Topological data analysis of the synchronization transition of coupled oscillators

Cristina Masoller
Departamento de Física, Universitat Politecnica de Catalunya

Alvaro Zabaleta-Ortega and Lev Guzmán-Vargas
Unidad Profesional Interdisciplinaria en Ingeniería y Tecnologías Avanzadas,
Instituto Politécnico Nacional, Ciudad de México

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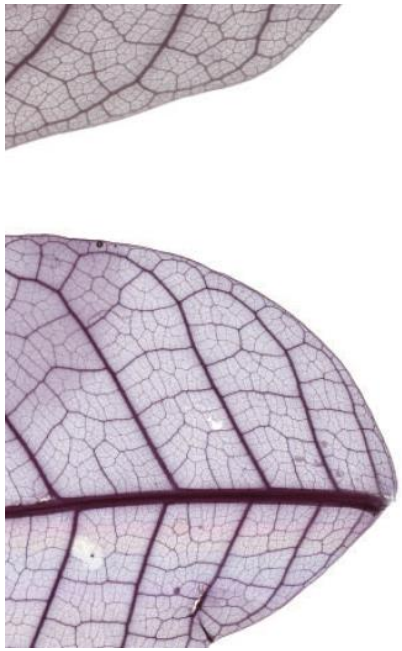
cristina.masoller@upc.edu

@cristinamasoll1

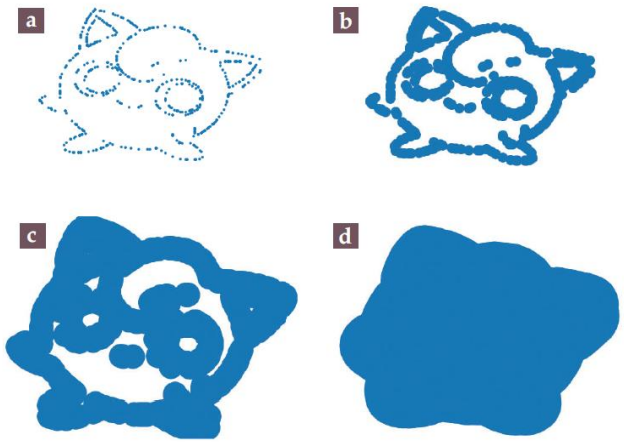
Physics Today, January 2023



THE TOPOLOGY OF DATA

Mason A. Porter, Michelle Feng, and Eleni Katifori

Topological data analysis, which allows systematic investigations of the "shape" of data, has yielded fascinating insights into many physical systems.



Thickening the dots of a point cloud and observing the changes in topological structure gives info about the topology of the data.

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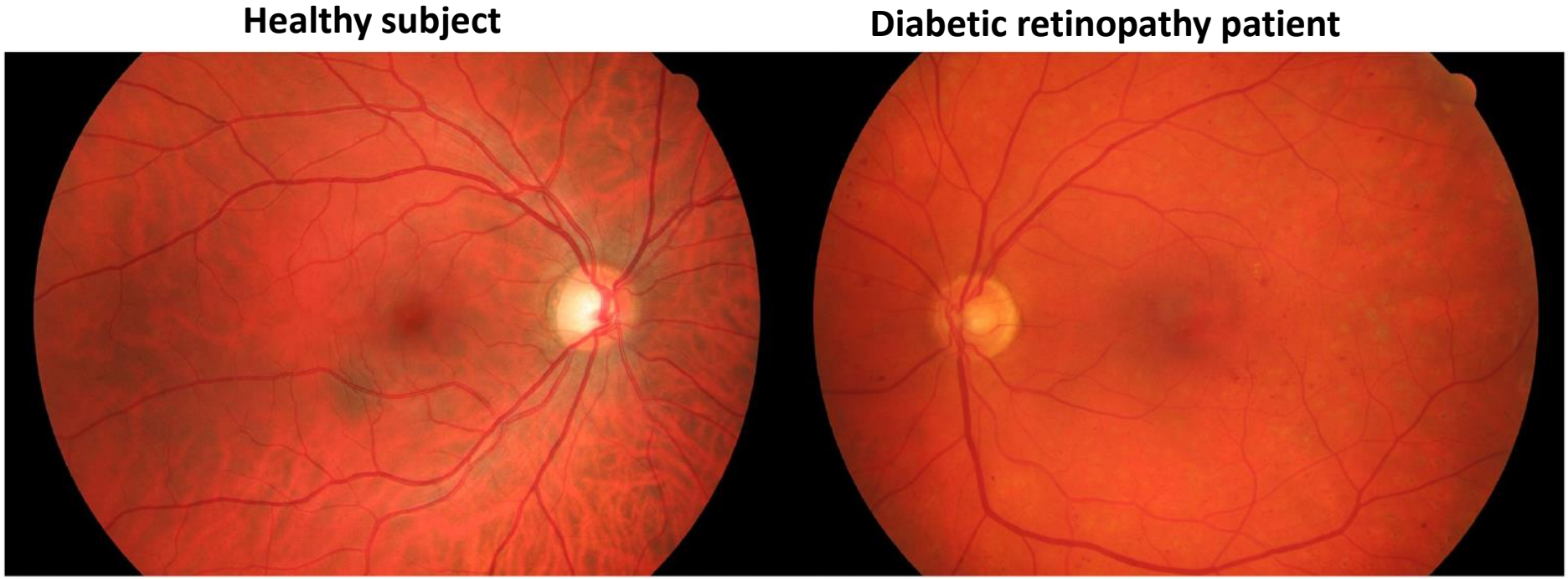
An example of TDA application



