MSc in Photonics & Europhotonics

Laser Systems and Applications 2018/2019

Laser Models and Dynamics

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Outline

- Models
  - Intensity (Photon) rate equation model
  - Optical field rate equation model

- Dynamics
  - Current modulation
  - Optical injection
  - Optical feedback
  - Polarization instability

Semiconductor Lasers : Stability, Instability and Chaos, J. Ohtsubo
(Springer, 3er ed. 2013, **ebook available** in UPC library)

Laser dynamics, T. Erneux & P. Glorieux (Cambridge University Press 2010)

Learning objectives

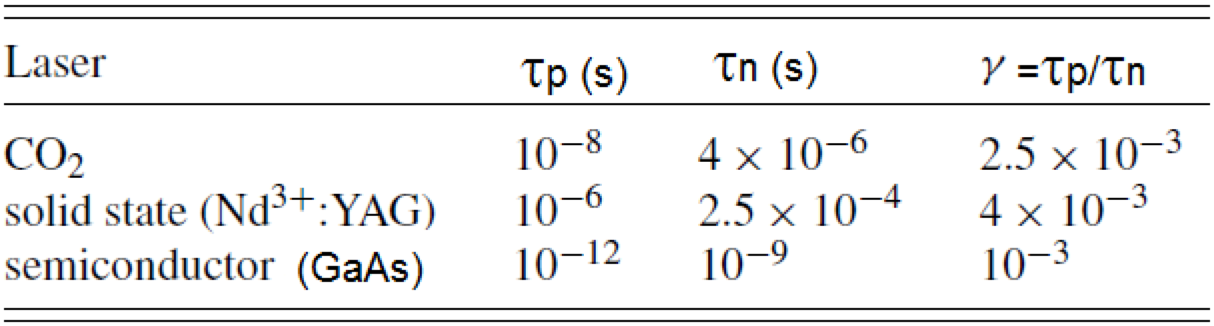
- Acquire a basic knowledge of the simplest laser rate equation model, for the photon and carrier densities.
- Understand the relaxation oscillations and dynamics during the laser turn on.
- Understand the small and large signal modulation response.
- Perform simple numerical simulations.
- Become familiar with the single-mode equation for the complex optical field.
- Understand the effects of optical perturbation.
- Acquire a basic knowledge of multi-mode models.

Semiconductor lasers are **class B** lasers

- Governed by **two rate-equations**: one for the photons (**S**) and one for the carriers (**N**) .
- Display a stable output (with only transient relaxation oscillations).
- Single-mode “conventional” EELs diode lasers are class B lasers.
- Other class B lasers are ruby, Nd:YAG, and CO2 lasers.
- Because of the α-factor (a specific feature of diode lasers, more latter) diode lasers display complex dynamics when they are optically perturbed.

τ_n = Carrier lifetime
τ_p = Photon lifetime

Characteristic times for common class B lasers



Laser	τ_p (s)	τ_n (s)	$\gamma = \tau_p/\tau_n$
CO_2	10^{-8}	4×10^{-6}	2.5×10^{-3}
solid state (Nd^{3+}:YAG)	10^{-6}	2.5×10^{-4}	4×10^{-3}
semiconductor (GaAs)	10^{-12}	10^{-9}	10^{-3}

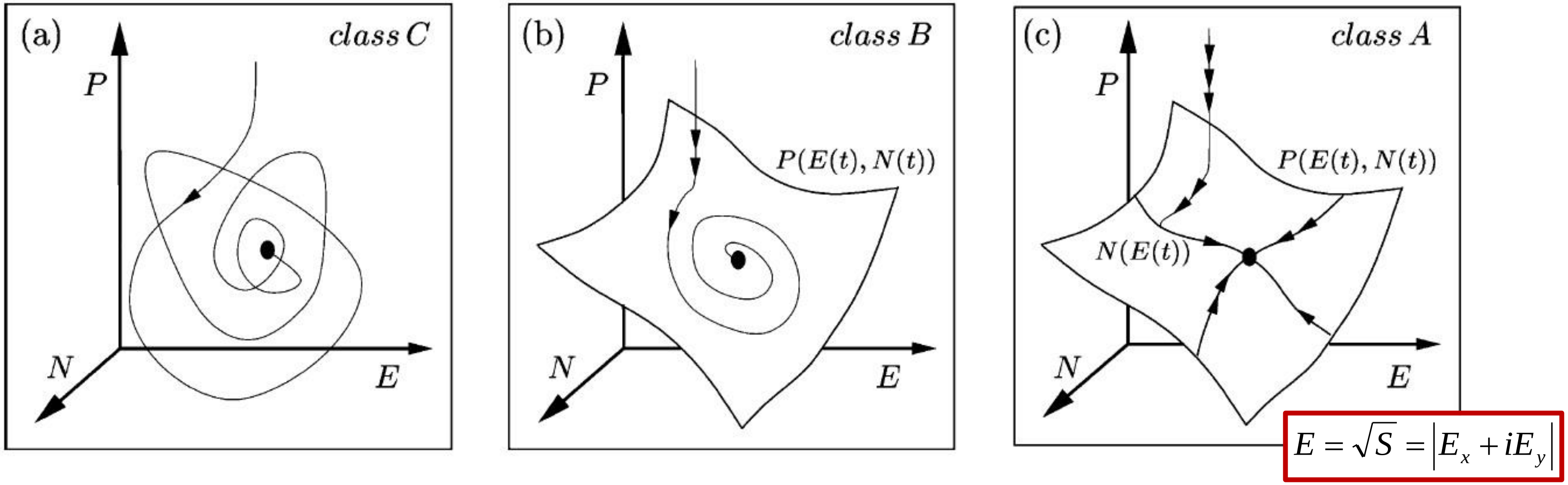
Other types of lasers

- **Class A** (Visible He-Ne lasers, Ar-ion lasers, dye lasers): governed by one rate equation for the optical field (the material variables can be adiabatically eliminated), no oscillations.

- **Class C** (infrared He-Ne lasers): governed by three rate equations (N, S, P=macroscopic atomic polarization), display sustained oscillations and even a chaotic output. No commercial applications.

Dynamics of Class C, B and A lasers

S. Wieczorek et al. / Physics Reports 416 (2005) 1–128



$$E = \sqrt{S} = \left|E_x + iE_y\right|$$

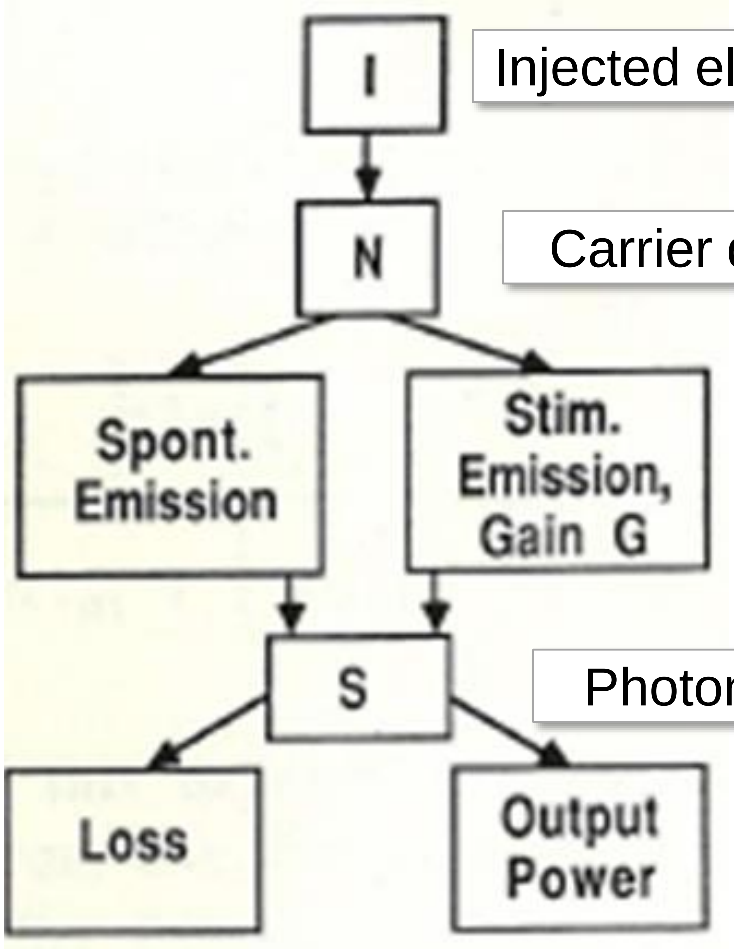
Fig. 1. Sketches of a typical trajectory approaching a stable fixed point in class-C, class-B, and class-A free-running lasers.

