Characterizing and detecting regime transitions by using ordinal analysis

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Classical ordinal patterns and beyond (COPAB 2025)
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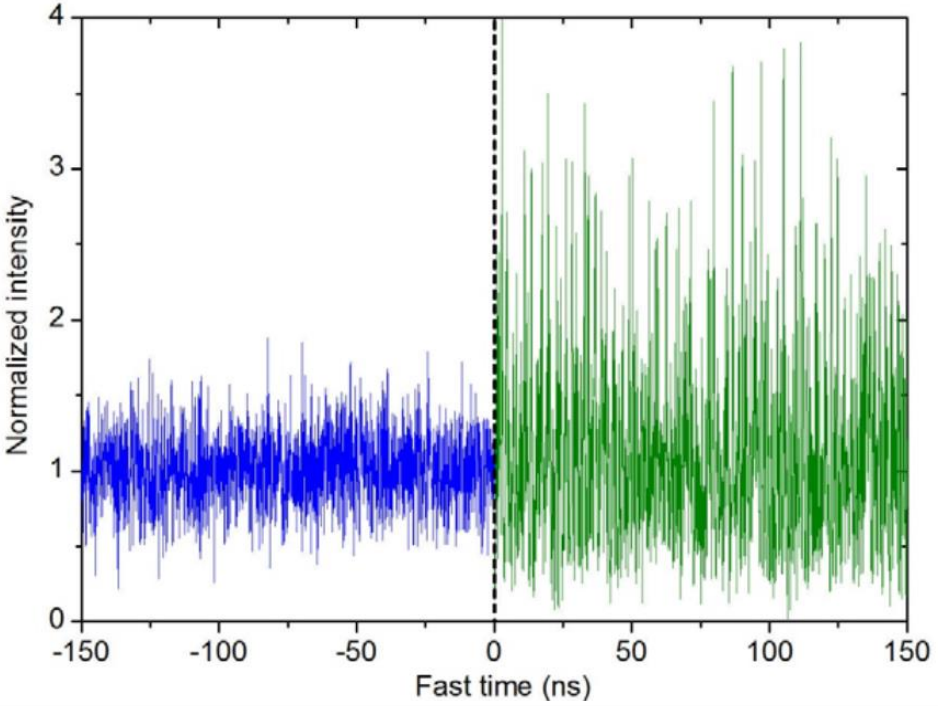
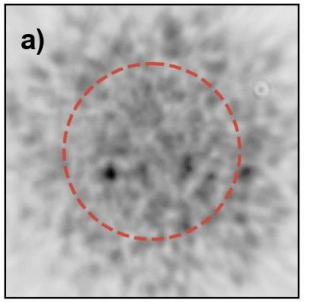
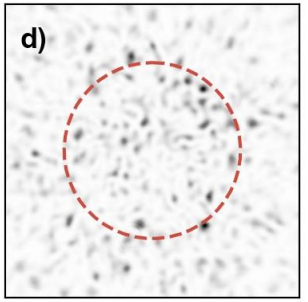
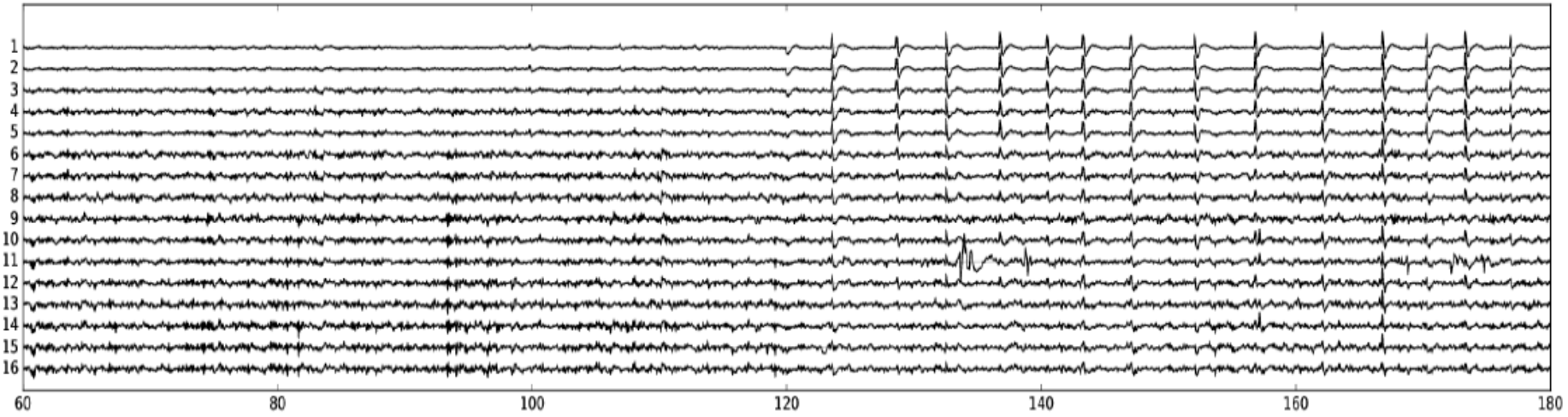
Regime transitions in complex systems









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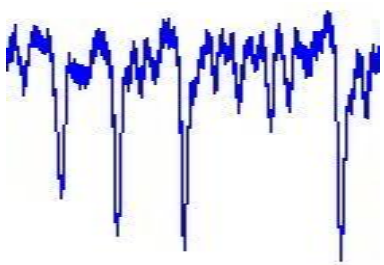
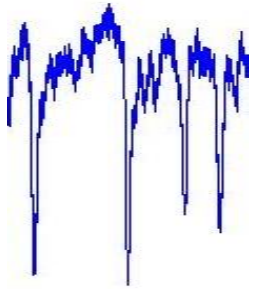
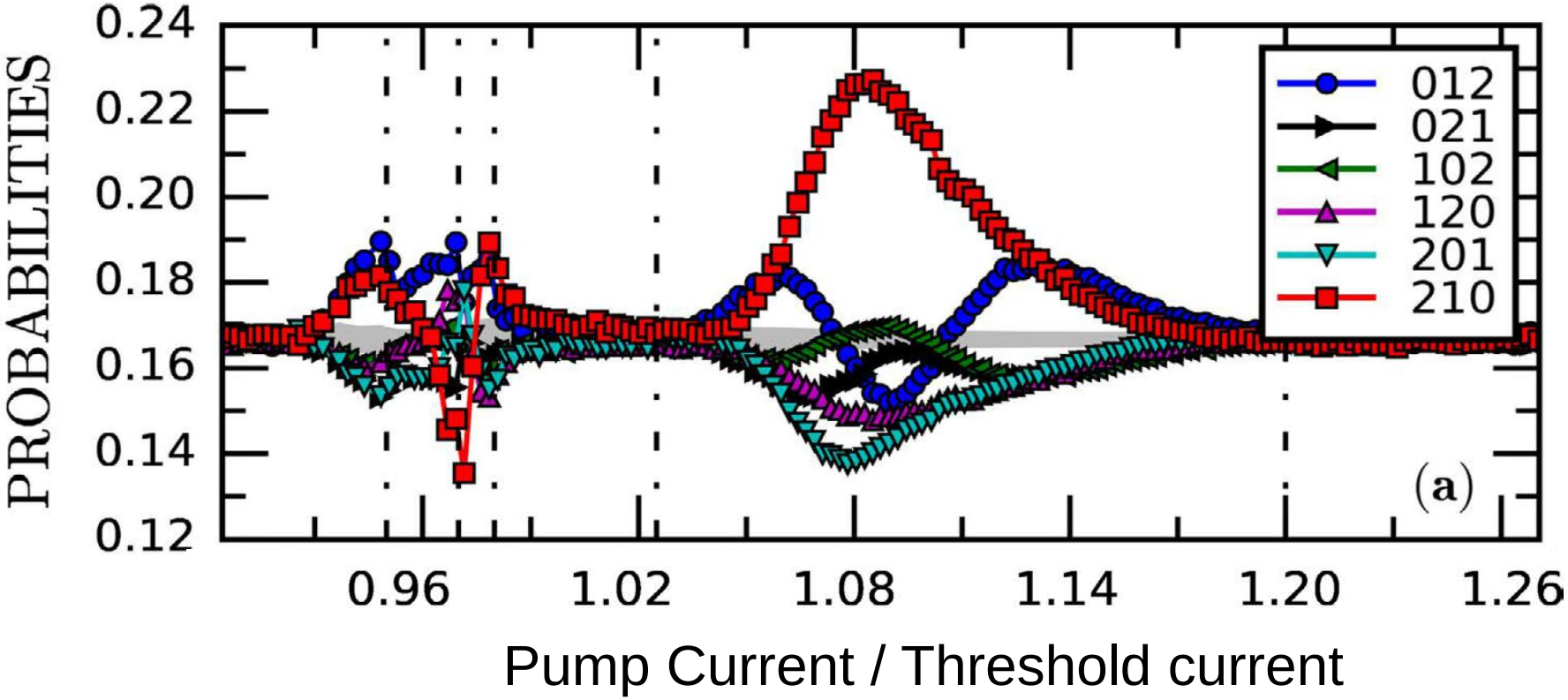
Probabilities of D=3 ordinal patterns computed from sequences of inter-spike-intervals



⇒012



⇒210



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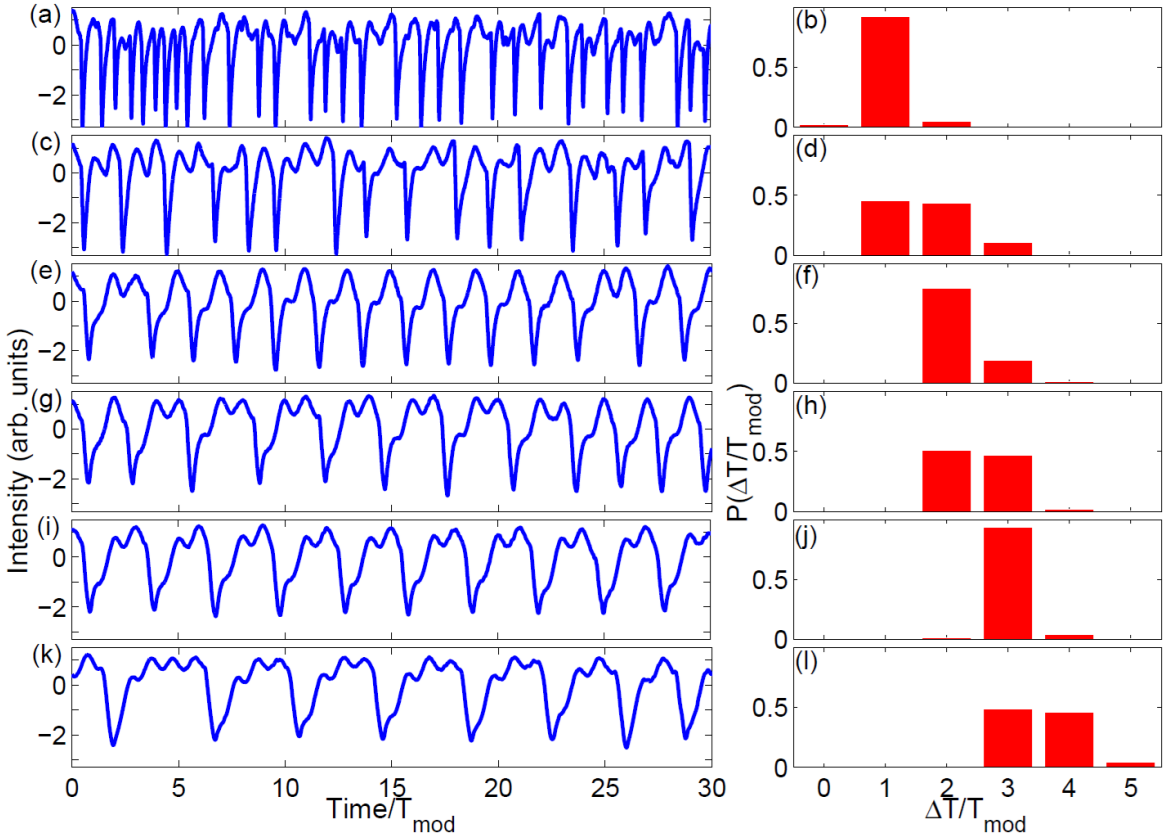
And when the laser is periodically modulated

Slow modulation

Fast modulation



T. Sorrentino et al, "*Effects of periodic forcing on the temporally correlated spikes of a semiconductor laser with feedback*", Opt. Express 23, 5571 (2015).

