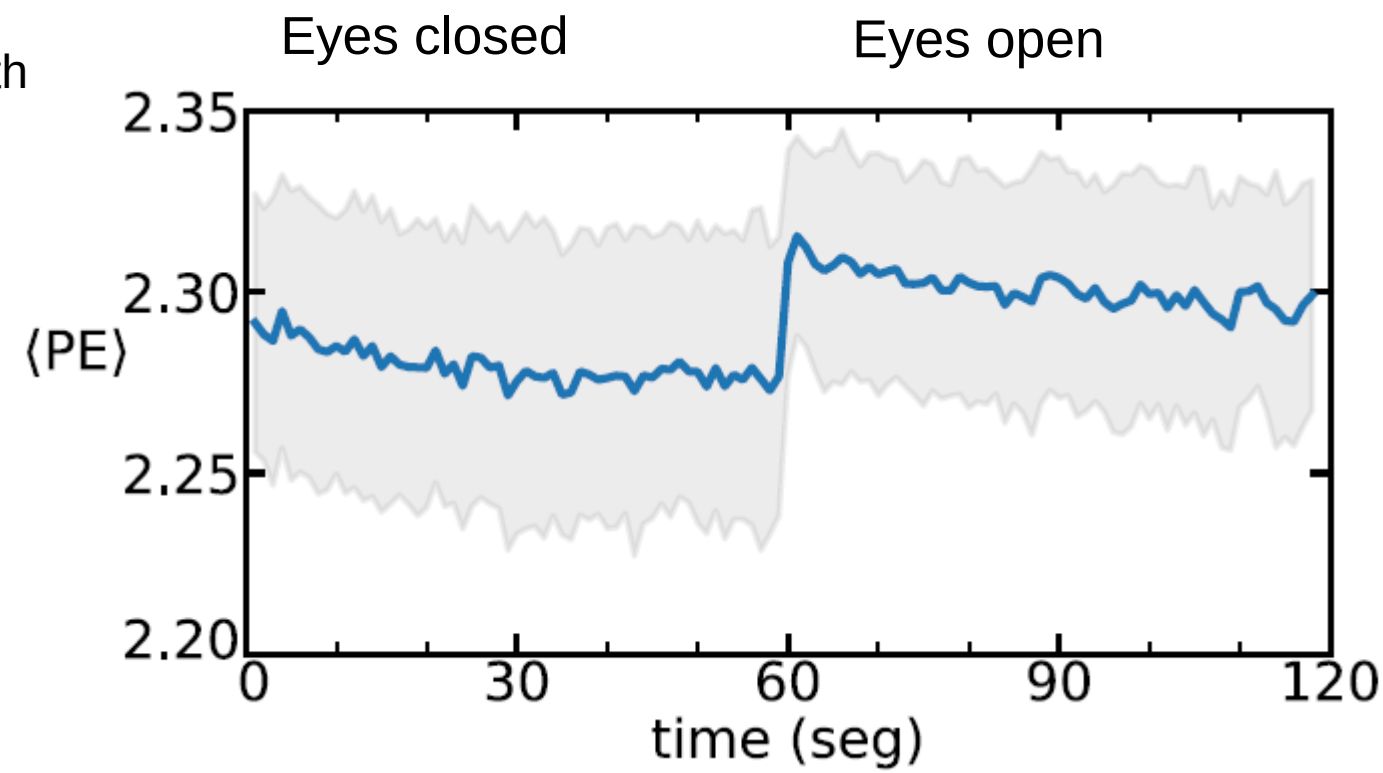
# The Permutation Entropy increases in the eyes open state

$$\langle \mathrm{PE} \rangle = \frac{1}{N[\mathrm{electrodes}]} \sum_i \mathrm{PE}^i$$

PE was calculated with patterns of length 4 (# of possible patterns 24) in time windows containing >4000 patterns

Gray region: Standard deviation of ⟨PE⟩ across subjects




C. Quintero-Quiroz, L. Montesano, A. Pons, M.Torrent, J. García-Ojalvo, C. Masoller, *"Differentiating resting brain states using ordinal symbolic analysis"*, Chaos 28, 106307 (2018).



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# Spatial data ⇒ Spatial Permutation Entropy (SPE)

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 8 | 3 | 4 |
| 6 | 7 | 5 |

| | |
|---|---|
| 1 | 2 |
| 8 | 3 |

1283

⇒0132

| | |
|---|---|
| 2 | 1 |
| 3 | 4 |

2134

⇒1023

| | |
|---|---|
| 8 | 3 |
| 6 | 7 |

8367

⇒3012

| | |
|---|---|
| 3 | 4 |
| 7 | 5 |

3475

⇒0132

$$H = -\frac{1}{\ln N!}\sum_{i=1}^{N} p_i \ln p_i$$

2x2 pixels:
24 possible
patterns

*Haroldo Ribeiro, Luciano Zunino and coworkers, PLoS ONE 7, e40689 (2012)*



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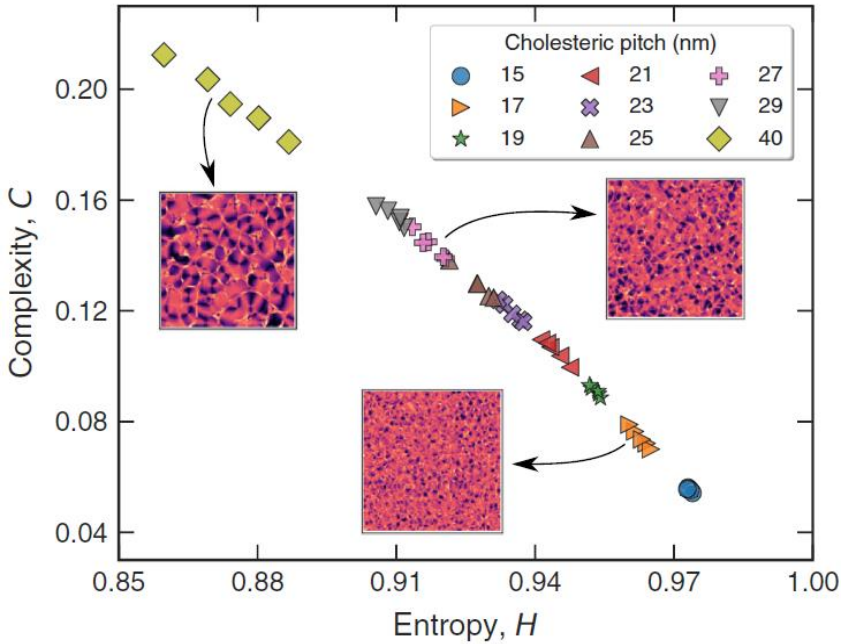
# The “spatial” permutation entropy was proposed to characterize two dimensional patterns and images.

PHYSICAL REVIEW E **99**, 013311 (2019)

**Estimating physical properties from liquid crystal textures via machine learning and complexity-entropy methods**

H. Y. D. Sigaki,[1] R. F. de Souza,[1] R. T. de Souza,[1,2] R. S. Zola,[1,2,*] and H. V. Ribeiro[1,†]
[1]*Departamento de Física, Universidade Estadual de Maringá, Maringá, PR 87020-900, Brazil*
[2]*Departamento de Física, Universidade Tecnológica Federal do Paraná, Apucarana, PR 86812-460, Brazil*



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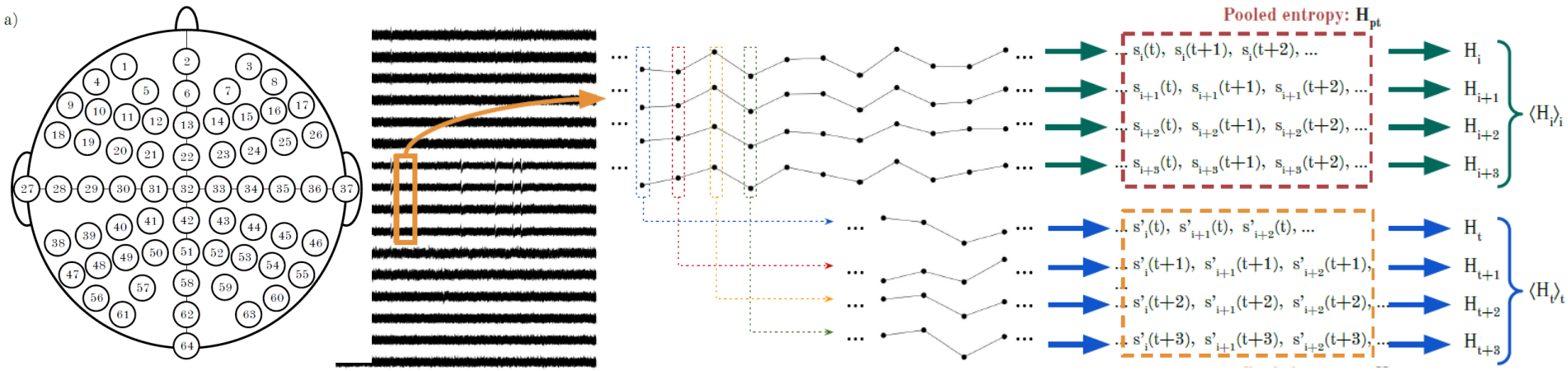
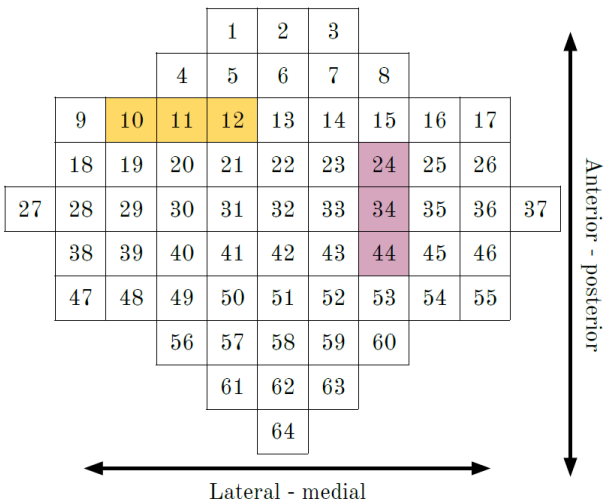
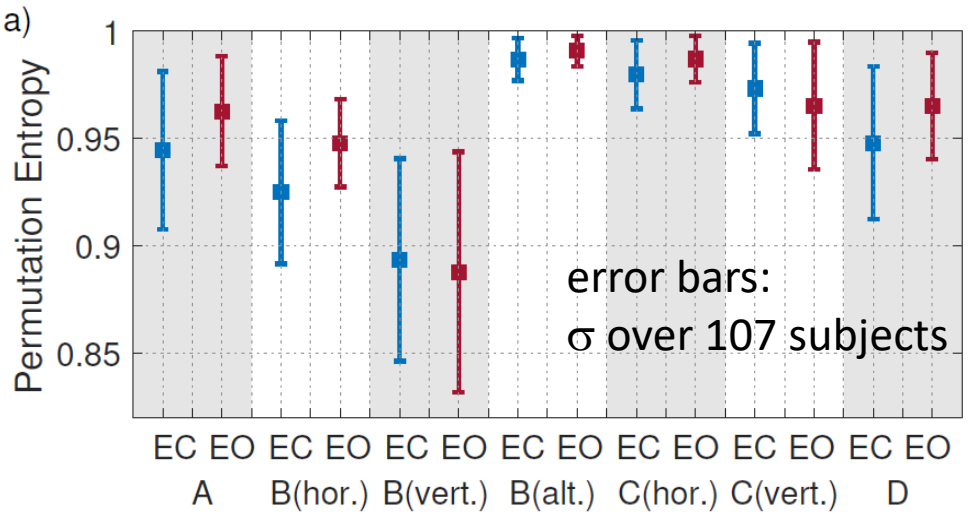
# Different approaches to calculate permutation entropy

Juan Gancio




J. Gancio, C. Masoller, G. Tirabassi, Chaos 34, 043130 (2024).

A: temporal coding
B: spatial coding (horizontal, vertical, alt.)
C: spatial pooling (horizontal and vertical symbols)
D: temporal pooling



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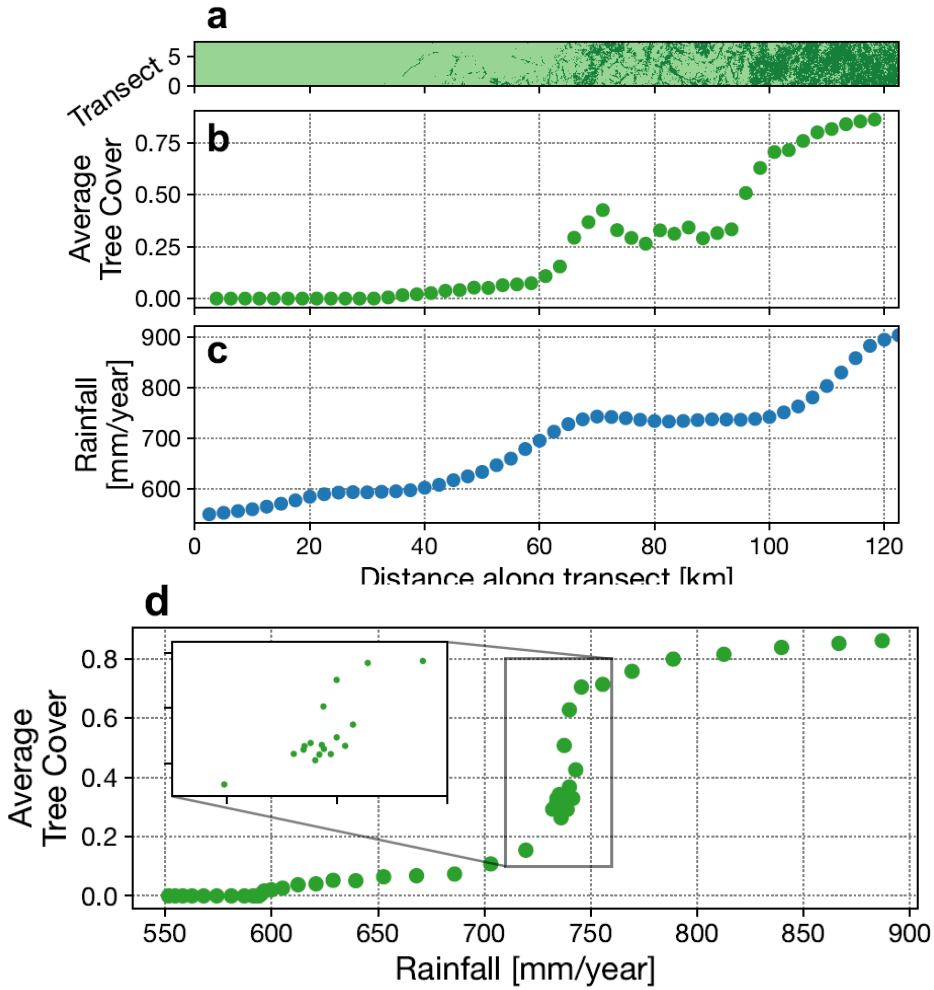
# Spatial permutation entropy (SPE) can be an early warning indicator of a desertification transition (tipping point)

Giulio Tirabassi




G. Tirabassi and C. Masoller, "*Entropy-based early detection of critical transitions in spatial vegetation fields*", PNAS 120, e2215667120 (2023).