infrared He-Ne lasers

Semiconductor, ruby, Nd:YAG, CO2 lasers

Visible He-Ne lasers, Ar-ion lasers, dye lasers

Diode lasers : electrical to power conversion



Injected electrical current

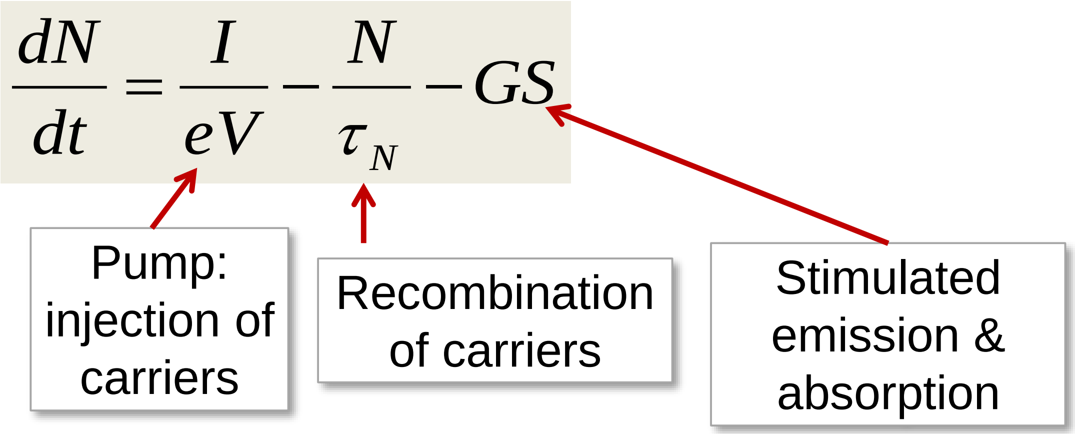
Carrier density

Net gain: emission – absorption

Photon density

Cavity losses

Rate equation for the carrier density N



$$\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau_N} - GS$$

Pump: injection of carriers

Recombination of carriers

Stimulated emission & absorption

I : **Injection current** (I/eV is the number of electrons injected per unit volume and per unit time).

τ_N : **Carrier lifetime**.

G (N,S) : **Net gain**

Rate equation for the photon density S

$$\frac{dS}{dt} = GS - \frac{S}{\tau_p} + \frac{\beta_{sp} N}{\tau_N}$$

Stimulated emission - absorption

Cavity losses

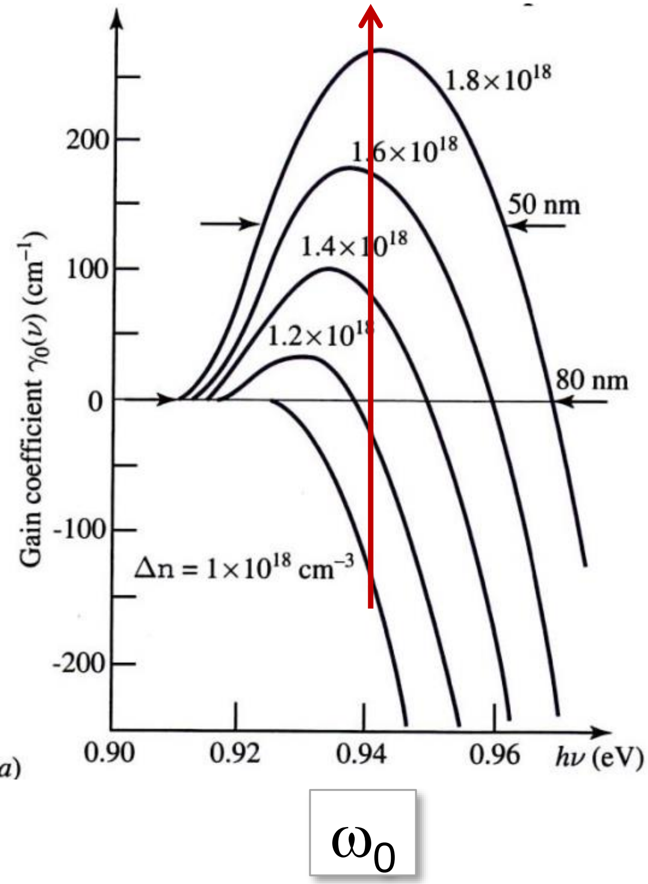
Spontaneous emission

τ_p : **Photon lifetime**.

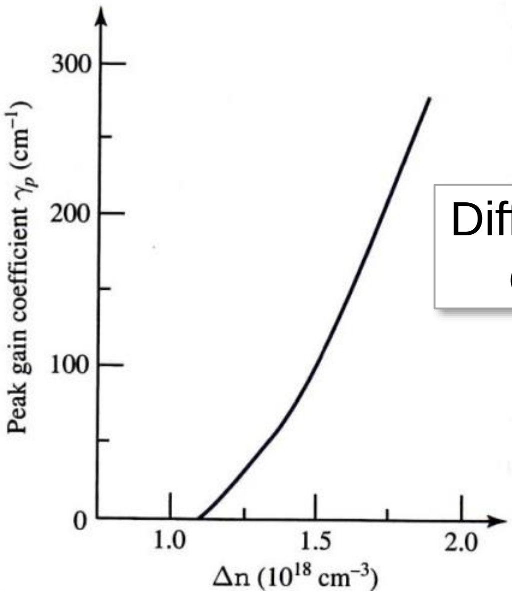
G (N,S) : **Net gain**

β_{sp} : **Spontaneous emission rate**

Simple model for the semiconductor gain



RT InGaASP laser



$$G = a(N - N_0)$$ DH

$$G = aN_0 \ln(N / N_0)$$ QW

Differential gain coefficient

Carrier density at **transparency**

We assume **single-mode emission** at λ_0. The differential gain coefficient **a** depends on λ_0 (multi-mode model latter).