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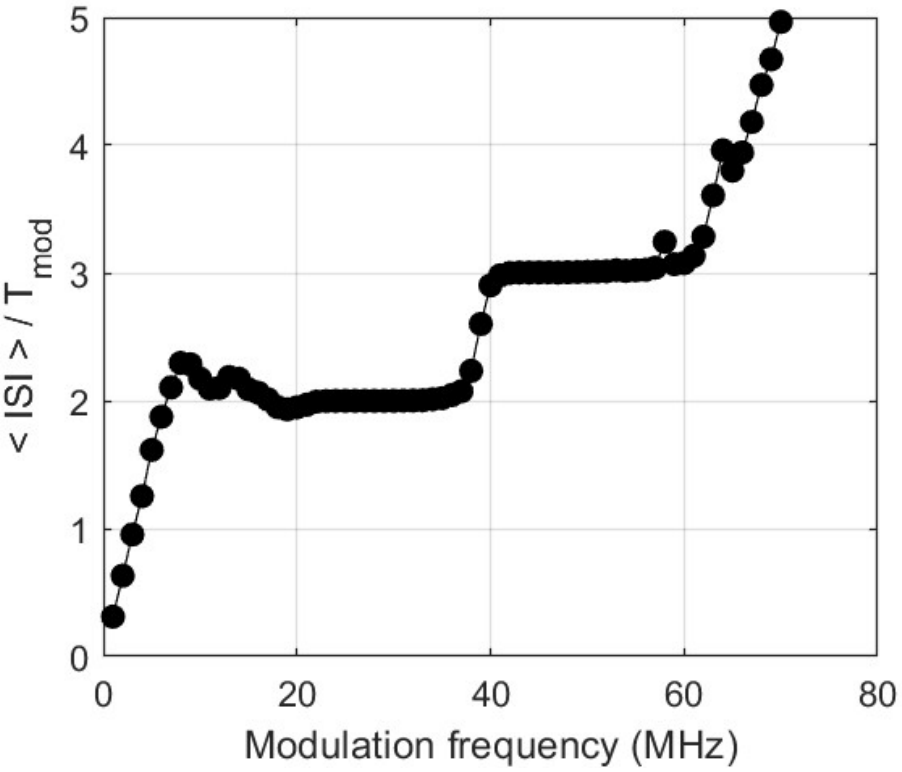
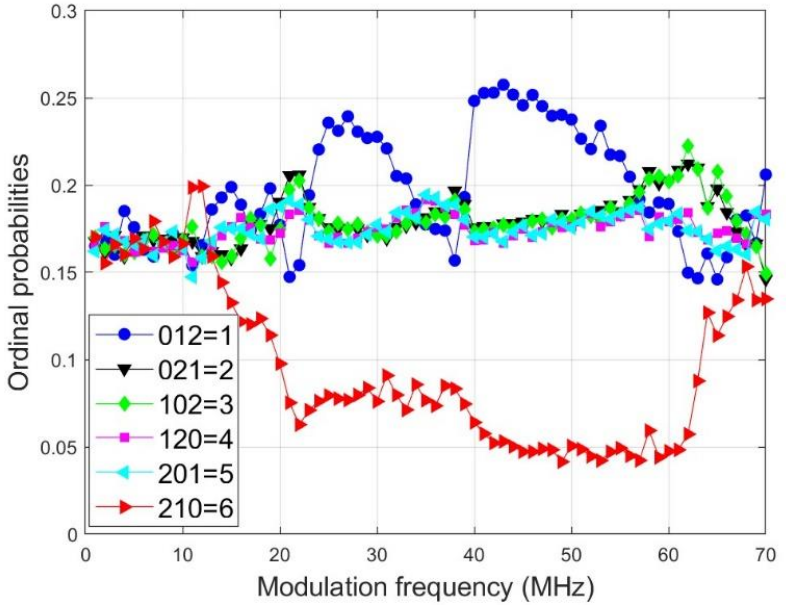
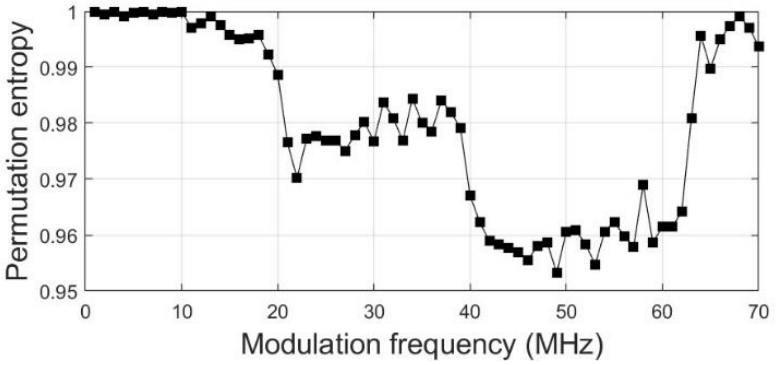
Ordinal analysis detects the locking transitions







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Ordinal network

Ordinal patterns are the nodes of the network and the transition probabilities are the weights the links.

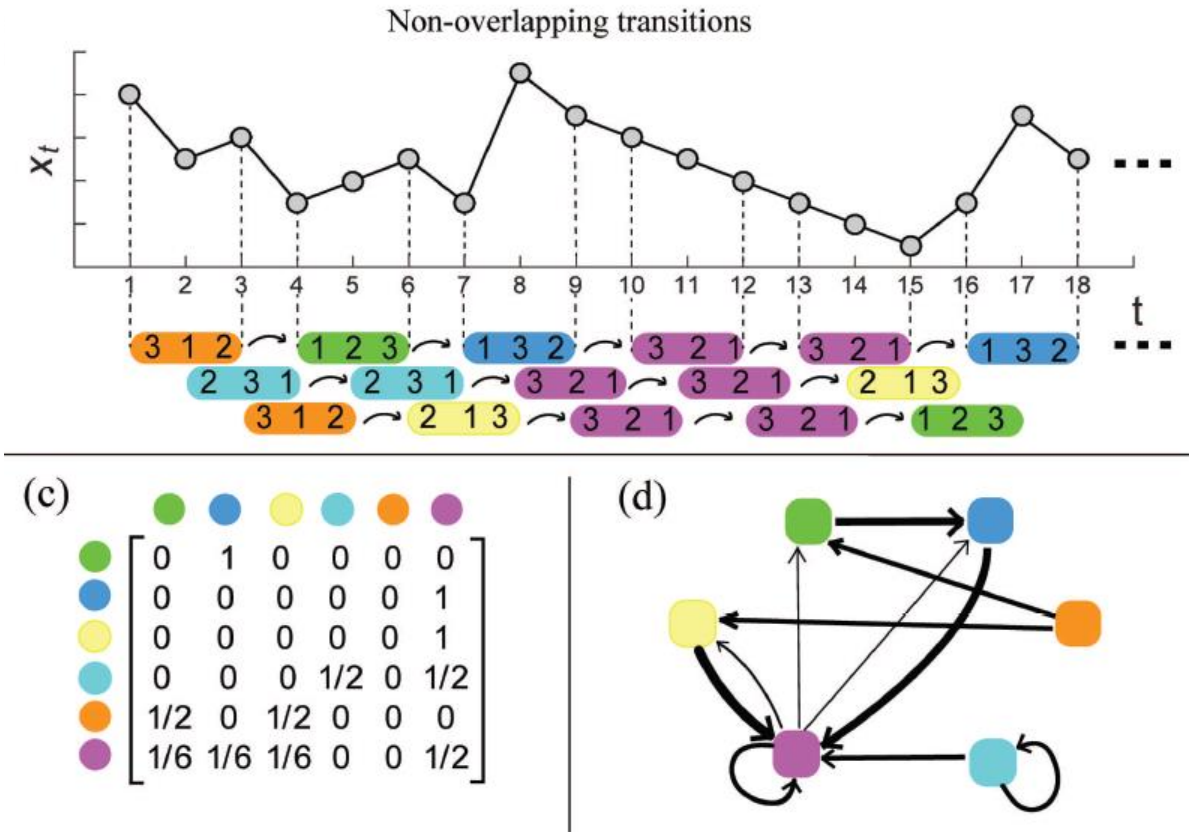
$$w_{ij} = TP(i \rightarrow j)$$

- In each node *i*:

$$\sum_j w_{ij} = 1$$

- Weigh of node i: probability of pattern *i*

$$(\sum_i p_i = 1)$$



F. Olivares et al., Chaos 30, 063101 (2020).

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Network-based diagnostic tools