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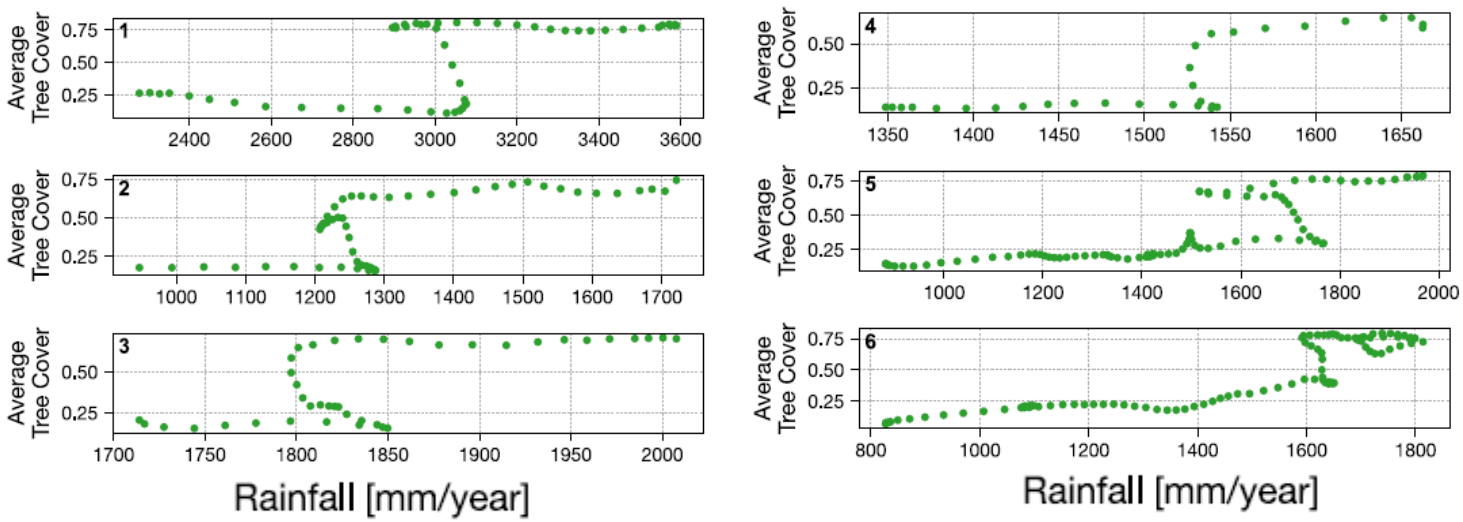
# Satellite vegetation data combined with rainfall data from the Tropical Rainfall Measuring Mission (TRMM)


*G. Tirabassi, C. Masoller, "Entropy-based early detection of critical transitions in spatial vegetation fields", PNAS 120, e2215667120 (2023).*



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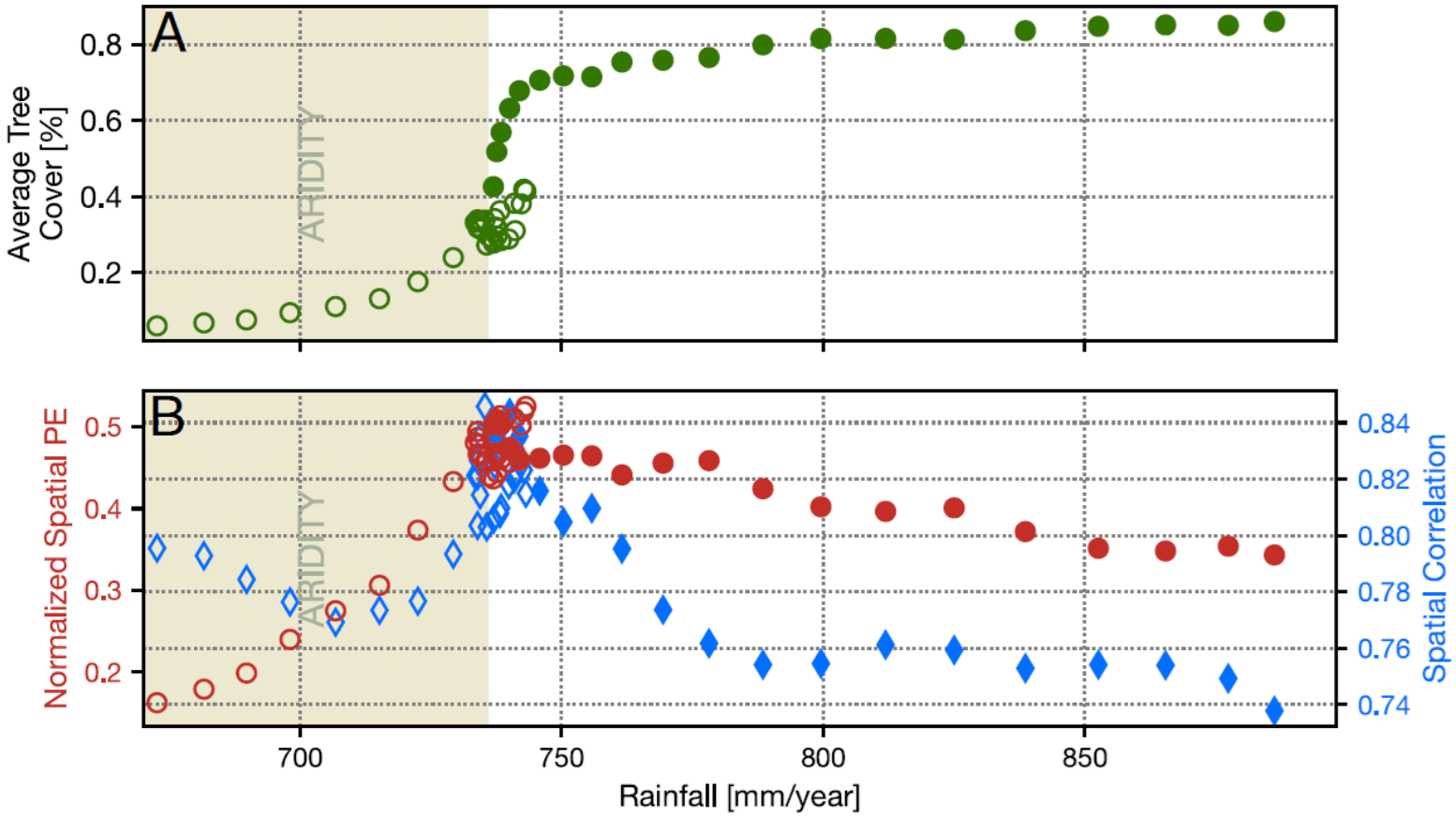


**Can we test this indicator in controlled experimental data?**

G. Tirabassi and C. Masoller, "*Entropy-based early detection of critical transitions in spatial vegetation fields*", PNAS 120, e2215667120 (2023).



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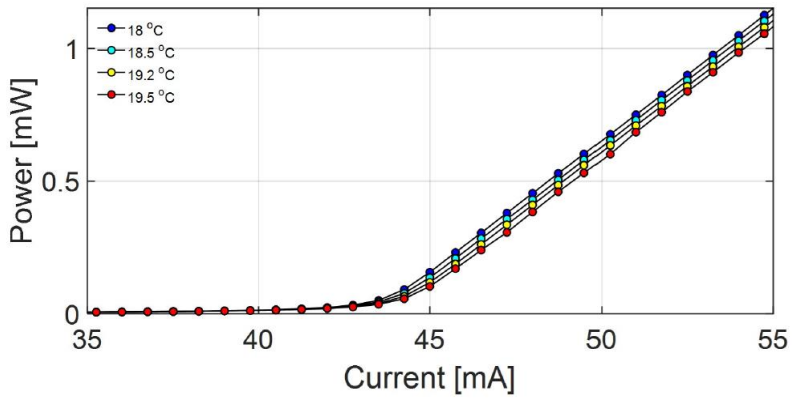
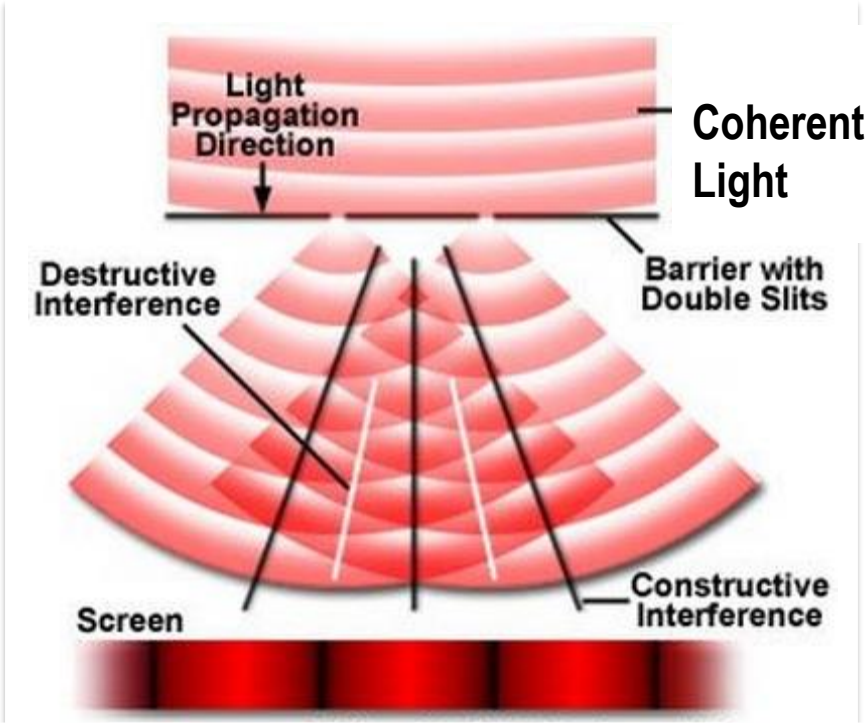
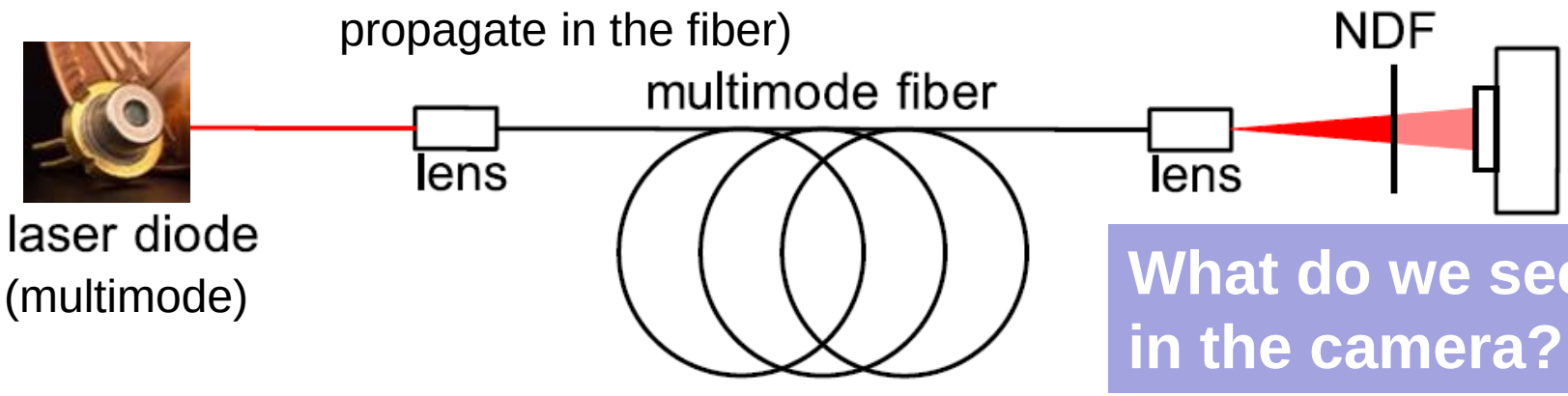
# Quick reminder: laser threshold and interference of coherent waves




(many modes can propagate in the fiber)




**What do we see in the camera?**



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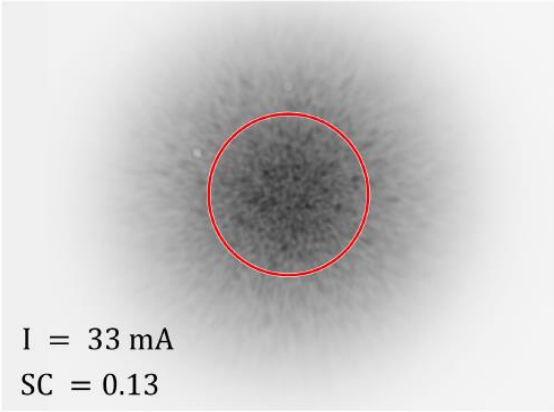
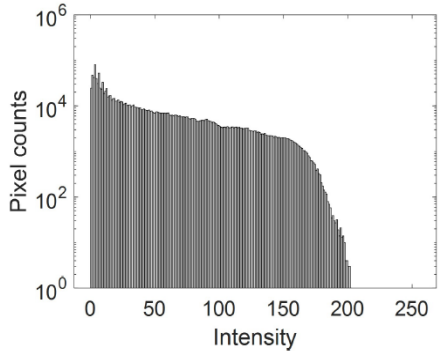
# Speckle pattern