https://www5.cs.fau.de/research/data/fundus-images/

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RESEARCH ARTICLE

PLoS ONE 14, e0217413 (2019)

Topological data analysis of high resolution diabetic retinopathy images

Kathryn Garside[1☯]*, Robin Henderson[1☯], Irina Makarenko[1☯], Cristina Masoller[2☯]

1 School of Mathematics, Statistics and Physics, Newcastle University, Newcastle upon Tyne, United Kingdom, **2** Department of Physics, Universitat Politecnica de Catalunya, Barcelona, Spain

☯ These authors contributed equally to this work.
* K.A.Garside2@newcastle.ac.uk

- TDA identifies topological features (connected components and holes) in high resolution retinal images.
- TDA describes the extent to which the components and holes persist in *persistence diagrams* and *barcodes*.
- TDA features can be used to discriminate between patients with diabetic retinopathy and healthy subjects.

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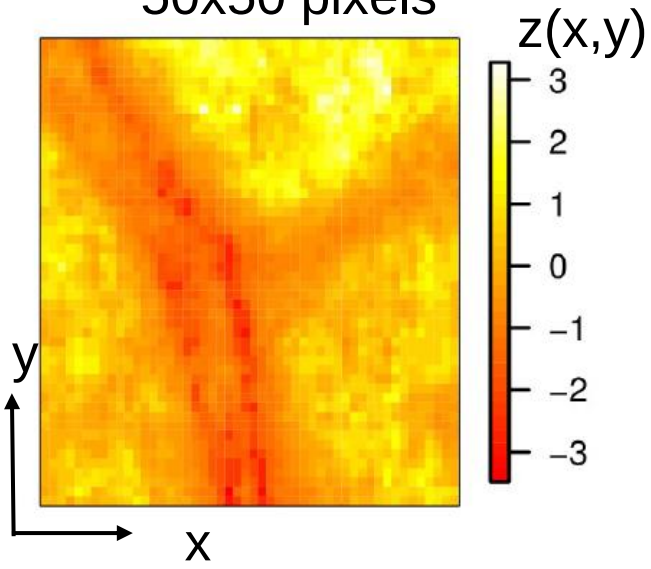
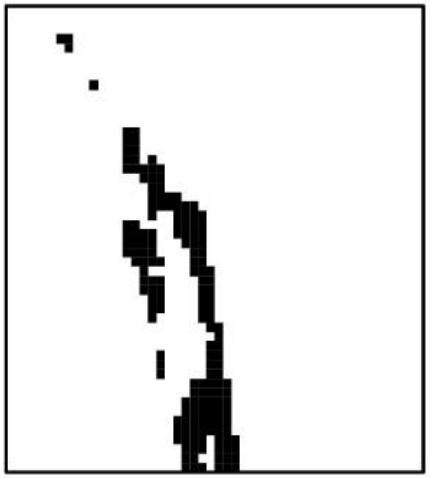
Methodology