- Entropy computed from node weights (**permutation entropy**)

$$s_p = -\sum p_i \log p_i$$

- Entropy of each node, computed from the node's transition probabilities, averaged over *M=D!* nodes (**node entropy**)

$$s_i = -\sum w_{ij} \log w_{ij}$$

$$s_n = \frac{1}{M} \sum_{i=1}^{M} s_i$$

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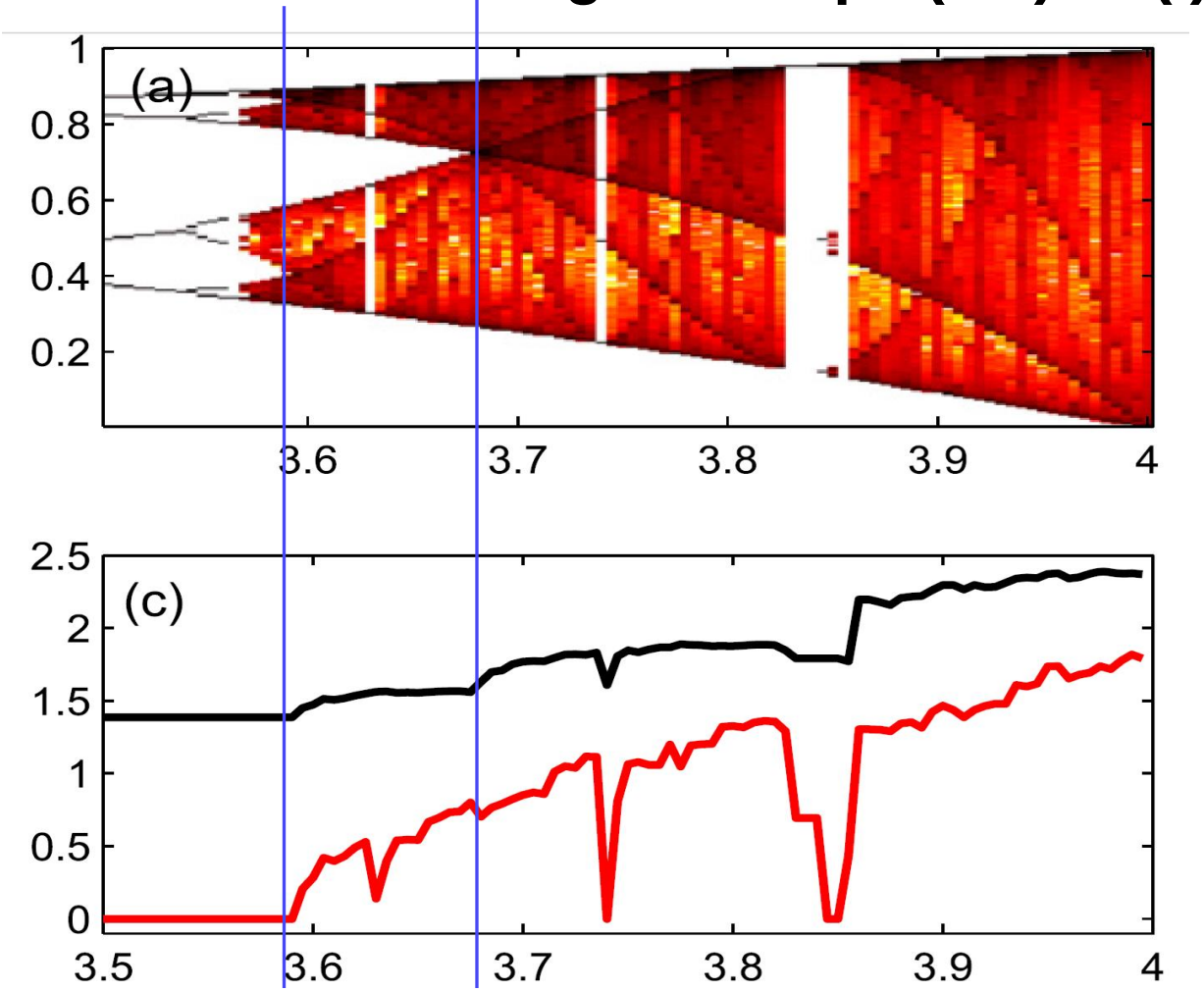
Chaos-chaos transitions in the Logistic Map x(i+1)=rx(i)[1-x(i)]



Permutation entropy (D=4)

Node entropy

C. Masoller, Y. Hong, et al. "*Quantifying sudden changes in dynamical systems using symbolic networks*", New J. of Phys. 17, 023068 (2015).

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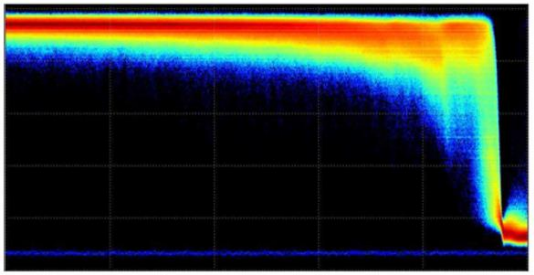
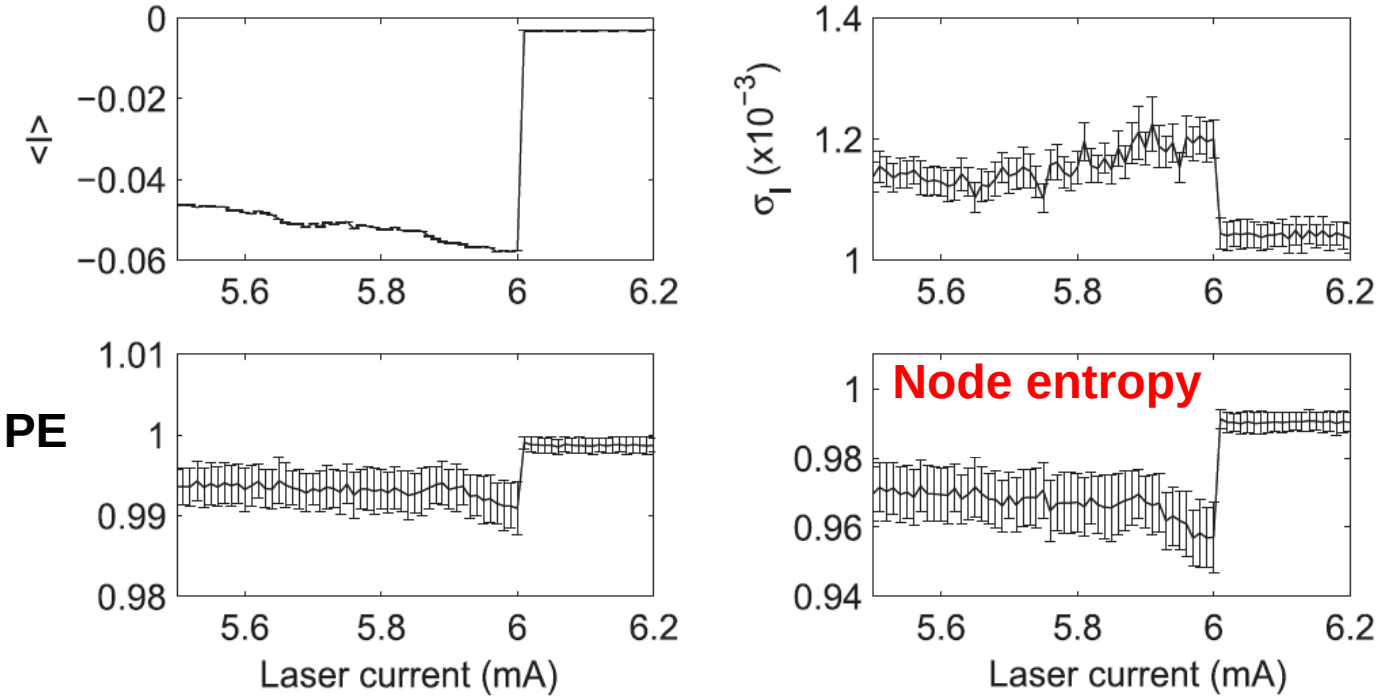
The entropies can anticipate an abrupt transition

Polarization switching: the polarization of the light emitted by a laser, abruptly changes when the current increases



Time / control parameter (current)



C. Masoller, Y. Hong, et al. "*Quantifying sudden changes in dynamical systems using symbolic networks*", New J. of Phys. 17, 023068 (2015).

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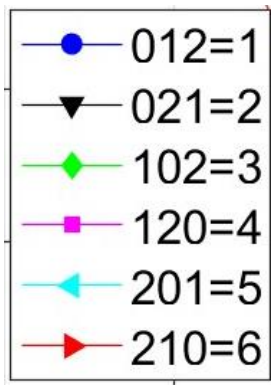
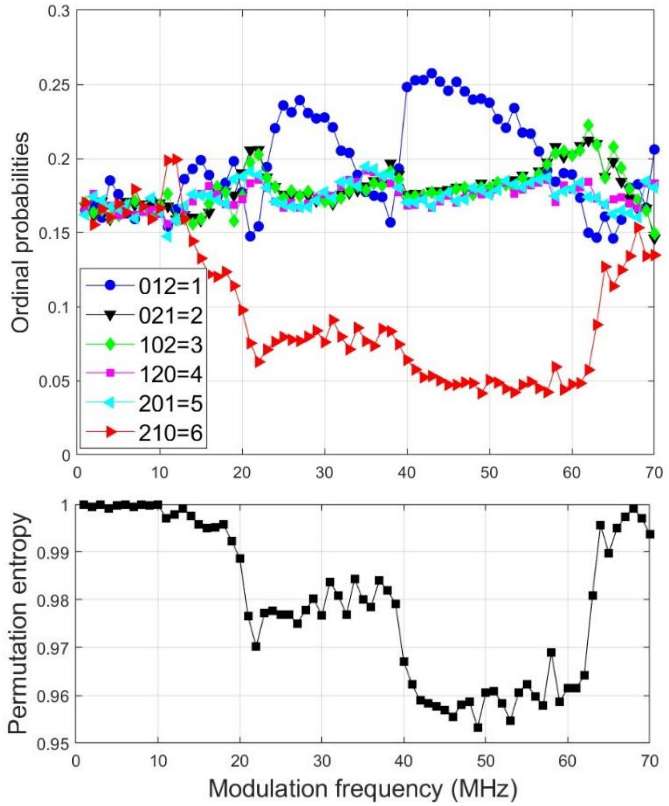
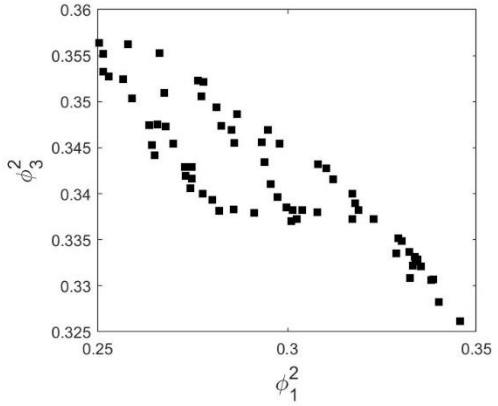
Other quantifiers for D=3 ordinal patterns

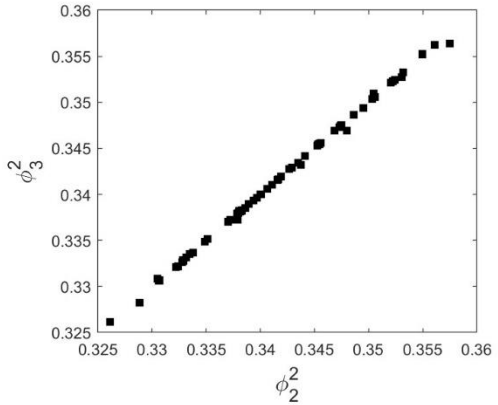
$$PE = -\frac{1}{\log 6}\sum_{i=1}^{i=6} p_i \log p_i$$

$$= \underbrace{-\frac{1}{\log 6} p_1 \log p_1 - \frac{1}{\log 6} p_6 \log p_6}_{\phi_1^2} \underbrace{-\frac{1}{\log 6} p_2 \log p_2 - \frac{1}{\log 6} p_4 \log p_4}_{\phi_2^2} \underbrace{-\frac{1}{\log 6} p_3 \log p_3 - \frac{1}{\log 6} p_5 \log p_5}_{\phi_3^2}$$