## Below lasing threshold

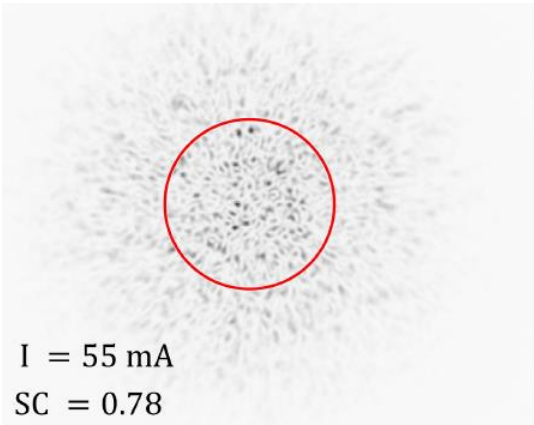
Low coherence → low speckle contrast

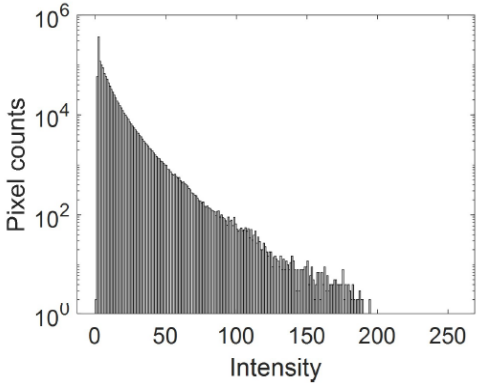



## Above lasing threshold

High coherence→ high speckle contrast




**How to quantify the degree of speckle?**

Speckle contrast
(coefficient of variation, CV)

$$SC = \sigma/\langle I \rangle$$



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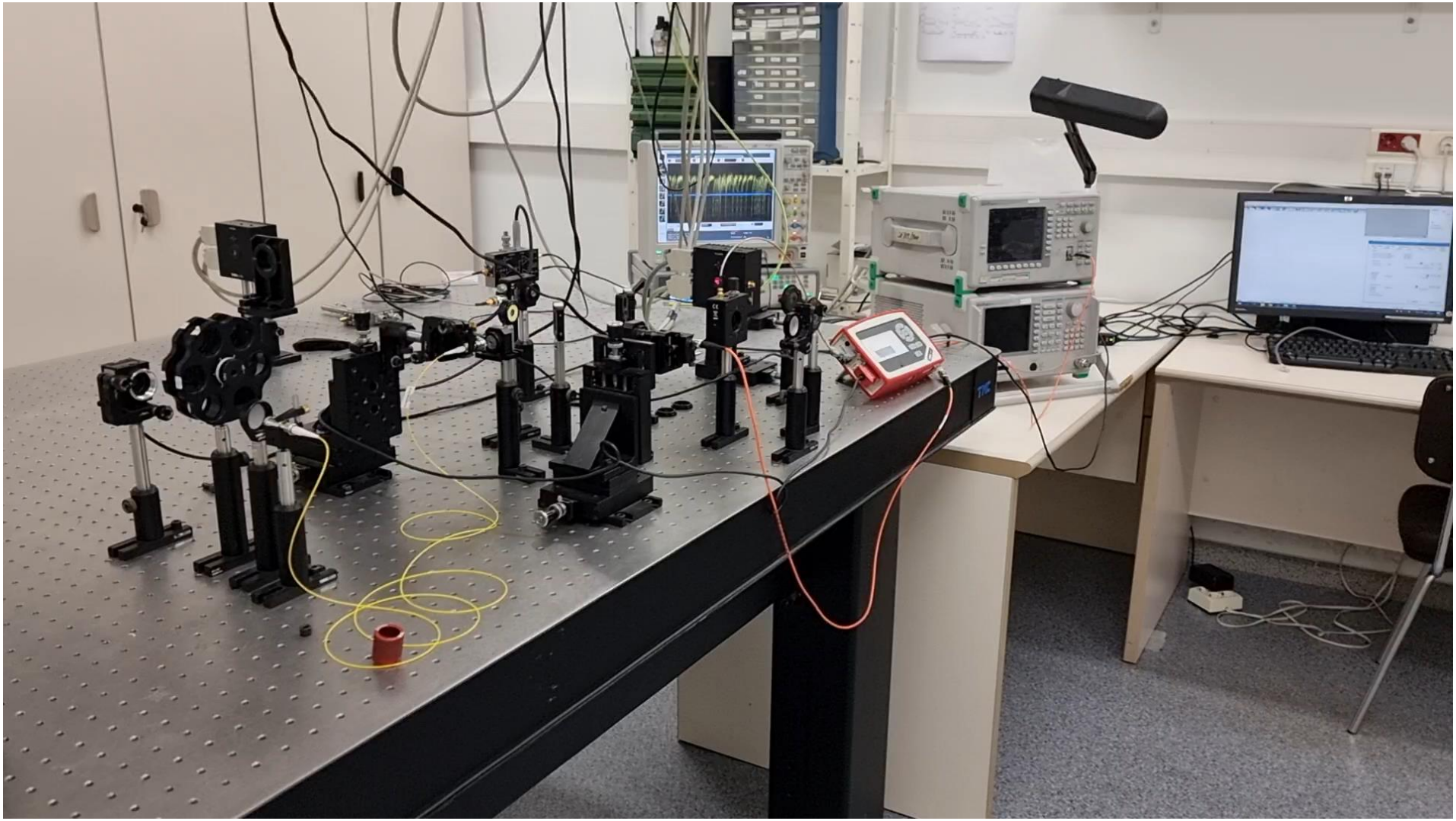
# Three different diffusive media are used to generate speckle:

Multimode fiber --- Multimode fiber and diffuser --- Single mode fiber and diffuser

Maria Duque





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**Can we identify, from the analysis of speckle images, the laser threshold? and infer which medium generated the speckles?**

Giulio Tirabassi

$$PE = -\frac{1}{\ln N!}\sum_{i=1}^{N} p_i \ln p_i$$

$$SC = \sigma/\langle I \rangle$$

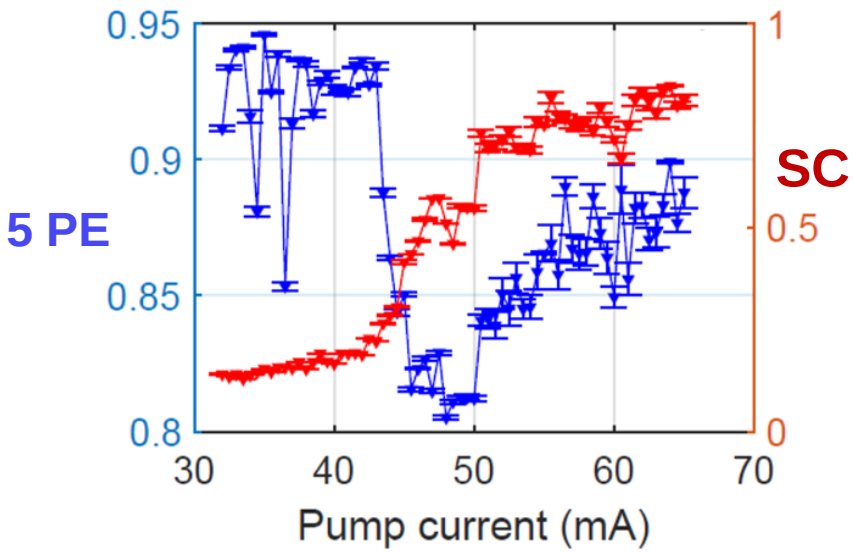
***Shape*** of the ordinal pattern:

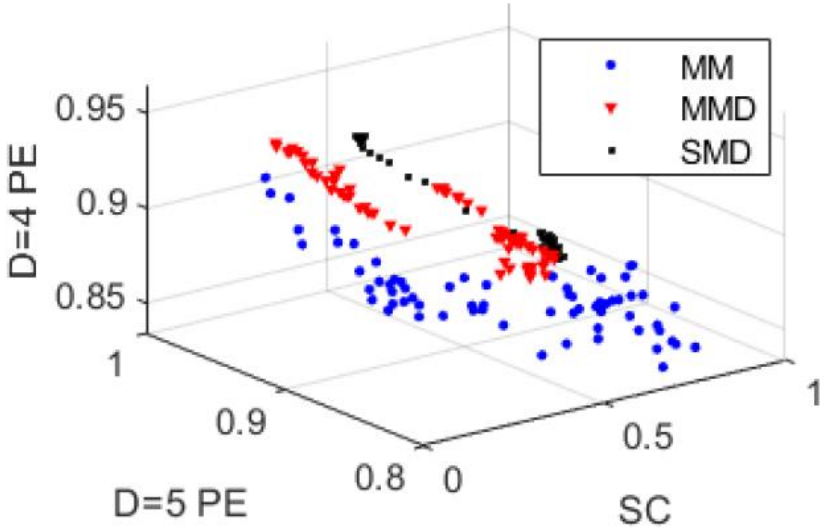
| | |
|---|---|
| "square" | |
| 4 PE : | x x |
| | x x |

| | |
|---|---|
| "cross" | x |
| 5 PE : | x x x |
| | x |

Diffusive medium:

- Multimode fiber
- Multimode fiber and diffuser
- Single mode fiber and diffuser




Accuracy of the random forest classifier: 99.4 % ± 0.4 %

G. Tirabassi, M. Duque-Gijon, J. Tiana-Alsina, C. Masoller, APL Photonics 8, 126112 (2023).



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# We also used permutation entropy to analyze climate data: sea surface temperature (SST) anomalies.

Earth Syst. Dynam., 17, 533–561, 2026
https://doi.org/10.5194/esd-17-533-2026


Earth System Dynamics
Open Access
EGU

## Detecting transitions and quantifying differences in two SST datasets using spatial permutation entropy

**Juan Gancio[1], Giulio Tirabassi[2], Cristina Masoller[1], and Marcelo Barreiro[3]**

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Rambla Sant Nebridi 22, Terrassa 08222, Barcelona, Spain
[2]Departament de Informàtica, Matemàtica Aplicada i Estadística,
Universitat de Girona, Carrer de la Universitat de Girona 6, Girona 17003, Spain
[3]Departamento de Ciencias de la atmósfera y Física de los Océanos, Facultad de Ciencias,
Universidad de la República, Montevideo, 11400, Uruguay

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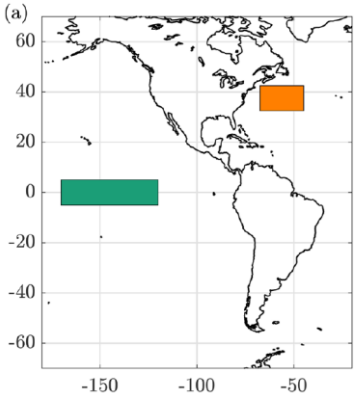
Received: 2 October 2025 – Discussion started: 15 October 2025
Revised: 23 April 2026 – Accepted: 28 April 2026 – Published: 12 May 2026




We analyzed two geographical regions, using “temporal” and “spatial” permutation entropy (horizontal & vertical orientation) and compared two datasets (USA and Europe)



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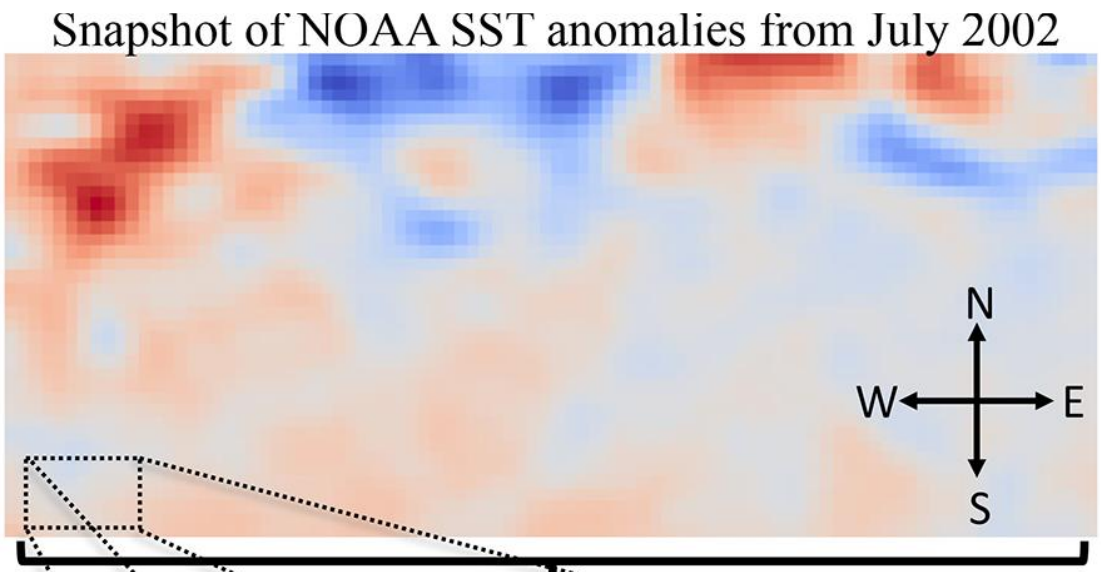
# Spatial ordinal patterns can be defined with lagged points: a spatial lag allows to tune the spatial scale of Permutation Entropy (PE) analysis




| | | | | | |
|---|---|---|---|---|---|
| -0.65 | -0.71 | -0.63 | -0.51 | -0.48 | -0.47 |
| -0.57 | -0.62 | -0.52 | -0.43 | -0.42 | -0.40 |
| -0.50 | -0.52 | -0.45 | -0.37 | -0.33 | -0.28 |

$\delta$=1 $\delta$=1 $\delta$=2 $\delta$=2

Ordinal patterns defined from values along each column, then, all columns "pooled" to calculate $\mathbf{PE_{NS}}$

Ordinal patterns defined from values along each row, then, all rows "pooled" to calculate $\mathbf{PE_{WE}}$

