



REUNIÓN DE LA ASOCIACIÓN FÍSICA ARGENTINA
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LA PLATA 2025
SEPTIEMBRE 15 - 18

Tercera Reunión Conjunta AFA-SUF
15-18 de septiembre de 2025
La Plata, Argentina



Metodologías de análisis de datos no lineales para la investigación de sistemas complejos

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UNIVERSITAT POLITÈCNICA
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ICREA



Presentation (1/2)



UNIVERSIDAD DE LA REPÚBLICA
URUGUAY



PHYSICS LETTERS A



- Bachelor and Master degrees in Montevideo, Uruguay (1986-1991).

- Master thesis co-supervised by Prof. Lilia Romanelli (UBA).

Physics Letters A 167 (1992) 185–190
North-Holland

Regular and chaotic behavior in the new Lorenz system

C. Masoller, A.C. Sicardi Schifino

*Instituto de Física de la Facultad de Ciencias, T. Narvaja 1674, Montevideo, Uruguay
and Instituto de Física de la Facultad de Ingeniería, J. Herrera y Reissig 565, CC30, Montevideo, Uruguay*

and

Lilia Romanelli

Departamento de Física, F.C.E. y N. (U.B.A.), Ciudad Universitaria (1428), Buenos Aires, Argentina

Received 21 August 1991; revised manuscript received 11 May 1992; accepted for publication 13 May 1992

Presentation (2/2)

- Bachelor and Master in Uruguay (1986-1991).
- PhD in Laser Physics (1999, Bryn Mawr College, PA, USA)
- Universitat Politècnica de Catalunya (2004 –to date; Prof. Catedratic since 2018)



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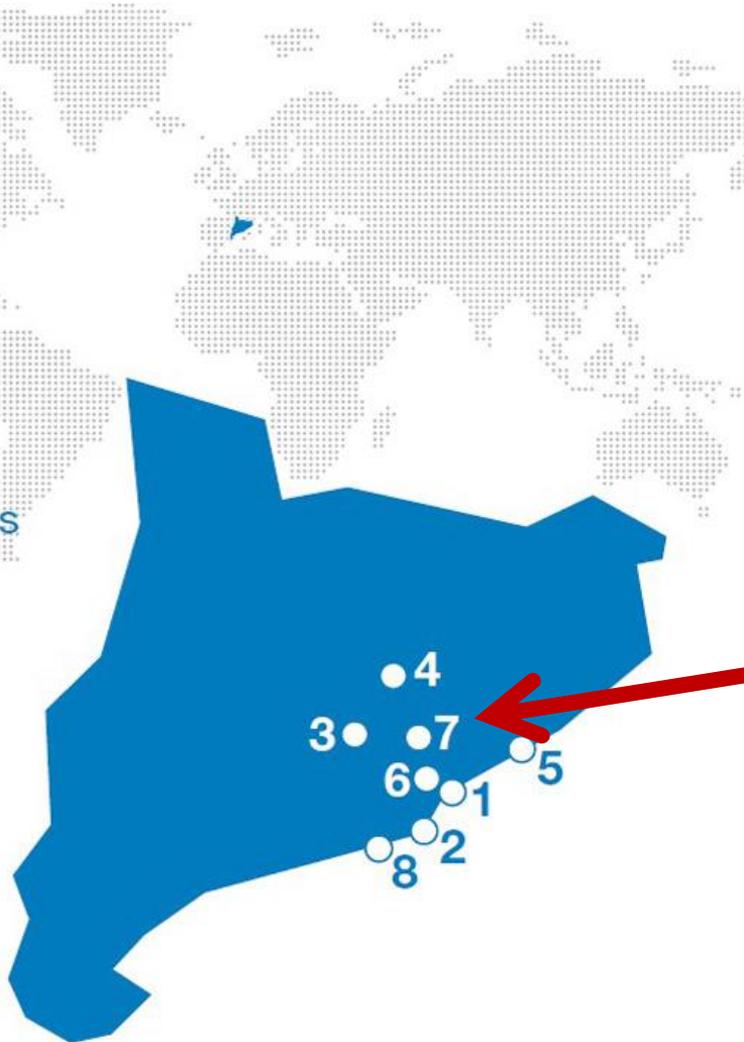
Marcelo Barreiro
*Universidad de la
República, Uruguay*

Where are we? Universitat Politècnica de Catalunya

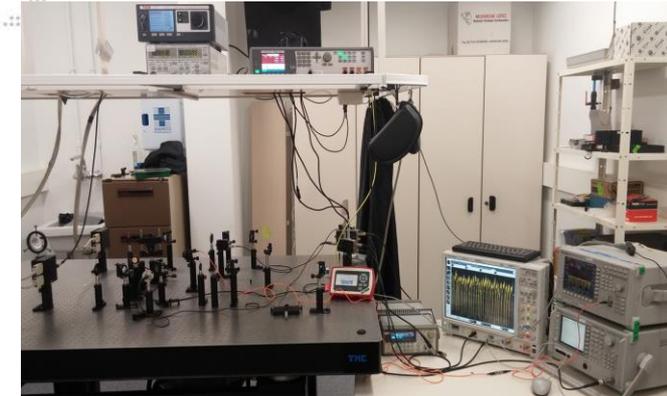
Campus Terrassa

Viernes, 25 de septiembre de 2009 Diari de Terrassa

1. Barcelona
2. Castelldefels
3. Igualada
4. Manresa
5. Mataró
6. Sant Cugat del Vallès
7. Terrassa
8. Vilanova i la Geltrú



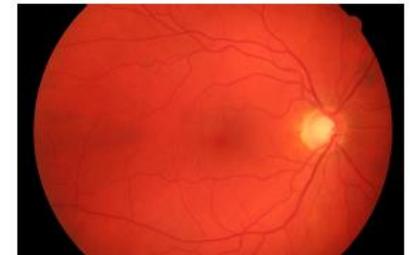
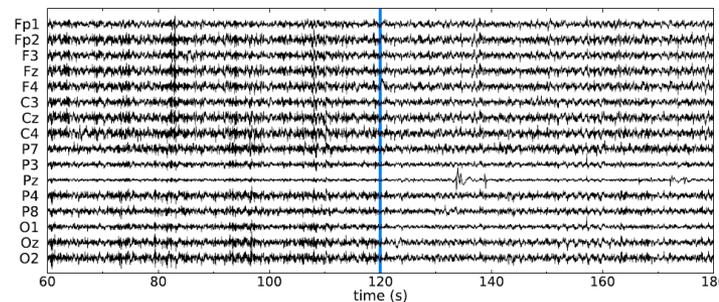
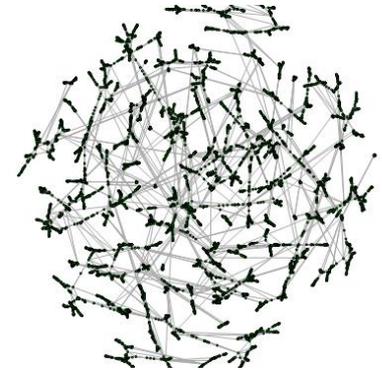
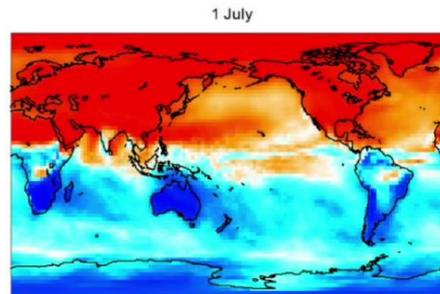
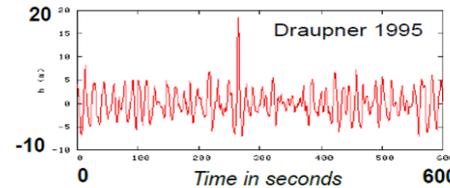
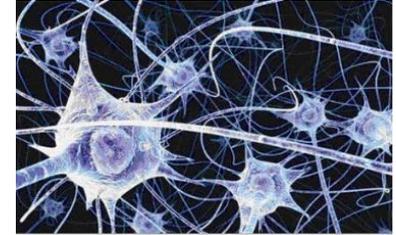
El edifici Gala centraliza grups científics consolidats i emergents.



Semiconductor laser lab

Research topics

- Nonlinear laser dynamics
- Neuronal dynamics
- Synchronization
- Complex networks
- Extreme events
- Climate data analysis
- Biomedical data analysis



The Nobel Prize in Physics 2021



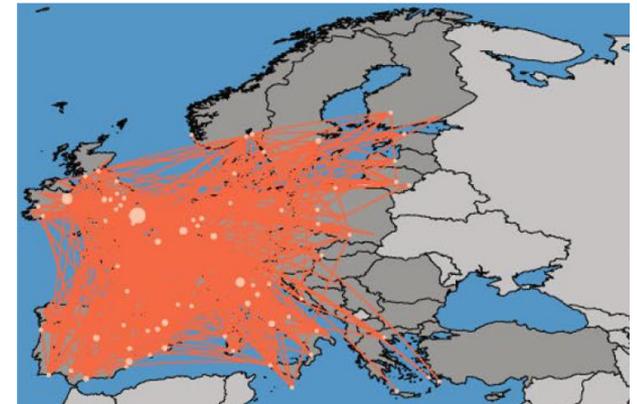
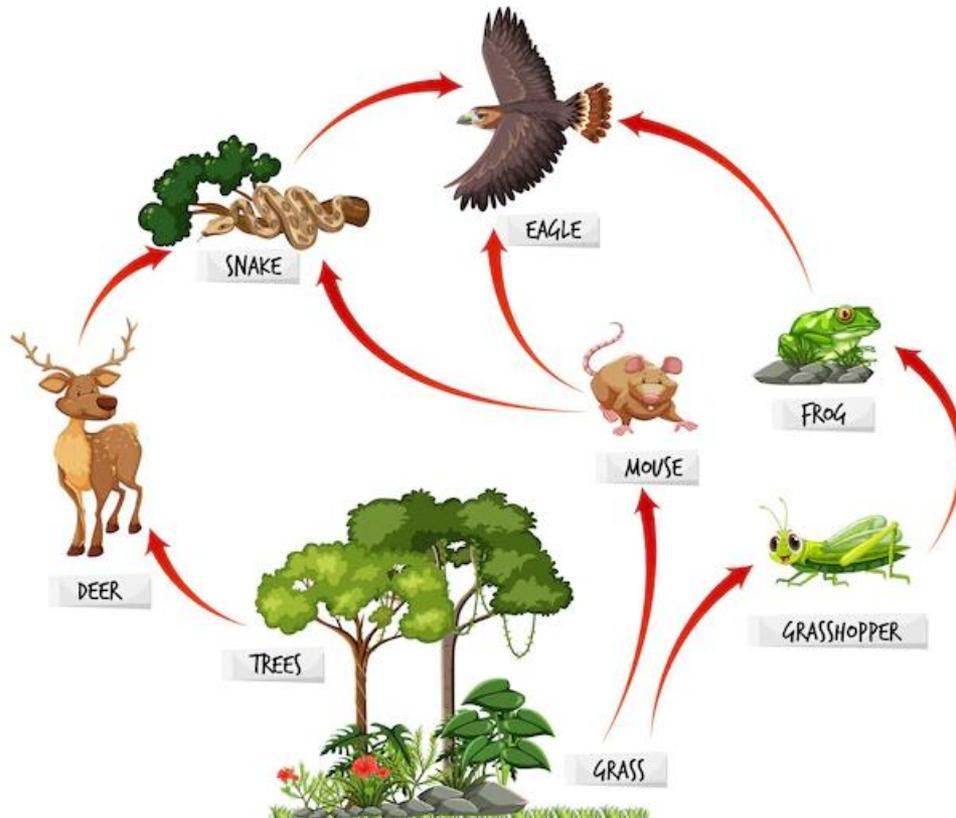
for groundbreaking contributions to our understanding of **complex systems**

½ Syukuro Manabe and Klaus Hasselmann ½ Giorgio Parisi

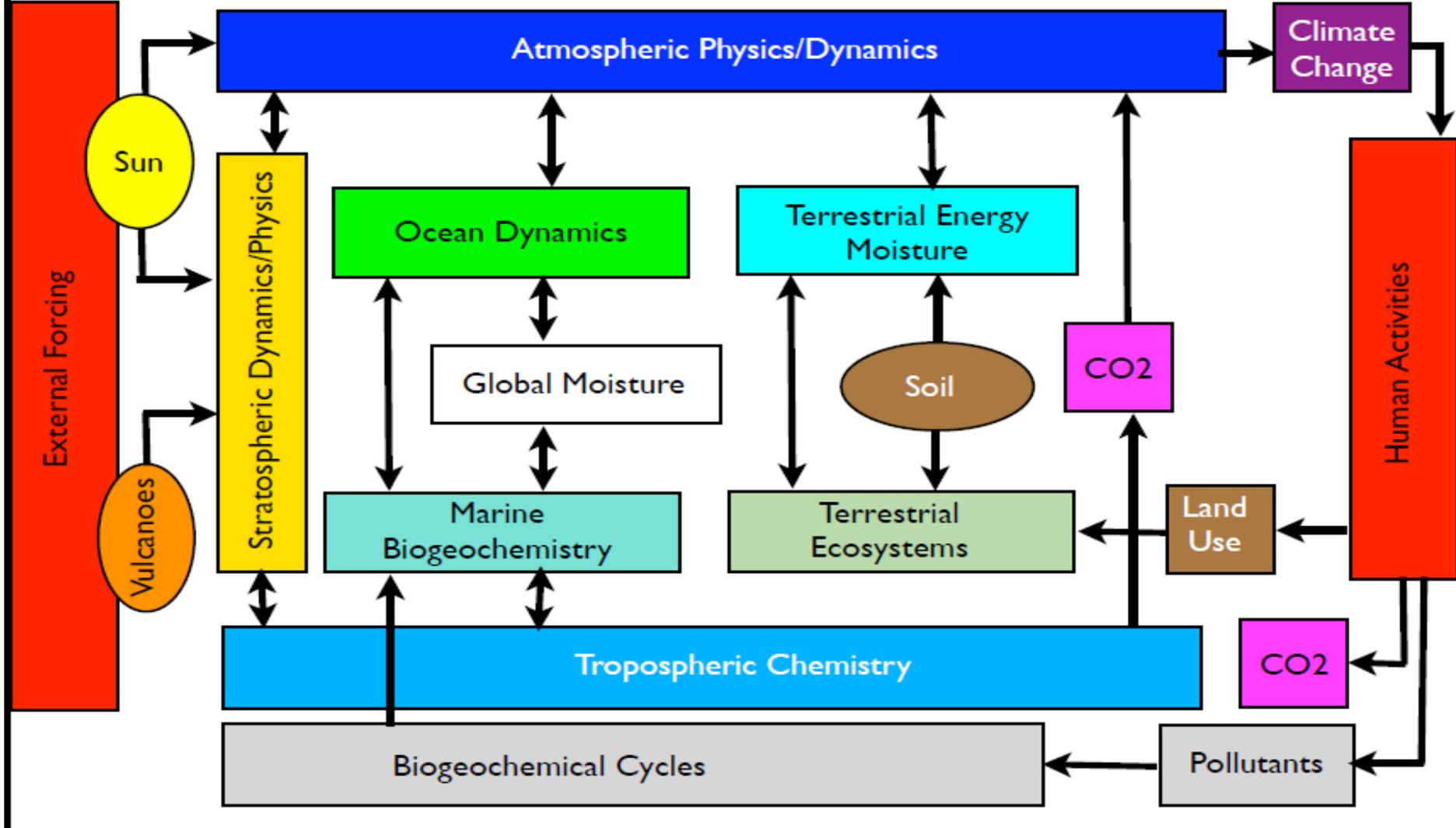
What is a complex system?

Complex systems are

- High-dimensional (huge number of variables or elements)
- Nonlinear (the elements and/or the interactions are nonlinear)
- Heterogeneous, multiscale, have memory and adapt
- Often modeled as complex networks / graphs

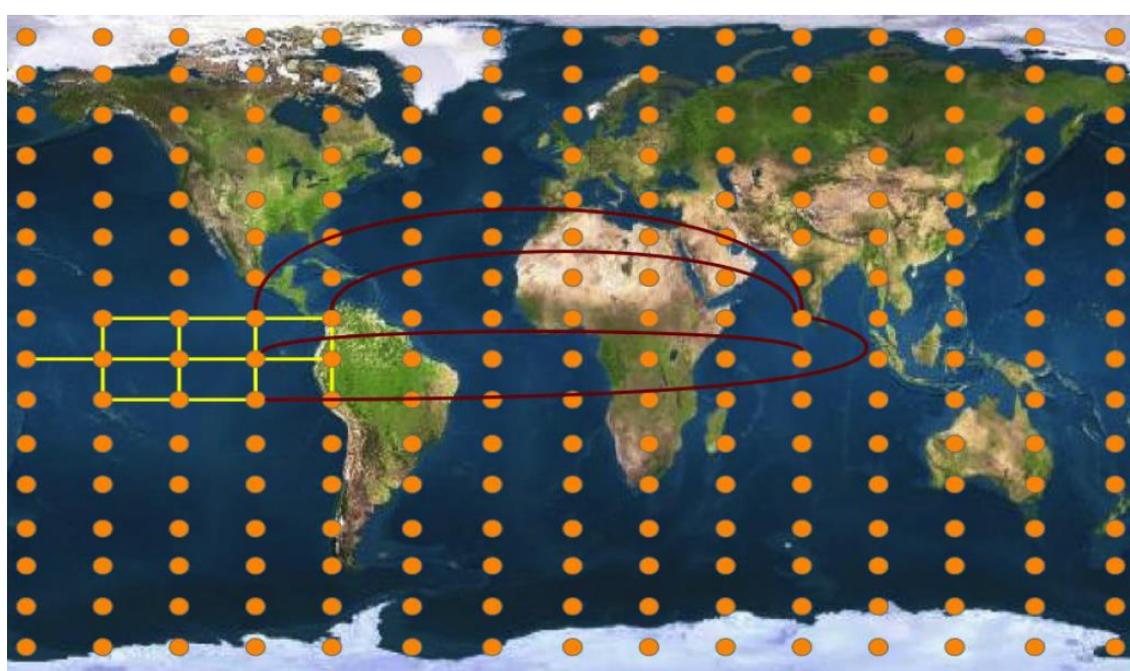


The Climate System



Courtesy of Henk Dijkstra (Universidad de Utrecht)

The analysis of interdependencies and causal interactions reveals networks of tele-connections



H. A. Dijkstra, E. Hernandez-Garcia, C. Masoller and M. Barreiro,
“*Networks in Climate*”, Cambridge University Press 2019

Are extreme weather events becoming more extreme? More frequent?