M. Duque-Gijon, J. Tiana-Alsina, C. Masoller, A. Aragoneses, submitted (2025)

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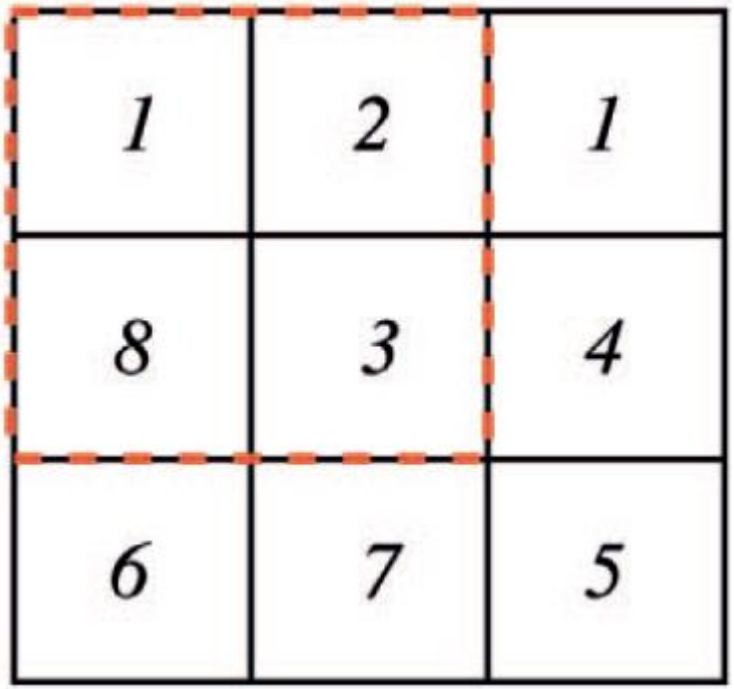
Ordinal analysis of two-dimensional data



1	2
8	3

1283 ⇒0132

2	1
3	4

2134 ⇒1023

8	3
6	7

8367 ⇒3012

3	4
7	5

3475 ⇒0132

Spatial permutation entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

2x2 pixels: 24 possible patterns

Haroldo V. Ribeiro and coworkers, PLoS ONE 7, e40689 (2012)

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Example of application: analysis of liquid crystal textures

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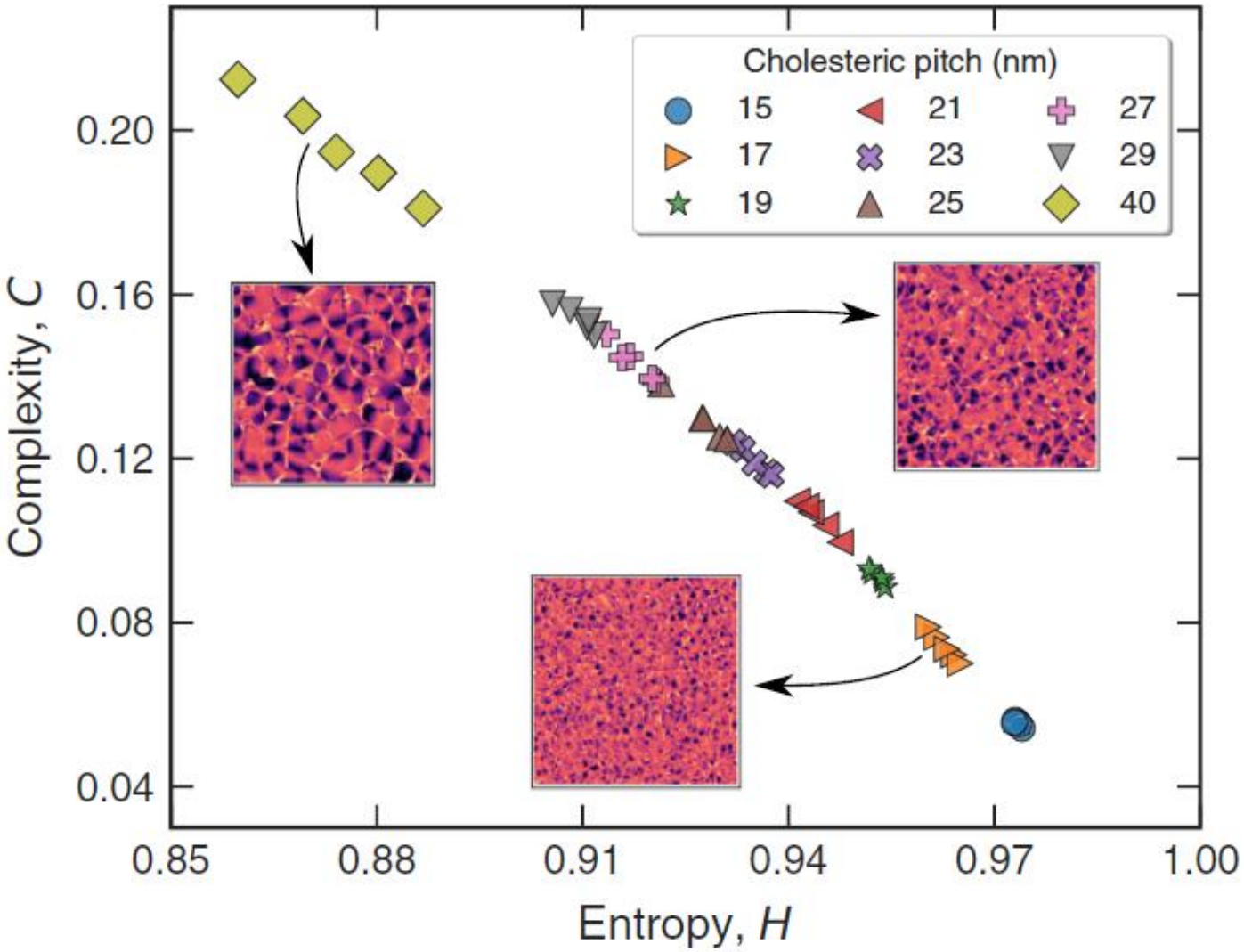

Haroldo V. Ribeiro and coworkers, PRE 99, 013311 (2019)

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Can the variation of the permutation entropy give an early indication of an approaching transition?

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SPE analysis of regime transitions

- Desertification: High $\rightarrow$ low biomass (vegetation 2D data – observational and simulated)
- Laser turn-on: Low $\rightarrow$ high coherence (speckle images)
- Eyes closed $\rightarrow$ eyes open (multichannel EEG data)

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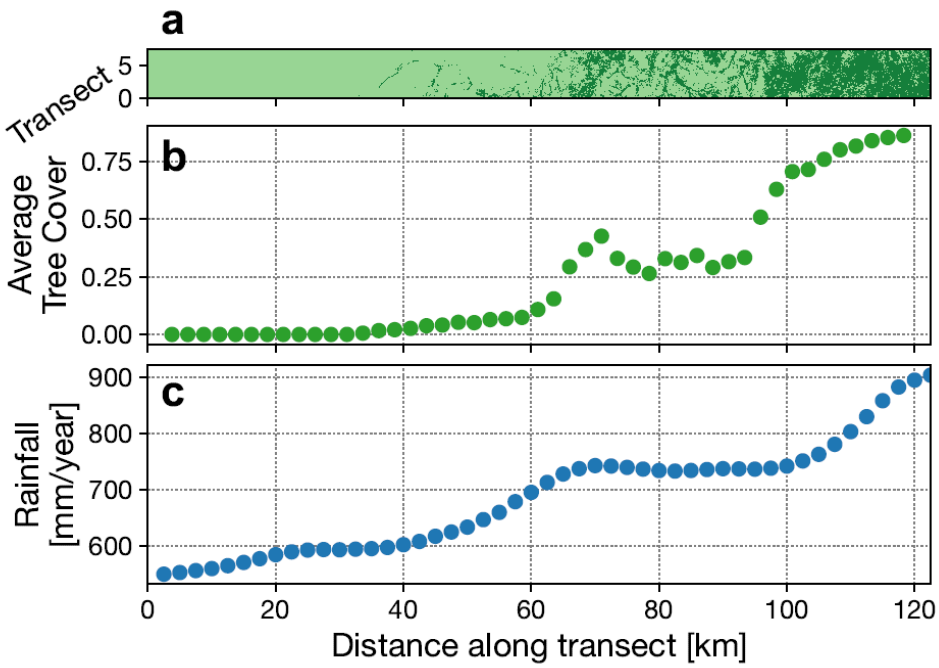
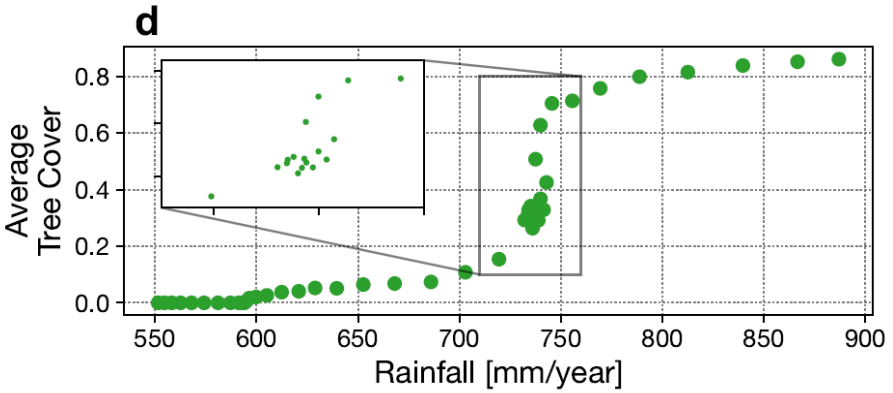
Analysis of high-resolution vegetation data





Data from the Serengeti–Mara ecosystem in northern Tanzania and southern Kenya.

G. Tirabassi, C. Masoller, "Entropy-based early detection of critical transitions in spatial vegetation fields", PNAS 120, e2215667120 (2022).