50x50 pixels

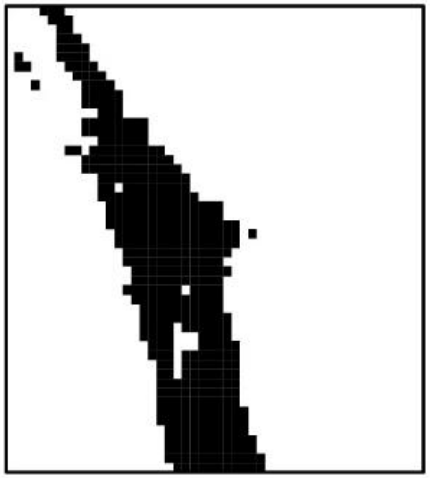


z(x,y) < -1.5



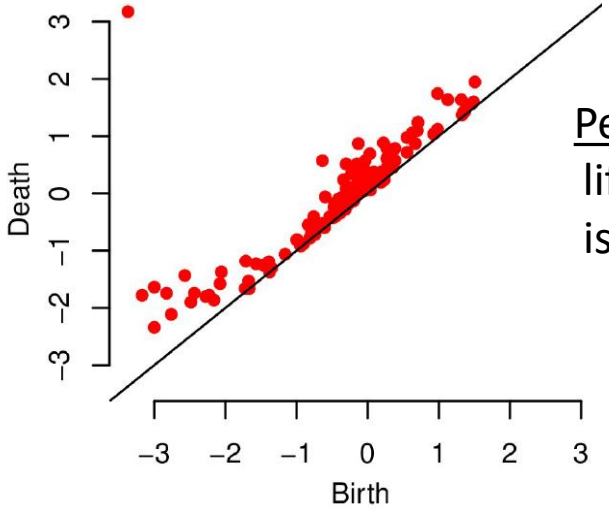
five groups of connected components but no holes

z(x,y) < -0.9

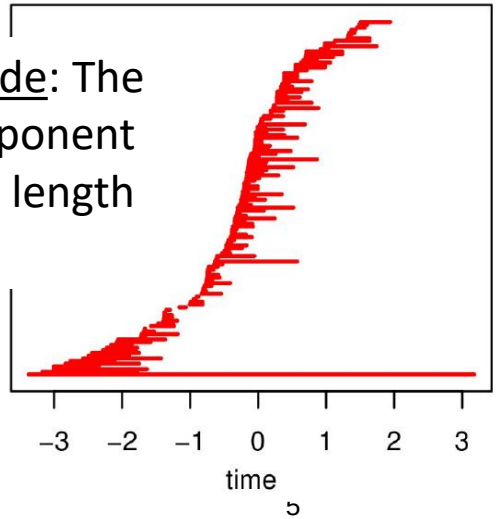


one large component, 4 new small components and 3 small holes

Persistence diagram of connected components (the first component to appear is the last one to die)



Persistence barcode: The lifetime of a component is encoded in the length of the bar



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Results

Table 1. Variables included in LASSO-informed SVM.

	SVM 1	SVM 2
Number of Components	x	x
Components 90% Convex Peel C_x	x	x
Components 90% Convex Peel P	x	
Components 3rd Landscape A		x
Components 3rd Landscape F	x	
Components Accumlative Persistence A	x	x
Holes 99% Convex Peel C_y	x	x
Holes 1st Landscape P	x	x
Holes 2nd Landscape C_y	x	
Holes 3rd Landscape C_y	x	

Conclusions

- ML using TDA features extracted from retinal fundus images may allow to detect diabetic retinopathy.
- Our results are at worst comparable and often better than those reported in the literature.
- Drawback: needs high-resolution images.

Test Set Size	SVM 1		SVM 2	
	Sensitivity	Specificity	Sensitivity	Specificity
1	1.000	1.000	1.000	1.000
2	1.000	0.996	1.000	1.000
3	0.998	0.987	0.998	0.998
4	0.995	0.983	0.995	0.995
5	0.993	0.980	0.991	0.993

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Can we use TDA to characterize the transition to sync of coupled oscillators?

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Surprisingly, we found few studies of sync and TDA