Threshold carrier density

Threshold condition: net gain = cavity loss

$$G(N_{th}) = \frac{1}{\tau_p}$$

$$G = a\left(N - N_0\right) \quad \Rightarrow \quad \frac{1}{\tau_p} = a\left(N_{th} - N_0\right)$$

$$\frac{dS}{dt} = GS - \frac{S}{\tau_p} + \frac{\beta_{sp} N}{\tau_N}$$

$$G - \frac{1}{\tau_p} = a\left(N - N_0\right) - a\left(N_{th} - N_0\right) = a\left(N - N_{th}\right)$$

$$\Rightarrow \frac{dS}{dt} = a\left(N - N_{th}\right)S + \frac{\beta_{sp} N}{\tau_N}$$

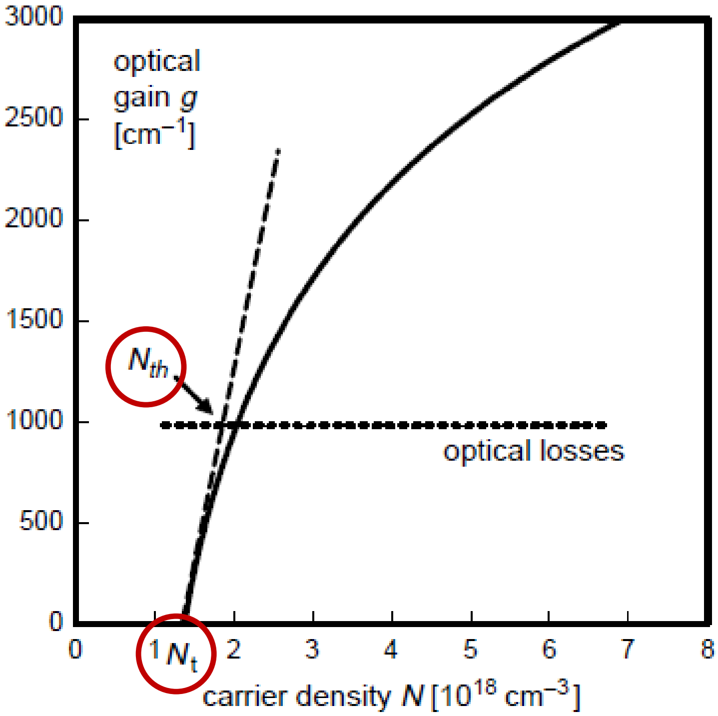


Fig. 1.20 Optical gain $g(N)$ vs. carrier density for an InGaAsP strained quantum well active layer (1.55 μm) at 20°C. The power gain is defined by $\Gamma G(N) = \Gamma v_g g(N)$, where v_g is the photon group velocity ($\sim 10^{10}$ cm s^{-1}) and Γ is the confinement factor (~ 0.1) (redrawn from Figure 3.1 of Piprek and Bowers [45]).

SIMPLEST RATE EQUATION MODEL

-STEADY-STATE SOLUTIONS & LI CURVE

-TIME-DEPENDENT SOLUTIONS WHEN THE INJECTION CURRENT VARIES

Two coupled nonlinear rate-equations

$$\frac{dS}{dt} = GS - \frac{S}{\tau_p} + \frac{\beta_{sp} N}{\tau_N}$$

$$\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau_N} - GS$$

- Ordinary differential equations (spatial effects neglected)
- Additional nonlinearities from carrier re-combination and gain saturation

$$\frac{1}{\tau_N} = A + BN + CN^2$$

$$G = \frac{a(N - N_0)}{1 + \varepsilon S}$$

- These equations allow to understand the LI curve and the modulation response.
- To understand the intensity noise and the line-width (the optical spectrum), we need a stochastic equation for the complex field E ($S=|E|^2$) (more latter).
- Spatial effects (diffraction, carrier diffusion) and thermal effects can be included phenomenologically.

Normalized equations

- Define the dimensionless variable:

$$N' = \frac{N - N_0}{N_{th} - N_0}$$

Threshold carrier density: gain = loss

$$a\left(N_{th} - N_0\right) = \frac{1}{\tau_p}$$

$$\frac{dS}{dt} = \frac{1}{\tau_p}\left(N' - 1\right)S + \frac{\beta_{sp} N'}{\tau_N}$$

$$\frac{dN'}{dt} = \frac{1}{\tau_N}\left(\mu - N' - N'S\right)$$

Pump current parameter: proportional to I/I_{th}

- Normalizing the equations eliminates two parameters (a, N_o)
- In the following we drop the " ' "

Role of spontaneous emission

$$\frac{dS}{dt} = \frac{1}{\tau_p}(N-1)S + \frac{\beta_{sp} N}{\tau_N}$$

- If at t=0 there are no photons in the cavity: S(0) = 0

- Then, without noise (β_{sp}=0): if S=0 at t=0 $\Rightarrow$ dS/dt=0

  $\Rightarrow$ S remains 0 (regardless the value of μ and N).

- Without spontaneous emission noise the laser does not turn.

Steady state solutions with β_{sp}=0

(Simple expressions if β_{sp} is neglected)

$$\frac{dS}{dt} = \frac{1}{\tau_p}(N-1)S$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}(\mu - N - NS)$$

$$\frac{dS}{dt} = 0 \Rightarrow \begin{cases} S = 0 \\ N = 1 \end{cases}$$

$$\frac{dN}{dt} = 0 \Rightarrow \begin{cases} S = 0 \rightarrow N = \mu \\ N = 1 \rightarrow S = \mu - 1 \end{cases}$$

Laser off

Stable if $\mu<1$

S=0
N=μ

Laser on

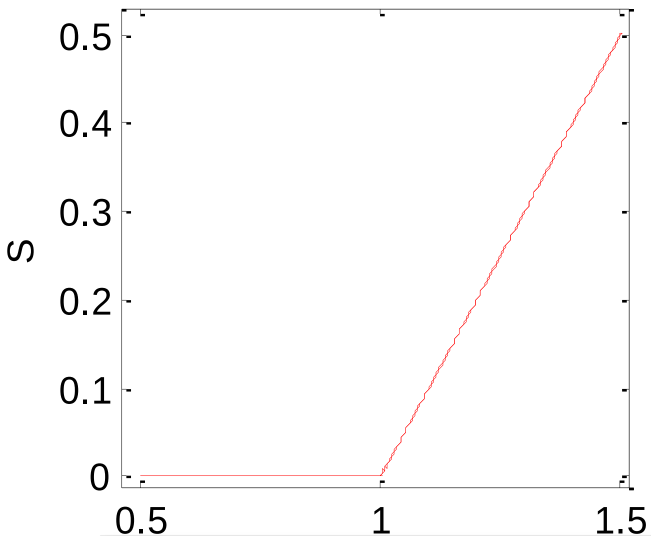
Stable if $\mu>1$

S = μ-1
N =1

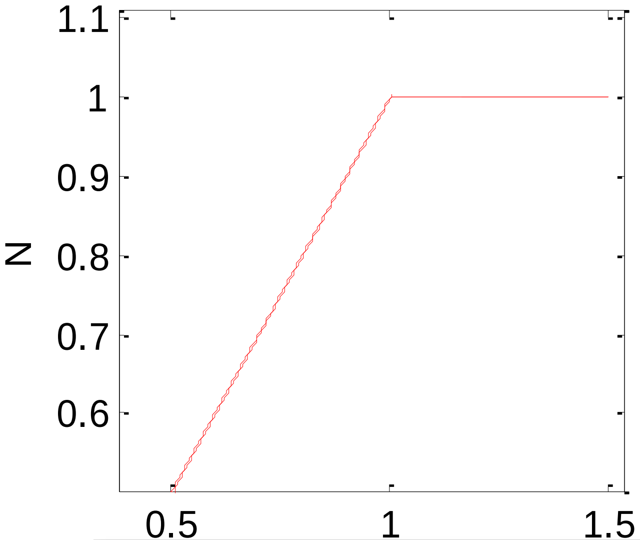
μ_{th} = 1

Above threshold the carriers are "**clamped**".