J. Gancio, G. Tirabassi, C. Masoller, M. Barreiro, Earth Syst. Dynam. 17, 533 (2026).



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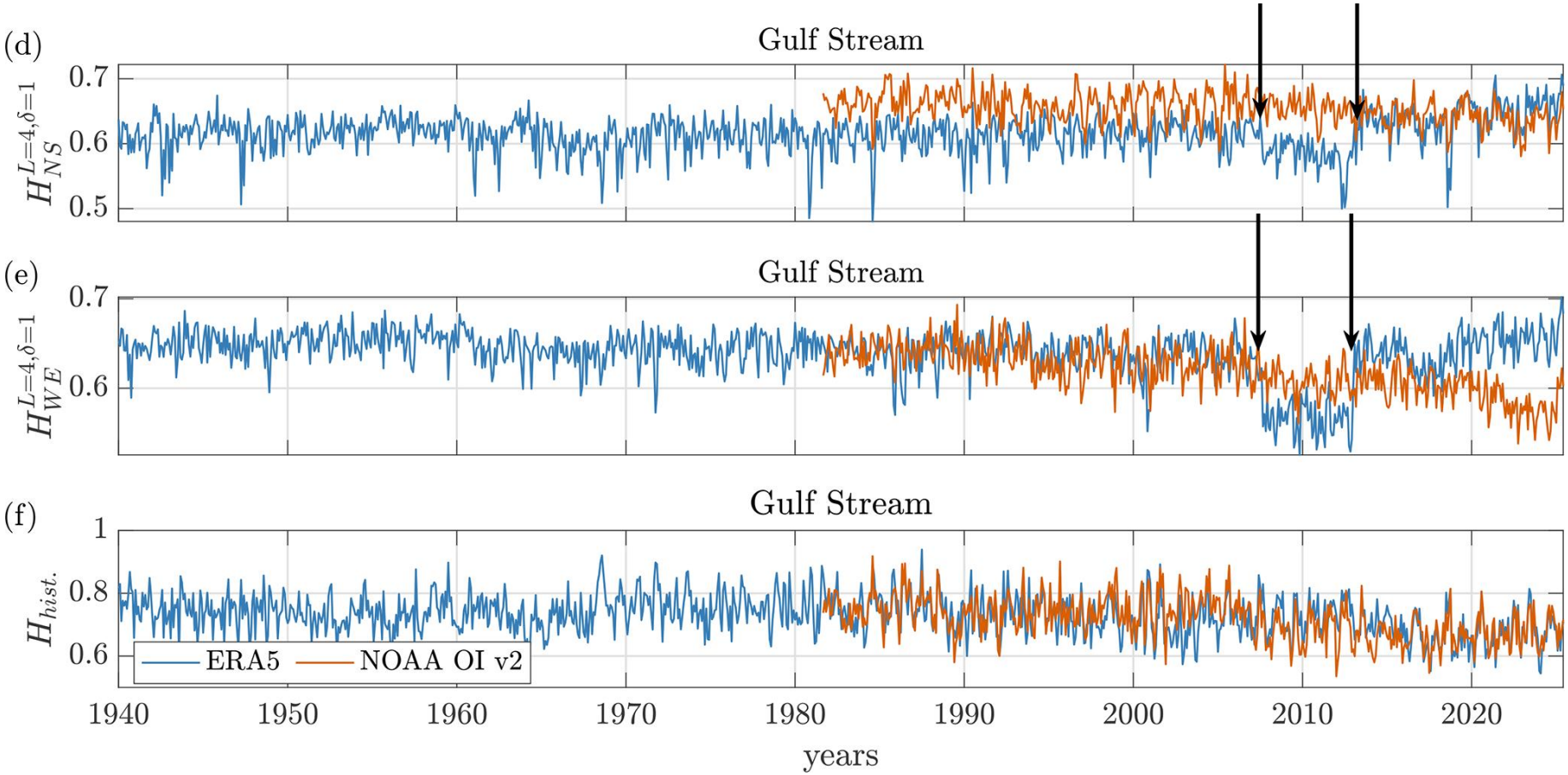
# Comparison of ERA5 (Europe) and USA (NOAA)




- Changes detected can be interpreted as due to advances in Earth observation systems (satellites) and/or new data processing methodologies.
- Entropy of distributions of data values (hist): no changes identified.

J. Gancio, G. Tirabassi, C. Masoller, M. Barreiro, Earth Syst. Dynam. 17, 533 (2026).



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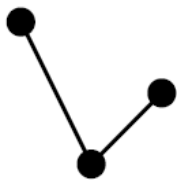
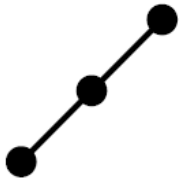
# Using lagged data points to define patterns allows to select the time scale of the analysis, very useful for climate data

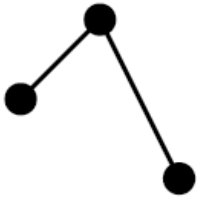
**Intra-season**

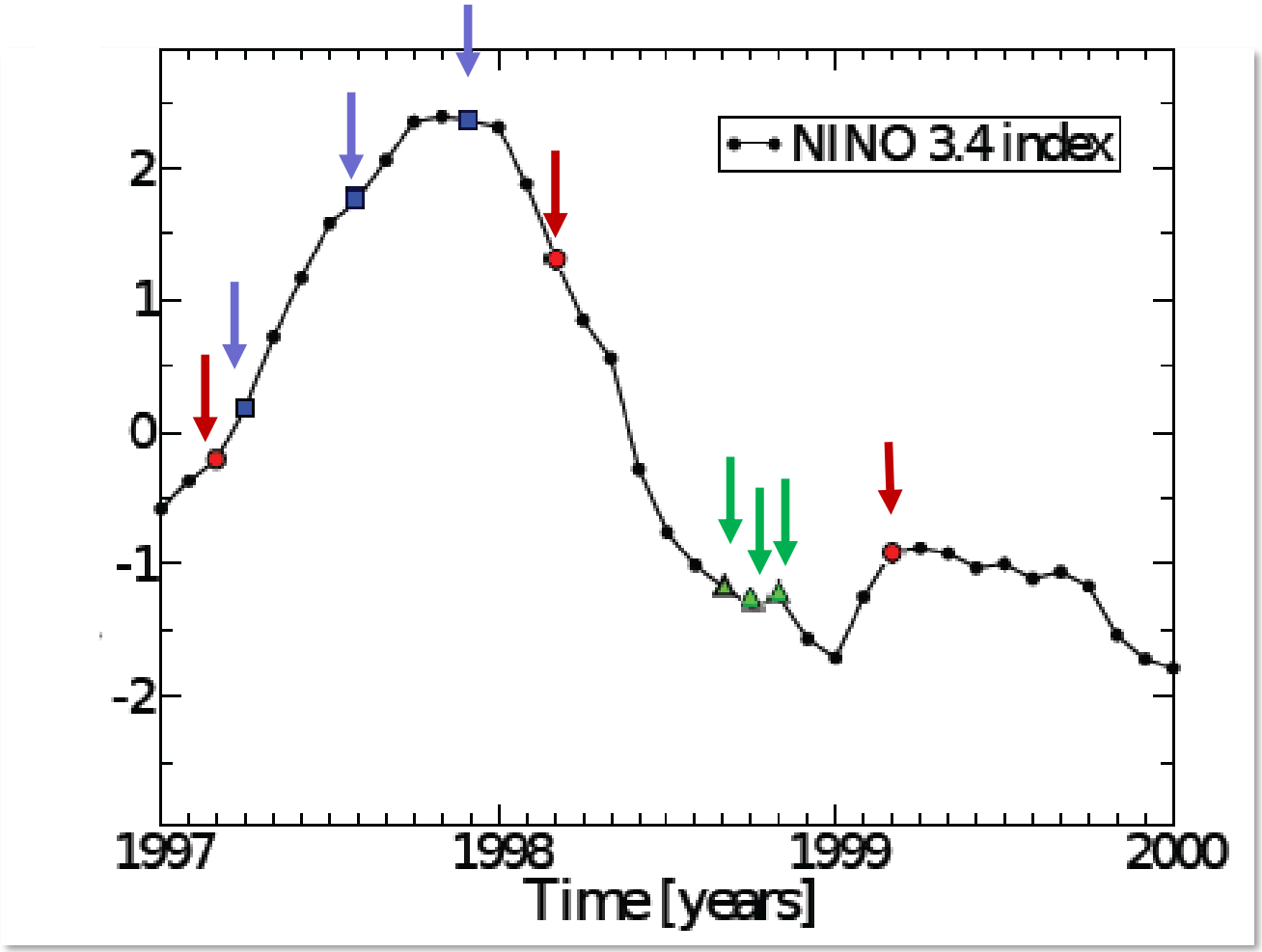


**Intra-annual**



**Inter-annual**





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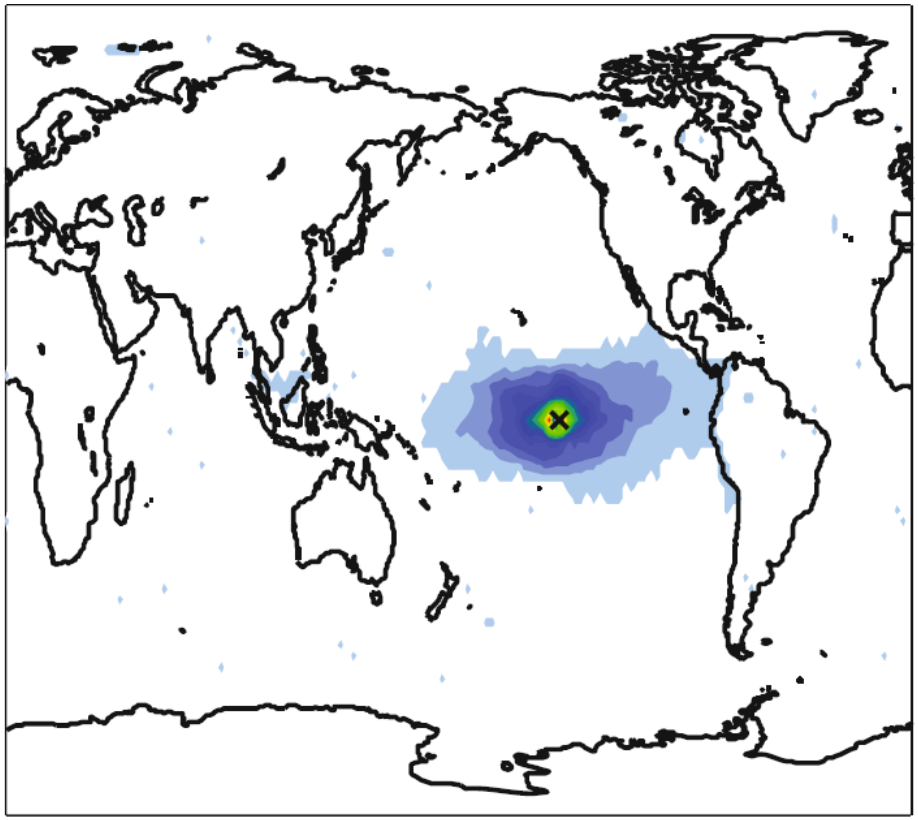
# Mutual Information (color code) from probabilities of ordinal patterns (white: MI not significant)

$$M_{ij} = \sum_{m,n} p_{ij}(m,n) \log \frac{p_{ij}(m,n)}{p_i(m)p_j(n)}$$

MI from probabilities of ordinal patterns defined by values of **surface air temperature anomalies** in 3 consecutive months.



*J. I. Deza, M. Barreiro, C. Masoller, Eur. Phys. J. ST 222, 511 (2013).*



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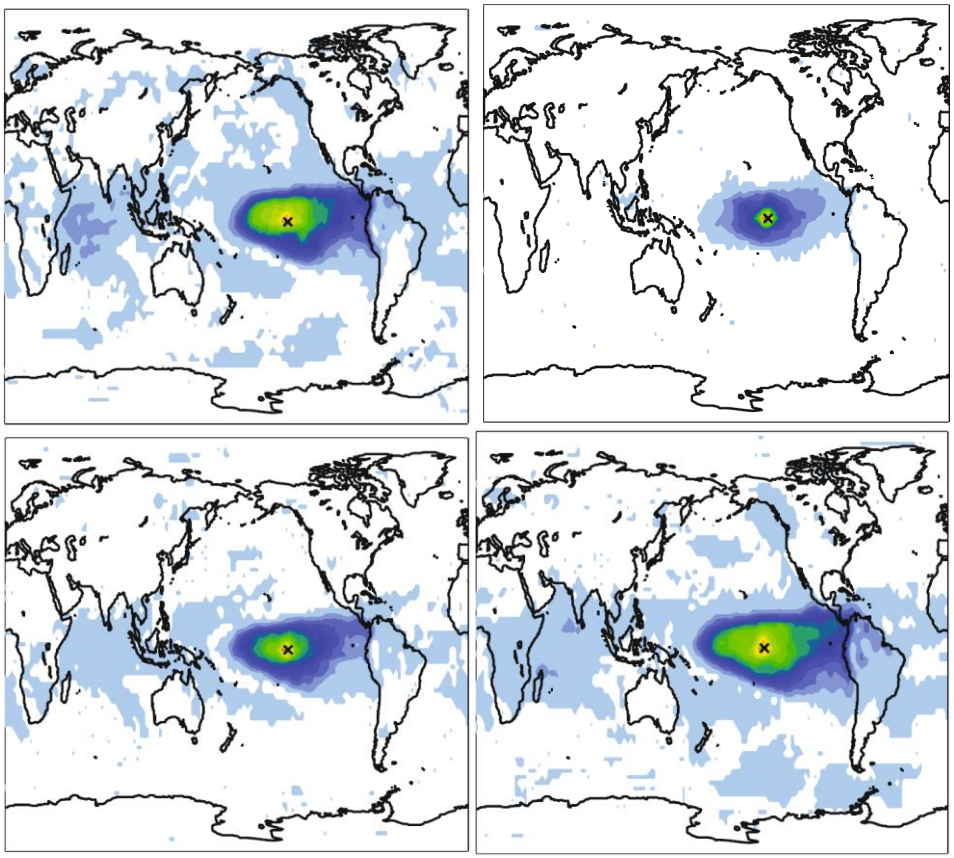
# Comparison

$$M_{ij} = \sum_{m,n} p_{ij}(m,n) \log \frac{p_{ij}(m,n)}{p_i(m)p_j(n)}$$

Probabilities of data values



Probabilities of ordinal patterns defined by values in 3 consecutive months.

Probabilities of patterns defined by 3 values within a year.

Probabilities of patterns defined by values in 3 consecutive years.

*J. I. Deza, M. Barreiro, C. Masoller, Eur. Phys. J. ST 222, 511 (2013).*



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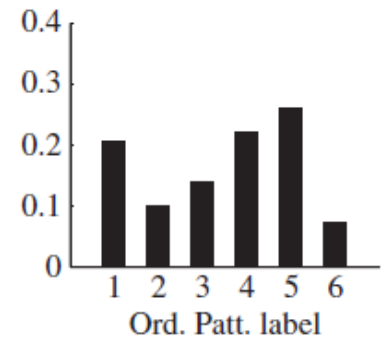
# Are the ordinal probabilities also informative?

Are the patterns equally probable?



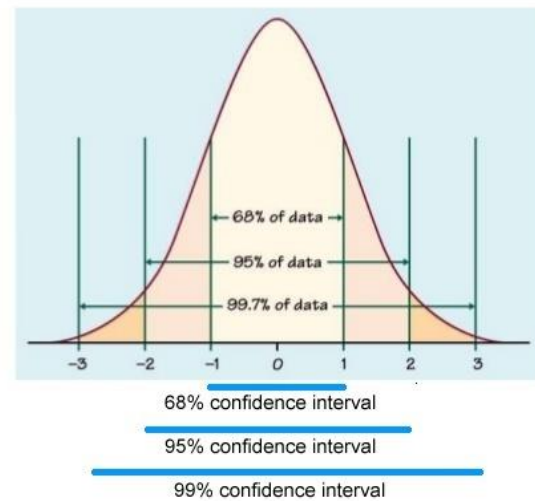

- **Null hypothesis**:

  $p_i = p = 1/\text{number-of-possible-patterns}$

If at least one probability **is not** in the interval $p \pm 3\sigma$ with $\sigma = \sqrt{p(1-p)/N}$

and $N$ the number of ordinal patterns:

We **reject** the NH with 99.74% confidence level.