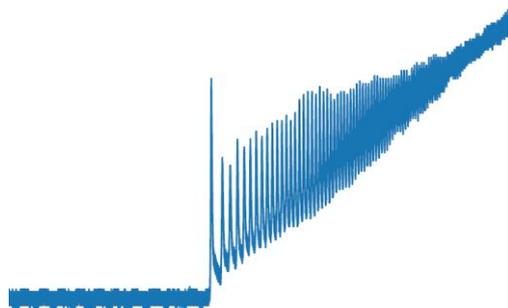
ECMWF



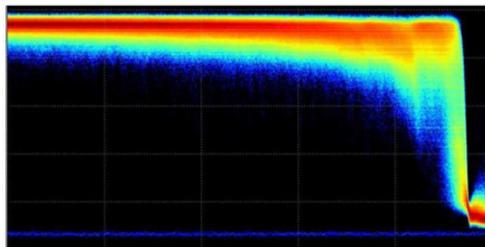
Credit: Richard Williams, North Wales, UK

Complex systems typically display:

- Emergent phenomena synchronization
- Gradual or abrupt transitions
- Extreme fluctuations



Time



Time

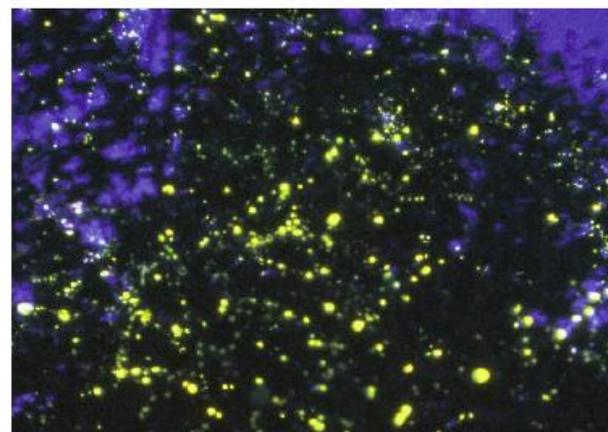
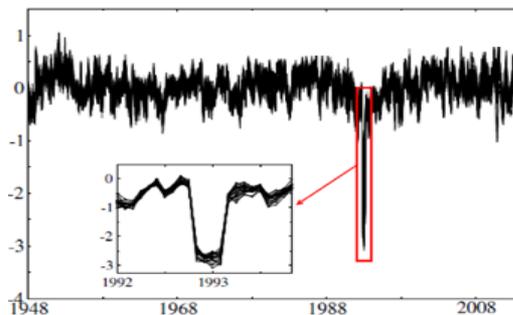
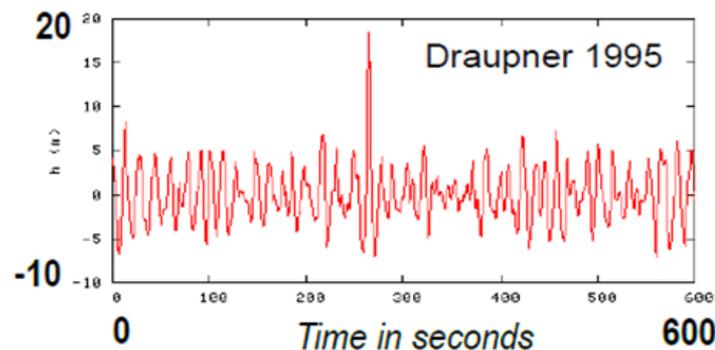


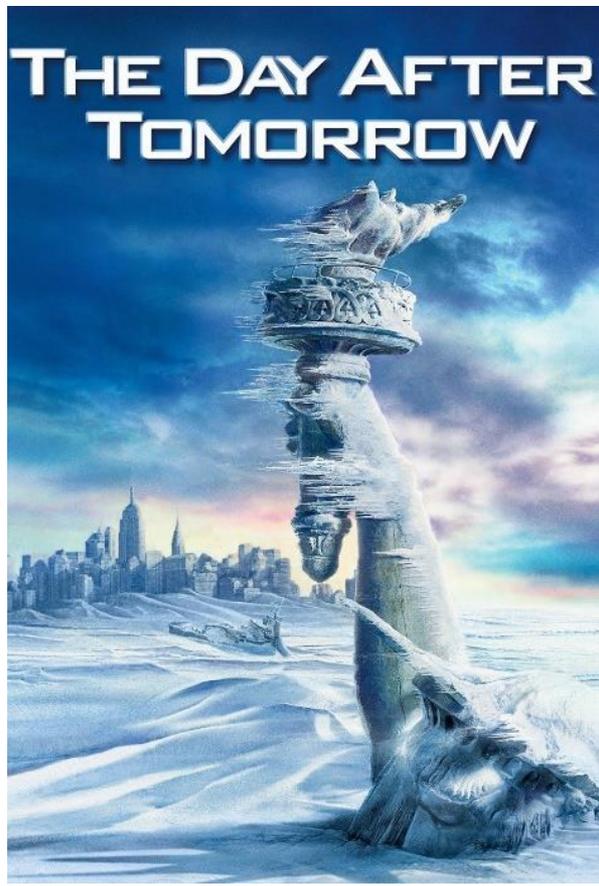
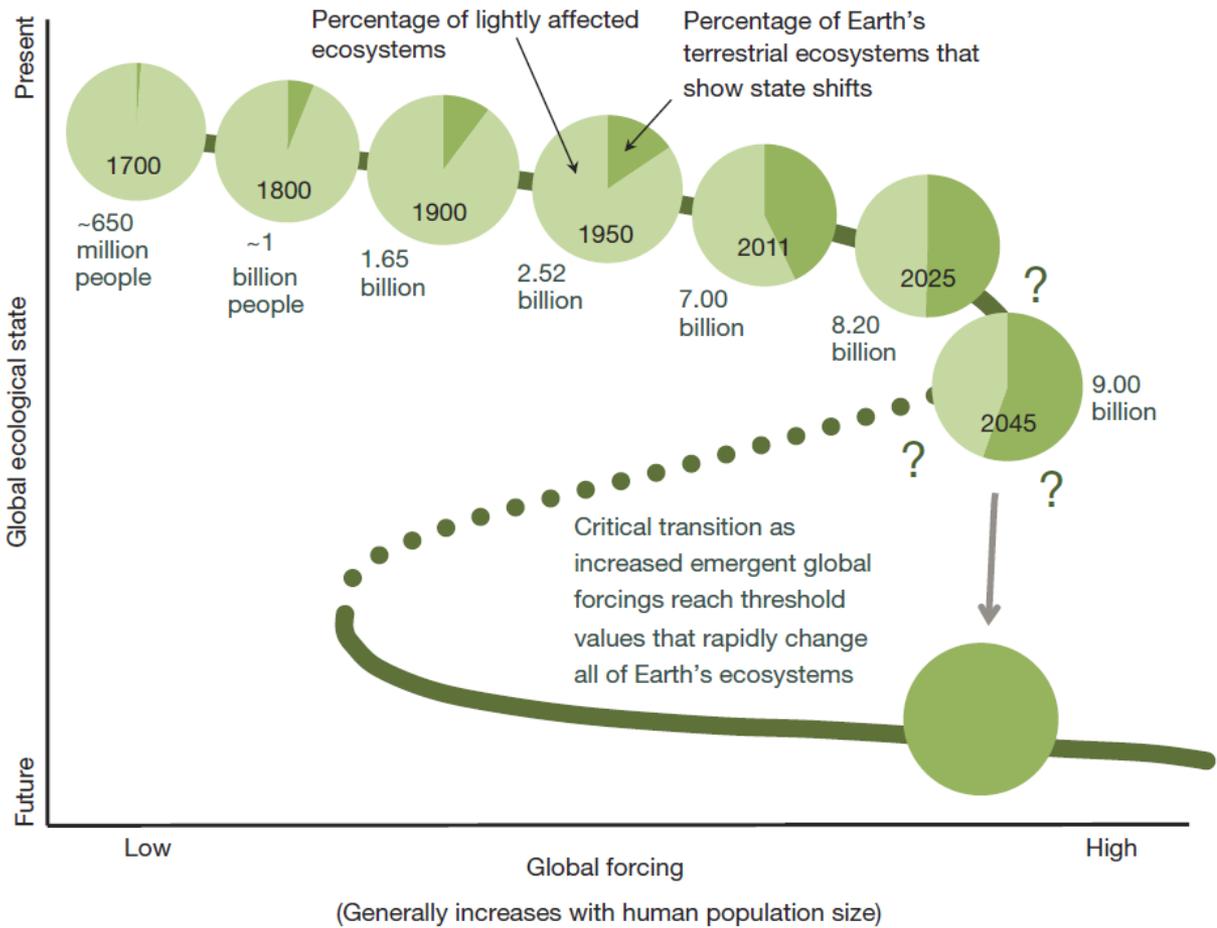
Figure 1 | Fireflies, fireflies burning bright. In the forests of the night, certain species of firefly flash in perfect synchrony — here *Pteroptyx malacciae* in a mangrove apple tree in Malaysia. Kaka *et al.*² and Mancoff *et al.*³ show that the same principle can be applied to oscillators at the nanoscale.



For the sake of clarity, what is NOT, in my view, a complex system:
Any linear system (no matter how big).
Low dimensional system.

A lot of research of “early warning signals” of critical transitions

Are we approaching a planetary-scale critical transition?



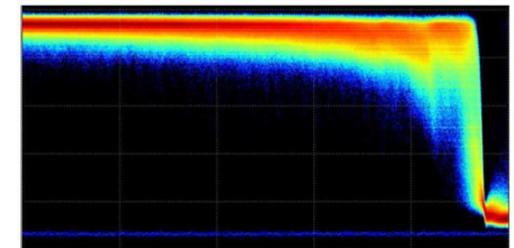
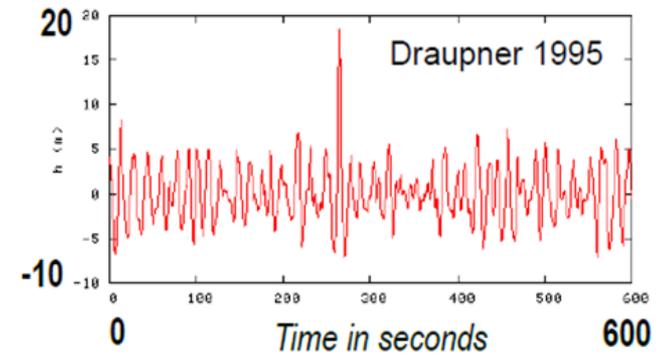
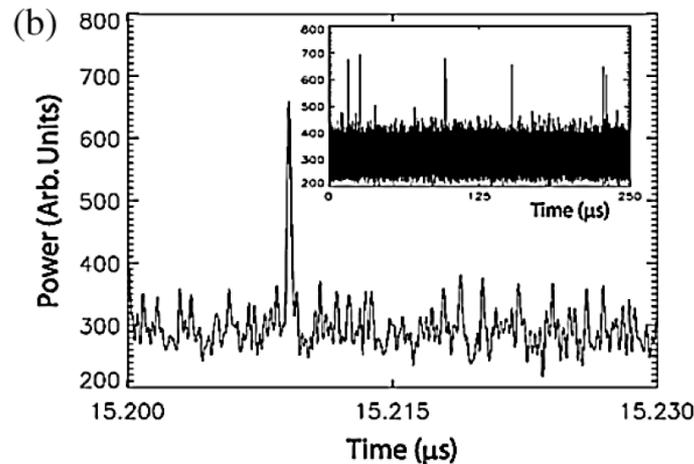
A. D. Barnosky et al, “Approaching a state shift in Earth’s Biosphere”, Nature 52, 486 (2012).

Controlled experiments with complex systems are rare

Time scales of photonic systems allow recording long data sets in short time.

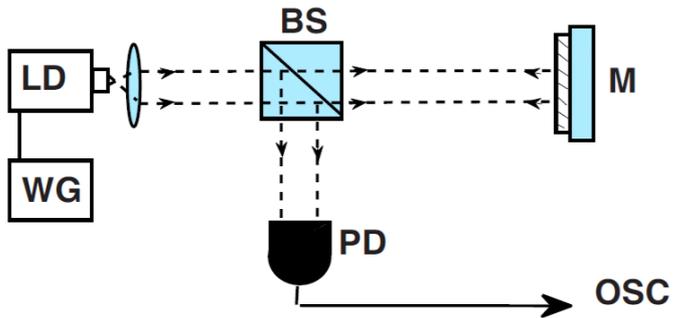
The power of analogies NATURE PHOTONICS | VOL 8 | JANUARY 2014 |

Black holes, gravitational lenses, turbulence, chaotic flow and rogue waves are just a few examples of complex physical phenomena that can be conveniently modelled using photonics.

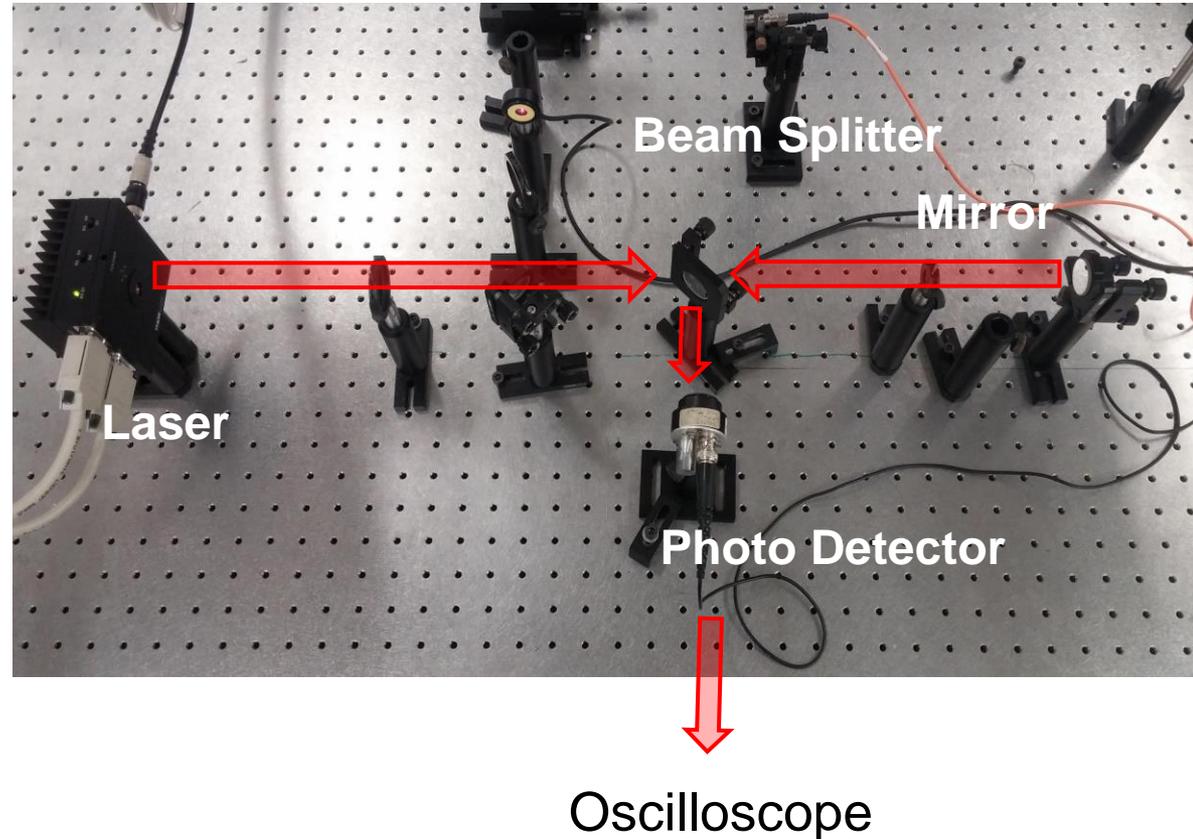


C. Bonatto et. al, Phys. Rev. Lett. 107, 053901 (2011)

In our lab: experiments with diode lasers with optical feedback



WG: wave generator used to modulate the laser current.

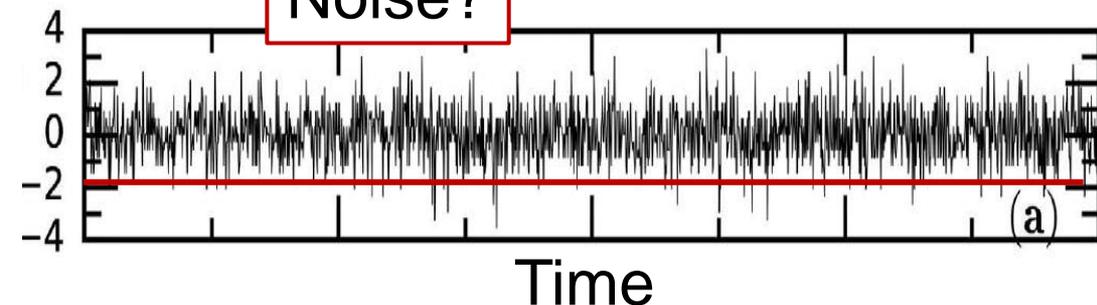


What do we see in the oscilloscope?
(with 1 GHz resolution)

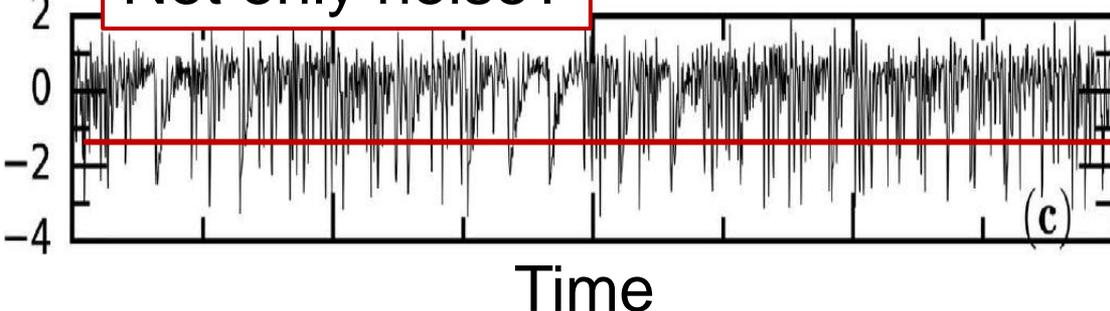
By counting the number of “threshold crossings” we can distinguish dynamical regimes

$$Z = \frac{X - \mu}{\sigma}$$

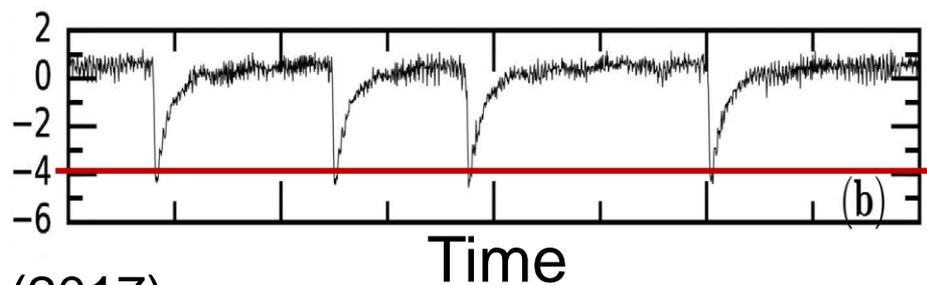
Noise?