Graphical representation



Pump current, μ



Pump current, μ

$\mu_{th} = 1$

off

$S=0$

on

$S=\mu-1$

off

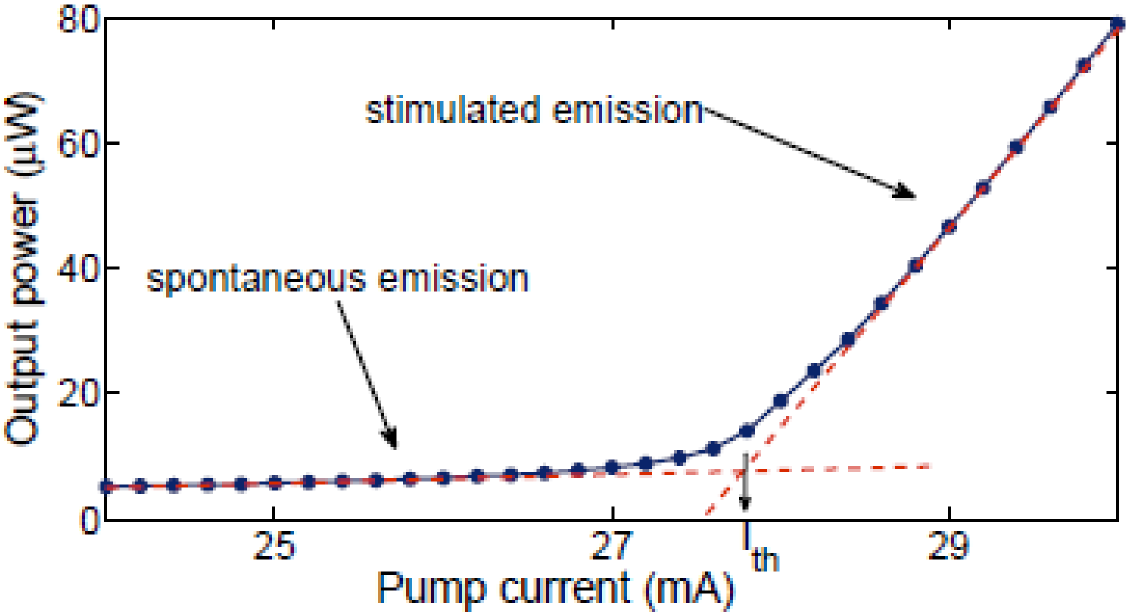
$N=\mu$

on

$N= 1$

Above threshold the carriers are "**clamped**".

Experimental LI curve



Hitachi Laser Diode HL6724MG (A. Aragoneses PhD thesis 2014)

$$\frac{dS}{dt} = a(N - N_{th})S + \frac{\beta_{sp} N}{\tau_N}$$

This LI curve is obtained from model simulations when β_{sp} is not neglected.

LI curve with $\beta_{sp} \neq 0$

$$\frac{dS}{dt} = \frac{1}{\tau_p}(N-1)S + \frac{\beta_{sp} N}{\tau_N}$$

$$\gamma = \frac{\tau_p}{\tau_N}$$

$$t' = \frac{t}{\tau_p}$$

$$\frac{dS}{dt'} = (N-1)S + bN$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}(\mu - N - NS)$$

$$b = \frac{\beta_{sp}\tau_p}{\tau_N}$$

$$\frac{dN}{dt'} = \gamma(\mu - N - NS)$$

Steady-state solution:

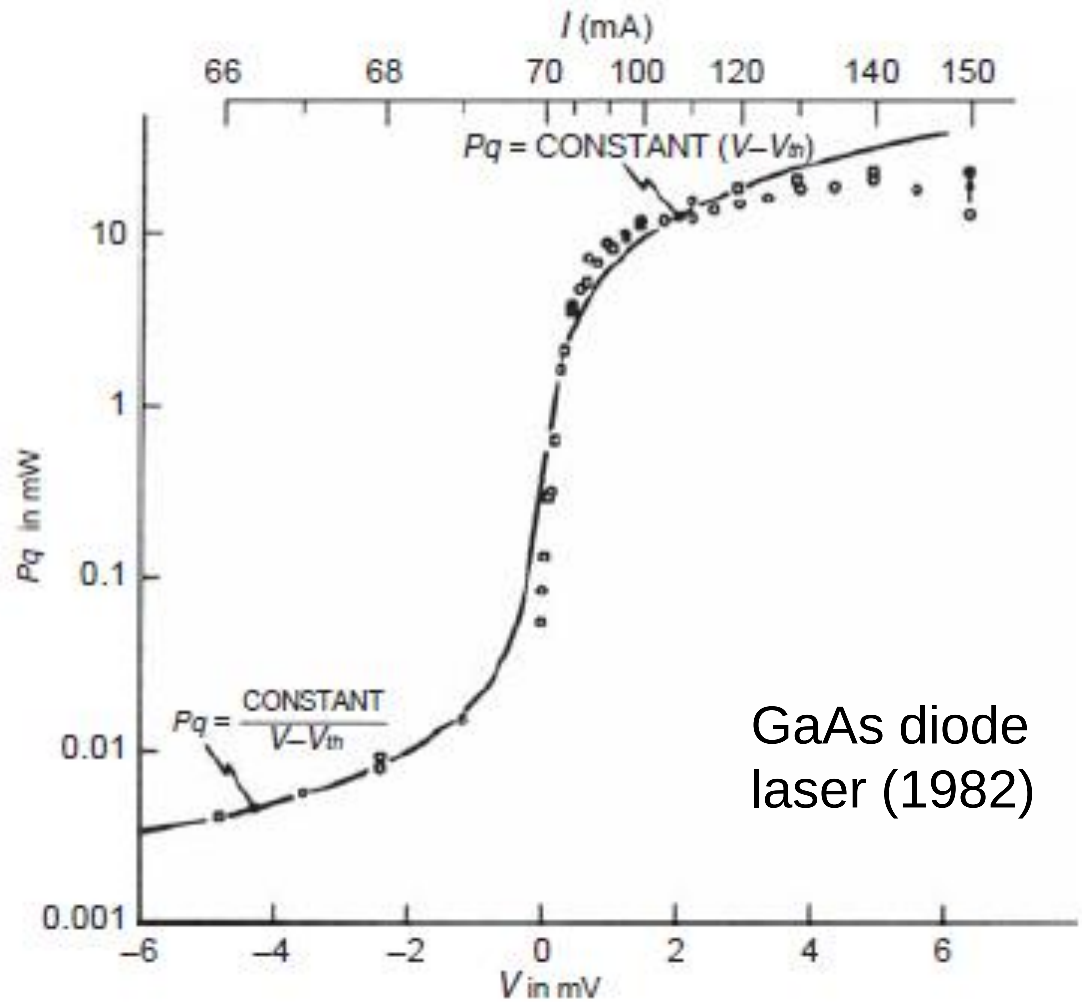
$$S = \frac{1}{2}\left[(\mu - 1) + \sqrt{(\mu-1)^2 + 4b\mu}\right]$$