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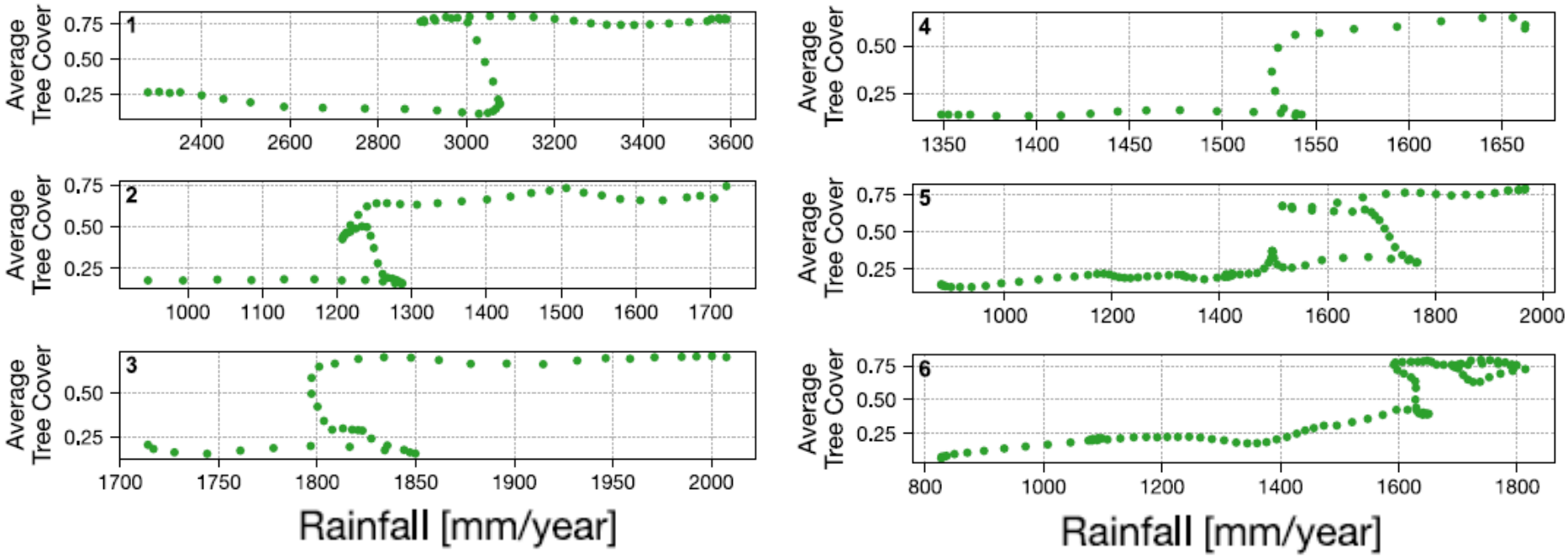
Analysis of low-resolution satellite vegetation data



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Two possible indicators of the desertification transition

Spatial Permutation Entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

Ordinal patterns defined by the values of 2x2 pixels

Spatial correlation

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(u_i - \bar{u})(u_j - \bar{u})}{\sum_i (u_i - \bar{u})^2}$$

w_{ij}=1 if pixels I and j first neighbors, else w_{ij}=0

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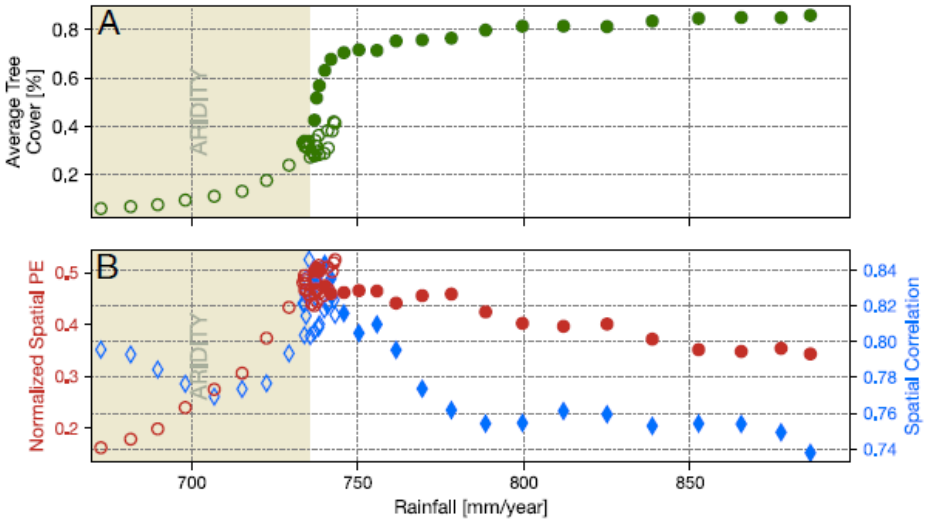
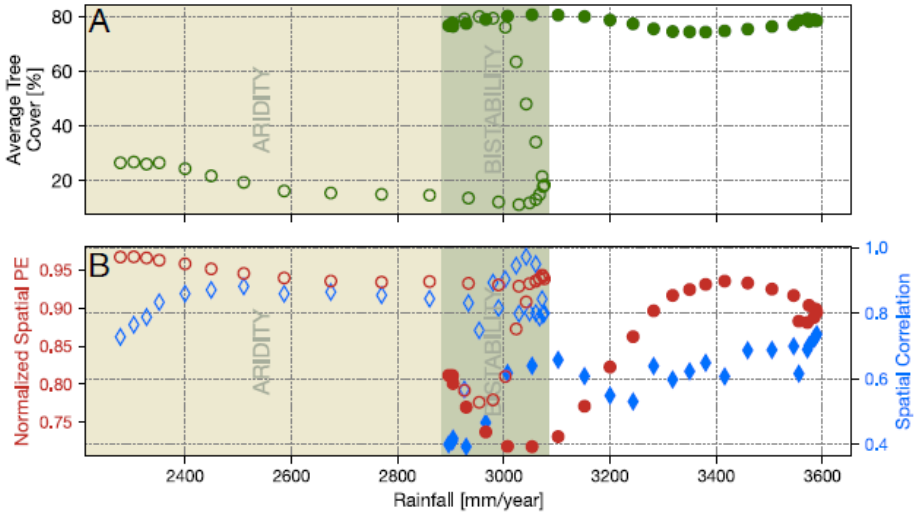
Results

High-resolution data



Low-resolution data

(transect 1; large variability across transects)



G. Tirabassi, C. Masoller, "Entropy-based early detection of critical transitions in spatial vegetation fields", PNAS 120, e2215667120 (2022).

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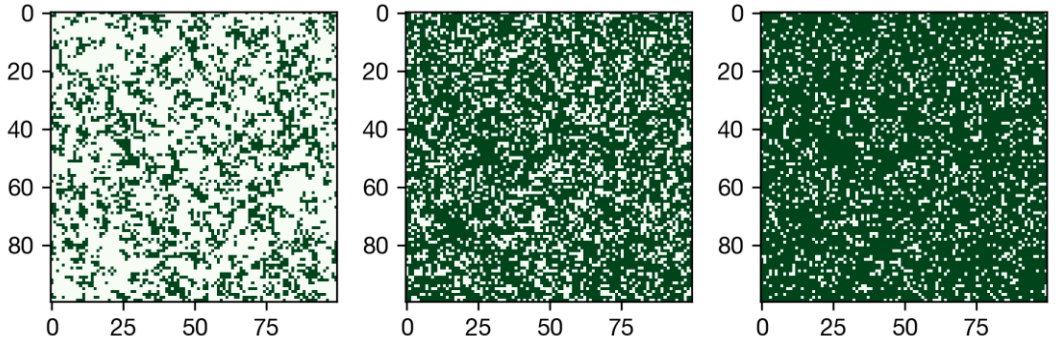
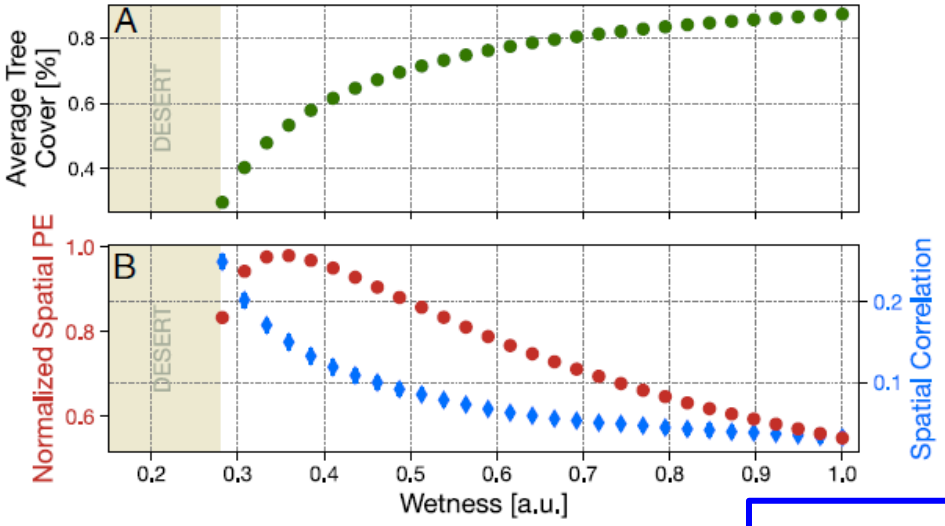
To gain insight: simulations of vegetation models

A) Cellular automata model





$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(u_i - \bar{u})(u_j - \bar{u})}{\sum_i (u_i - \bar{u})^2}$$

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Can we test the indicator in controlled experimental data?

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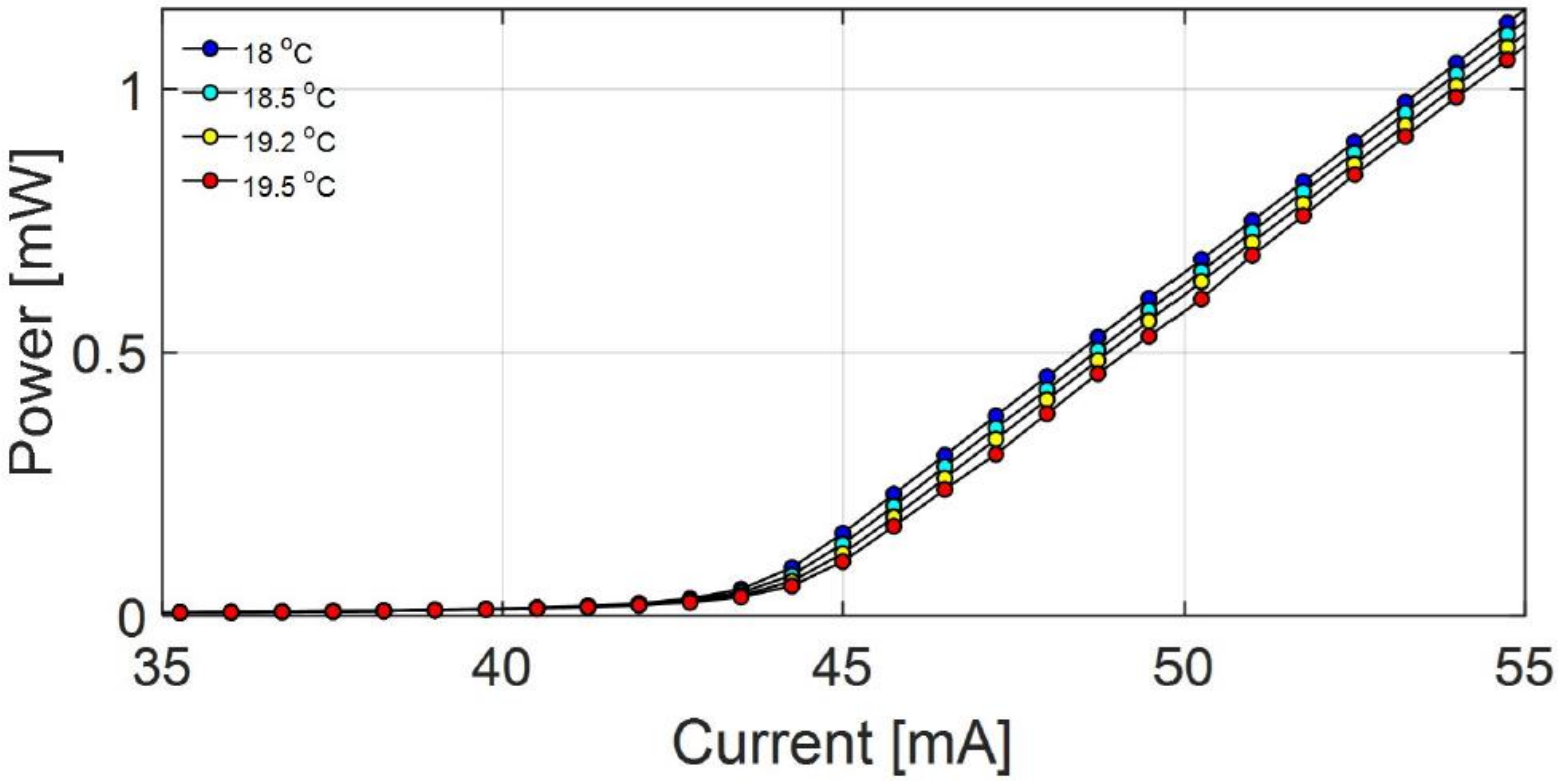
Analysis of the spatial coherence of the light emitted by a semiconductor laser