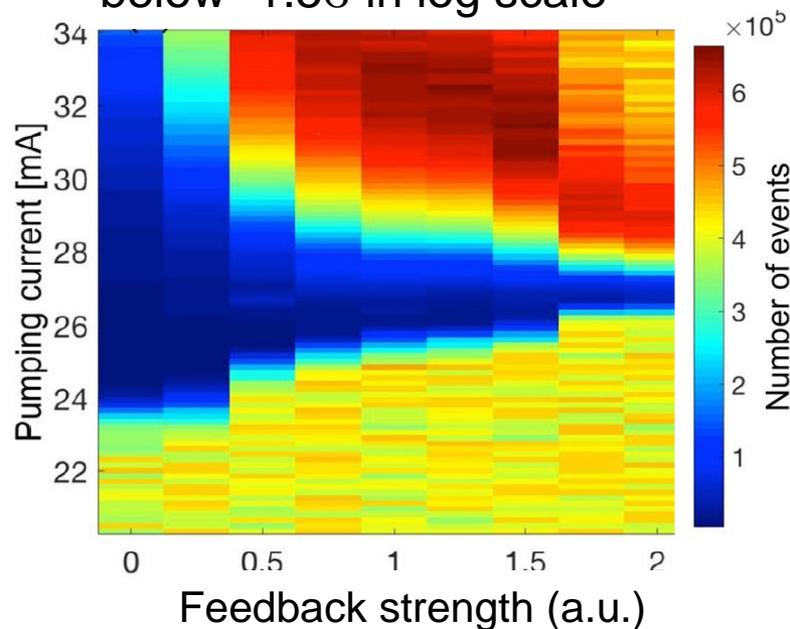
Not only noise?



Blue region? Optical Spikes

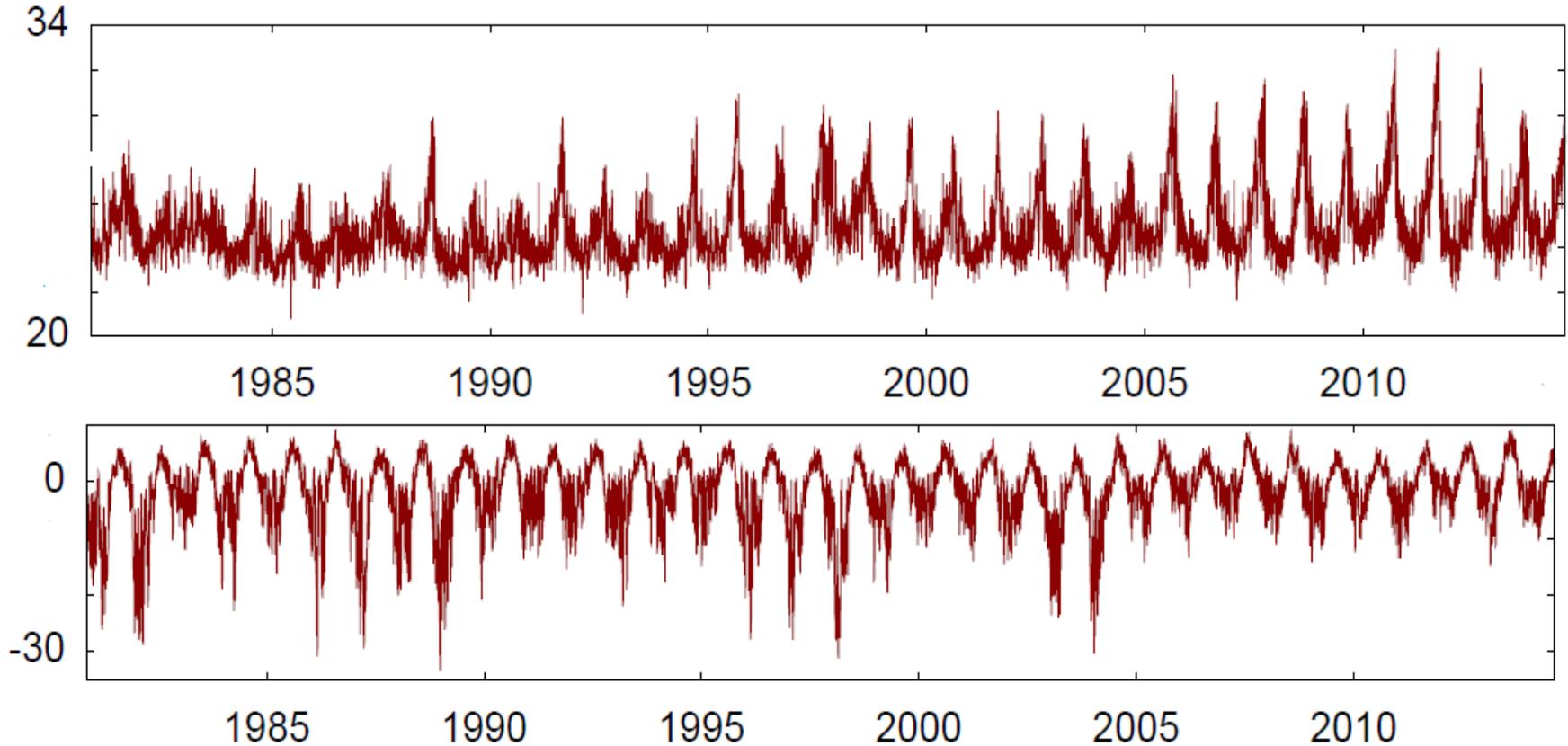


Number of crossings below -1.5σ in log scale



M. Panozzo et al, Chaos 27, 114315 (2017).

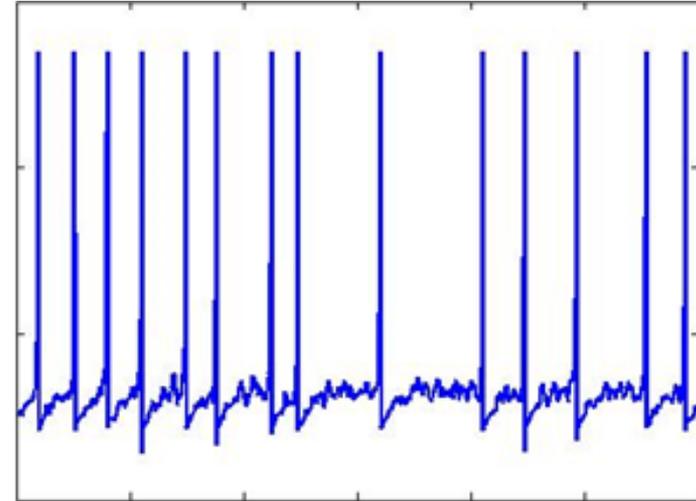
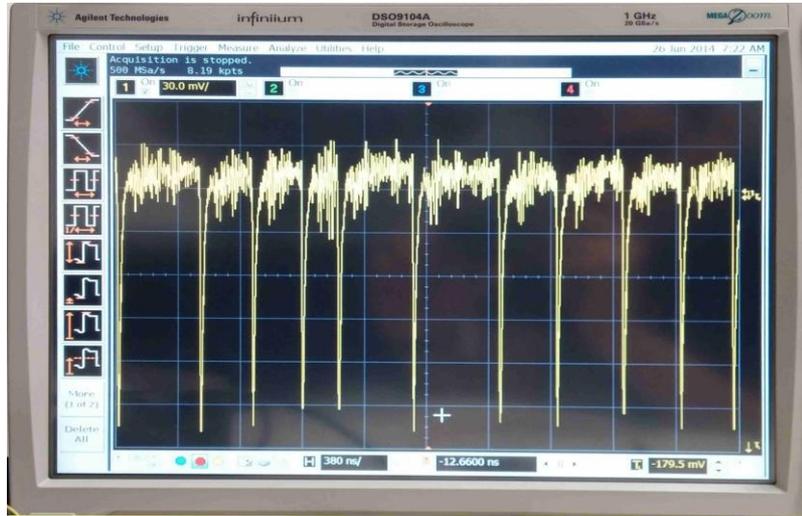
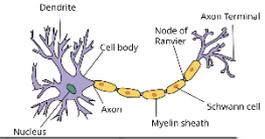
An example of a gradual change in behavior: Surface Air Temperature (SAT) in two geographical regions



How to quantify the differences?

Dario A. Zappala, M. Barreiro and C. Masoller, *Earth Syst. Dynam.* 9, 383 (2018)

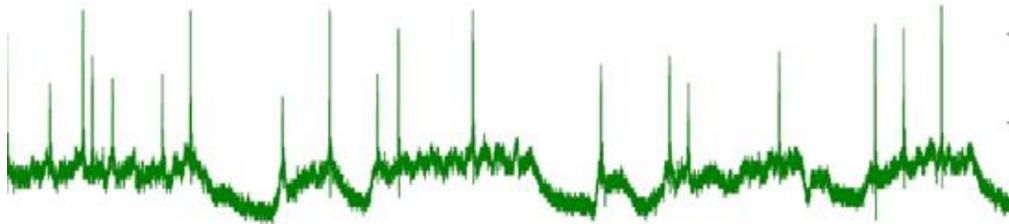
Intensity emitted by a diode laser and simulated spikes



Time 10^{-9} s

Time 10^{-3} s

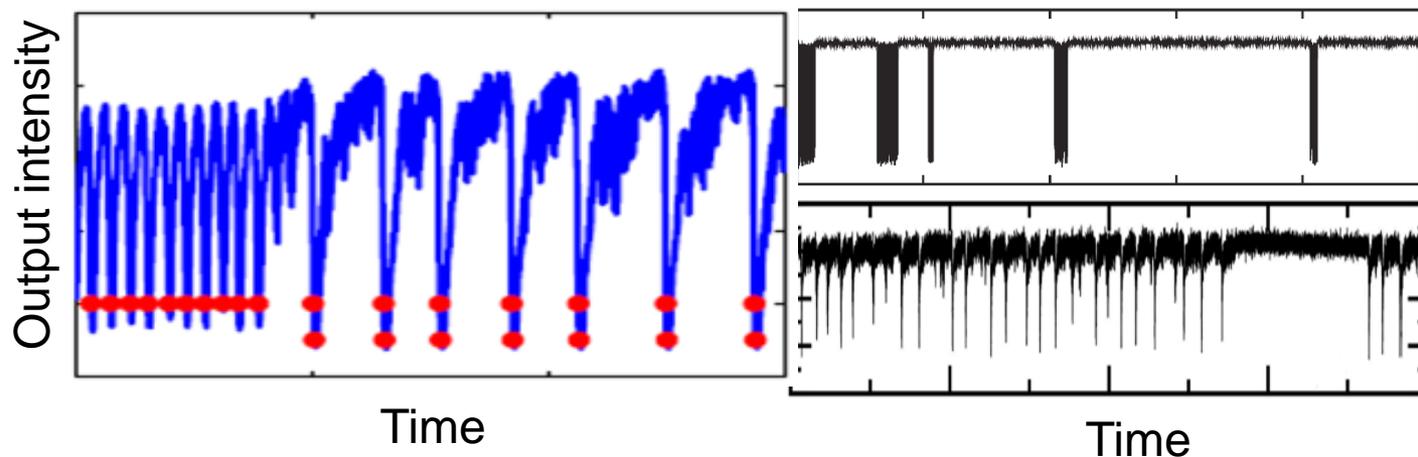
How neurons encode information?



Uncovering similarities between neuron and lasers...

Interesting but relevant?

- Data centers, AI and HPC systems consume a lot of power.
- Big concern in the context of climate change.
- The human brain works with only 20 Watts.
- **Laser-based neurons** should work as neurons, but
 - Much faster
 - With much less energy consumption.

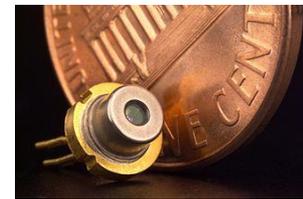


Statistically similar to neurons?

A. Aragonese et al., Sci. Rep. **4**, 4696 (2014).

C. Quintero-Quiroz et al., Sci. Rep. **6** 37510 (2016).

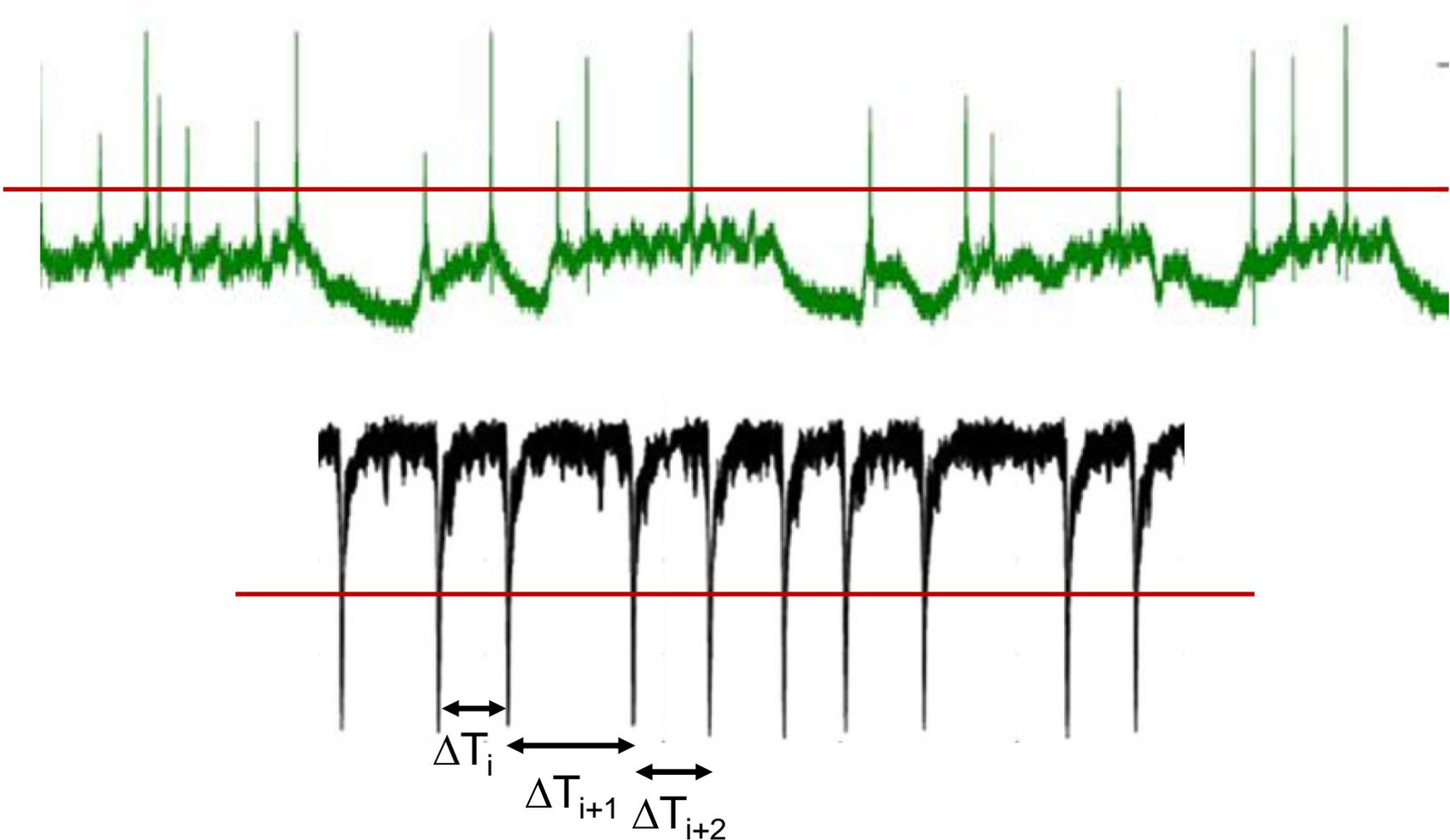
Why photonic neurons with semiconductor diode lasers?



- They are inexpensive, compact and energy-efficient.
- They emit a wide range of wavelengths.
- Nanolasers emit ultra low powers; large arrays can be integrated on chips (photronics integrated chips =PICs).
- There are several configurations in which a diode laser emits (“fires”) well-defined pulses of light (“optical spikes”).