- If $\mu > 1$ $S \approx \mu - 1$
- If $\mu < 1$ $S \approx 1/|\mu - 1|$

“kink” of the LI curve

- If $\mu > 1$ $S \approx \mu - 1$
- If $\mu < 1$ $S \approx 1/|\mu - 1|$

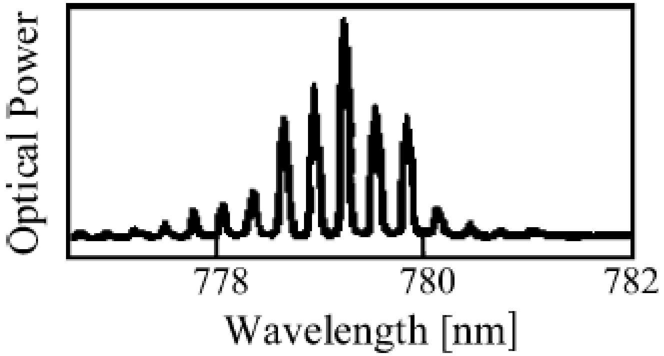
$\boxed{\mu_{th} = 1}$

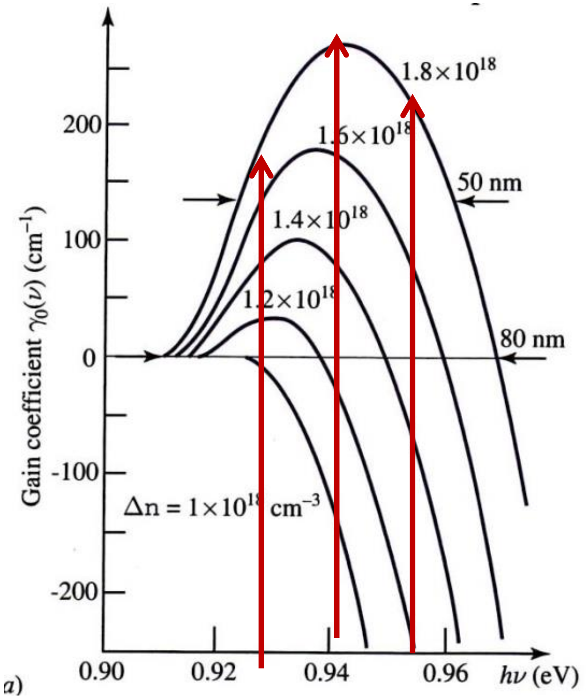


GaAs diode laser (1982)

Source: T. Erneux and P. Glorieux, *Laser Dynamics* (2010)

Model for a multi-mode laser





Gain coefficient for mode *j*:

$$G_{\mathrm{n},j} = G_{\mathrm{n}}\left\{1 - \left(\frac{j}{M}\right)^2\right\}$$

$$\frac{\mathrm{d}S_j(t)}{\mathrm{d}t} = [G_{\mathrm{n},j}\{n(t) - n_{\mathrm{th},j}\}]S_j(t) + R_{\mathrm{sp}}(\omega_j)$$

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = \frac{J(t)}{ed} - \frac{n(t)}{\tau_{\mathrm{s}}} - \sum_{j=-M}^{M} G_{\mathrm{n},j}\{n(t) - n_0\}S_j(t)$$

Carrier density (n) + several photon densities (for each longitudinal mode)

SIMPLEST RATE EQUATION MODEL

-STEADY-STATE SOLUTIONS & LI CURVE

-TIME-DEPENDENT SOLUTIONS WHEN THE INJECTION CURRENT VARIES

Time variation of the injection current

$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - NS\right)$$

$$\frac{dS}{dt} = \frac{1}{\tau_p}\left(N - 1\right)S + \frac{\beta_{sp} N}{\tau_N}$$

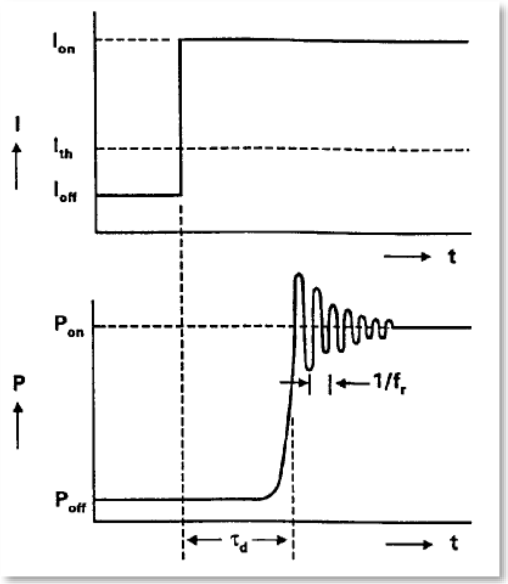
μ (t)

Parameter values

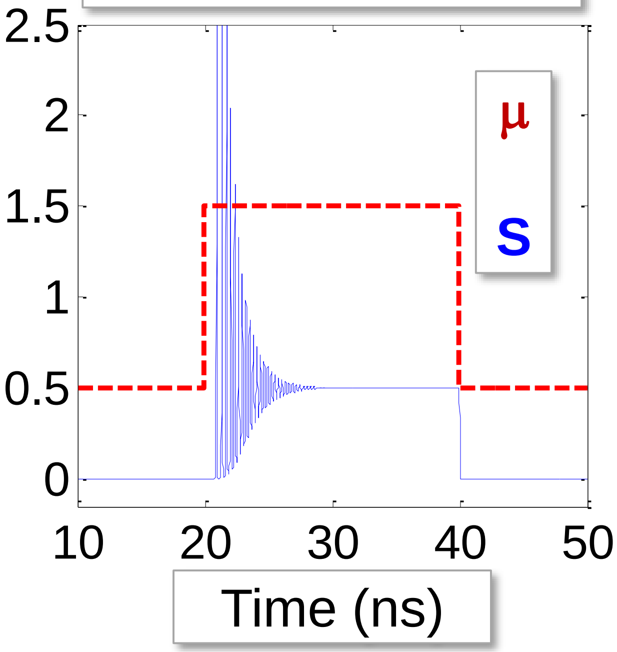
τ_p	**1 ps**
τ_N	1 ns
β_{sp}	10^{-4}

- **Step** (laser turn on): μ_{off}, μ_{on}
- **Triangular** (dynamic LI curve): μ_{min}, μ_{max}, T_{ramp}
- **Sinusoidal** (modulation response): μ_{dc}, A, T_{mod}

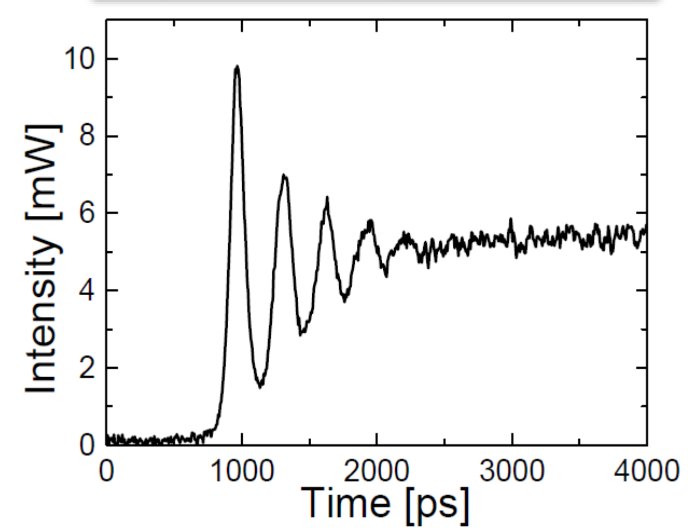
Current step: turn-on delay & relaxation oscillations



Simulations



Experiments (diode laser)