- Else, we **fail to reject** the NH with 99.74% confidence level.

**Back to question: How to detect similar temporal order in spike sequences of neurons and lasers?**



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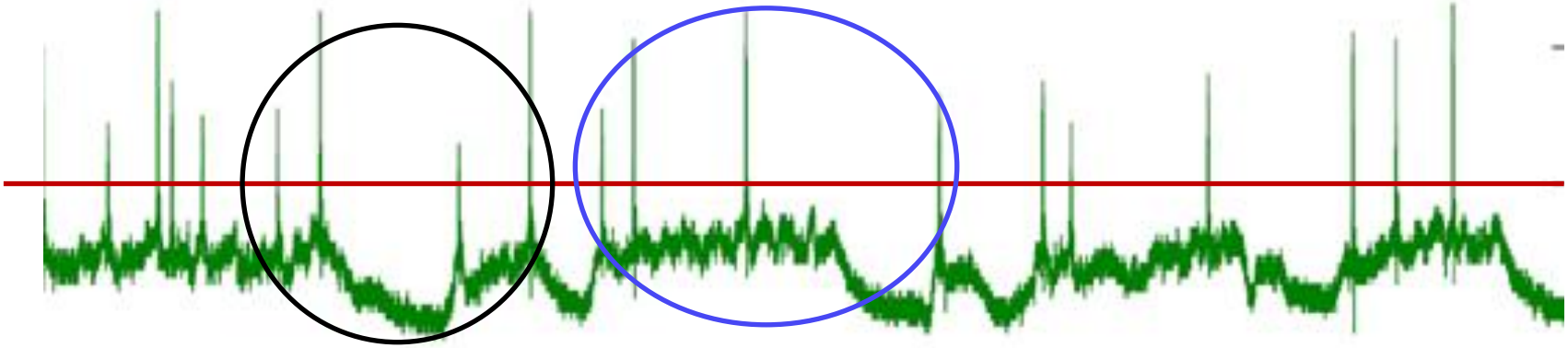
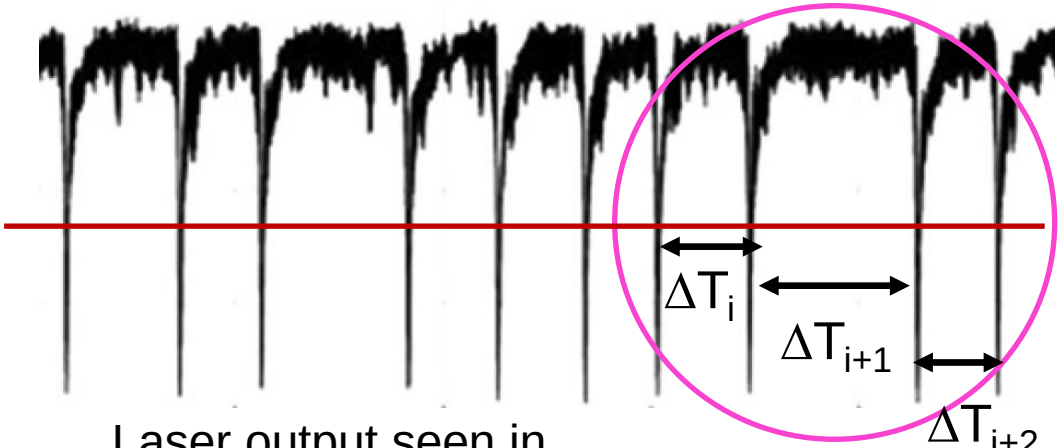
# Sequence of inter-spike-intervals (ISIs) ⇒ sequence of ordinal patterns

Intracellular recording



**021** **012**




Laser output seen in
1 GHz oscilloscope

**120**

**012 021 102 120 201 210**

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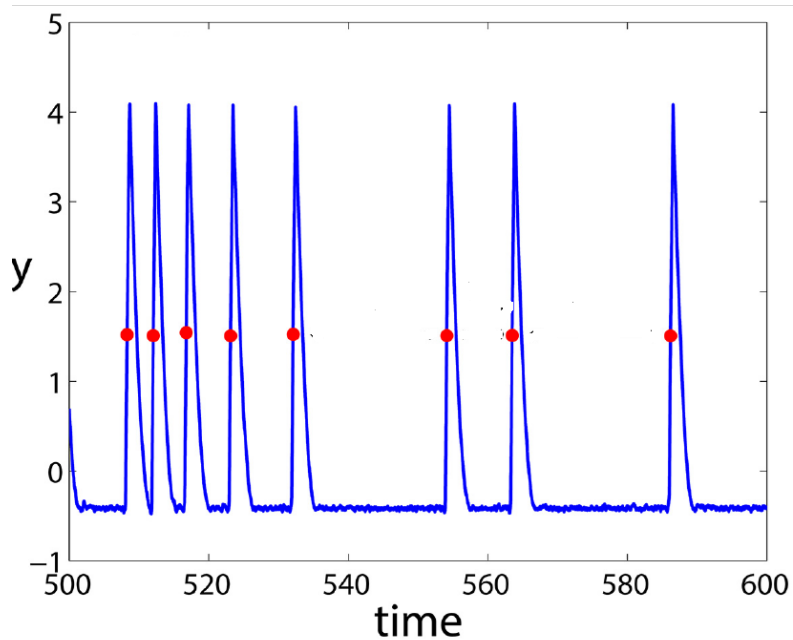
# Analysis of inter-spike-sequences (ISIs) simulated with a simple neuron model (FitzHugh-Nagumo)

$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + \boxed{a_o \cos(2\pi t/T)} + D\xi(t),$$

Weak, **subthreshold** input

Gaussian white noise




$a_0$=0
Noise-induced spikes

J. A. Reinoso, M. C. Torrent, C. Masoller, Phys. Rev. E. 94, 032218 (2016).



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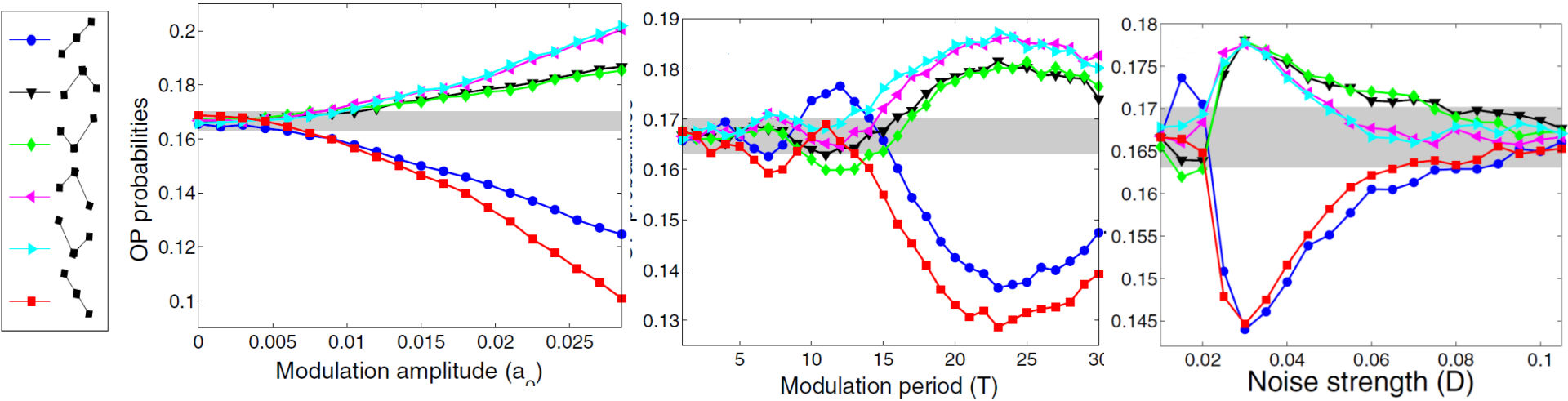
# Role of modulation amplitude and period and level of noise

$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$
$$\frac{dy}{dt} = x + a + \boxed{a_o \cos(2\pi t/T)} + D\xi(t),$$




$\Rightarrow$ The probabilities of the patterns depend on the modulation period, the amplitude and the level of noise.

Gray region: null-hypothesis, 99.74% confidence level

J. A. Reinoso, M. C. Torrent, C. Masoller, Phys. Rev. E. 94, 032218 (2016).



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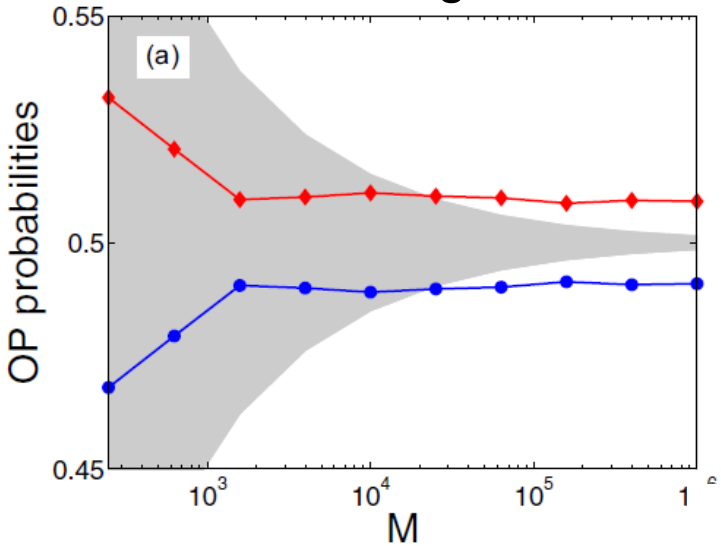
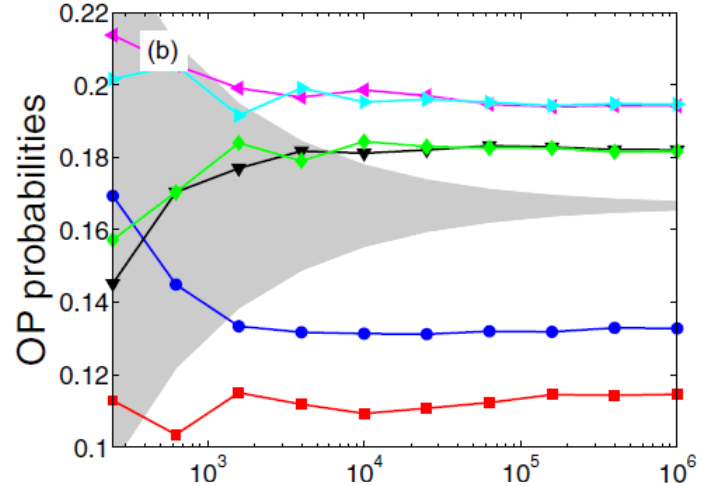
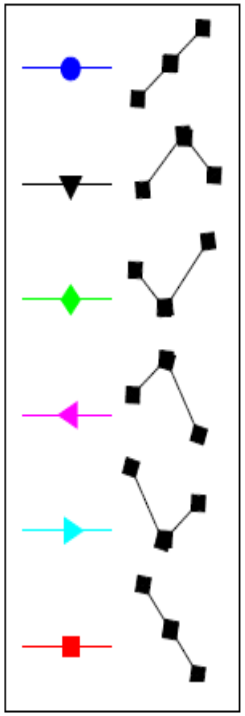
# How many spikes do we need to estimate the probabilities?

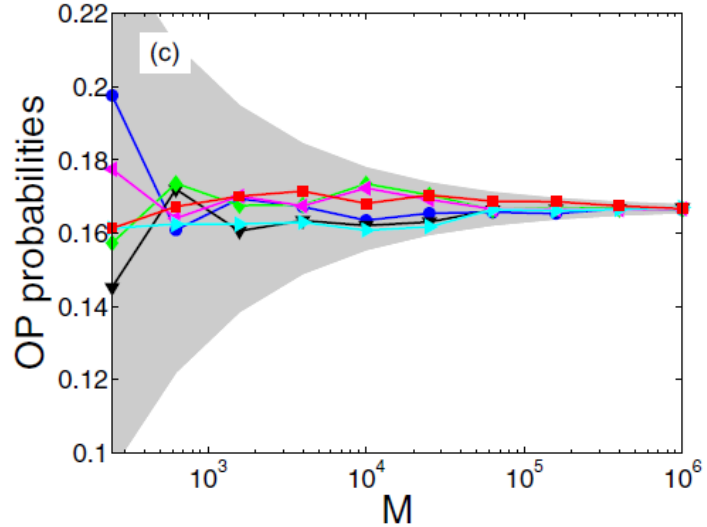
With external signal **01** **10**



Without external signal




Gray region: null-hypothesis, 99.74% confidence level

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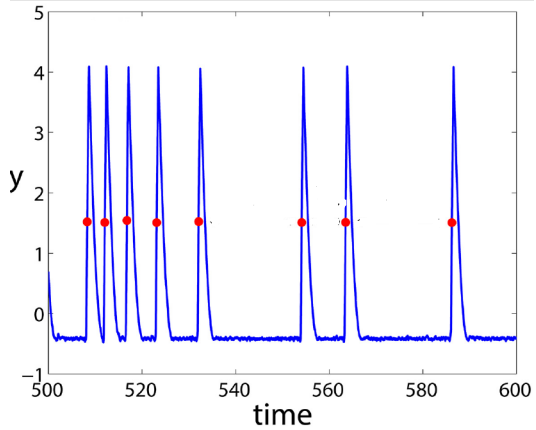
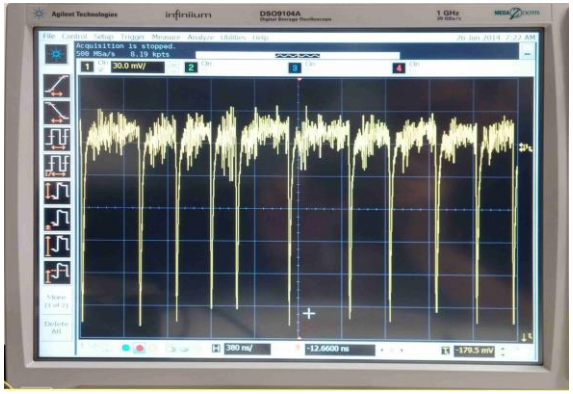
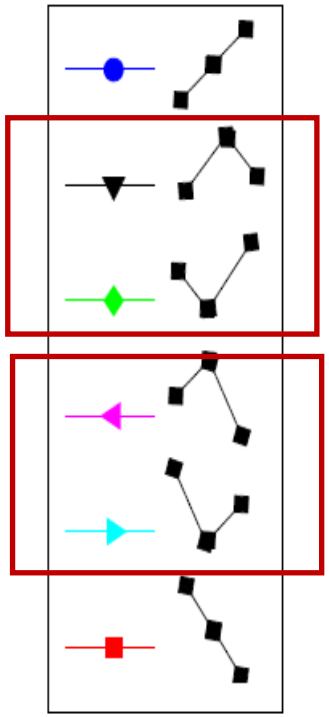
# Comparing the spike timing of a neuron and a laser when they are weakly forced with a sinusoidal input