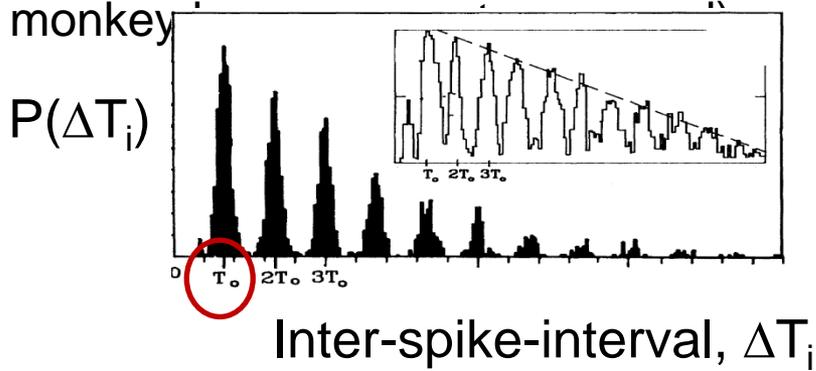


Thresholding detects the spike times \Rightarrow Point Process \Rightarrow sequence of inter-spike-intervals (ISIs)



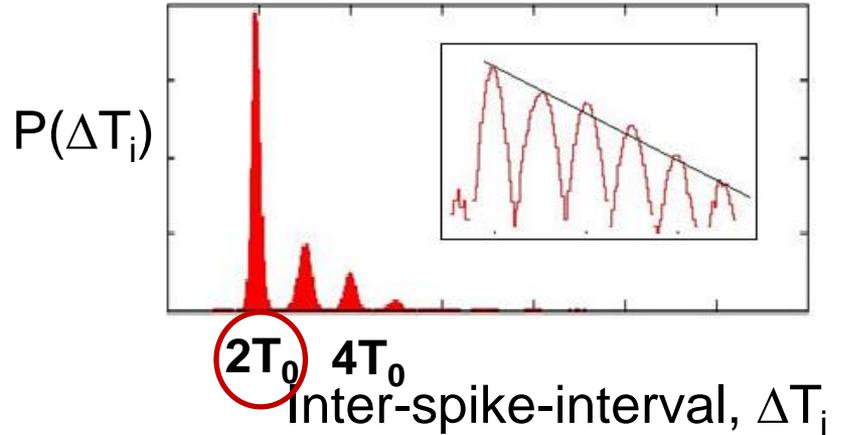
Neuron inter-spike-intervals (ISI) distribution

(spikes in the auditory nerve when a monkey)

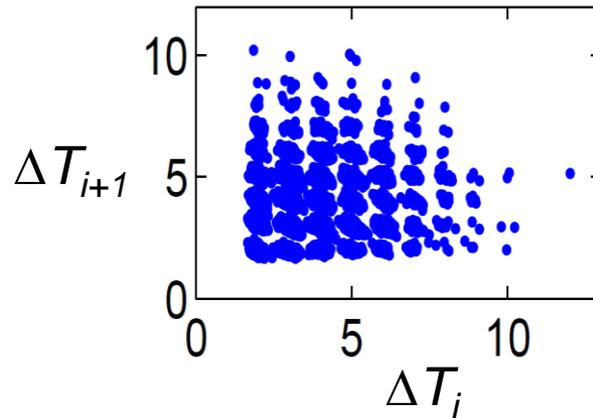
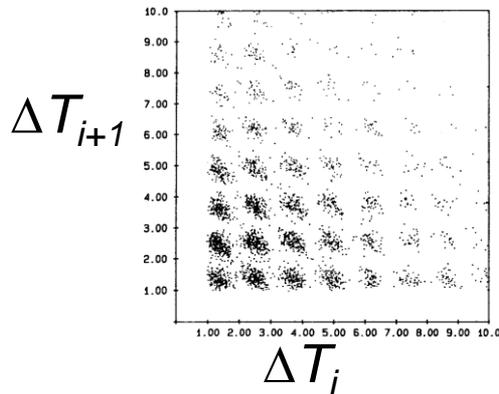


Diode laser ISI distribution

(when the laser is sinusoidally modulated)



Auditory nerve when a cat hears a pure tone sound:



How to detect similar temporal order in the ISI sequences?

A. Longtin et al. PRL (1991), IJBC (1993).

A. Aragoneses et al. Opt Express (2014).

First analysis method: ordinal analysis

$$\{\dots X_i, X_{i+1}, X_{i+2}, \dots\}$$

Possible order relations among three numbers (e.g., 2, 5, 7)

$\{\dots 2, 5, 7 \dots\}$

$\{\dots 5, 2, 7 \dots\}$

$\{\dots 7, 2, 5 \dots\}$



$\{\dots 2, 7, 5 \dots\}$

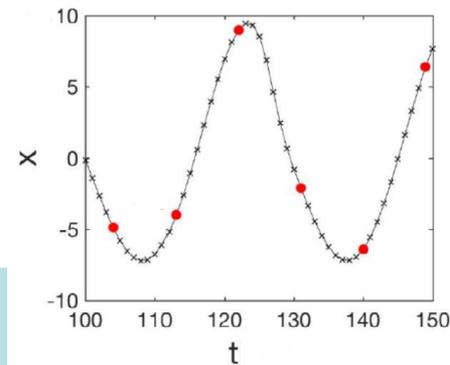


$\{\dots 5, 7, 2 \dots\}$



$\{\dots 7, 5, 2 \dots\}$

Which is the code?



A B F C

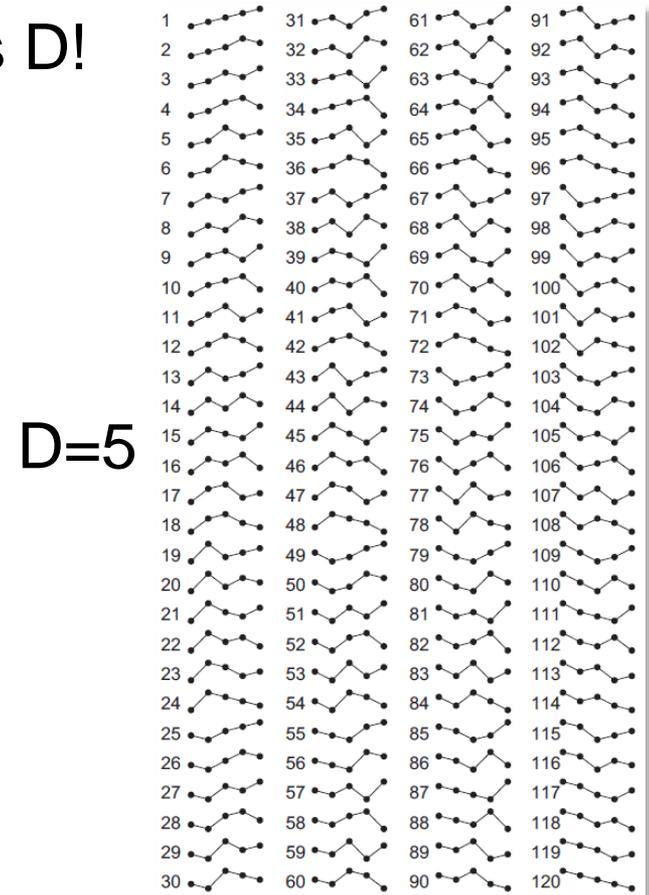
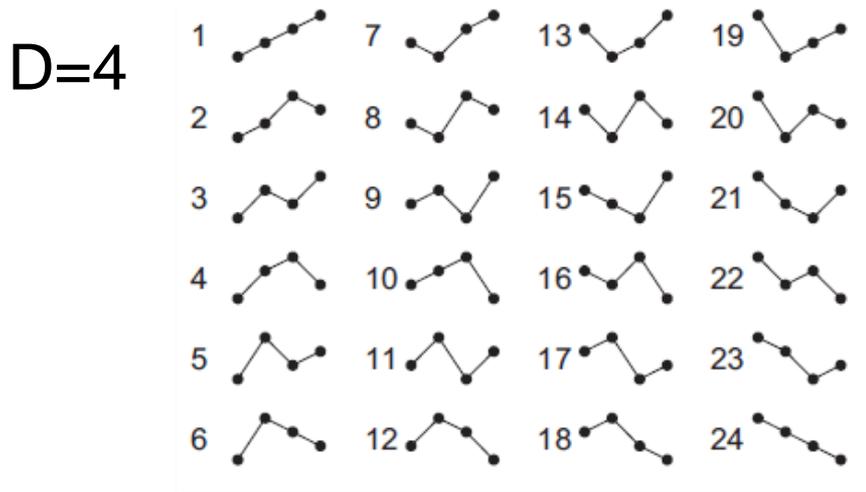
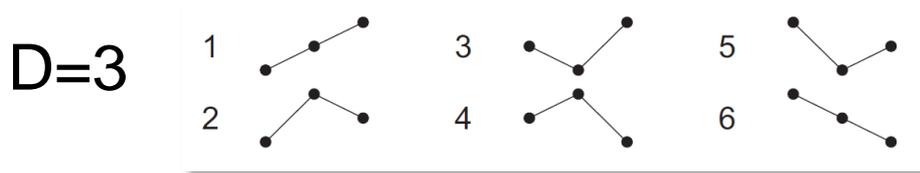
C. Bandt and B. Pompe, Phys. Rev. Lett. 88, 174102 (2002).

Drawbacks

1. Information about the data values is lost.

(5,7,2) and (5,70,2) are both represented by the same symbol.

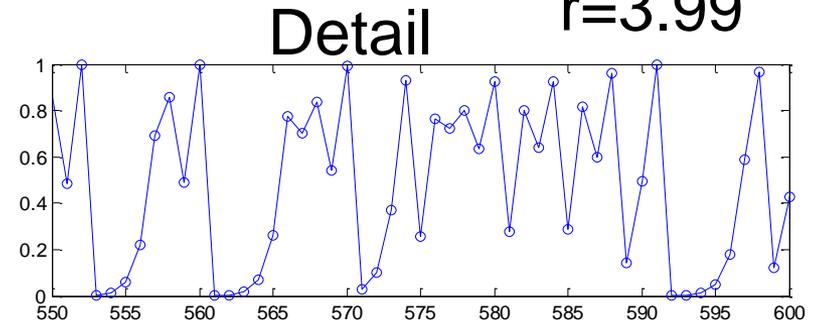
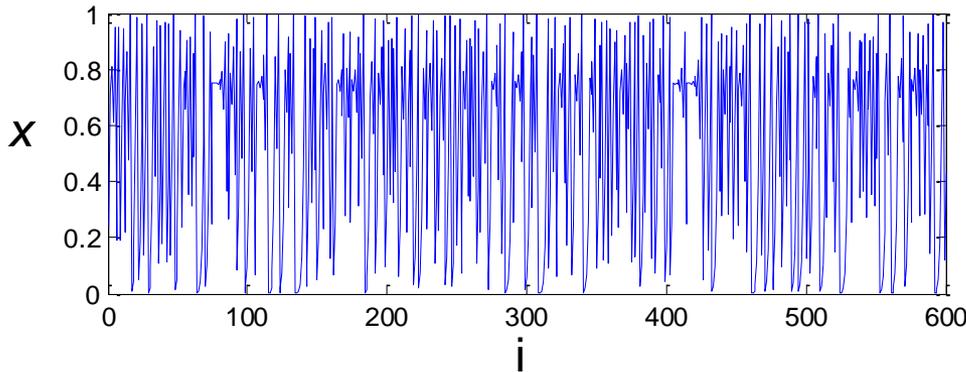
2. The number of possible patterns (N) grows with the length (D) of the pattern as $D!$



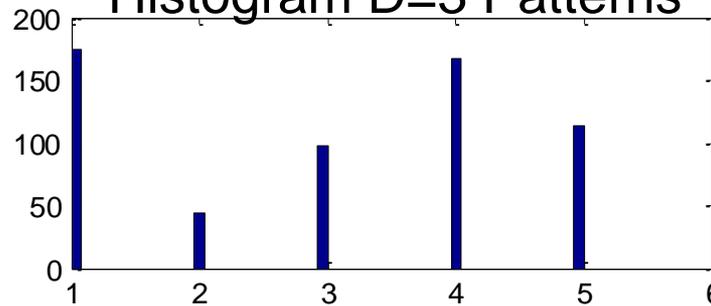
Example: chaotic time series generated with the Logistic map

$$x(i+1) = r x(i)[1 - x(i)] \quad x(1) \in (0,1)$$

$r=3.99$

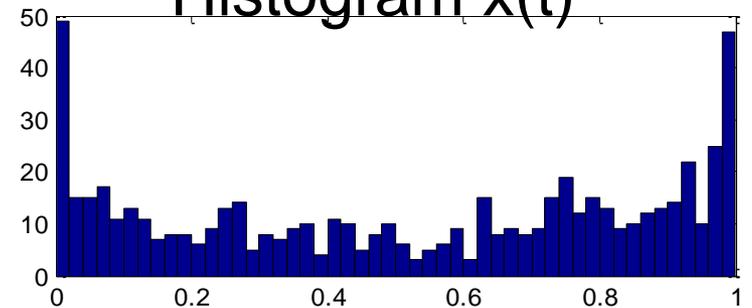


Histogram D=3 Patterns



forbidden

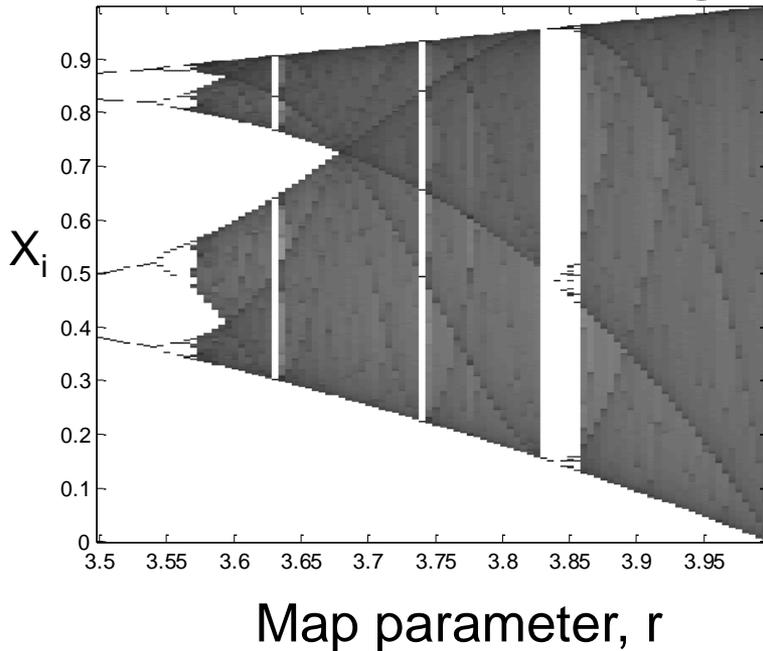
Histogram x(t)



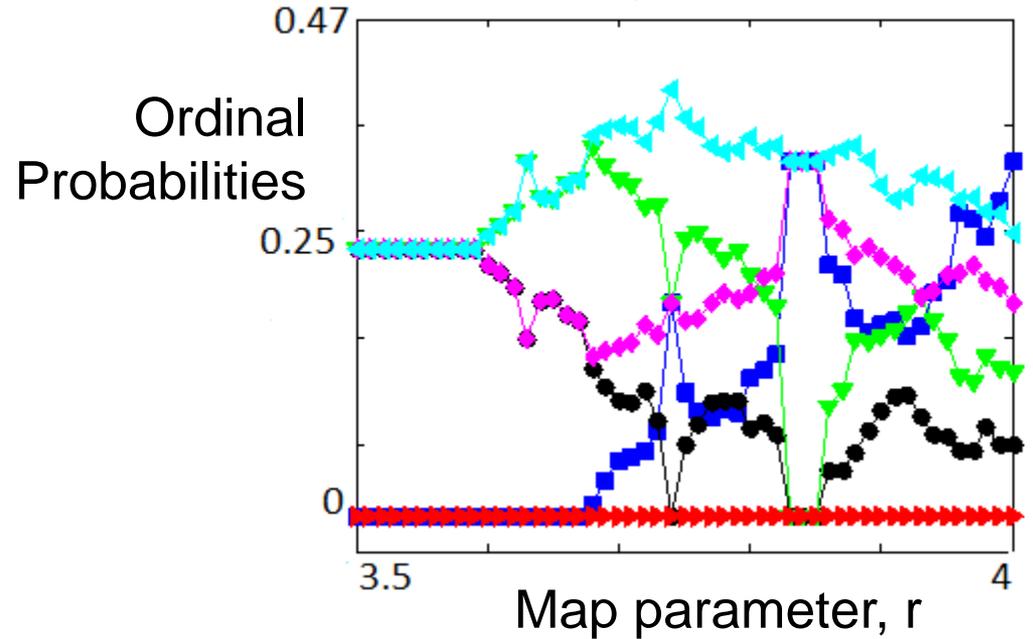
“Normal” and “Ordinal” bifurcation diagrams of the Logistic map

$$x(i+1) = r x(i)[1 - x(i)]$$

Normal bifurcation diagram



Ordinal diagram with D=3



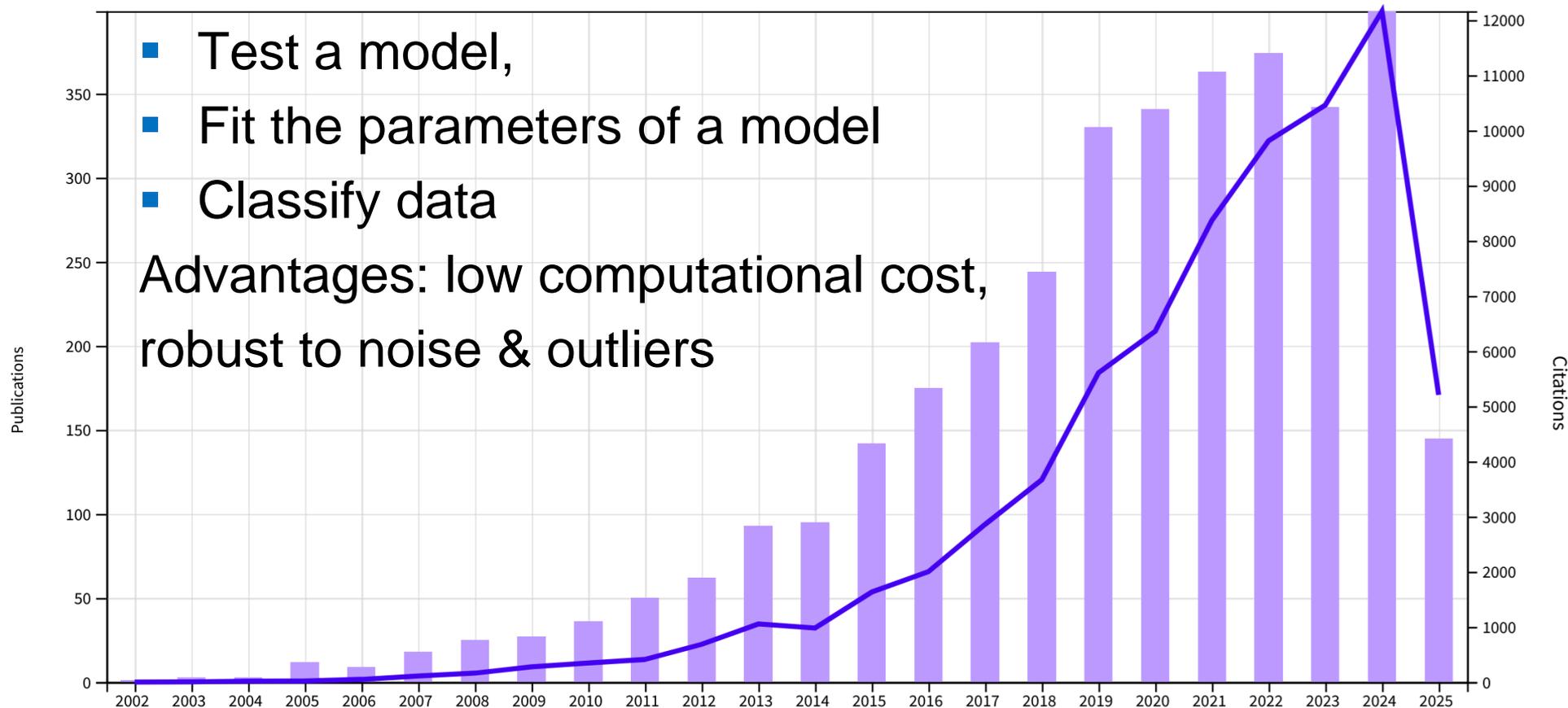
012 021 102 120 201 210

Pattern **210** is always forbidden; pattern **012** becomes more probable as r increases

Ordinal analysis is a popular technique to:

- Test a model,
- Fit the parameters of a model
- Classify data

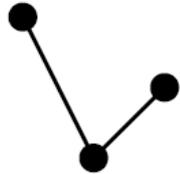
Advantages: low computational cost,
robust to noise & outliers



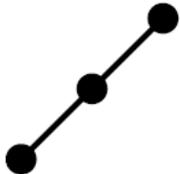
I. Leyva, J. M. Martinez, C. Masoller, O. A. Rosso, M. Zanin, “20 Years of Ordinal Patterns: Perspectives and Challenges”, EPL 138, 31001 (2022).