A linear stability analysis of the rate equations allows to calculate the RO frequency

$$\omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}}$$

Source: J. Ohtsubo, *Semiconductor lasers*

Variation of the relaxation oscillation frequency with the output power

$$\omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}}$$

Laser on: $S \approx \mu - 1$

$$\omega_{RO} \approx \sqrt{\frac{S}{\tau_p \tau_N}}$$

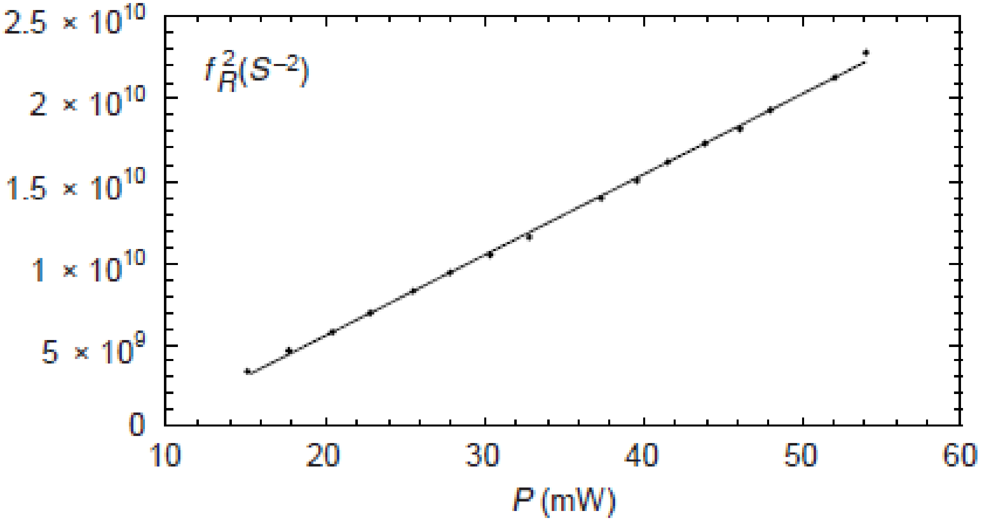
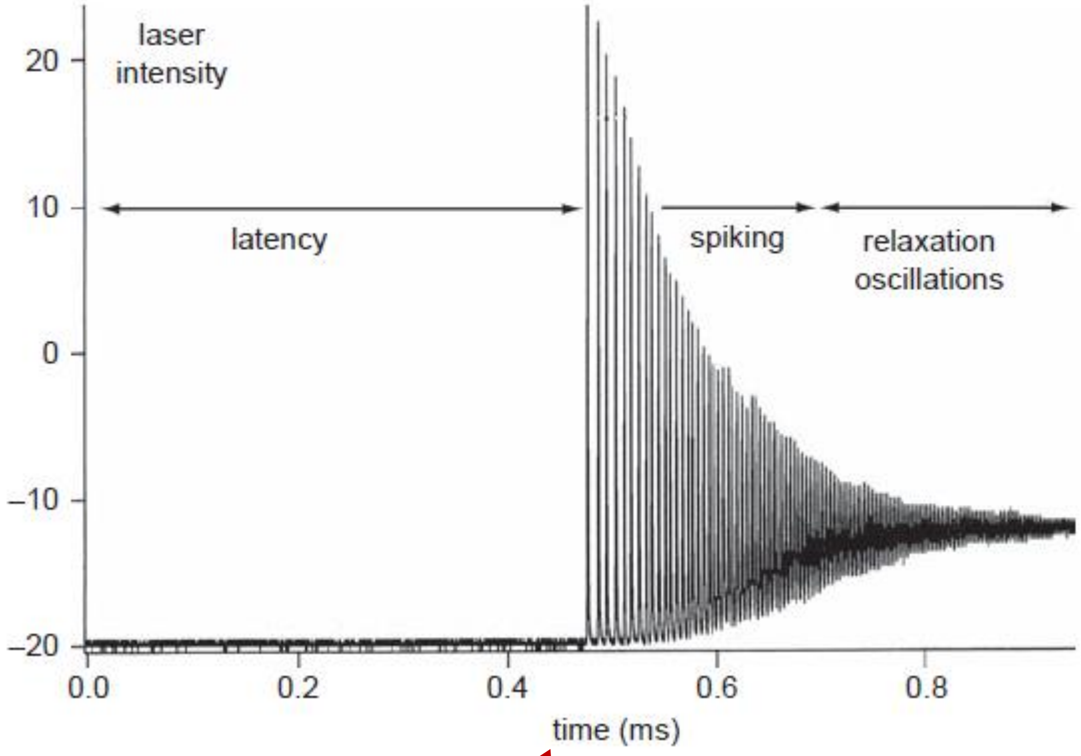


Fig. 1.5 Square of the relaxation oscillation frequency f_R vs. pump power P for an erbium doped fiber laser. Adapted Figure 4 from Sola *et al.* [35] with

Source: T. Erneux and P. Glorieux, *Laser Dynamics* (2010)

Turn-on transient of a Nd^{3+}:YAG laser

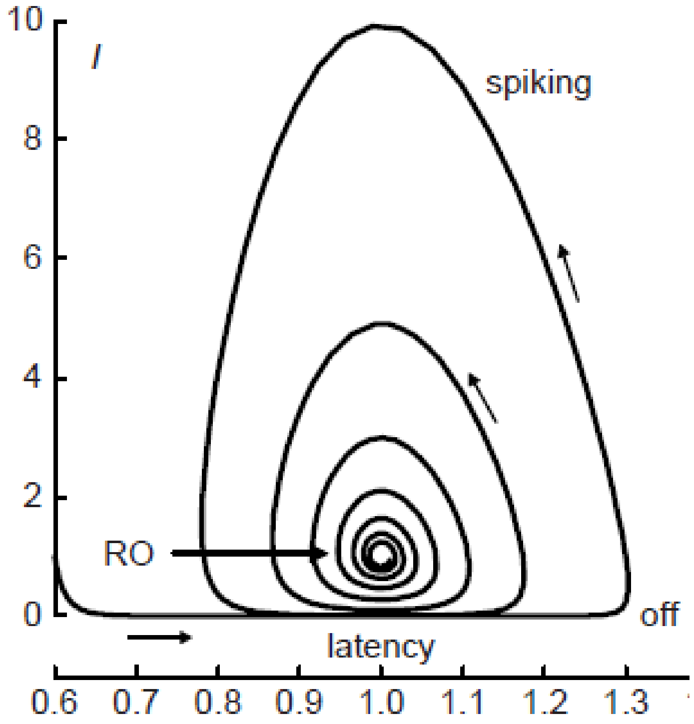


Note the time-scale: for diode lasers is a few ns

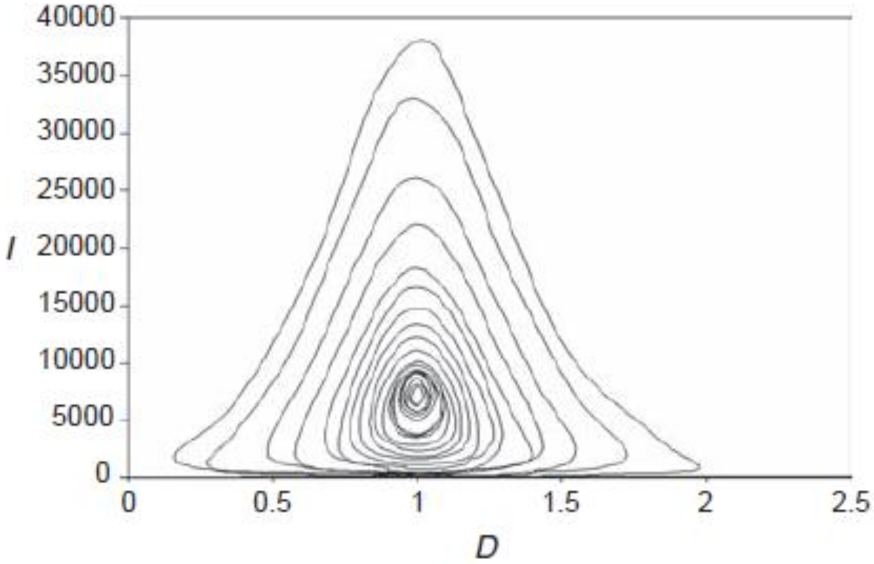
Source: T. Erneux and P. Glorieux, *Laser Dynamics* (2010)