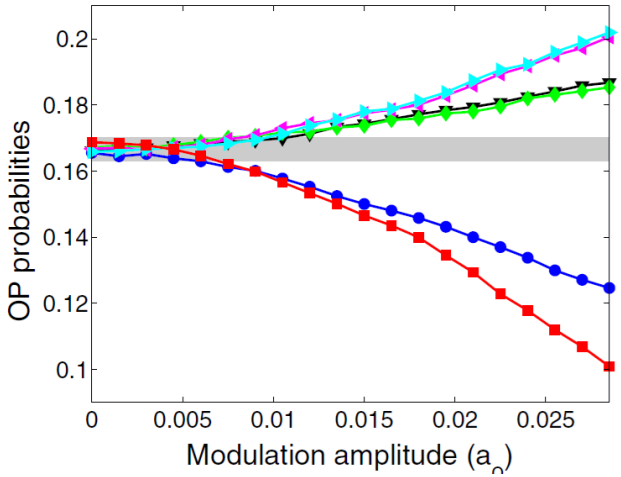
## Neuron model

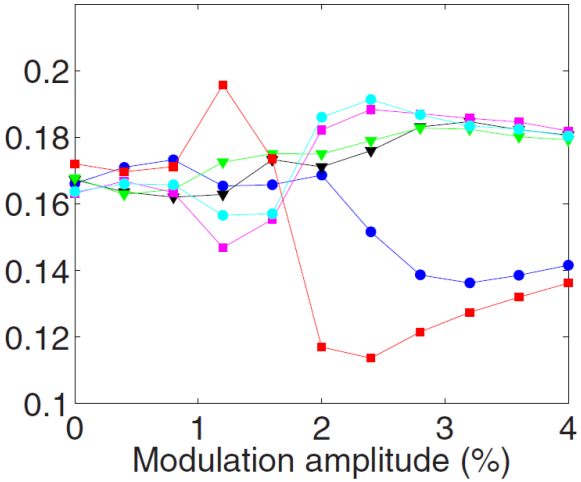



## Diode laser with feedback








J. M. Aparicio-Reinoso et al PRE 94, 032218 (2016)  A. Aragoneses et al, Sci. Rep. **4**, 4696 (2014)



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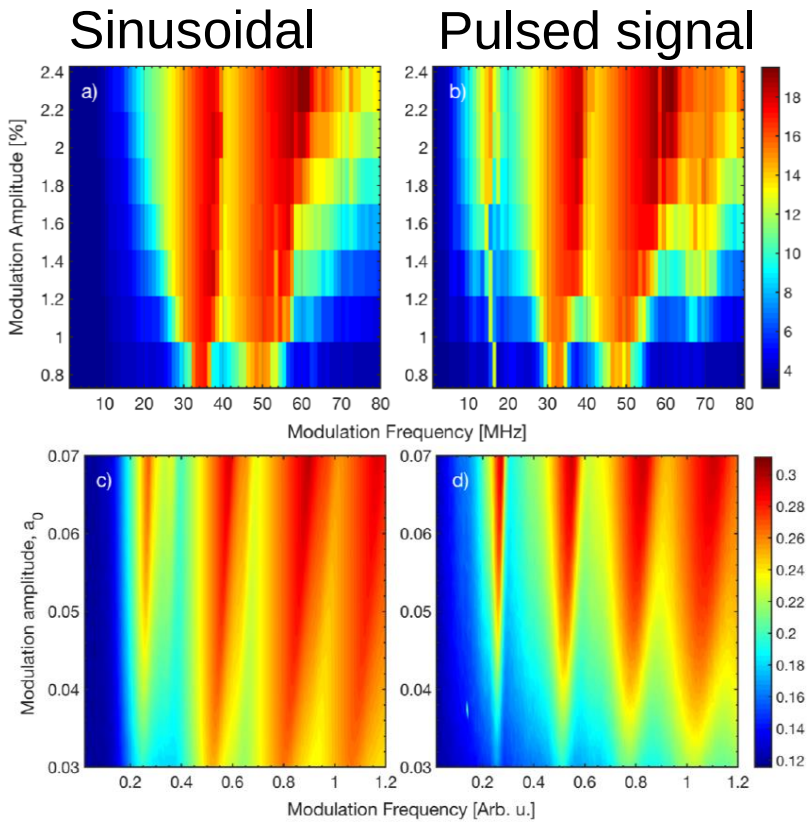
# Laser-neuron comparison: spike rate

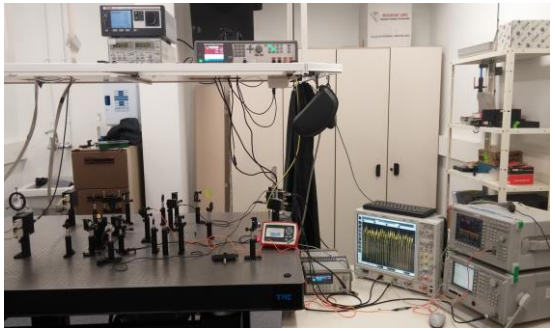
Jordi Tiana

Spike rate in color code

Experiments modulating the laser current

Neuron model with the same (re-scaled) input



**How about temporal order in the ISI sequences?**

J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, "*Comparing the dynamics of periodically forced lasers and neurons*", New J. of Phys. 21, 103039 (2019) (2019).

J. Tiana-Alsina and C. Masoller, Sci. Rep. 12, 4914 (2022).



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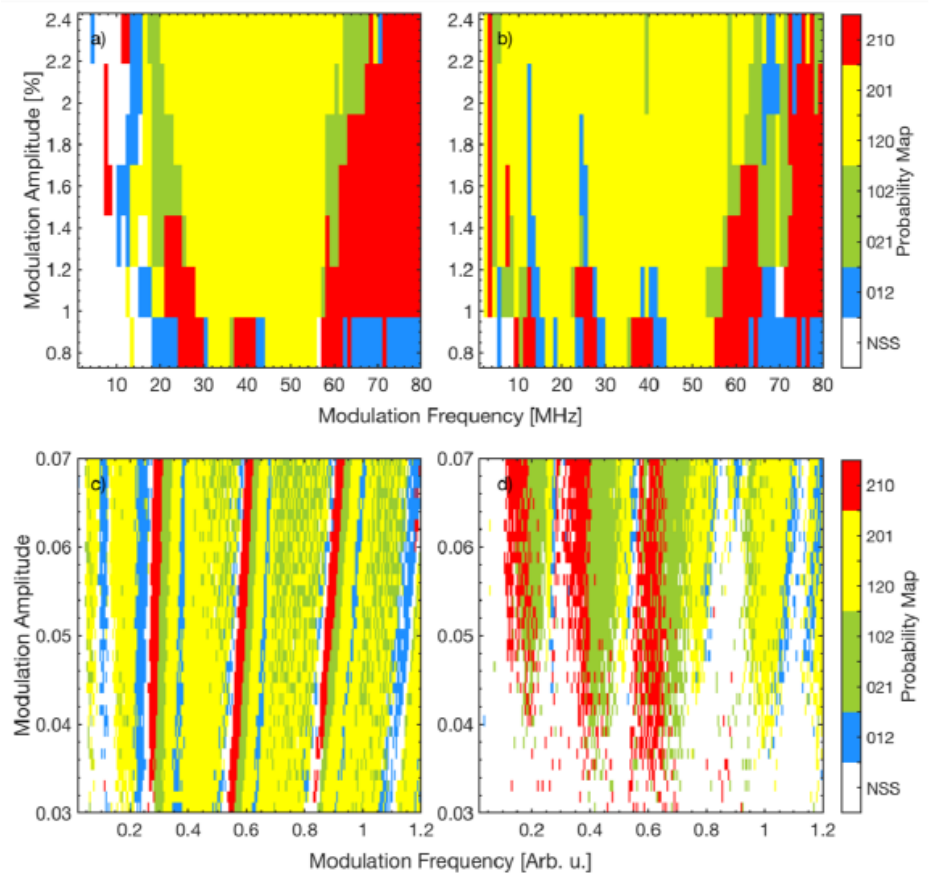
# Ordinal analysis uncovers differences in ordinal probabilities

Most probable pattern in color code

Sinusoidal Pulsed signal

Diode laser with optical feedback

FitzHugh-Nagumo model




J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, New J. of Phys. 21, 103039 (2019).



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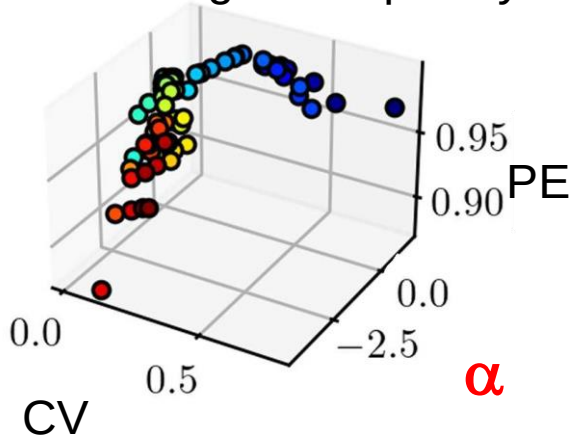
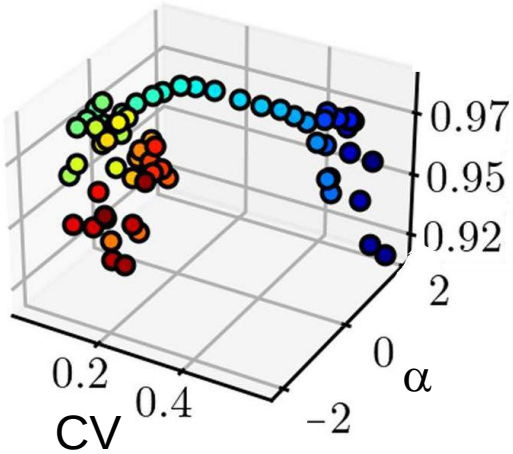
# Can we "recover" information of the signal applied to the laser current, from the analysis of the spike timing?

Bruno Boaretto

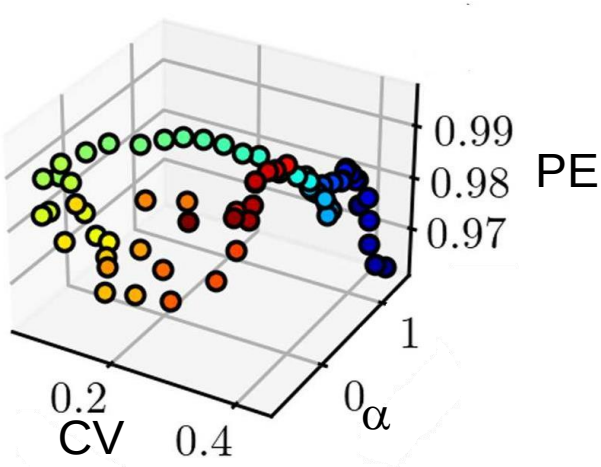
$I_{dc}$=26.5 mA
Color: signal frequency




$I_{dc}$=27 mA




$I_{dc}$=27.5 mA




CV=σ/⟨ISI⟩

Machine learning fit of the spectrum of the ISI time series to flicker noise, $P(f)=1/f^{\alpha}$


Bruno R. R. Boaretto, R. C. Budzinski, K. L. Rossi, T. L. Prado, S. R. Lopes, C. Masoller, "*Discriminating chaotic and stochastic time series using permutation entropy and artificial neural networks*", Sci. Rep. 11, 15789 (2021).

Bruno R. R. Boaretto, E. E. N. Macau, C. Masoller, "*Characterizing the spike timing of a chaotic laser by using ordinal analysis and machine learning*", Chaos 34, 043108 (2024).

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# Single-neuron vs ensemble encoding

Maria Masoliver

- Single-neuron encoding: **slow** because long spike sequences are needed to estimate the ordinal probabilities.
- Ensemble encoding: can be much **faster** because, from the ISI sequences of all the neurons, few spikes per neuron can be enough to accurately estimate the probabilities.

subthreshold input

$$\epsilon \dot{u}_i = u_i - \frac{u_i^3}{3} - v_i + a_0 \cos(2\pi t/T) + \frac{\sigma}{k_i} \sum_j^N a_{ij}(u_j - u_i) + \sqrt{2D}\xi_i(t), \qquad i \neq j$$

$$\dot{v}_i = u_i + a.$$

$$k_i = \sum_j a_{ij}$$

$$a_{ij} = a_{ji} = 1$$
$$a_{ij} = a_{ji} = 0$$

Random with prob. ***p***

Maria Masoliver and C. Masoller, "*Neuronal coupling benefits the encoding of weak periodic signals in symbolic spike patterns*", Commun. Nonlinear Sci. Numer. Simulat. 88, 105023 (2020).