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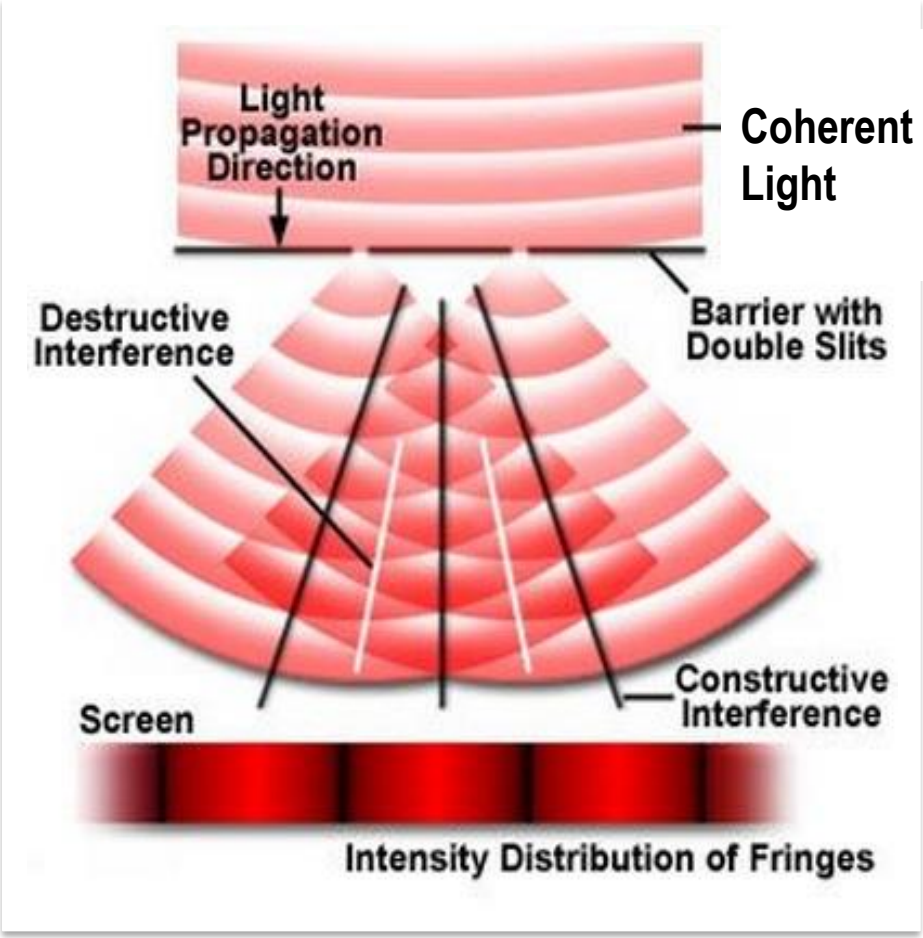
Quick reminder of the interference of coherent waves



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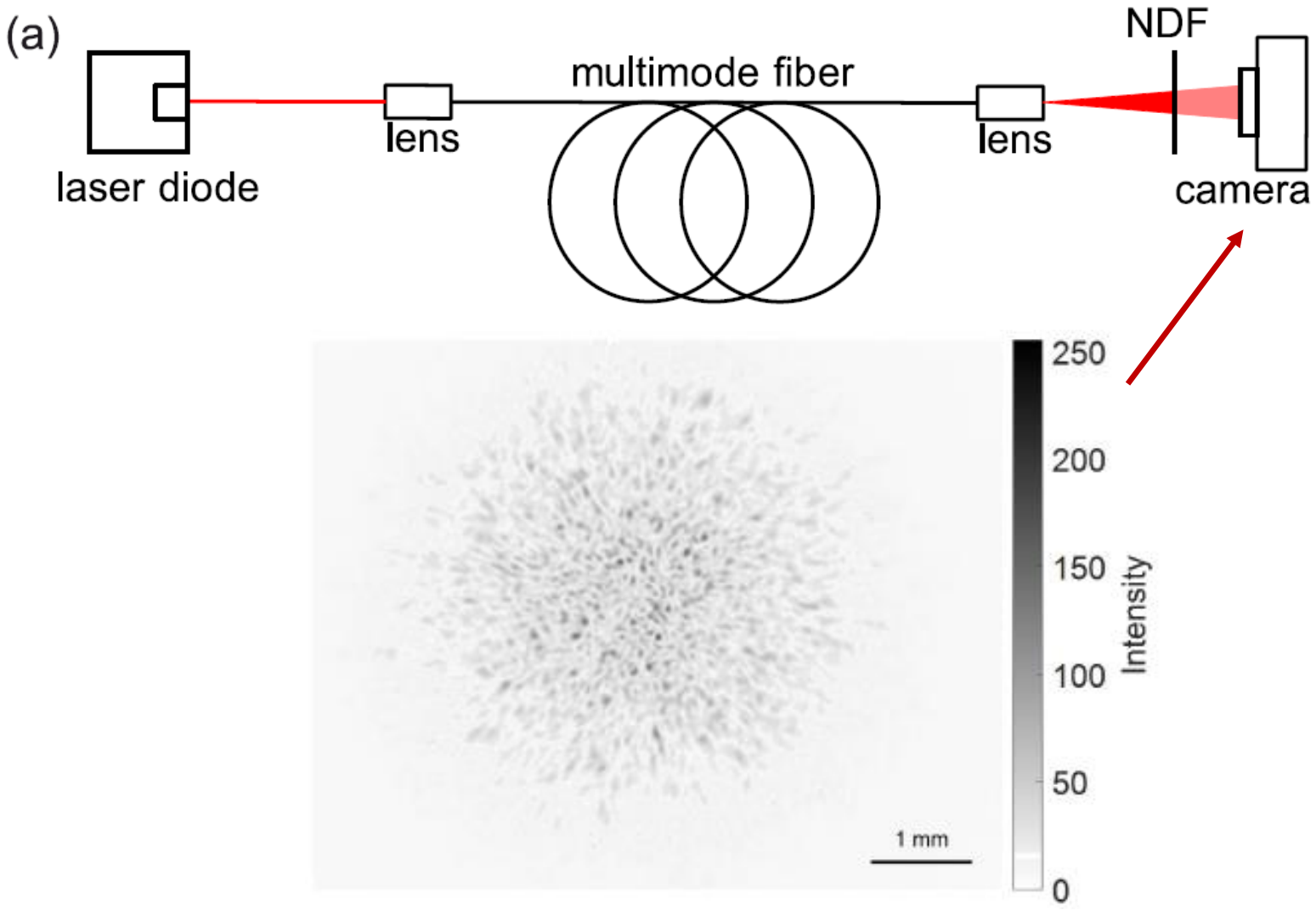
Speckle pattern: generated by the interference / scattering of coherent waves



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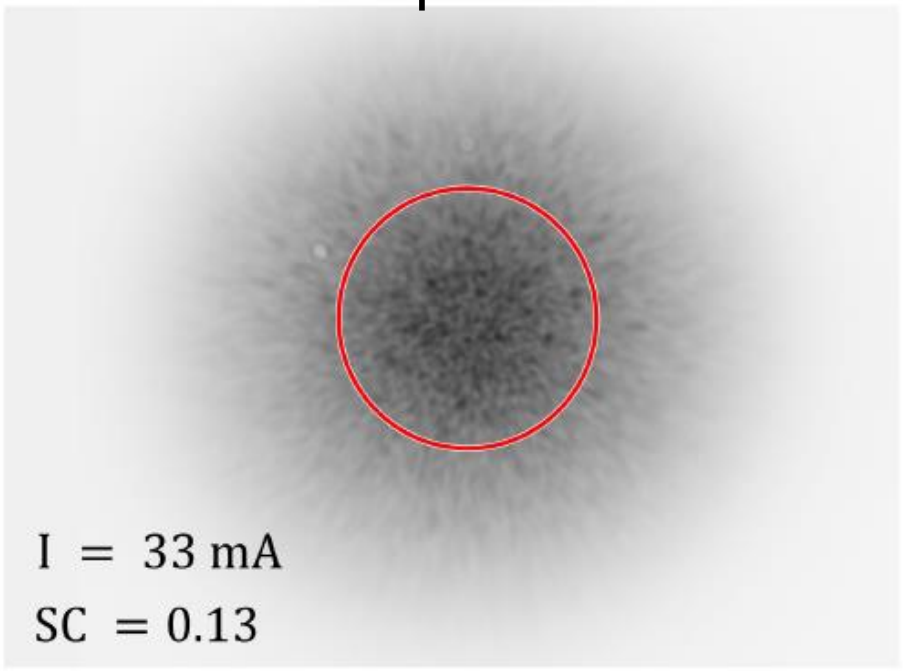
Speckle pattern depends on the coherence of the laser light