Phase-space representation

S, N plane



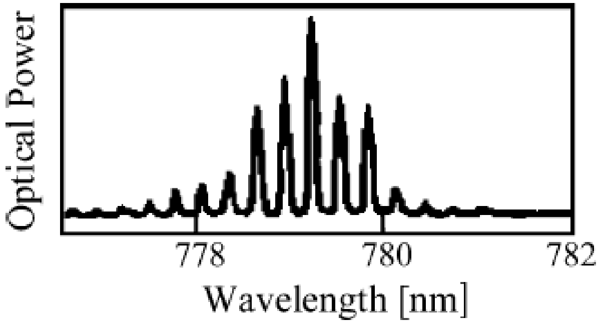
For the Nd^{3+}:YAG laser

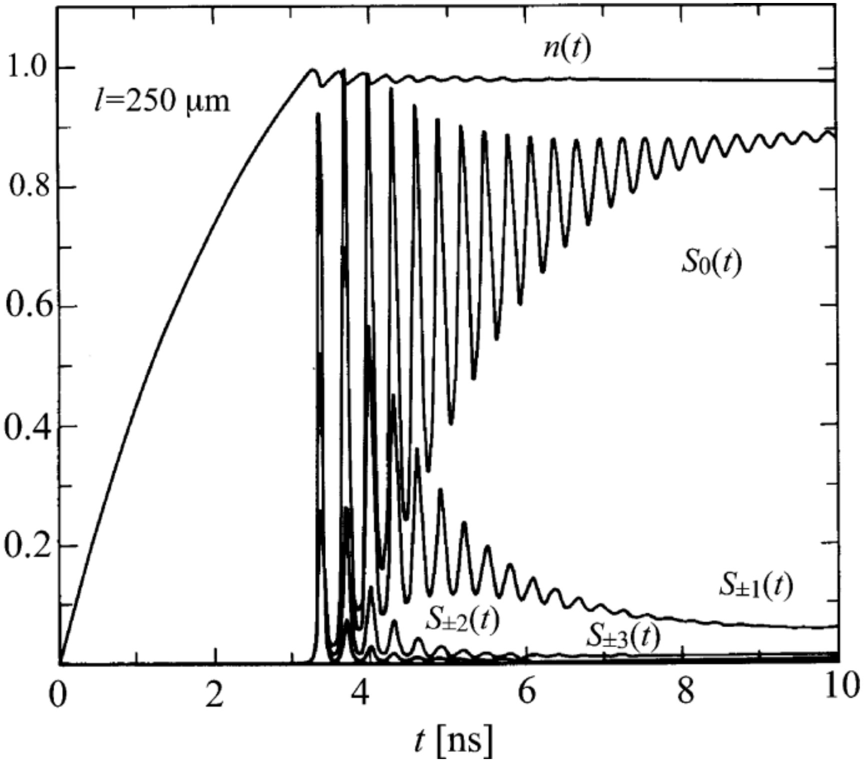


What is D? D=(dI/dt)/I+1

Turn-on of a multi-mode laser

Parabolic gain profile: $G_{n,j} = G_n \left\{ 1 - \left(\frac{j}{M} \right)^2 \right\}$