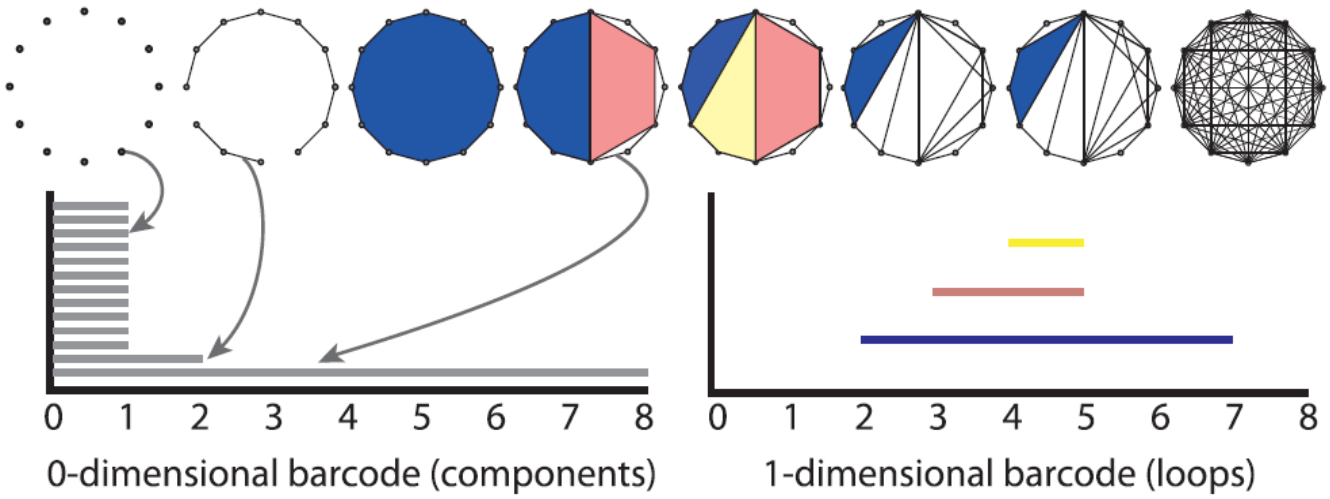
RESEARCH ARTICLE | APRIL 28 2017

Persistent homology of time-dependent functional networks constructed from coupled time series

Bernadette J. Stolz; Heather A. Harrington; Mason A. Porter

Chaos 27, 047410 (2017)
https://doi.org/10.1063/1.4978997

Rank all edge weights (w_{max} … w_{min}) at each step of the filtration threshold the graph to obtain a binary graph.



Step 0: every node is a component

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Chaos, Solitons and Fractals 192 (2025) 116054

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

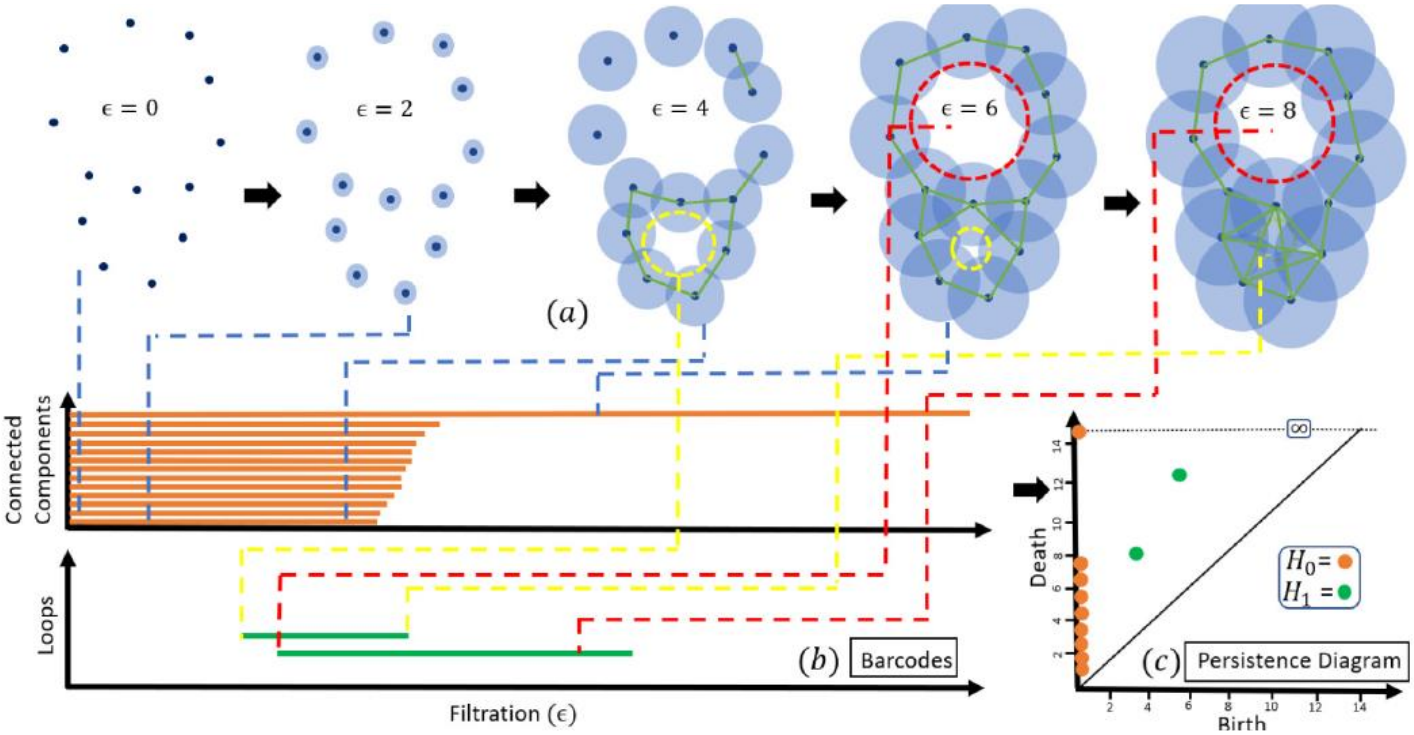
journal homepage: www.elsevier.com/locate/chaos