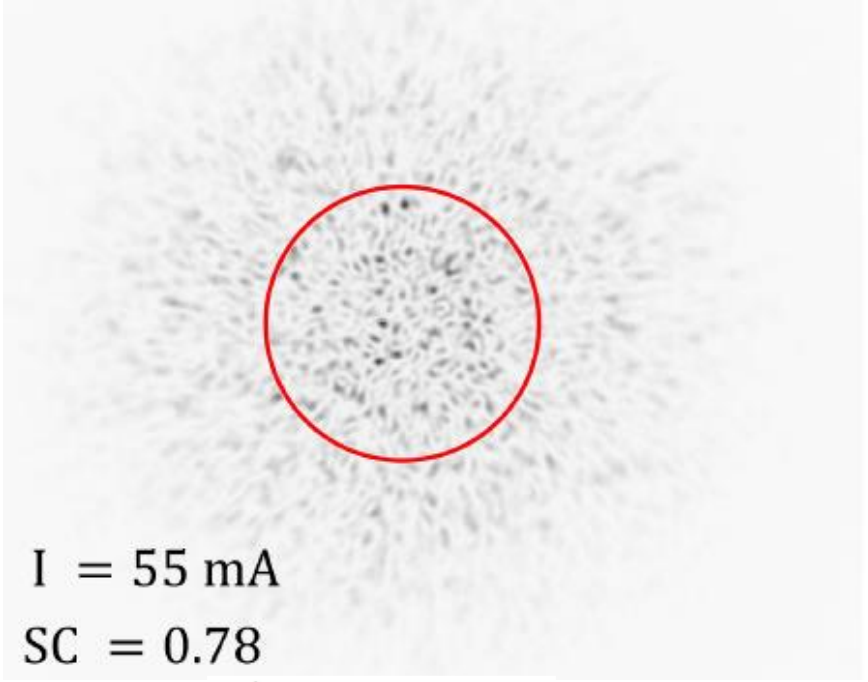
Below threshold

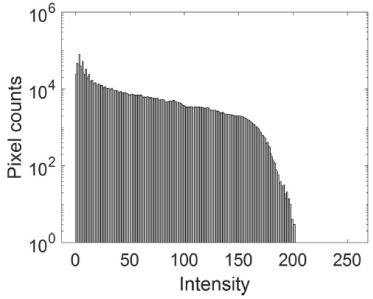
Low light coherence → low speckle contrast



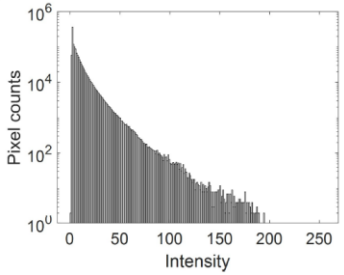
Above threshold

High light coherence→ high speckle contrast





$SC = \sigma/\langle I \rangle$



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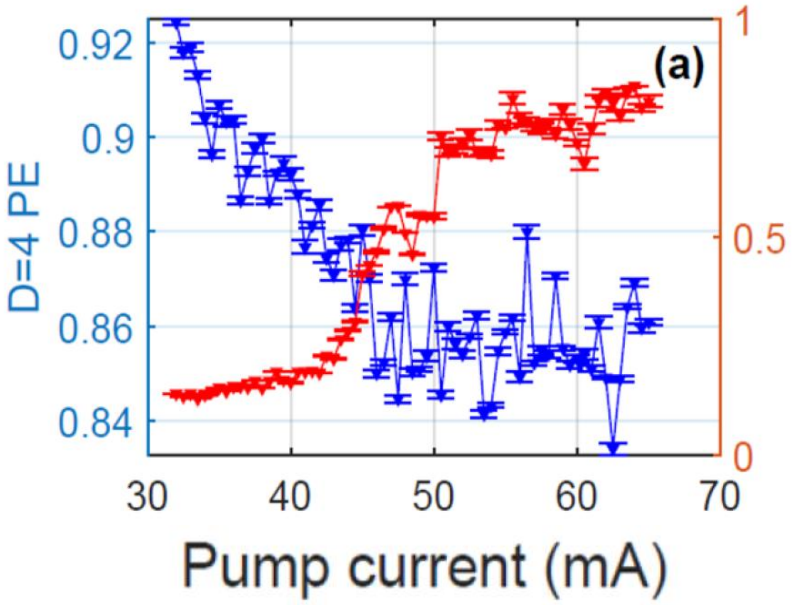
Results

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

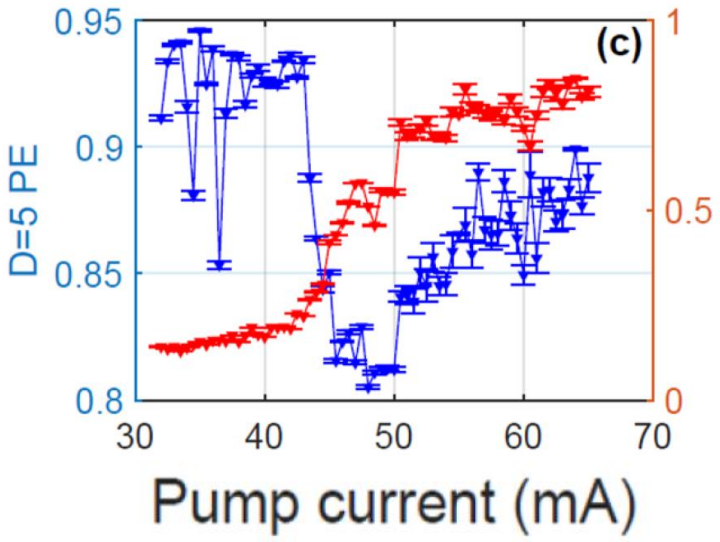
Speckle Contrast

SC = σ/⟨I⟩

Pattern: x x
x x



Pattern: x
x x x
x



G. Tirabassi et al., "Permutation entropy-based characterization of speckle patterns generated by semiconductor laser light", APL Photonics 8, 126112 (2023).

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Useful for the analysis of complex temporal data?

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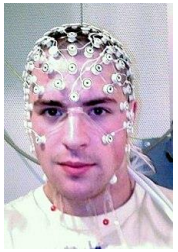
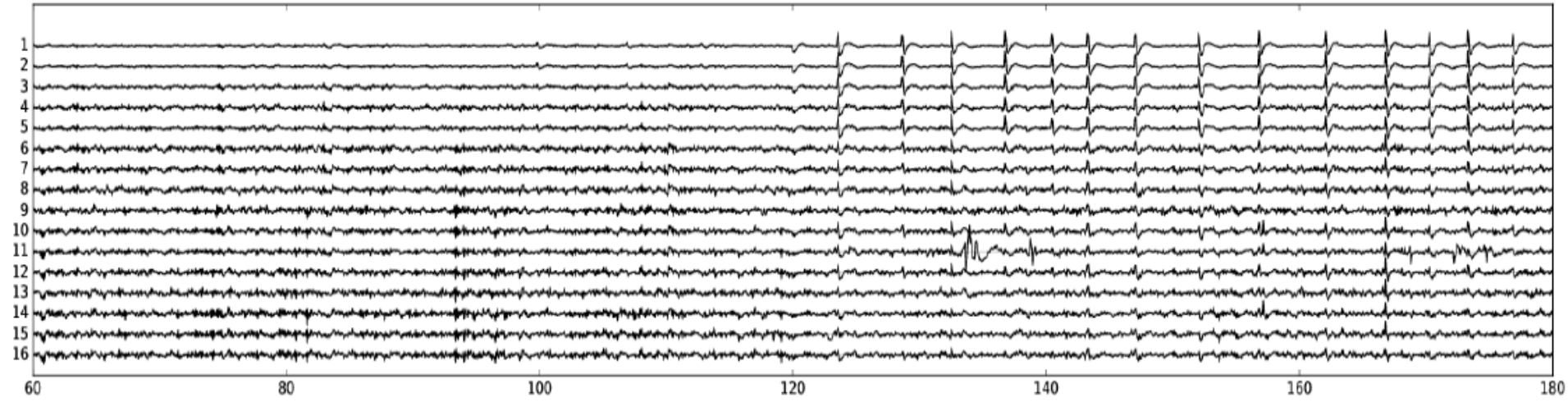
Analysis of EEG signals recorded from healthy subjects.



Eyes closed

Eyes open



Time (seconds)

TABLE I. Description of the datasets used.

	DTS1	DTS2
Sampling rate (Hz)	256	160
Time task (seg)	120	60
Total points	30 720	9600
Number of electrodes	16	64
Number of subjects	71	109

DTS1: Britbrain (Zaragoza)
DTS2: Physionet

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The Permutation Entropy increases in the “eyes open” state

$$\langle \mathrm{PE} \rangle = \frac{1}{N[\text{electrodes}]} \sum_i \mathrm{PE}^i$$

PE was calculated with patterns of length 4 (# of possible patterns 24) in time windows containing >4000 patterns

Gray region: σ of $\langle \mathrm{PE} \rangle$ values across subjects

C. Quintero-Quiroz et al., “Differentiating resting brain states using ordinal symbolic analysis”, Chaos 28, 106307 (2018).

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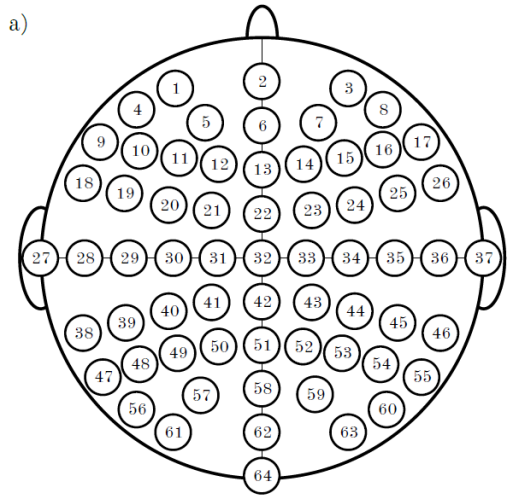
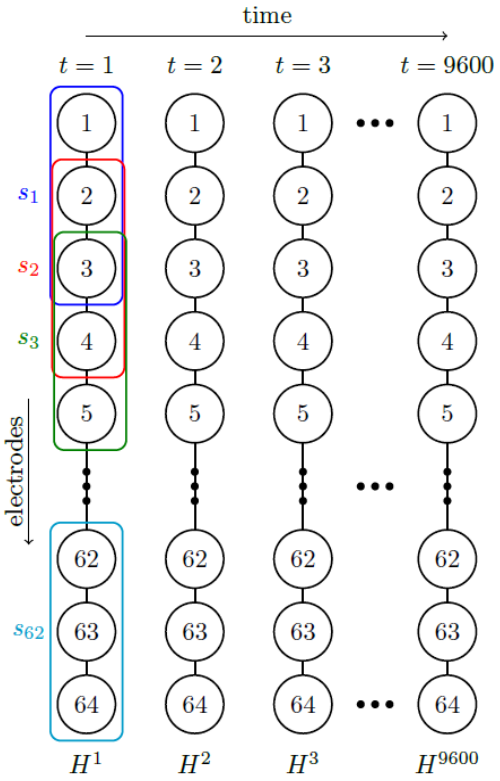
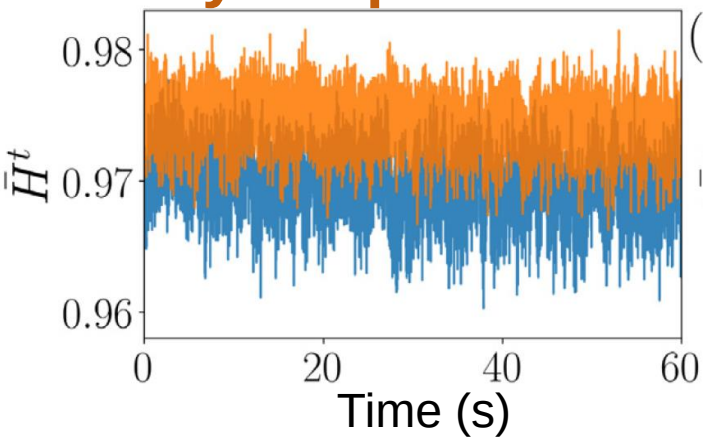
Spatial approach to compute the Permutation Entropy





Eyes closed

Eyes open



(H_t is averaged over subjects)

At each time: data values of 64 channels ⇒ 62 ordinal patterns to calculate 6 probabilities.