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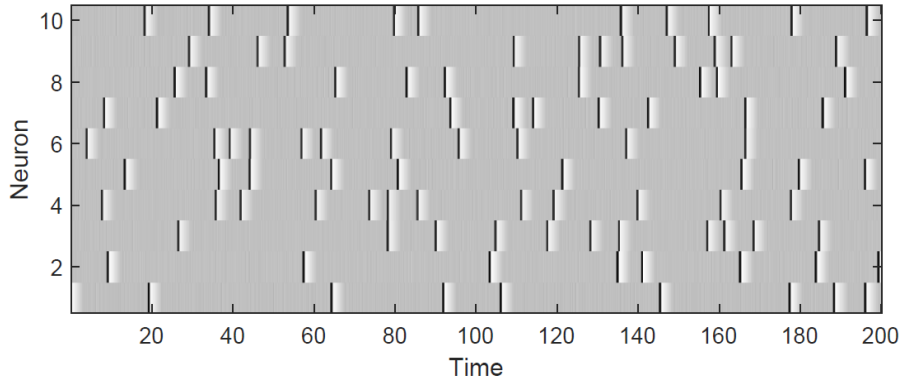
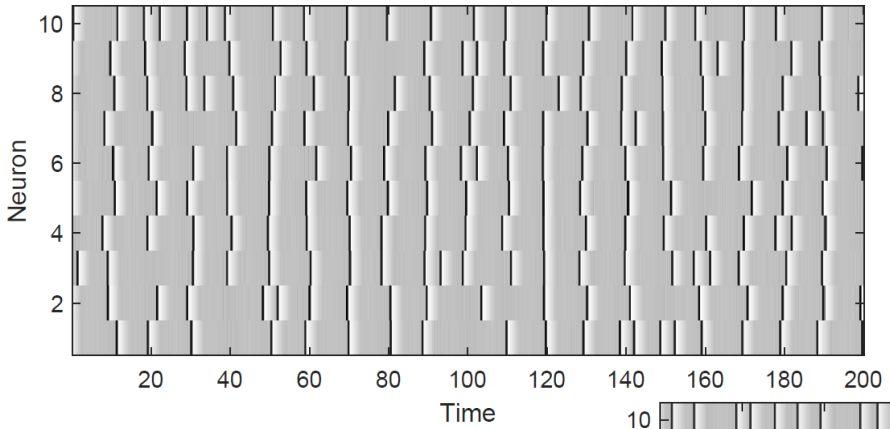
# Spiking dynamics with/without coupling, with/without external signal

No coupling
No external signal
(noise-induced spikes)

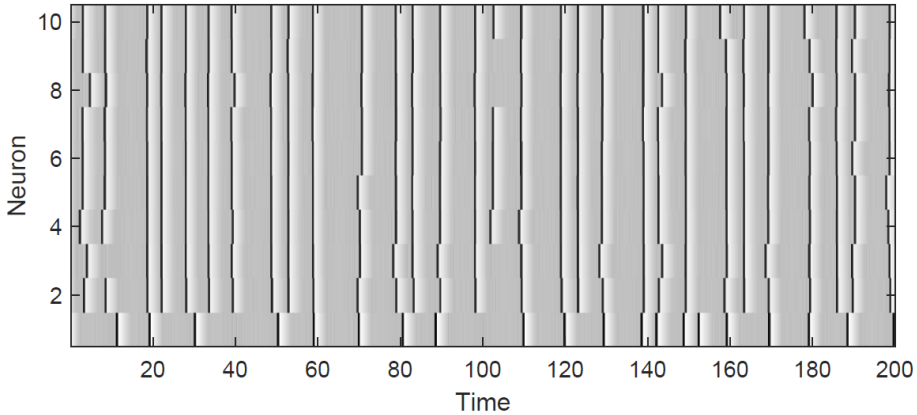
$\sigma=0$
$a_0=0$




$\sigma=0$
$a_0\neq0$




$\sigma\neq0$
$a_0\neq0$



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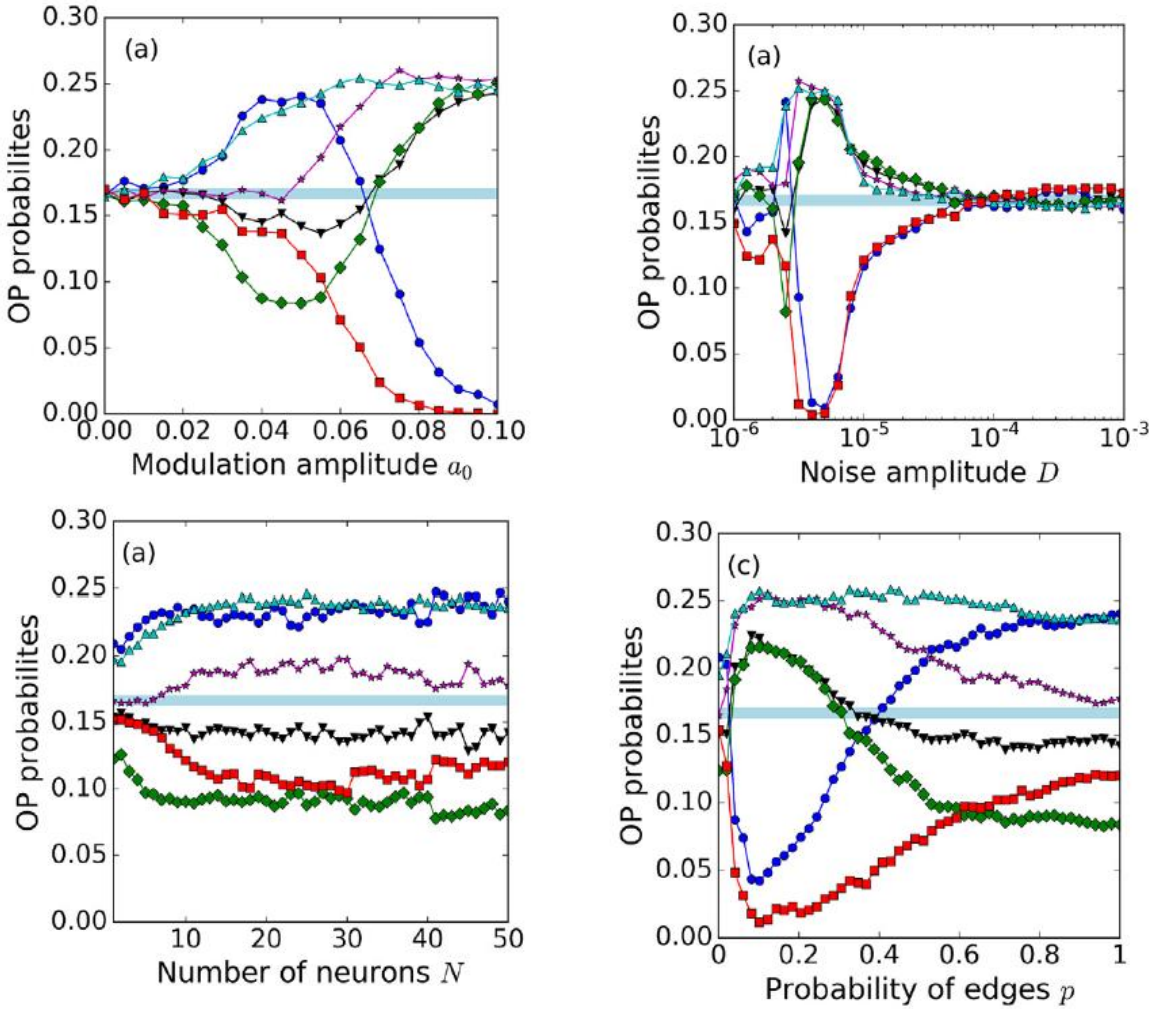
# Ensemble encoding of a weak sinusoidal signal in the frequencies of occurrence of ordinal patterns




M. Masoliver and C. Masoller, Commun. Nonlinear Sci. Numer. Simulat. 88, 105023 (2020).



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# Different symbolic methods can provide complementary information and can be combined.

PHYSICAL REVIEW RESEARCH **8**, 023165 (2026)

## Comparison of the performance of features based on symbolic dynamics and recurrence microstates for time series classification

H. C. Costa,[1,2] J. V. M. Silveira,[1,2] B. R. R. Boaretto,[3,4] C. Masoller,[3] S. R. Lopes,[1,2] and T. L. Prado[1,2,*]

[1]*Department of Physics, Federal University of Paraná, Curitiba 81531-980, Brazil*

[2]*Interdisciplinary Center for Science, Technology and Innovation CICTI, Federal University of Paraná, Curitiba, Brazil*

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[4]*Science and Technology Institute—ICT, Federal University of São Paulo, São José dos Campos, Brazil*

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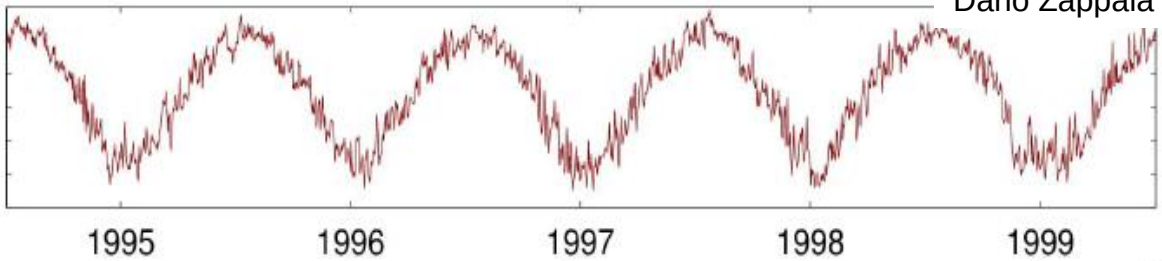
# Second data analysis method: Hilbert analysis

Dario Zappala

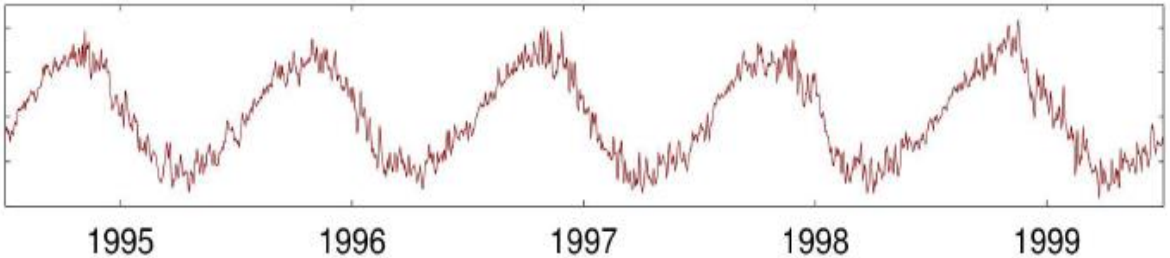
Surface Air Temperature (SAT) in a region

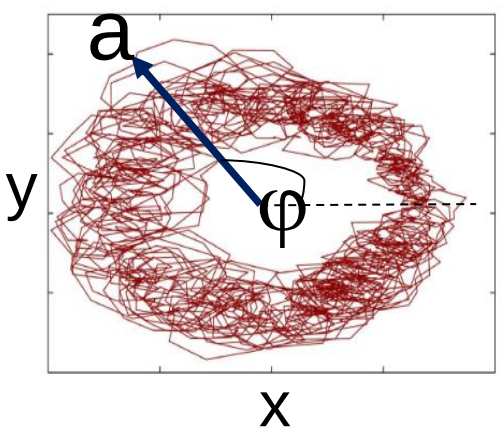
x(t)




HT[sin(x)]=cos(x)

y(t)= HT[x]




*Instantaneous* amplitude and phase

$$a(t) = \sqrt{[x(t)]^2 + [y(t)]^2}$$

$$\varphi(t) = \arctan[y(t)/x(t)]$$

Clear physical meaning only if *x(t)* is a narrow-band signal. Then, *a(t)* coincides with the envelope of *x(t)* and $\omega(t)=d\varphi/dt$, coincides with the main frequency in the spectrum.



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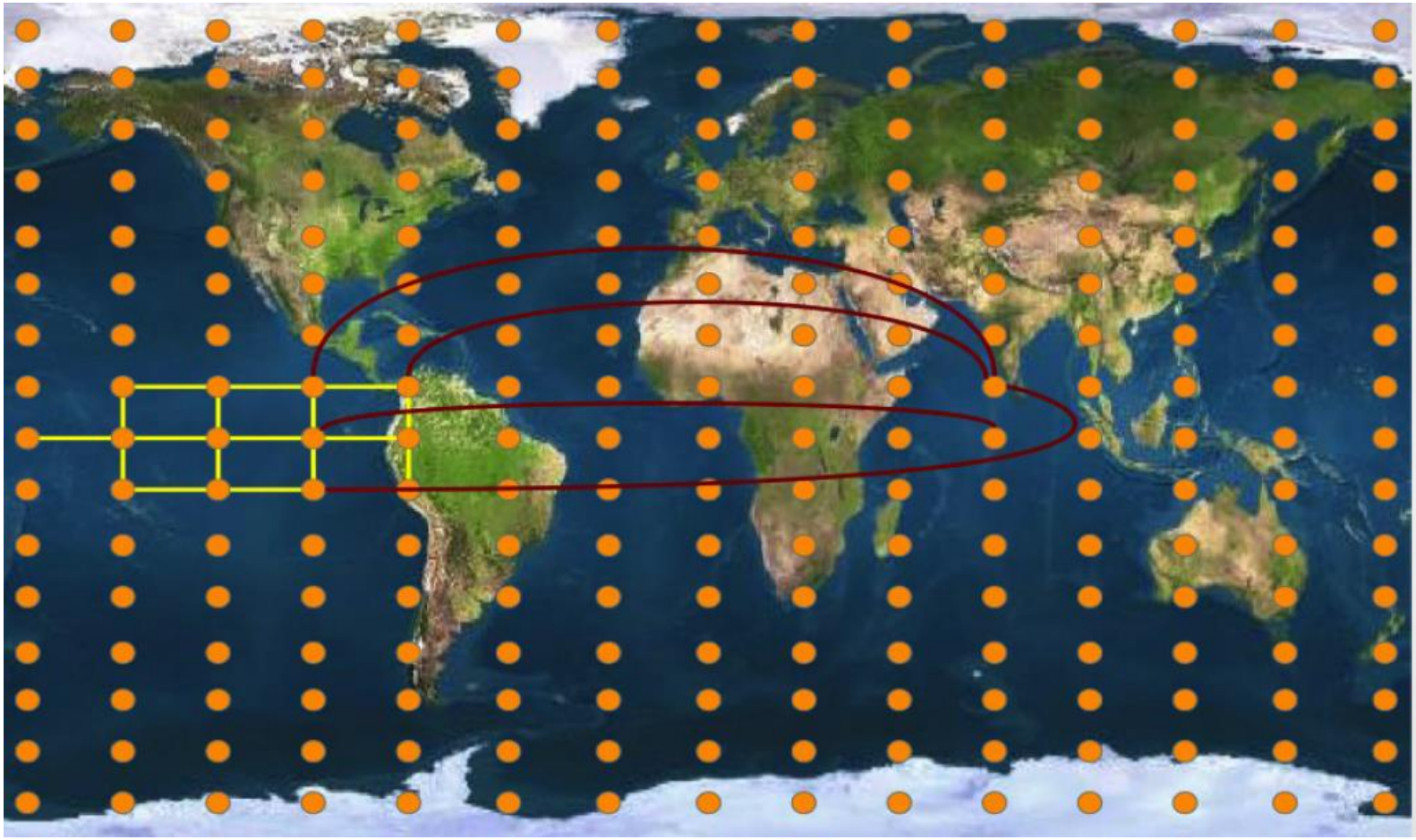
**Using the HT we analyzed "re-analysis data" from the *European Centre for Medium-Range Weather Forecasts*, with high spatial and temporal resolution in the period 1979-2016**



73 x 144 = 10 512 geographical sites, in each site the SAT time series has 13696 days