Persistent homology approach for uncovering transitions to Chaos

W. Hussain Shah [a], R. Jaimes-Reátegui [a,*], G. Huerta-Cuellar [a], J.H. García-López [a], A.N. Pisarchik [b,*]

[a] Departamento de Ciencias Exactas y Tecnología, Centro Universitario de los Lagos, Universidad de Guadalajara, Enrique Díaz de León 1144, Colonia Paseos de la Montaña, Lagos de Moreno, Jalisco, Mexico

[b] Center for Biomedical Technology, Universidad Politécnica de Madrid, Campus Montegancedo, 28223 Pozuelo de Alarcón, Madrid, Spain



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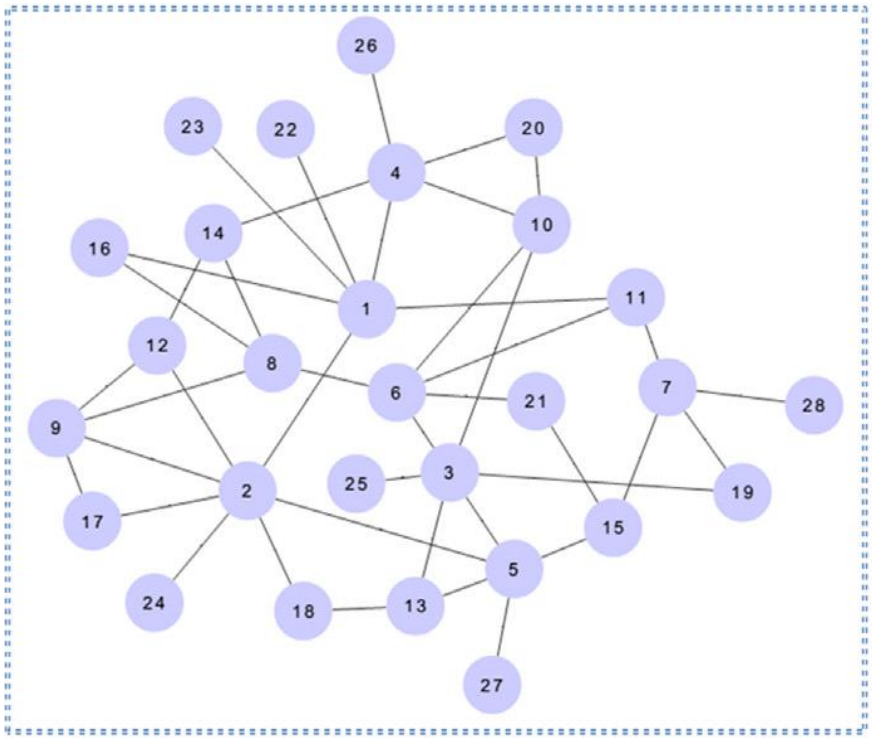
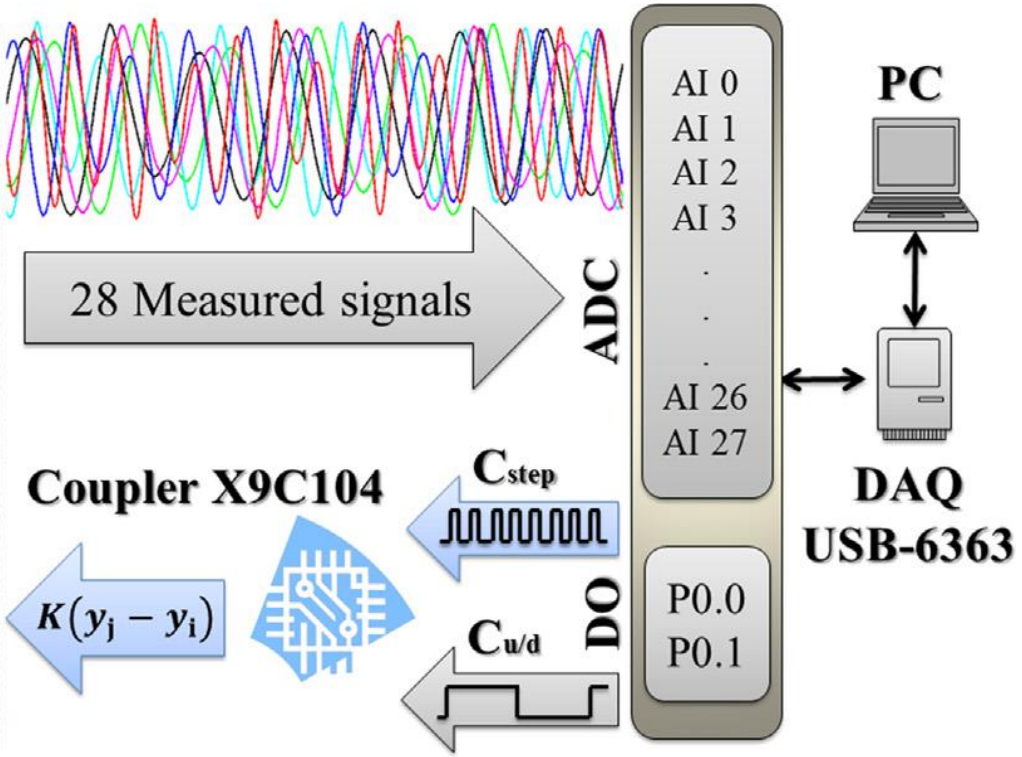
Data analyzed: recorded from a random network of 28 mutually coupled Rossler-like electronic circuits