Bruno Boaretto and coworkers, "Spatial permutation entropy distinguishes resting brain states", Chaos, Solitons & Fractals 171, 113453 (2023).

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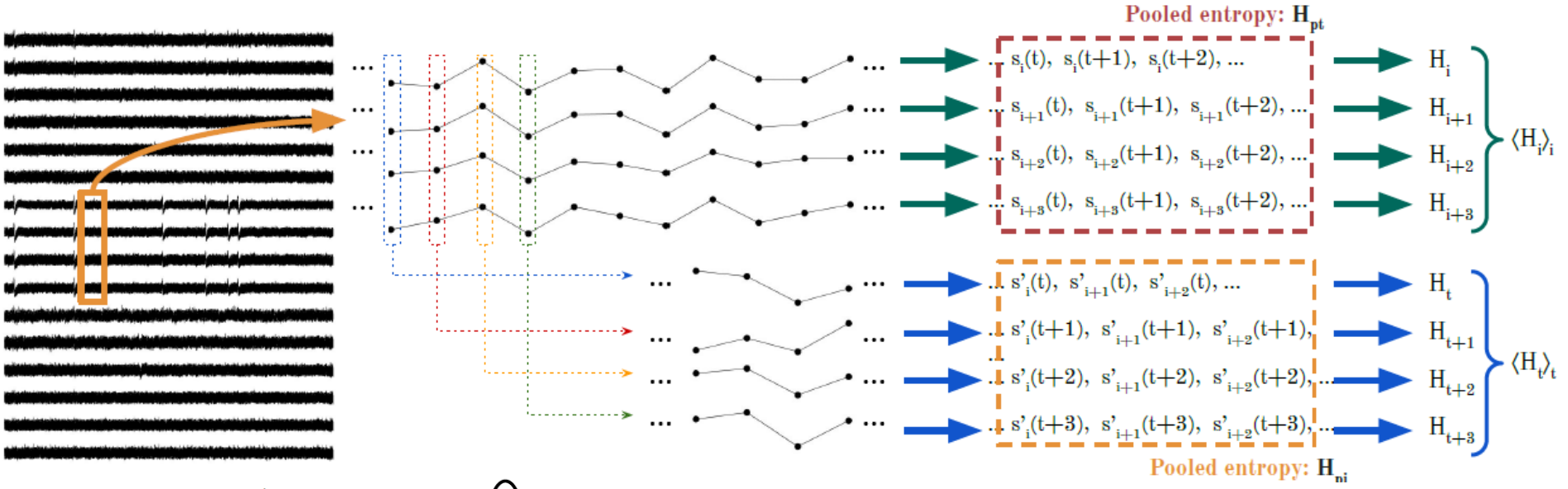
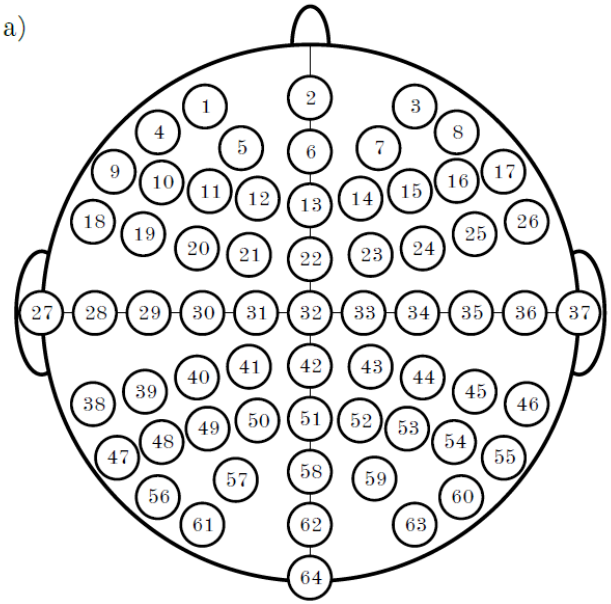
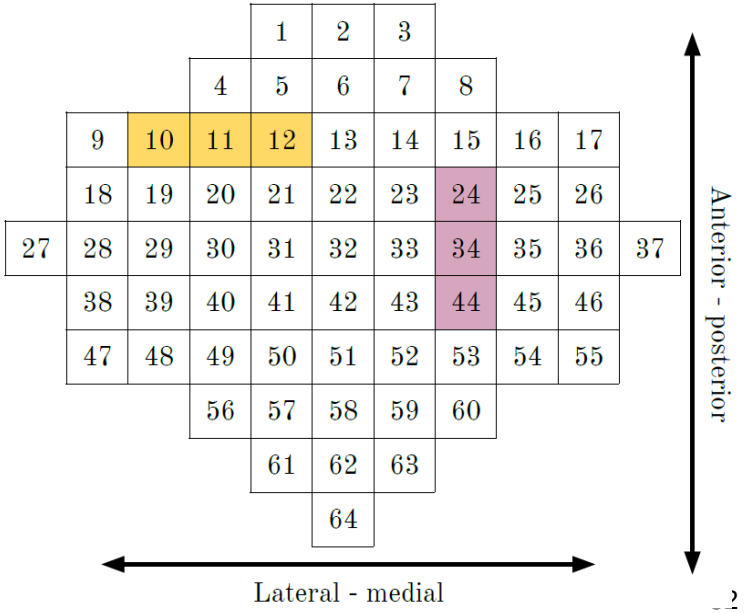
Four approaches to calculate the permutation entropy



a)





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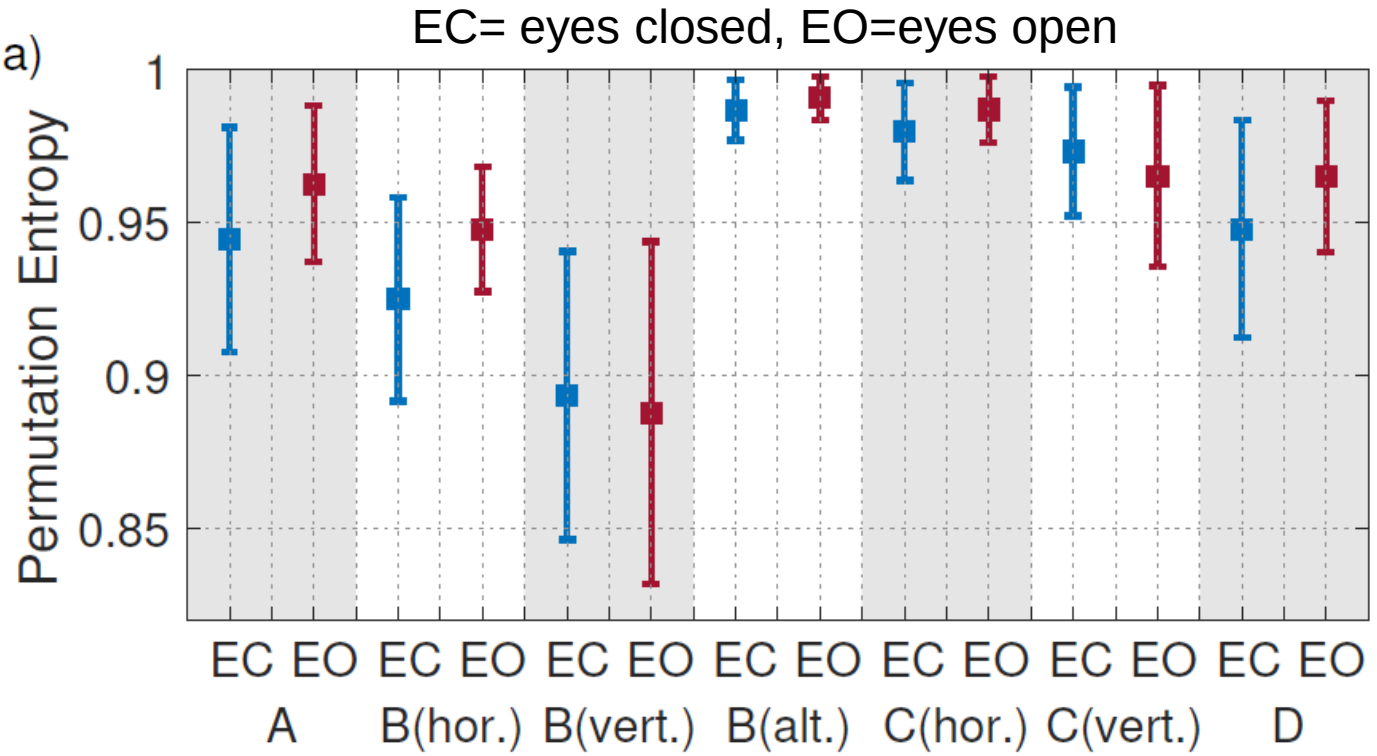
Results



error bars: σ over 107 subjects

A: temporal coding
B: spatial coding (horizontal, vertical, and alt. symbols)
C: spatial pooling (horizontal and vertical symbols)
D: temporal pooling

J. Gancio, C. Masoller, G. Tirabassi, "Permutation entropy analysis of EEG signals for distinguishing eyes-open and eyes-closed brain states: Comparison of different approaches", Chaos 34, 043130 (2024).

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Take home message and references

In simulated, experimental and real-world data, PE can separate different behaviors & can identify regime transitions.

Ongoing work: Performance of the other indicators? Transition to synchronization? Transitions in climate data?

- G. Tirabassi and C. Masoller, "*Entropy-based early detection of critical transitions in spatial vegetation fields*", PNAS 120, e2215667120 (2022).
- G. Tirabassi et. al,*"Permutation entropy-based characterization of speckle patterns generated by semiconductor laser light*", APL Phot. 8, 126112 (2023).
- J. Gancio et. al, "*Permutation entropy analysis of EEG signals for distinguishing eyes-open and eyes-closed brain states: Comparison of different approaches*", Chaos 34, 043130 (2024).

Thank you for your attention!

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