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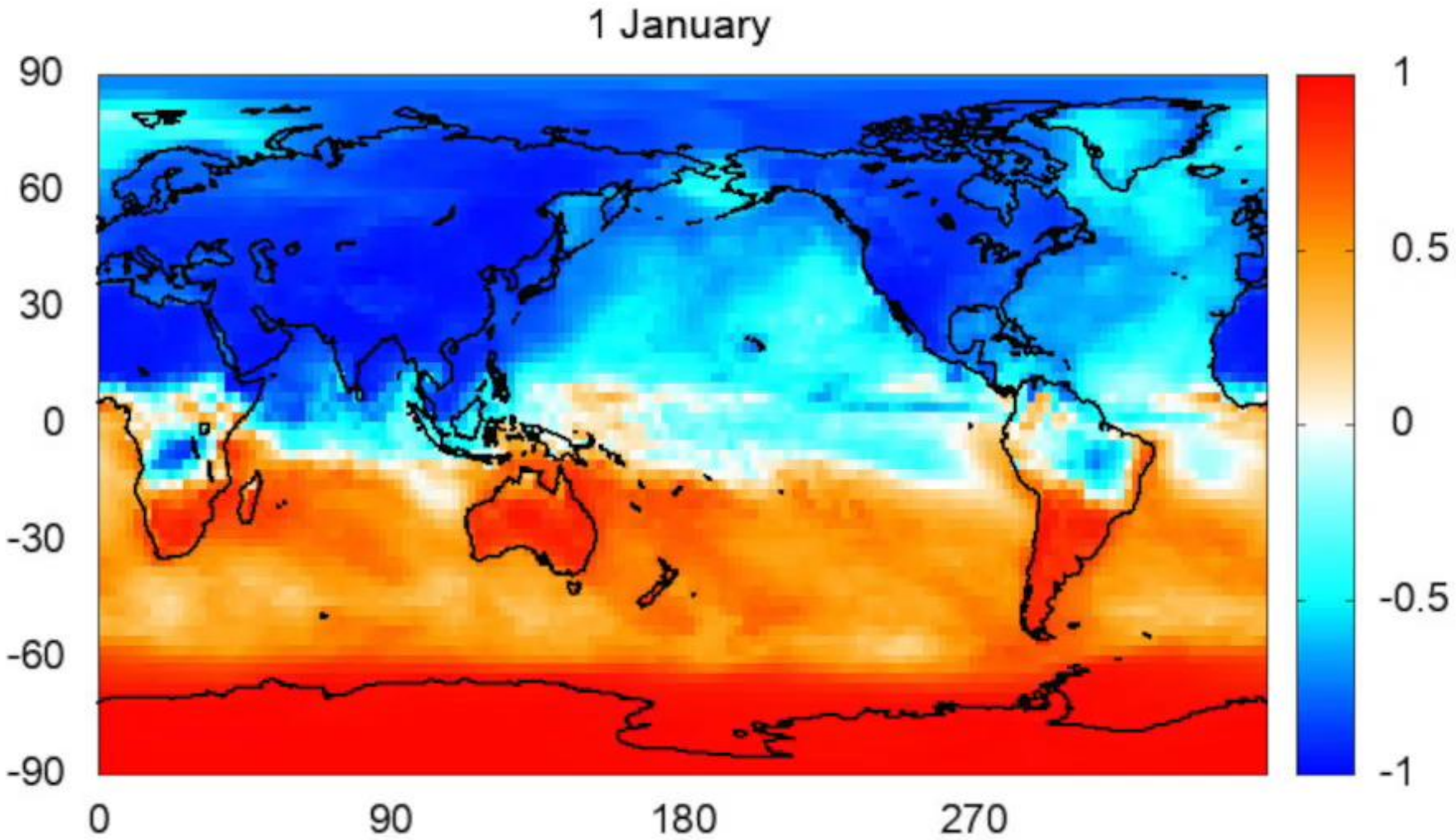
# Average of the cosine of Hilbert phase of surface air temp.

## Can we visualize the passing of the seasons?



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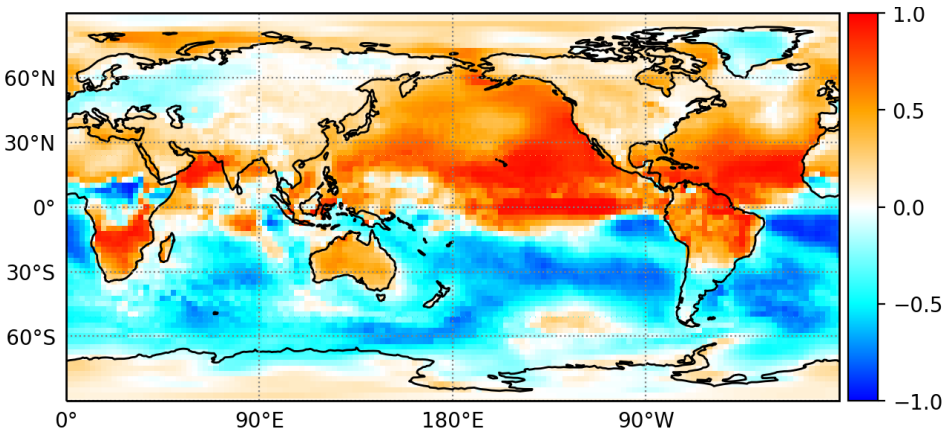
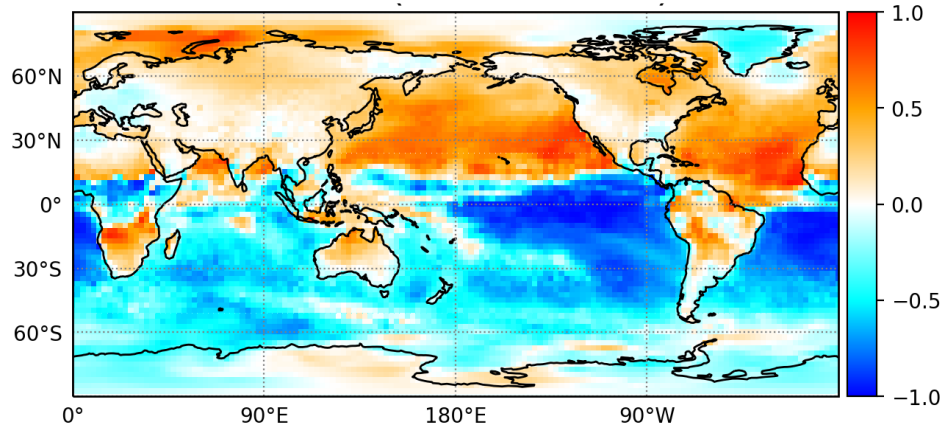
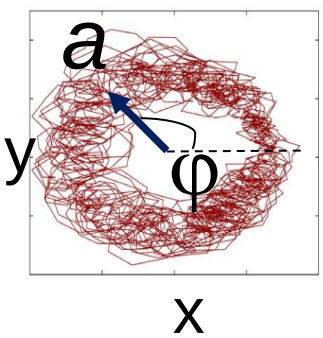
## Cosine of Hilbert phase during an *El Niño* year




## Cosine of Hilbert phase during a *La Niña* year




**Can we detect significant changes in the last decades in the amplitude of the oscillations of surface air temperature?**




In each grid point we calculate:

$$\Delta a = \langle a \rangle_{2016-2007} - \langle a \rangle_{1988-1979}$$

$$\frac{\Delta a}{\langle a \rangle_{2016-1979}}$$

Significant if: $\frac{\Delta a}{\langle a \rangle} \geq \langle . \rangle_s + 2\sigma_s$ or $\frac{\Delta a}{\langle a \rangle} \leq \langle . \rangle_s - 2\sigma_s$

(100 surrogates)

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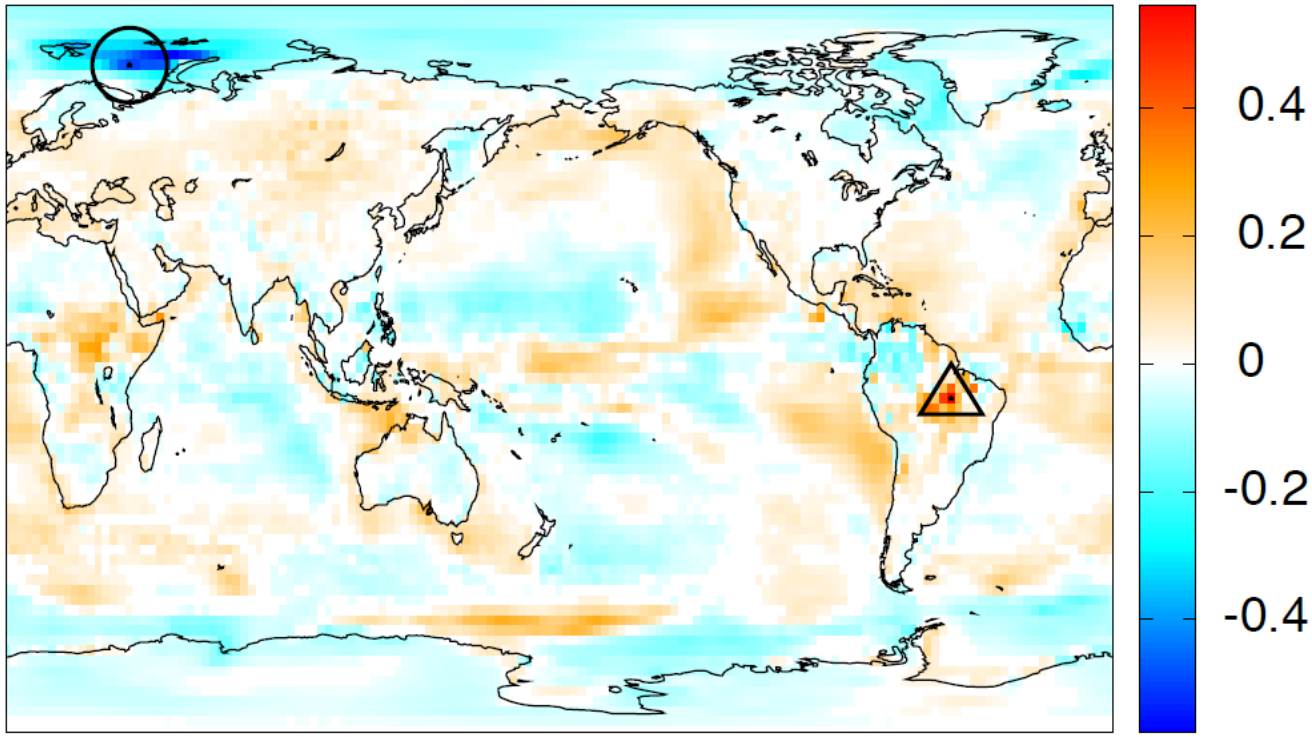
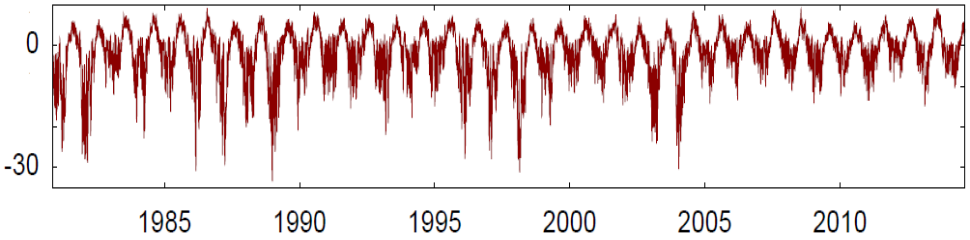
Color code:

$$\frac{\Delta a}{\langle a \rangle_{2016-1979}}$$

White: not significant

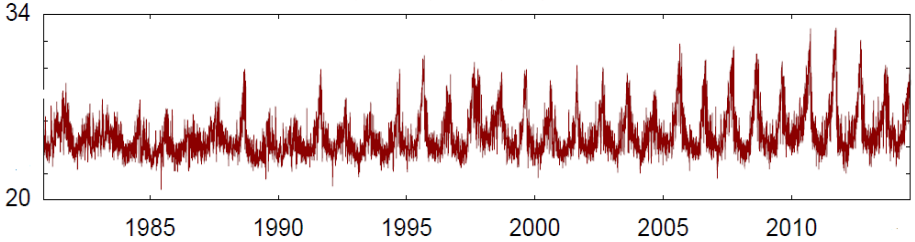



**Artic**




Sea ice melting

**Amazonas**




Deforestation

Dario A. Zappala, M. Barreiro and C. Masoller, Earth Syst. Dynam. 9, 383 (2018).



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## Take home messages

- Nonlinear data analysis techniques are useful to uncover patterns and relationships in data generated by complex systems.
- Different methods provide *complementary* information.
- “Surrogate” tests are needed to analyze statistical significance.
- Even when the data does not meet the mathematical or algorithmic requirements, the results can give useful info.
- Prof. Holger Kantz: “*Every data set bears its own difficulties: data analysis is never routine*”.



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REVIEWS OF MODERN PHYSICS, VOLUME 95, JULY–SEPTEMBER 2023

# Nobel Lecture: Multiple equilibria*

Giorgio Parisi†

*Dipartimento di Fisica, Università di Roma La Sapienza, INFN, Sezione di Roma I, CNR-NANOTEC UOS Roma Piazzale Aldo Moro 2, I-00185 Roma, Italy*

Extended version of Nobel Lecture, delivered on December 8, 2021.

Systems, to remain complex, must have an internal balance, and frustration should be high enough to avoid the formation of a ferromagnetic order. However, as stressed by Kondor (2021), it is possible that this delicate mechanism of co-operation, competition, stimulation, and inhibition is violated: in that case frustration decreases, the global behavior of it is that the system changes, the correlations between the sub-systems change, the overall system begins to be no longer complex, and something abnormal happens. Examples are a tumor in the cell, a disease in the nervous system, and a dictatorship in society. At this point, the complexity of the system degrades and the functioning of the system, as a whole, is severely damaged or even eliminated.



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So I would like to conclude with a quote from my friend Imre Kondor, who asserts that the loss of complexity is dangerous, and recall the warning attributed to a great 19th-century historian, Jacob Burckhardt, who studied political and social processes in depth: *The denial of complexity is the essence of tyranny* (Kondor, 2021).



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# Latin America Optics and Photonics Conference (LAOP)

09 - 12 November 2026

Universidad de los Andes, Bogota, COLOMBIA

- Optical Design, Instrumentation and Imaging Processing
- Atomic Physics, Metrology, Sensing and Spectroscopy
- Nonlinear & Quantum optics
- Laser Science and Technology
  *C. Masoller and Flavio C. Cruz, (UNICAMP, Campinas, Brasil)*
- Optical Coherence
- Optical Communications and Signal Processing
- Biophotonics and Medical Applications
- Materials and Integrated Photonics
- Other Multidisciplinary Approaches and Applications of Photonics

Submission deadline: 07 July 2026



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# Funding and references


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- J. Gancio, et al. "*Identifying and anticipating the threshold bifurcation of a complex laser with permutation entropy*", PRL 135, 093802 (2025).


Muito obrigada
pela sua atenção!



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