- 28 time series recorded for each coupling strength, K
- Each time series has 30000 data points
- K = 0 … 1, 100 values

R. Sevilla-Escoboza and J. M. Buldú, Data in Brief 7, 1185 (2016).

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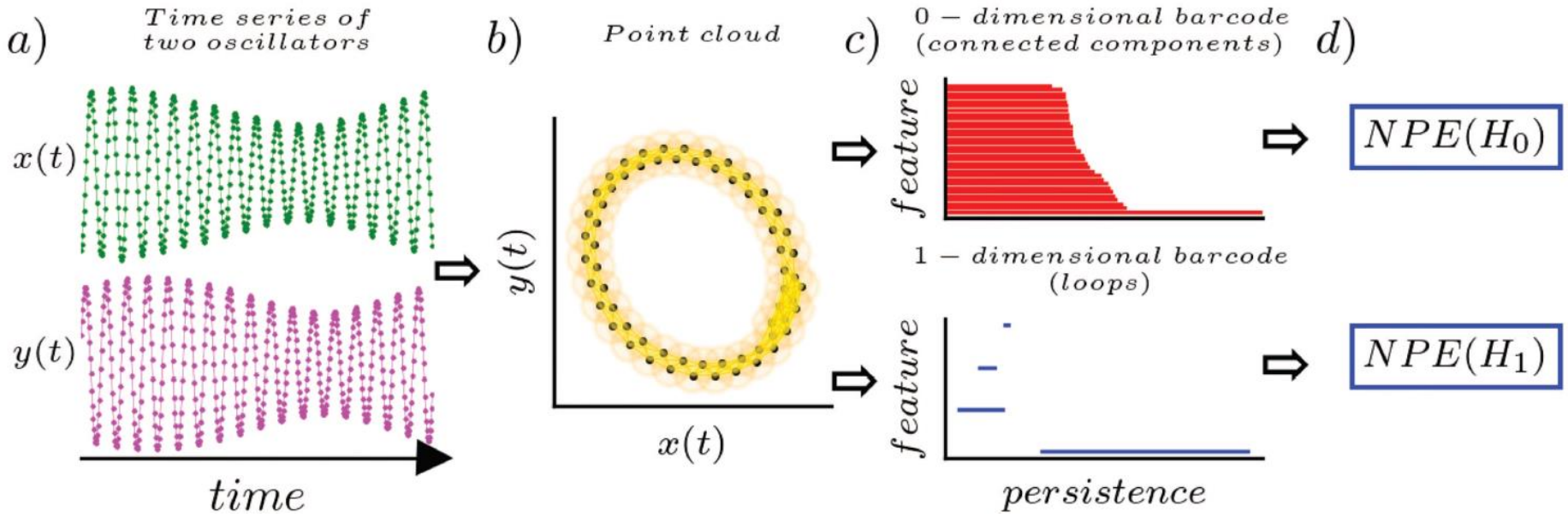
Normalized persistent entropy: a measure of the information content of a barcode



$$p(\ell_j) = \ell_j / L \qquad L = \sum_{i=1}^{N} \ell_i$$

$$PE = -\sum_{j=1}^{N} p(\ell_j) \log p(\ell_j)$$

$$NPE = \frac{PE}{L}$$

(We used Vietoris–Rips complexes, the *ripser* library & the *persistent_entropy* module in Python)