Using **lagged points** to define the patterns allows to select the time scale of the analysis, useful for climatological data

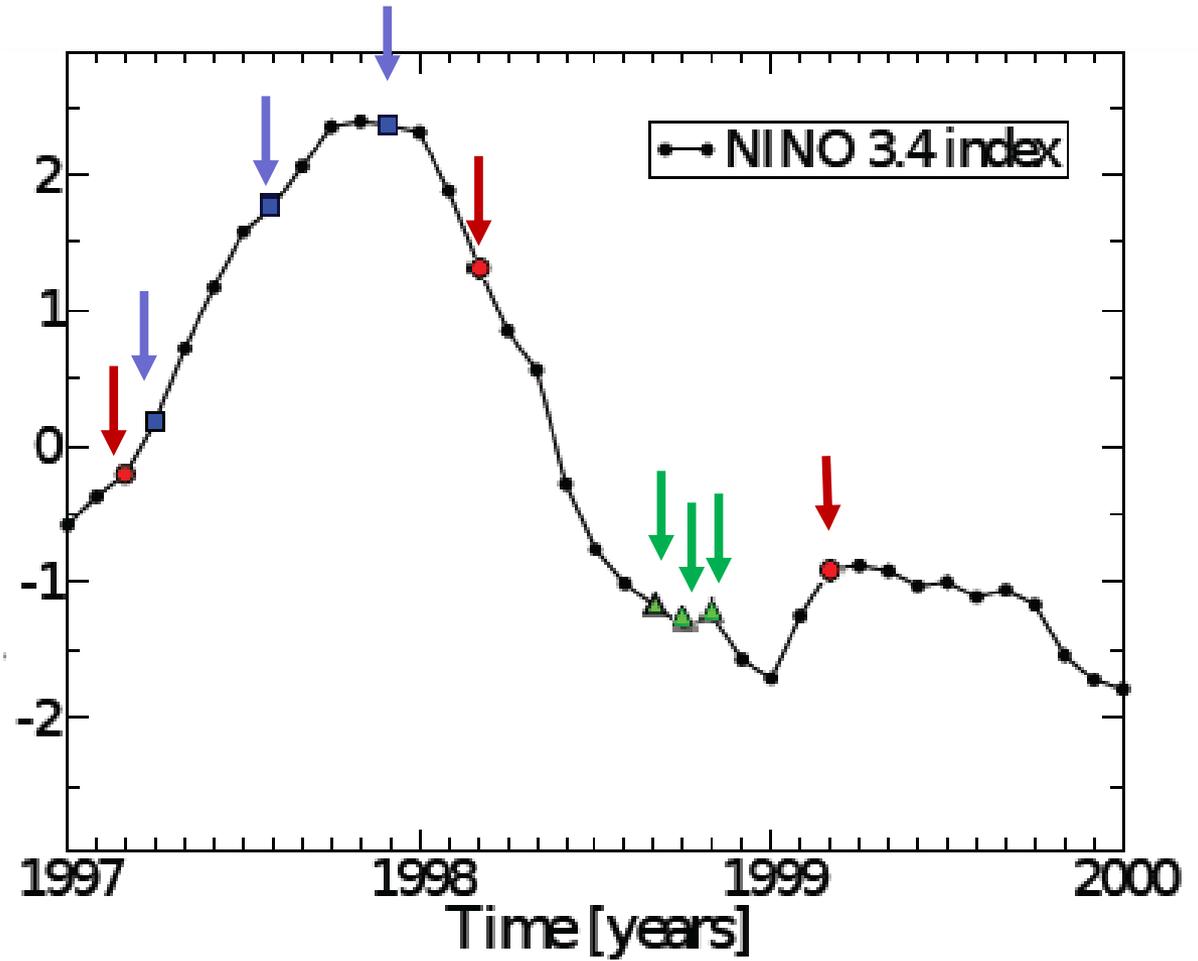
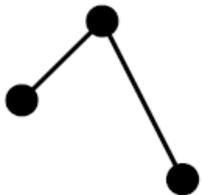
Intra-season



Intra-annual



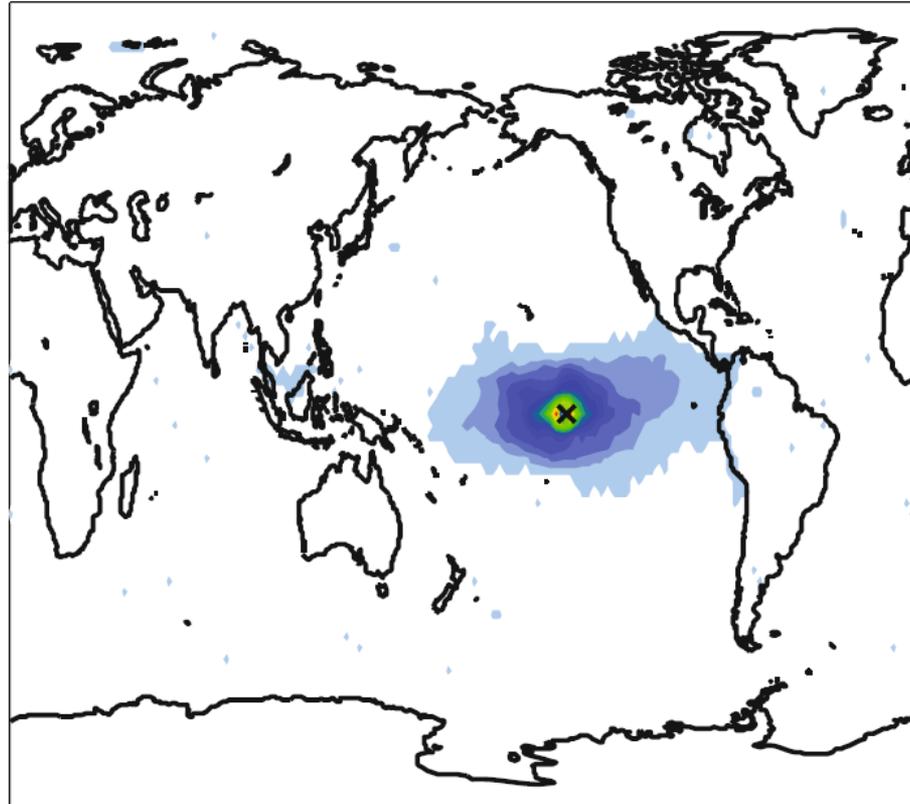
Inter-annual



Mutual Information (color code) from probabilities of ordinal patterns (white: MI not significant)

$$M_{ij} = \sum_{m,n} p_{ij}(m,n) \log \frac{p_{ij}(m,n)}{p_i(m)p_j(n)}$$

MI from probabilities of ordinal patterns defined by values of **surface air temperature anomalies** in 3 consecutive months.

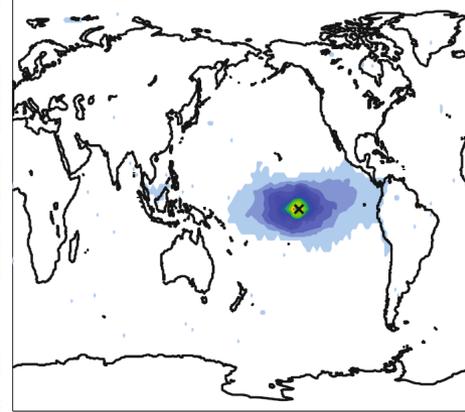
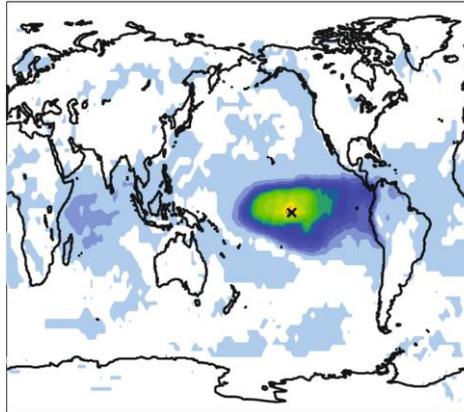


Juan Ignacio Deza, M. Barreiro, C. Masoller, Eur. Phys. J. ST 222, 511 (2013).

Comparison

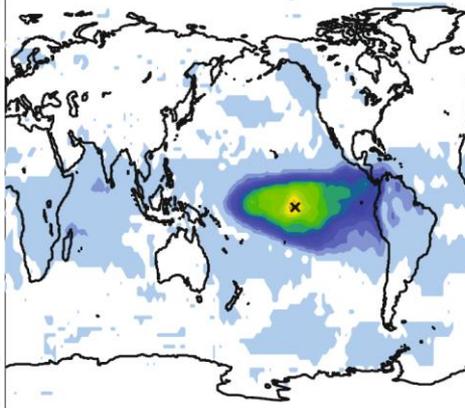
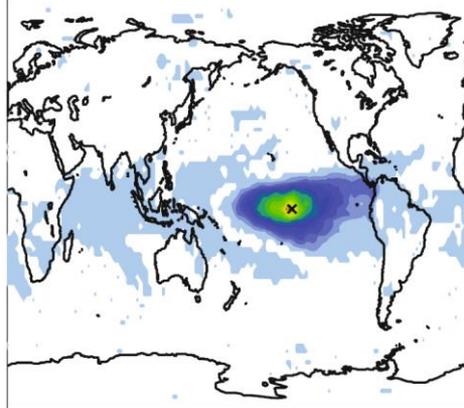
$$M_{ij} = \sum_{m,n} p_{ij}(m,n) \log \frac{p_{ij}(m,n)}{p_i(m)p_j(n)}$$

Probabilities of data values



Probabilities of ordinal patterns defined by values in 3 consecutive months.

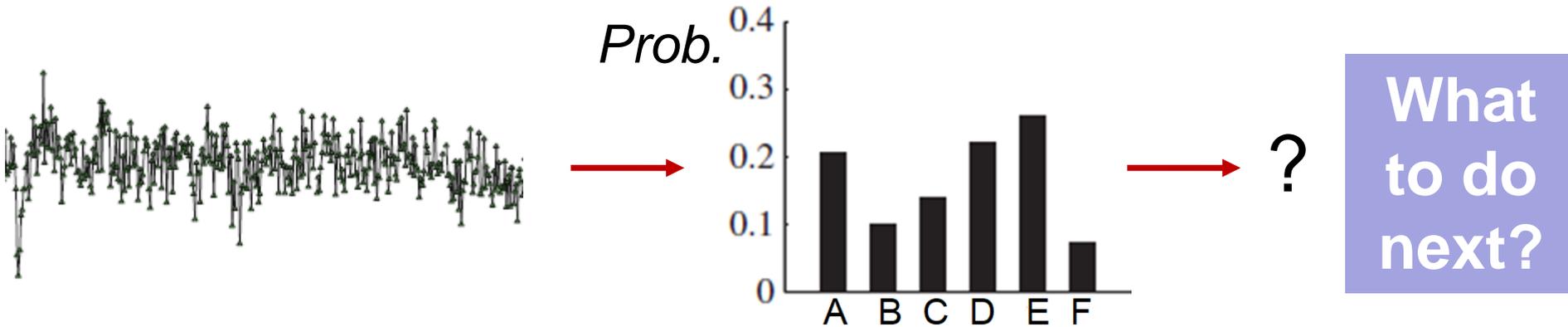
Probabilities of patterns defined by 3 values within a year.



Probabilities of patterns defined by values in 3 consecutive years.

J. I. Deza, M. Barreiro, C. Masoller, Eur. Phys. J. ST 222, 511 (2013).

From a sequence of data points, by counting the different patterns we can estimate the “ordinal probabilities”



1. Permutation Entropy:

$p_i = p_j$ for all $i, j \Rightarrow H=1$
 $p_i = 1, p_j = 0$ for all $j \neq i \Rightarrow H=0$

$$H = -\frac{1}{\ln N!} \sum_{i=1}^N p_i \ln p_i$$

(Nonlinear dimensionality reduction)

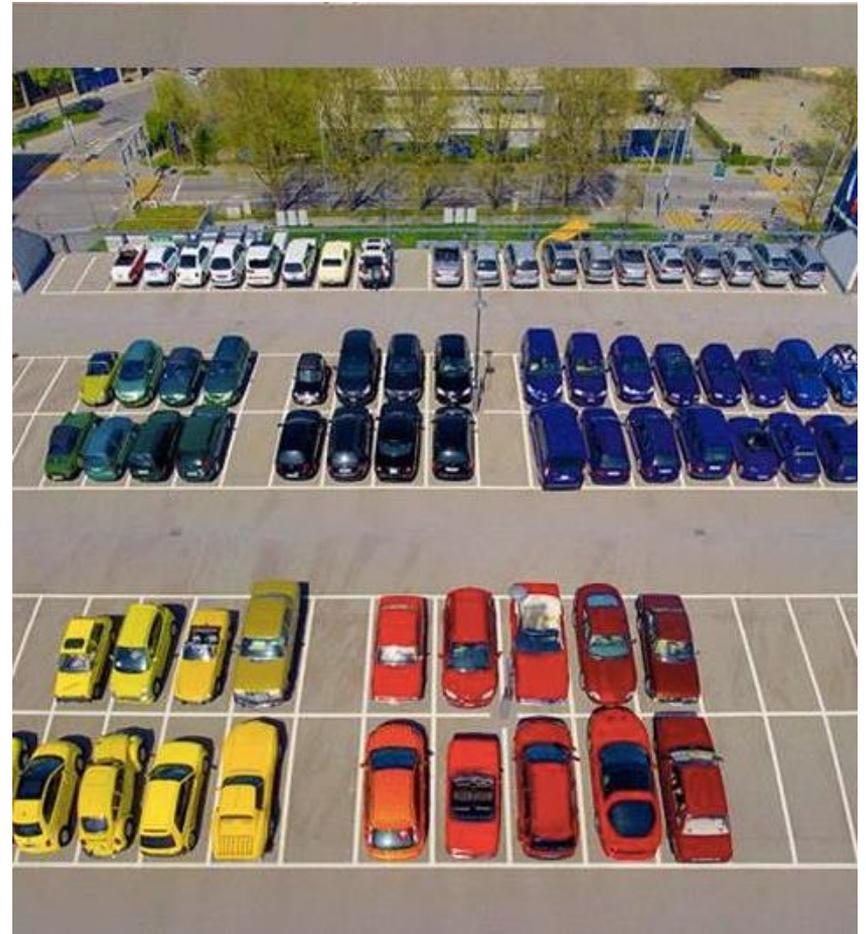
2. Analyze all the probabilities (features for machine learning algorithms)

Entropy (disorder) and information

High entropy low information



Low entropy high information



<https://imgur.com/gallery/Otg97>

Shannon entropy

$$H = -\sum_{i=1}^N p_i \ln p_i$$

$$\sum_{i=1}^N p_i = 1$$

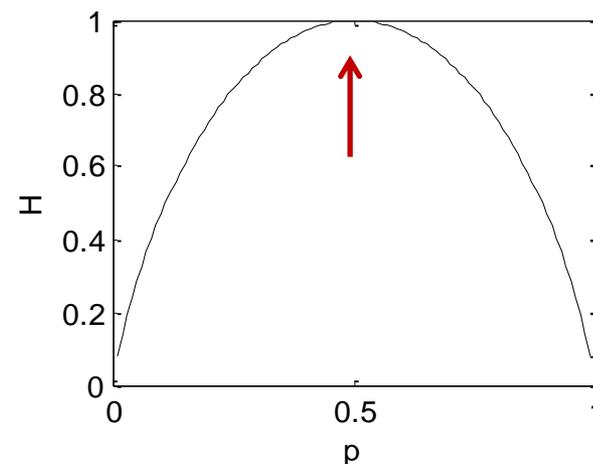
- Interpretation: “quantity of **surprise** one should feel upon reading the result of a measurement”.

- Example: a random variable takes values 0 or 1 with probabilities:

$$p(0) = p, \quad p(1) = 1 - p.$$

$$H = -p \ln(p) - (1 - p) \ln(1 - p).$$

⇒ $p=0.5$: Maximum **unpredictability**.



C. Shannon, "A Mathematical Theory of Communication",
Bell System Technical Journal. 27 (3): 379–423 (1948).
Bell System Technical Journal. 27 (4): 623–656 (1948).

First example: entropy analysis of EEG recordings can distinguish eyes-closed and eyes-open states?