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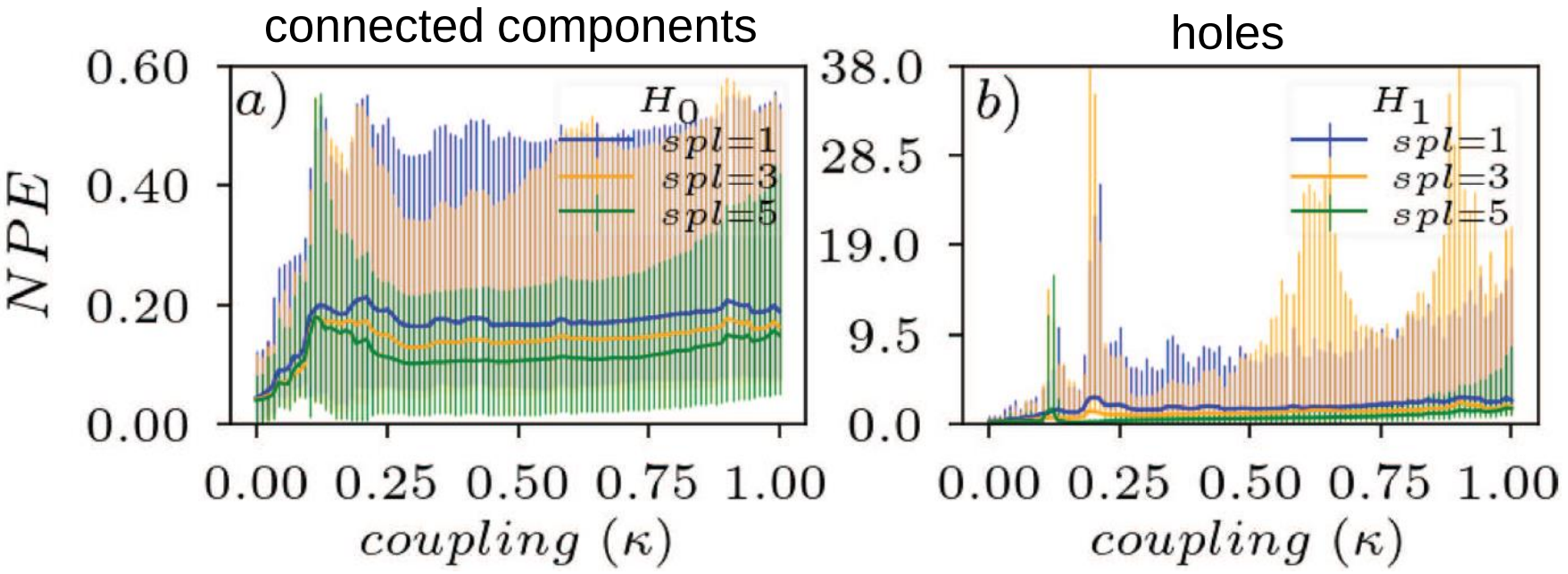
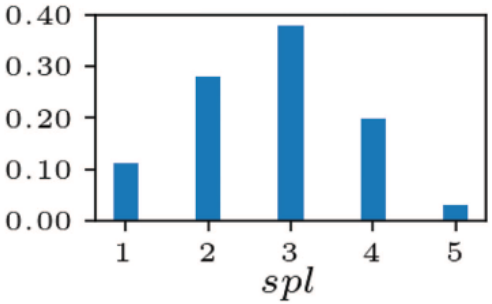
The persistence entropy of the cloud of points of two oscillators tends to decrease with the oscillators' distance



Distribution of shortest path length (spl)



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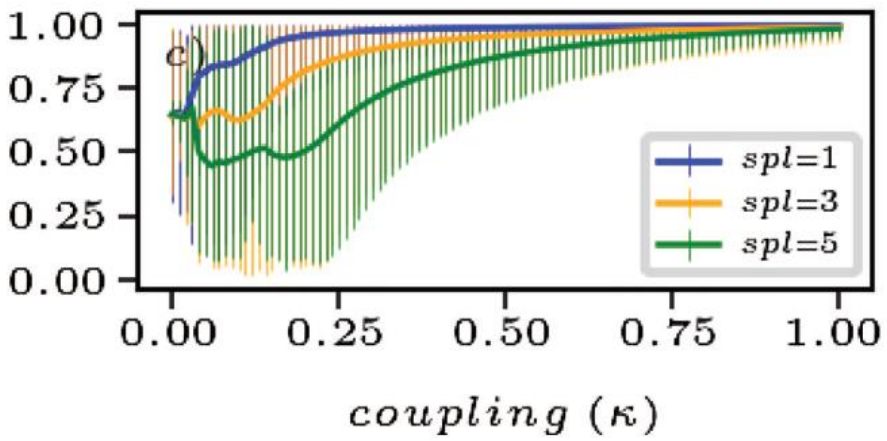
Comparison with a well-known sync measure: the phase locking value

$$PLV_{ik} = \langle | \exp(j\phi_{ik}(t)) | \rangle_t$$

instantaneous phase difference of oscillators *i* and *k* at time *t*, calculated with the Hilbert transform.