Eyes closed

Eyes open

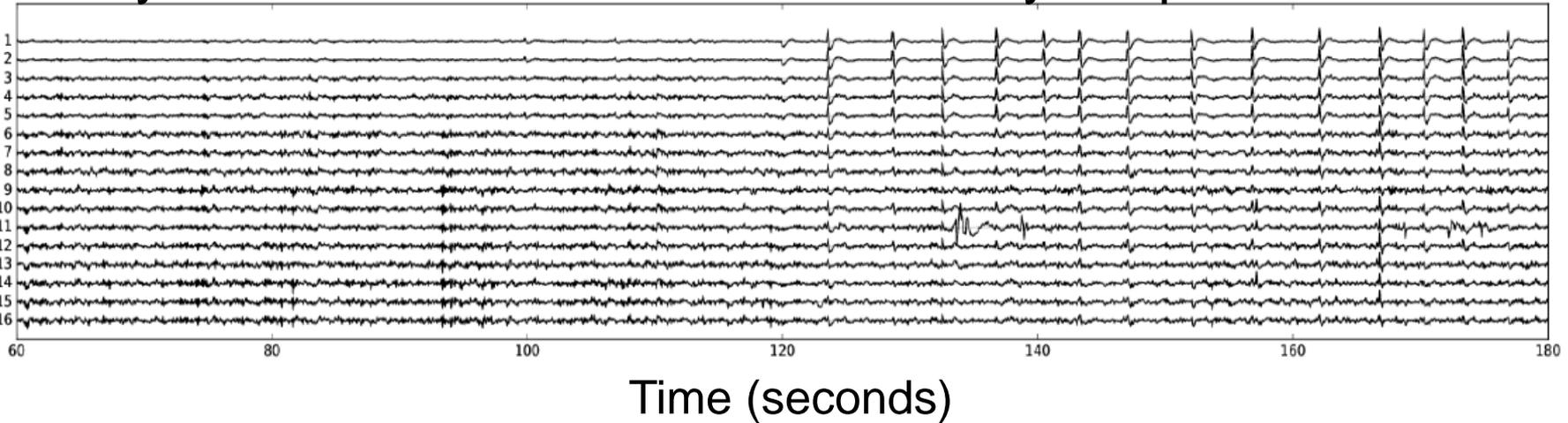


TABLE I. Description of the datasets used.

	DTS1	DTS2
Sampling rate (Hz)	256	160
Time task (seg)	120	60
Total points	30 720	9600
Number of electrodes	16	64
Number of subjects (healthy)	71	109

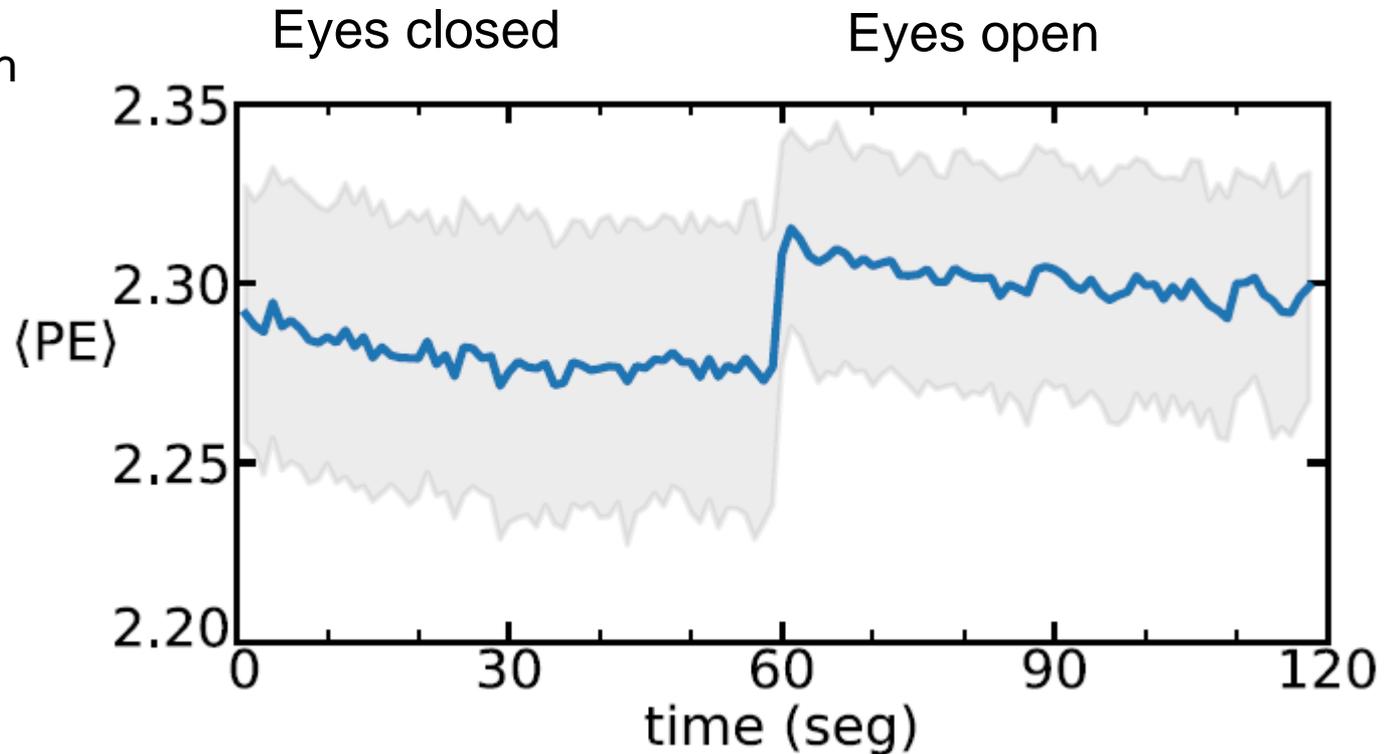
DTS1: Bitbrain (Zaragoza)
DTS2: Physionet

The Permutation Entropy increases in the eyes open state

$$\langle PE \rangle = \frac{1}{N[\text{electrodes}]} \sum_i PE^i$$

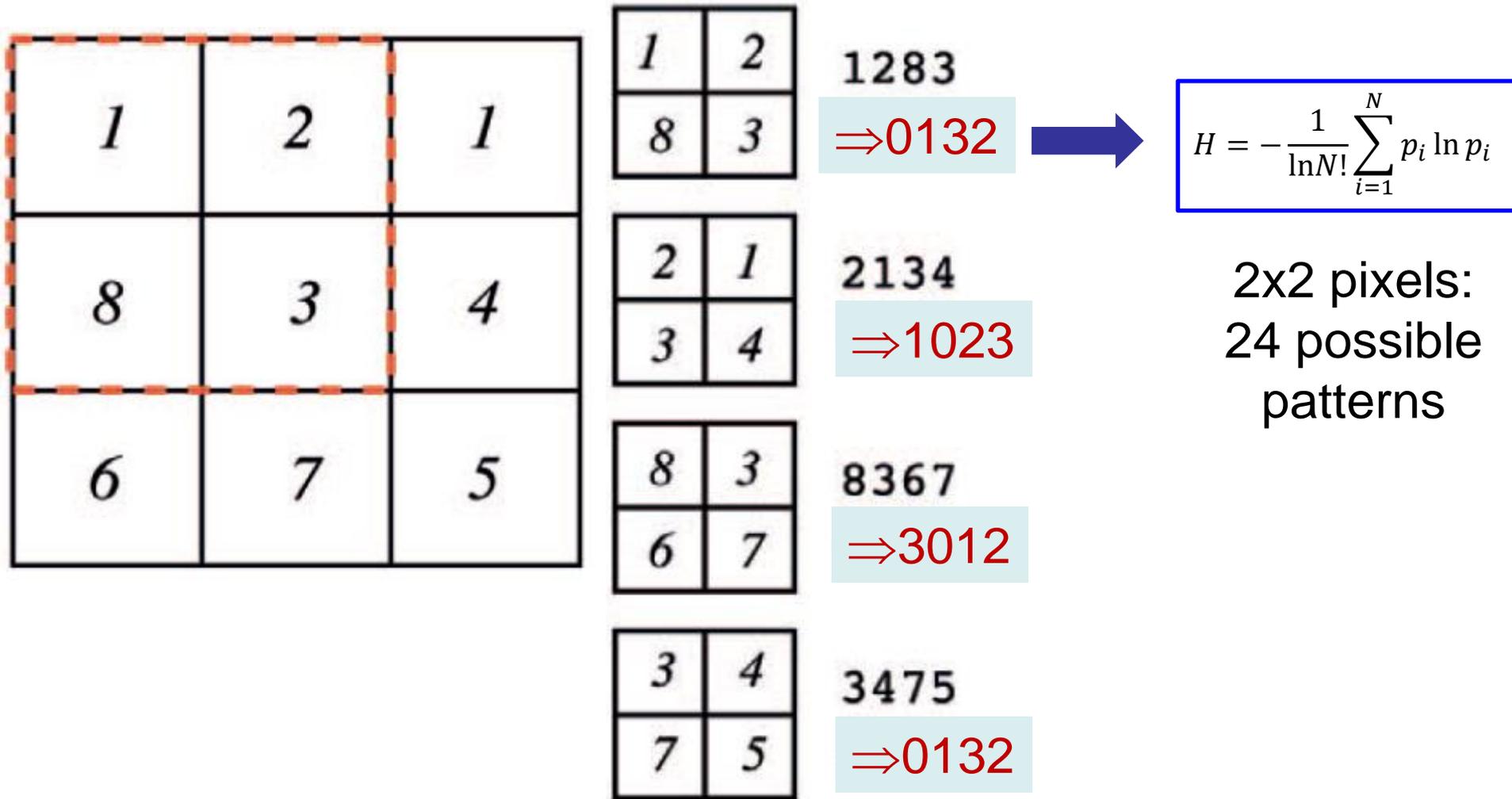
PE was calculated with patterns of length 4 (# of possible patterns 24) in time windows containing >4000 patterns

Gray region:
Standard deviation of $\langle PE \rangle$ across subjects



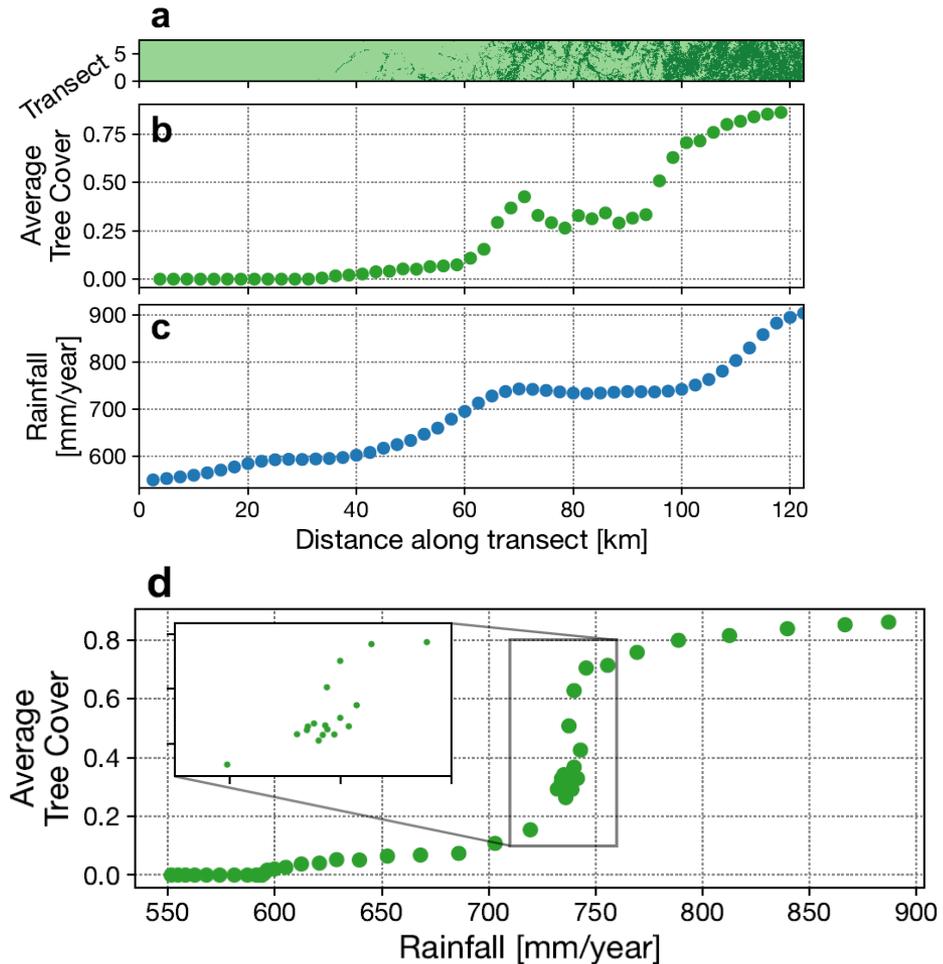
C. Quintero-Quiroz et al., "Differentiating resting brain states using ordinal symbolic analysis", Chaos 28, 106307 (2018).

Spatial data \Rightarrow Spatial Permutation Entropy (SPE)

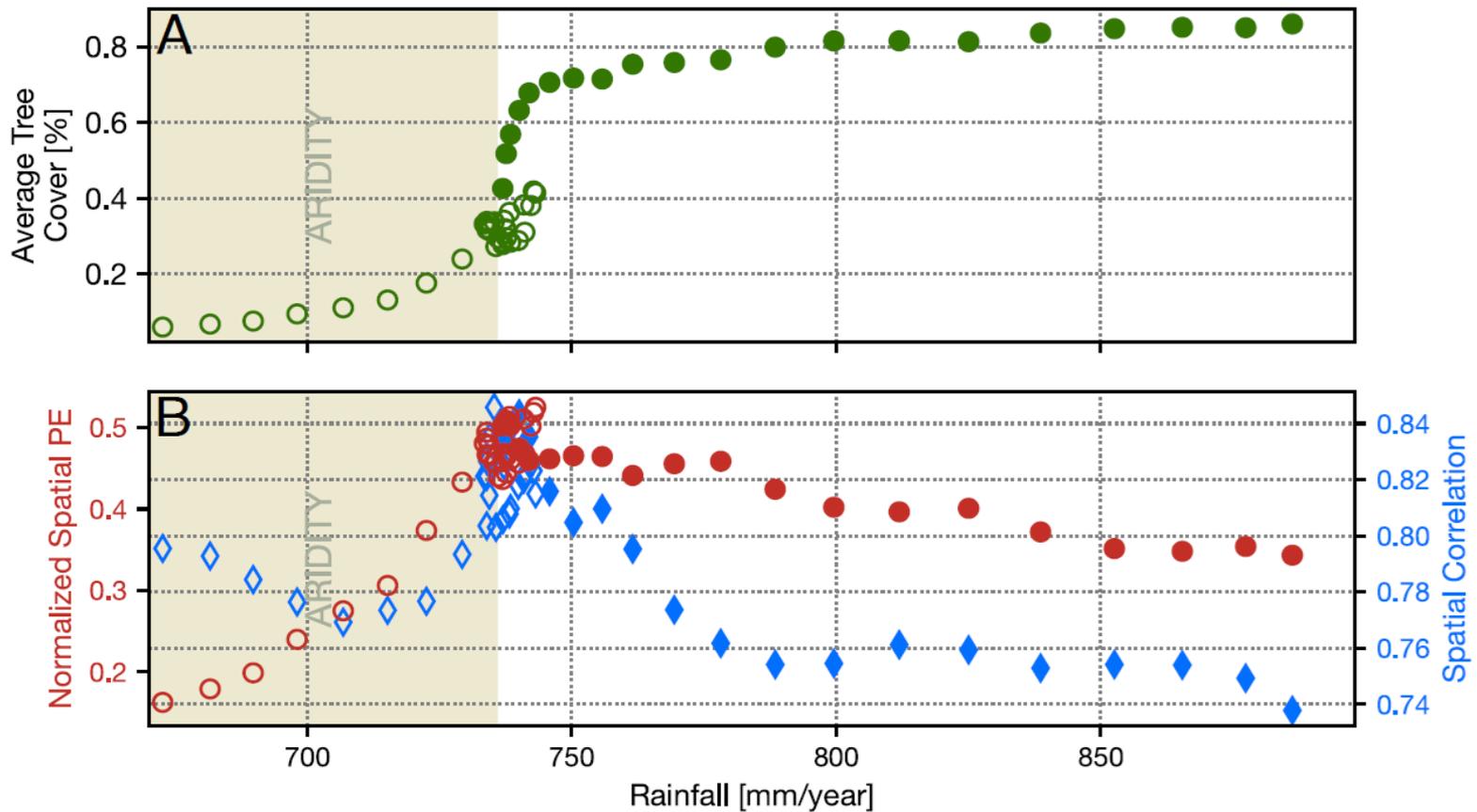


Haroldo Ribeiro, **Luciano Zunino** and coworkers, PLoS ONE 7, e40689 (2012)

The spatial permutation entropy (SPE) can be an early indicator of a vegetation transition (tipping point)



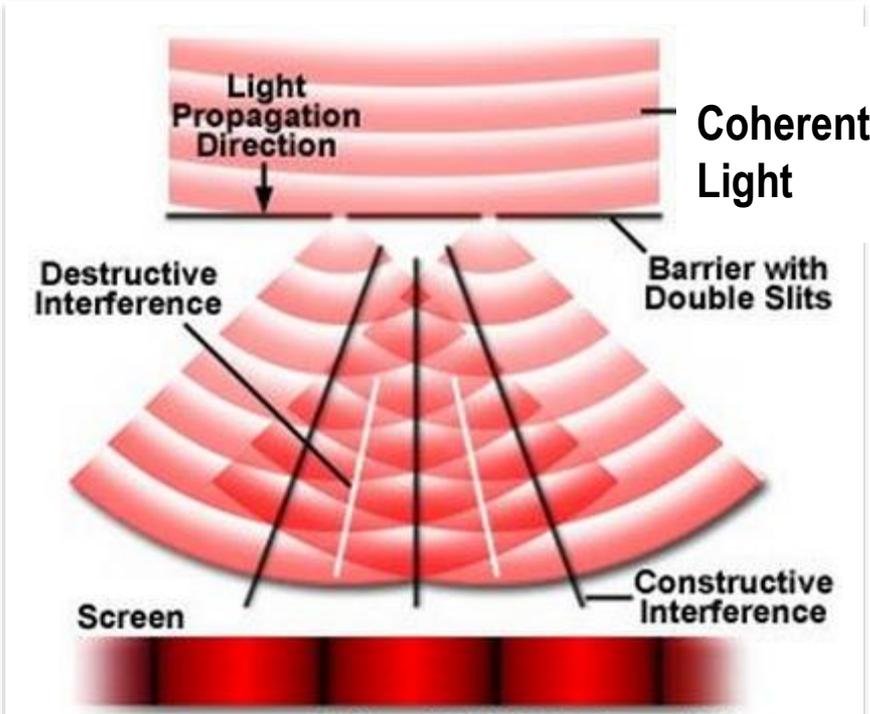
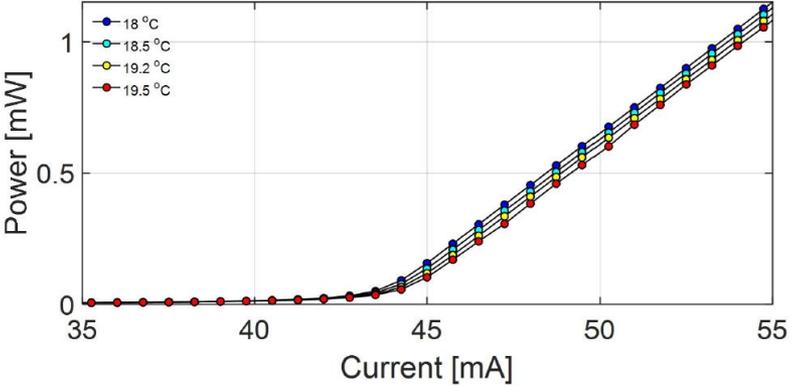
Giulio Tirabassi and C. Masoller, “*Entropy-based early detection of critical transitions in spatial vegetation fields*”, PNAS 120, e2215667120 (2023).



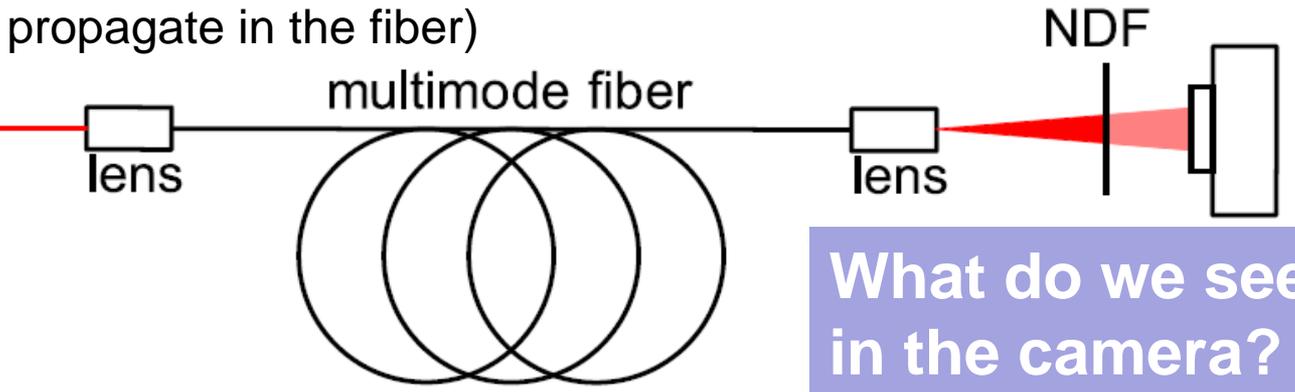
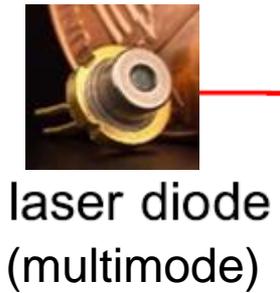
Can we test this indicator in controlled experimental data?

Giulio Tirabassi and C. Masoller, “*Entropy-based early detection of critical transitions in spatial vegetation fields*”, PNAS 120, e2215667120 (2023).

Quick reminder: laser threshold and interference of coherent waves



(many modes can propagate in the fiber)

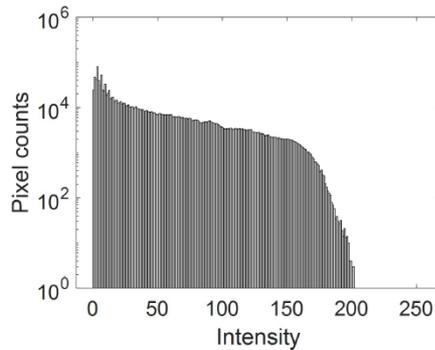
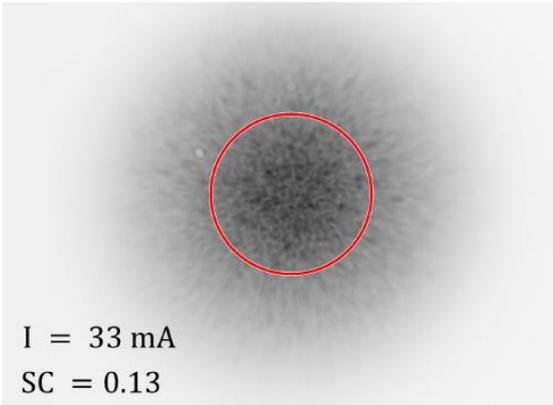


What do we see in the camera?

Speckle pattern

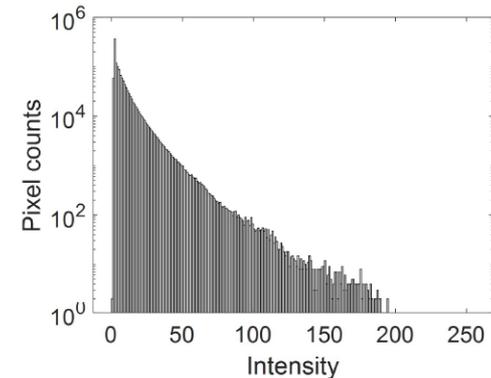
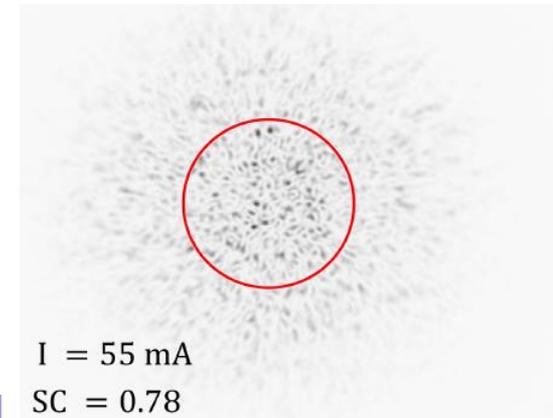
Below lasing threshold

Low coherence → low speckle contrast



Above lasing threshold

High coherence → high speckle contrast



How to quantify the degree of speckle?

Speckle contrast
(coefficient of variation, CV)

$$SC = \sigma / \langle I \rangle$$

Can we identify, from the entropy of the patterns, the laser threshold? and infer which medium generated the speckles?

$$PE = -\frac{1}{\ln N!} \sum_{i=1}^N p_i \ln p_i$$

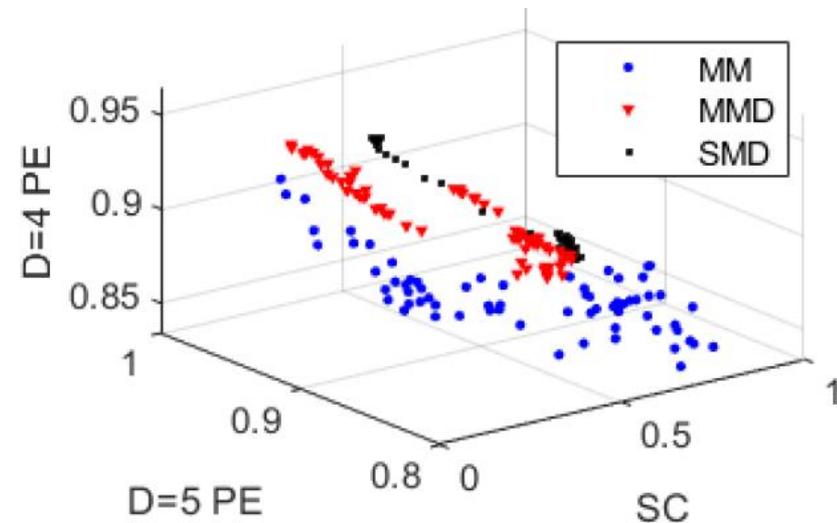
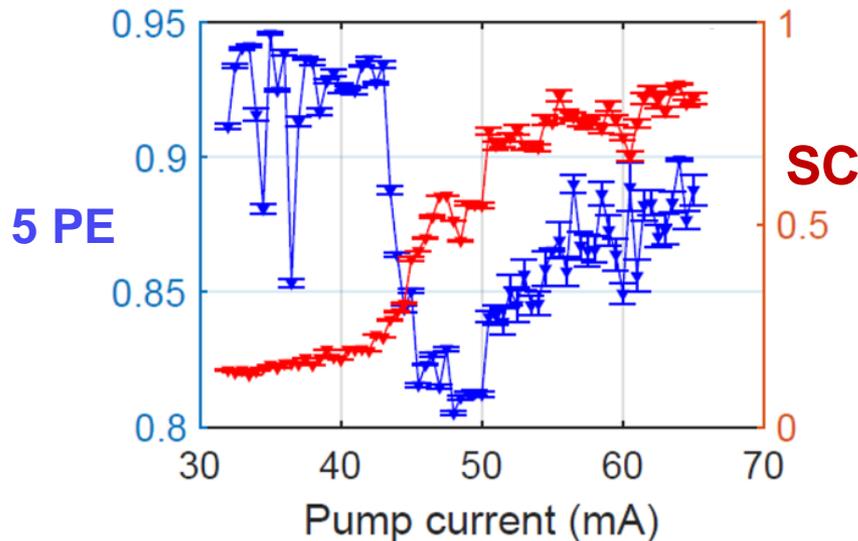
$$SC = \sigma / \langle I \rangle$$

4 PE : x x
x x

5 PE : x
x x x
x

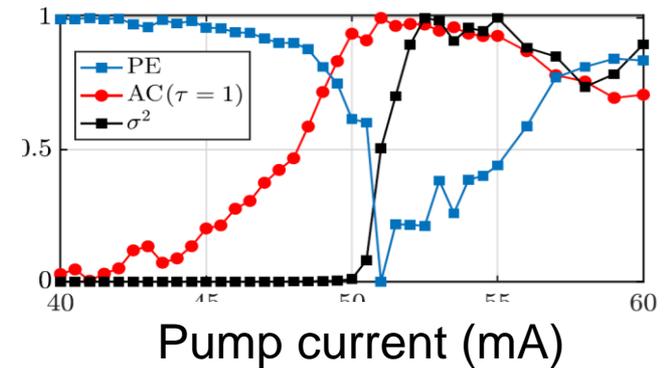
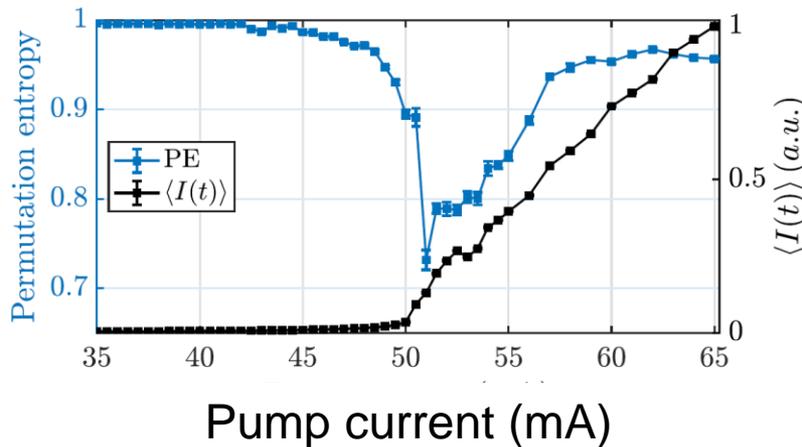
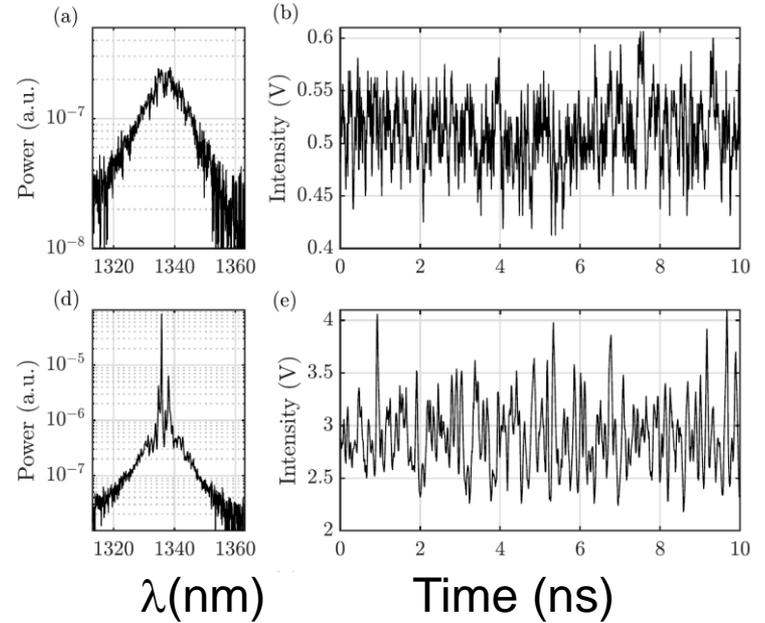
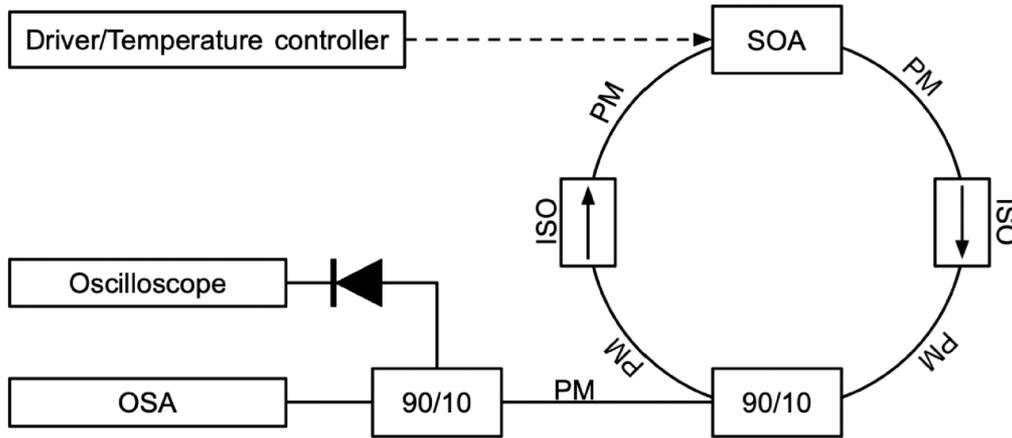
Diffusive Medium that generates speckle:

- Multimode fiber
- Multimode fiber and diffuser
- Single mode fiber and diffuser



Accuracy of the random forest classifier: 99.4 % \pm 0.4 %

Another example: the turn-on of a highly multimode laser

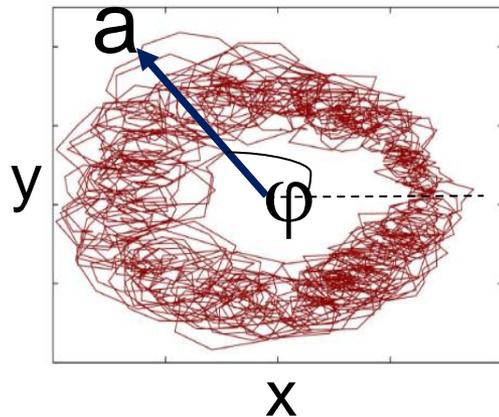
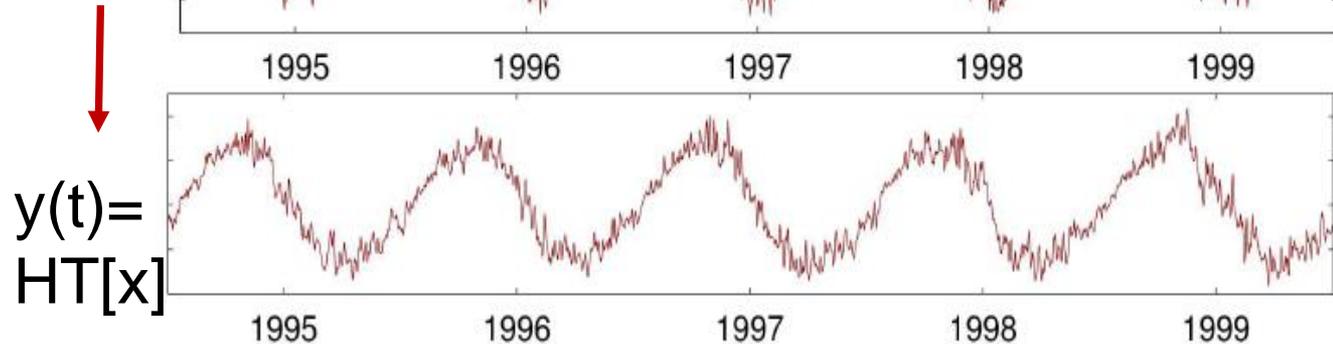
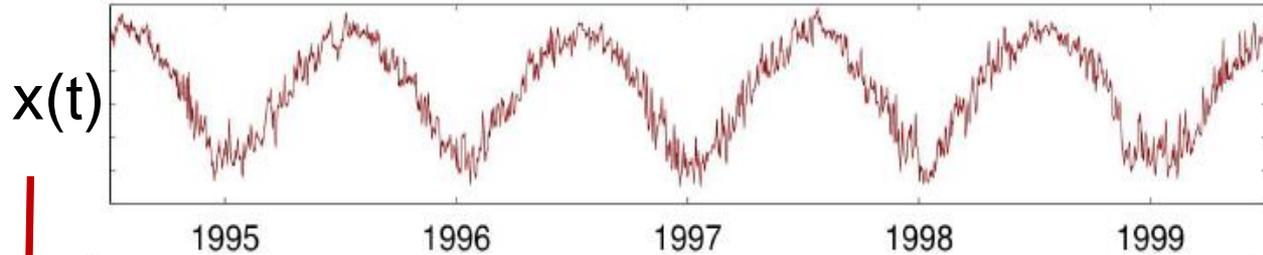


J. Gancio, C. Masoller, M. Marconi, "Identifying and anticipating the threshold bifurcation of a complex laser with permutation entropy", Phys. Rev. Lett. 135, 093802 (2025).