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Is this effect specific to mutually coupled Rossler-like oscillators?

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Kuramoto phase oscillators

$$\dot{\theta}_i = \omega_i + \lambda \sum_j^N A_{ij} \sin(\theta_j - \theta_i)$$

Order Parameter:

$$R = \frac{1}{N}\left\langle \left| \sum_{j=1}^{j=N} e^{i\theta_j(t)} \right| \right\rangle_t$$

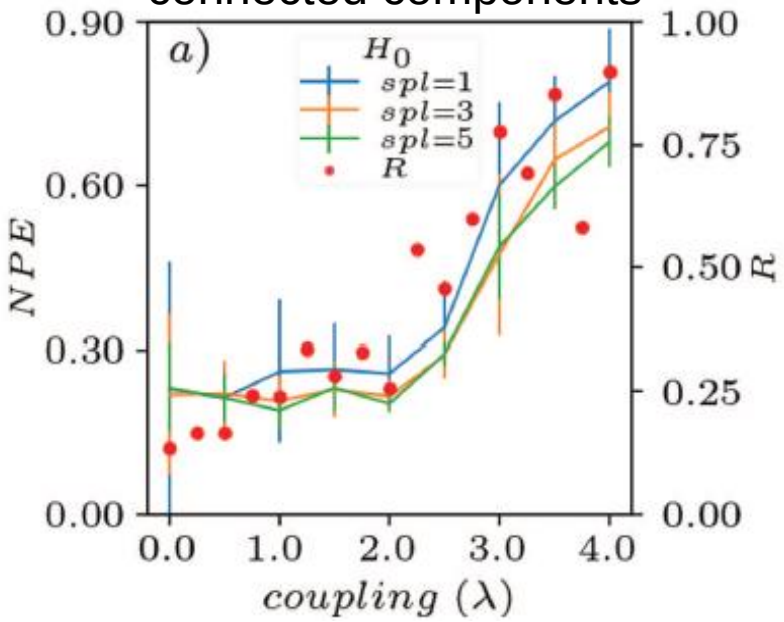
ω_i: Gaussian, zero mean and σ=1

A_{ij}: same random matrix as the circuits

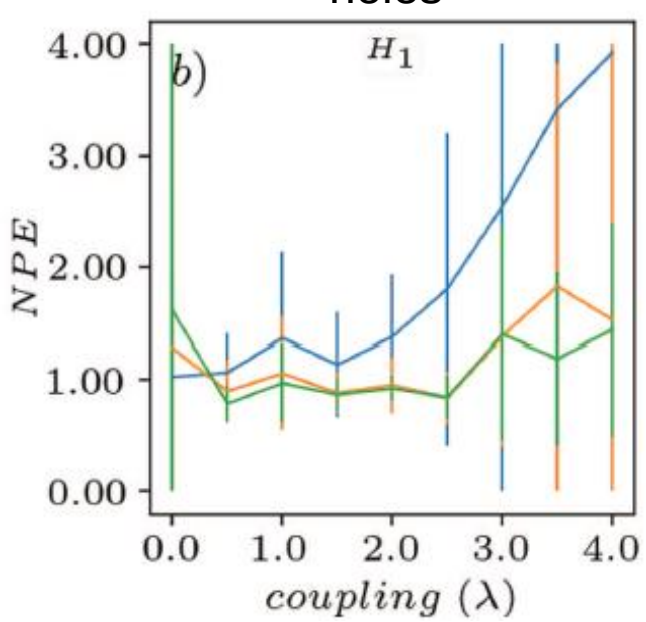
Point cloud is defined by

$x_i(t)=\cos(\theta_i(t))$ and $y_j(t)= \cos(\theta_j(t))$

connected components



holes



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Take home message and ongoing work

- Holes (H1) significantly capture the level of coupling between pairs of oscillators, while connected components (H0) may eventually reveal complementary information that needs to be explored.
- Can we distinguish triangles from triplets?

References

- K. Garside, R. Henderson, I. Makarenko, C. Masoller, "*Topological data analysis of high resolution diabetic retinopathy images*", PLoS ONE 14, e0217413 (2019).
- A. Zabaleta-Ortega, C. Masoller, L. Guzman-Vargas, "*Topological data analysis of the synchronization of a network of Rössler chaotic electronic oscillators*", Chaos 33, 113110 (2023).

THANK YOU FOR YOUR ATTENTION

cristina.masoller@upc.edu @cristinamasoll1