Second data analysis method: Hilbert analysis

Surface Air
Temperature (SAT)
in a region

$$\text{HT}[\sin(x)] = \cos(x)$$



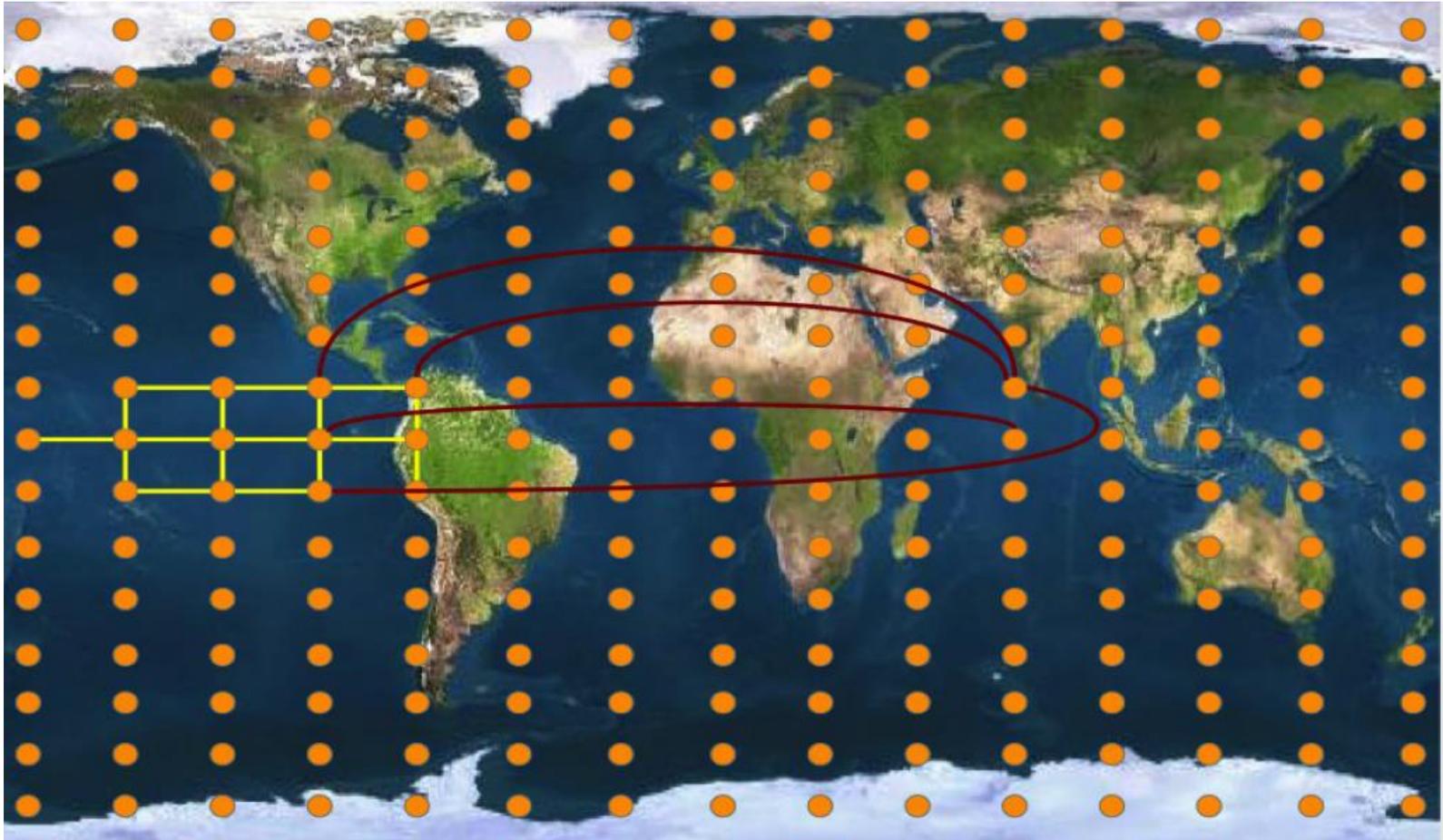
Instantaneous amplitude and phase

$$a(t) = \sqrt{[x(t)]^2 + [y(t)]^2}$$

$$\varphi(t) = \arctan[y(t)/x(t)]$$

Clear physical meaning only if $x(t)$ is a narrow-band signal. Then, $a(t)$ coincides with the **envelope** of $x(t)$ and $\omega(t) = d\varphi/dt$, coincides with the **main frequency** in the spectrum.

Using the HT we analyzed “re-analysis data” from the *European Centre for Medium-Range Weather Forecasts*, with high spatial and temporal resolution in the period 1979-2016

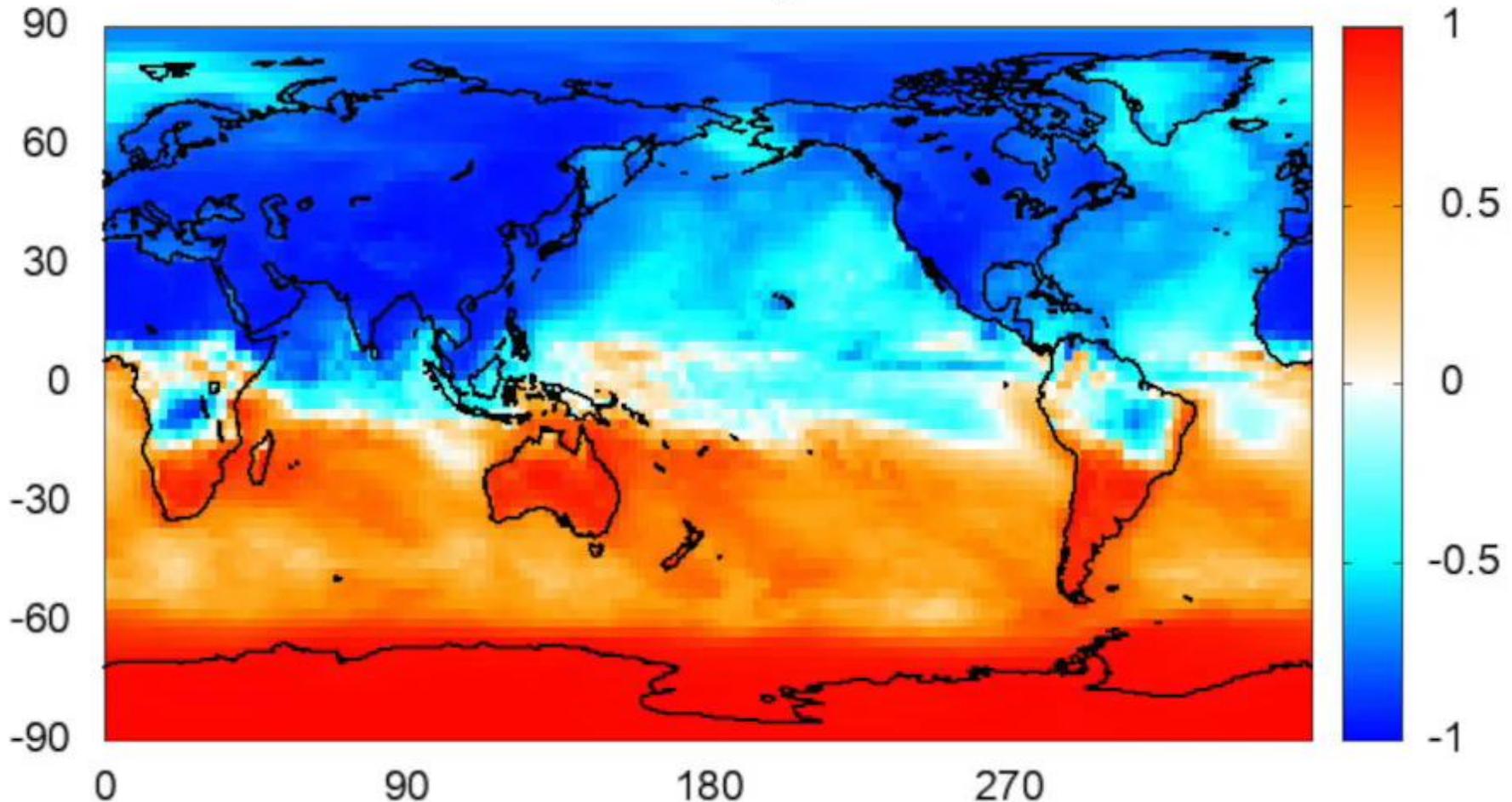


$73 \times 144 = 10\,512$ geographical sites, in each site the SAT time series has 13696 days

Average of the cosine of Hilbert phase of surface air temp.

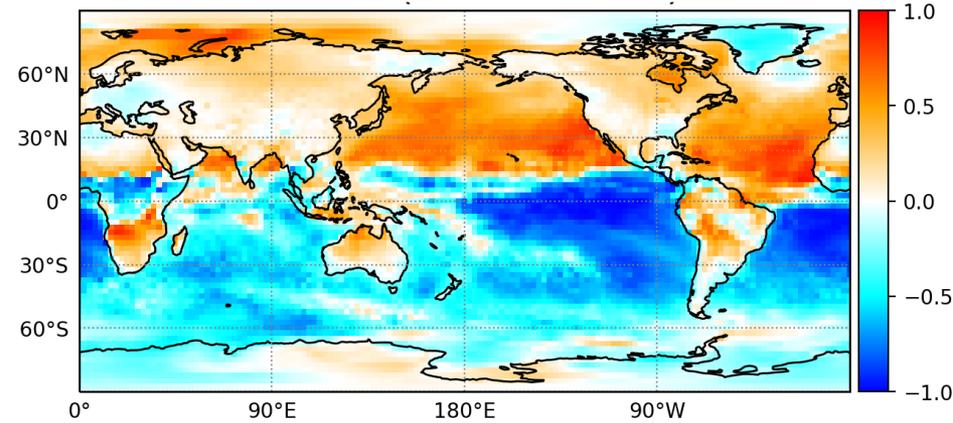
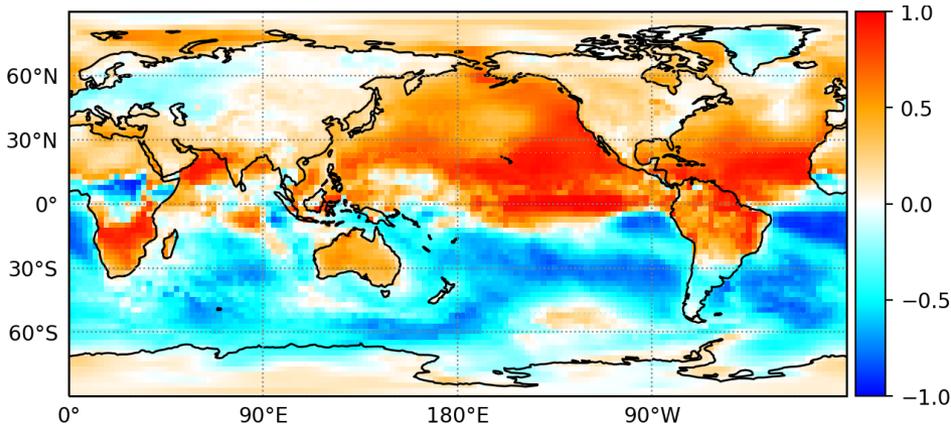
Can we visualize the passing of the seasons?

1 January



Cosine of Hilbert phase during an *El Niño* year

Cosine of Hilbert phase during a *La Niña* year



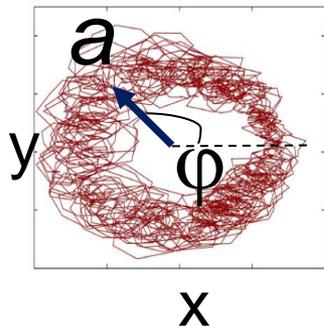
Can we detect significant changes in the last decades in the amplitude of the oscillations of surface air temperature?

In each grid point we calculate:

$$\Delta a = \langle a \rangle_{2016-2007} - \langle a \rangle_{1988-1979} \quad \frac{\Delta a}{\langle a \rangle_{2016-1979}}$$

$$\text{Significant if: } \frac{\Delta a}{\langle a \rangle} \geq \langle \cdot \rangle_s + 2\sigma_s \quad \text{or} \quad \frac{\Delta a}{\langle a \rangle} \leq \langle \cdot \rangle_s - 2\sigma_s$$

(100 surrogates)

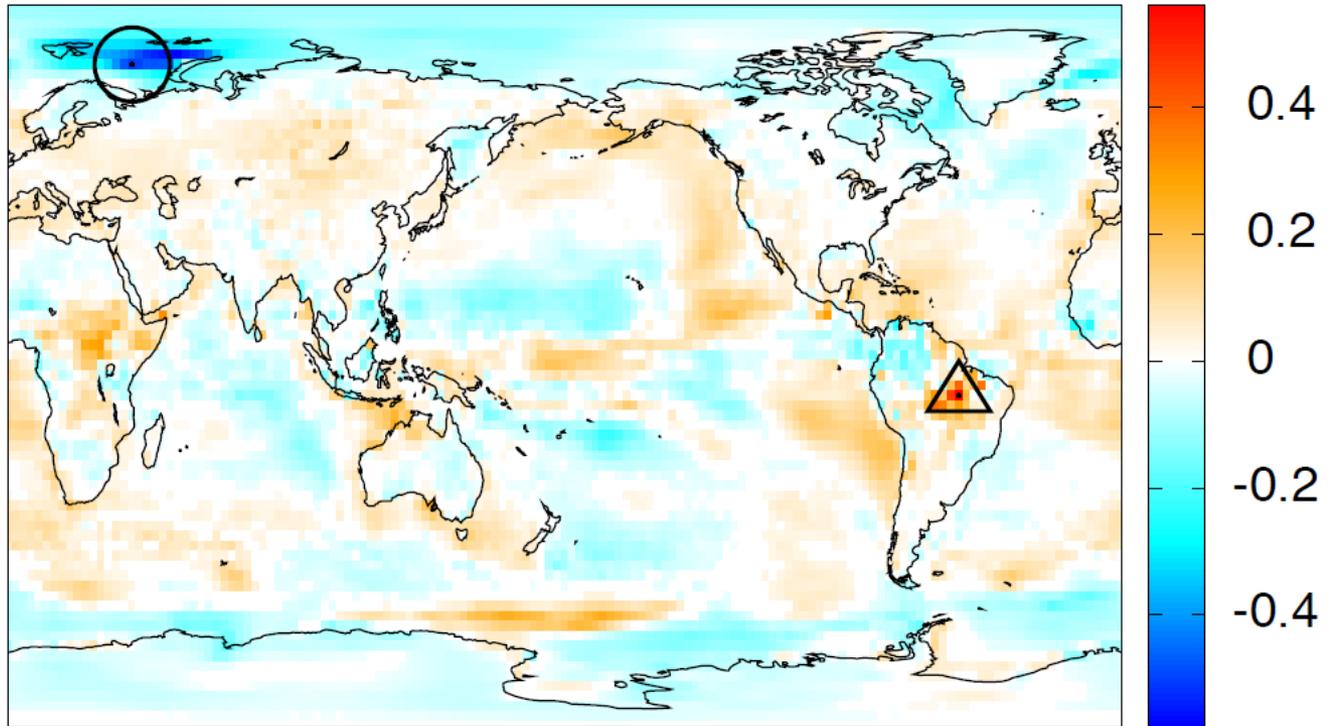


Color code:

$$\Delta a$$

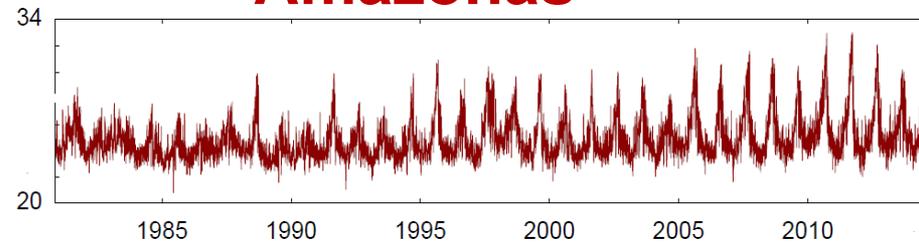
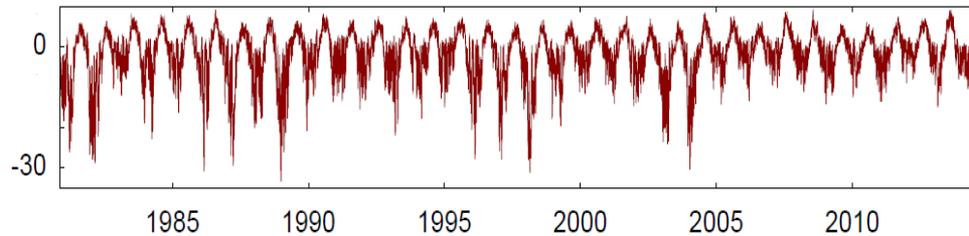
$$\langle a \rangle_{2016-1979}$$

White: not significant



Arctic

Amazonas



Sea ice melting

Deforestation

Dario A. Zappala, M. Barreiro and C. Masoller, Earth Syst. Dynam. 9, 383 (2018).

Take home messages

- Nonlinear data analysis techniques are useful to uncover patterns and relationships in data generated by complex systems.
- Different methods often provide *complementary* information.
- “Surrogate” tests are needed to analyze statistical significance.
- Even when the data does not meet the mathematical or algorithmic requirements, the results can give useful info.
- Prof. Holger Kantz: “*Every data set bears its own difficulties: data analysis is never routine*”.

Funding and references



ICREA



- B. R. R. Boaretto, E. E. N. Macau, C. Masoller, “*Characterizing the spike timing of a chaotic laser by using ordinal analysis and machine learning*”, Chaos 34, 043108 (2024).
- J. Gancio, C. Masoller, G. Tirabassi, “*Permutation entropy analysis of EEG signals for distinguishing eyes-open and eyes-closed brain states: Comparison of different approaches*”, Chaos 34, 043130 (2024).
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- D. A. Zappala et. al, “*Quantifying changes in spatial patterns of surface air temperature dynamics over several decades*”, Earth Syst. Dy. 9, 383 (2018).
- J. Gancio, C. Masoller, M. Marconi, “*Identifying and anticipating the threshold bifurcation of a complex laser with permutation entropy*”, Phys. Rev. Lett. 135, 093802 (2025).

Muchas gracias por su atención!