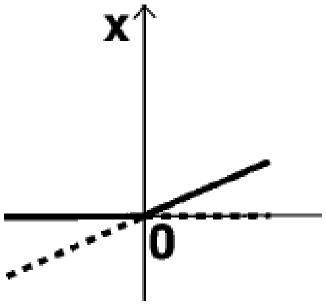


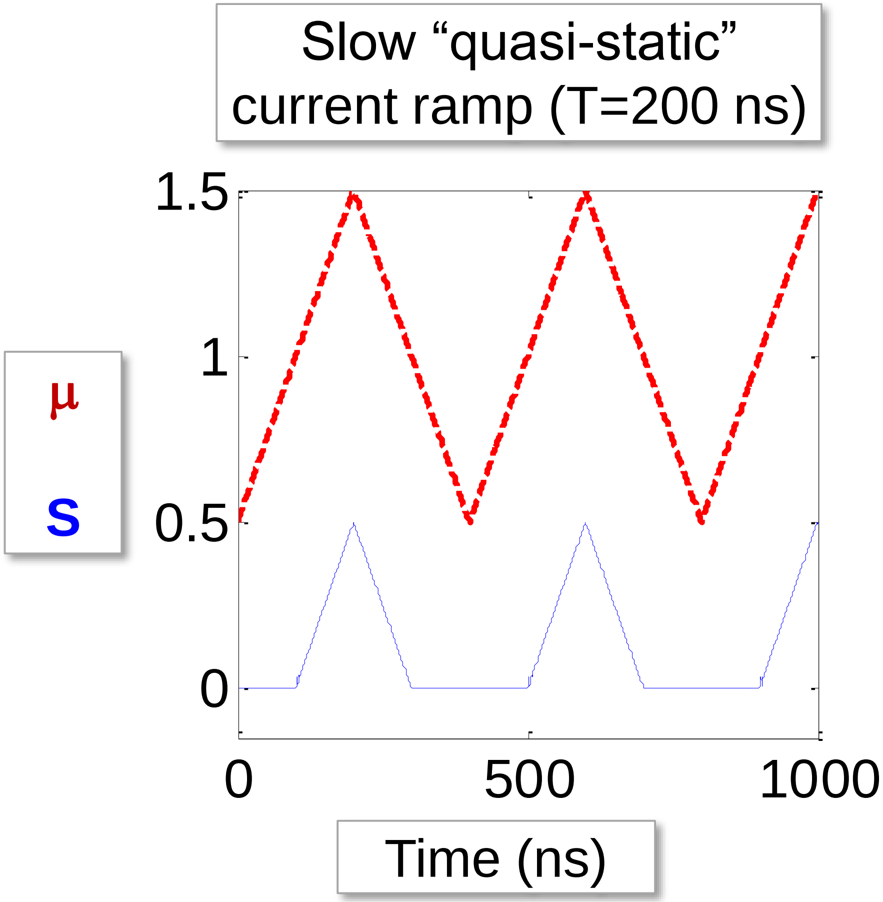


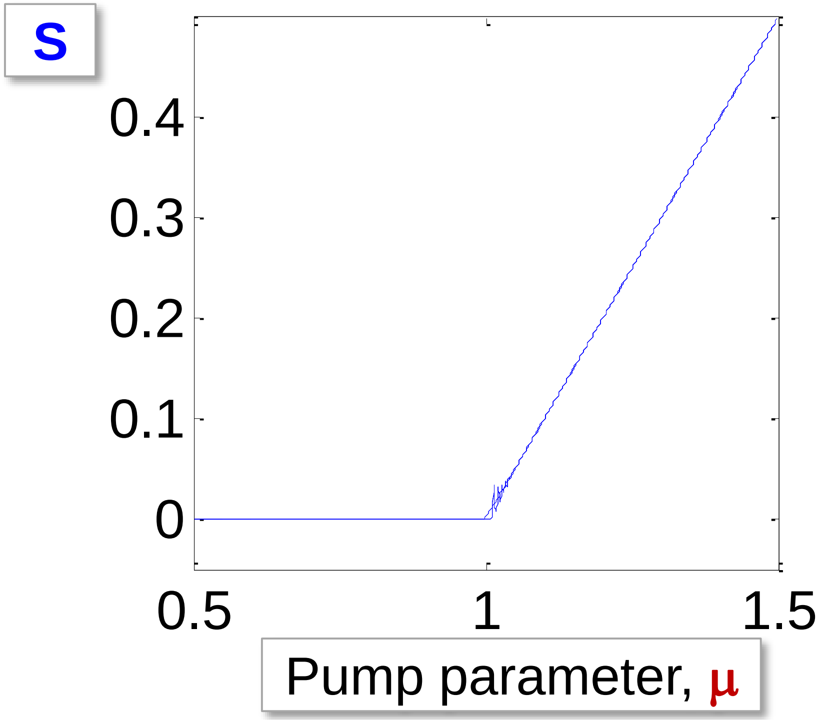
- <u>Winner takes all</u>: after transient “mode-competition”, the mode with maximum gain coefficient wins.
- But non-transient competition has been observed.
- More advanced gain models allow explaining non-transient competition.

Source: J. Ohtsubo, *Semiconductor lasers*

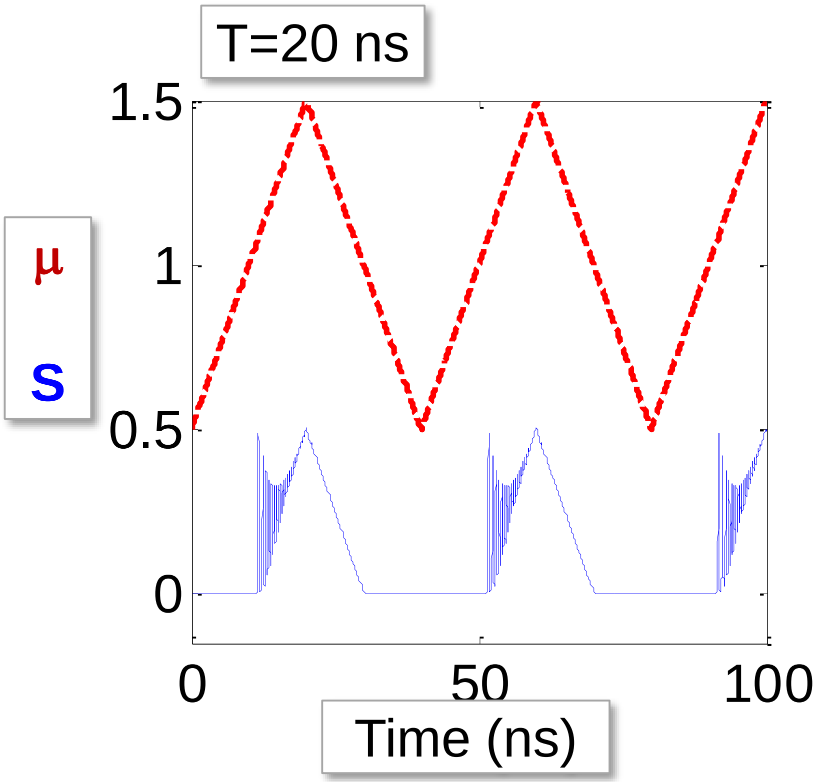
Triangular signal: LI curve



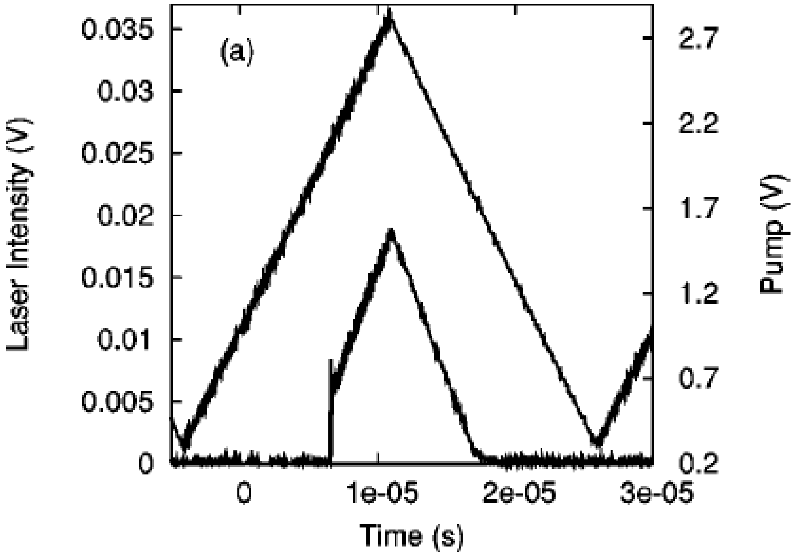

Slow “quasi-static” current ramp (T=200 ns)

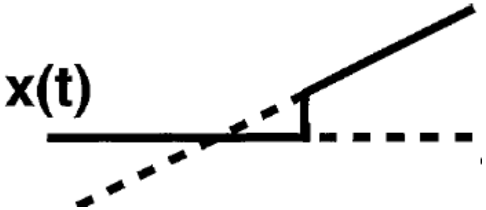


With a fast ramp: turn-on delay



Experiments

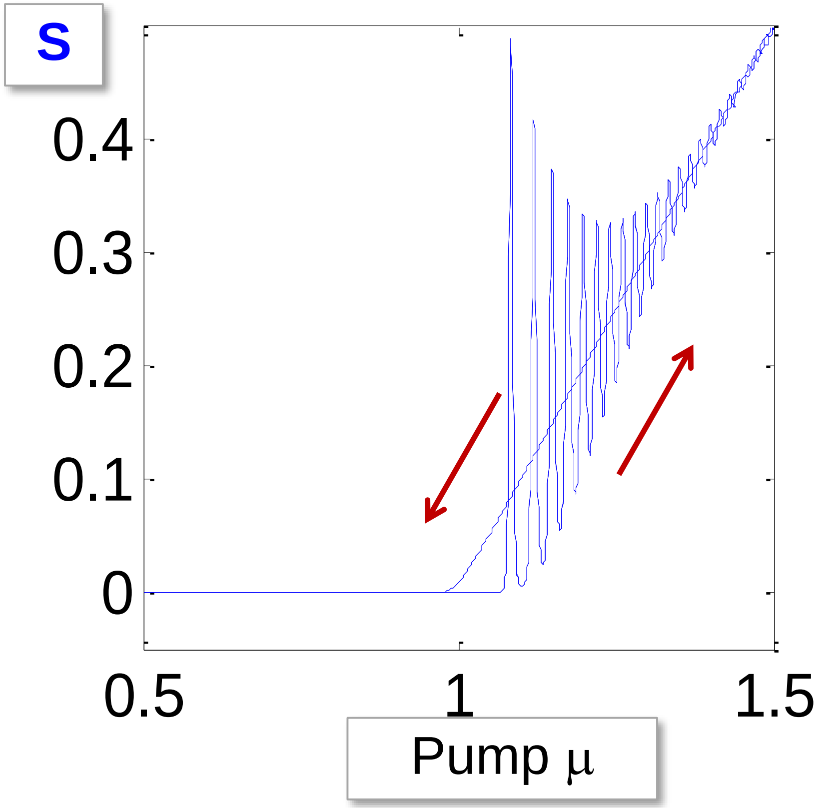




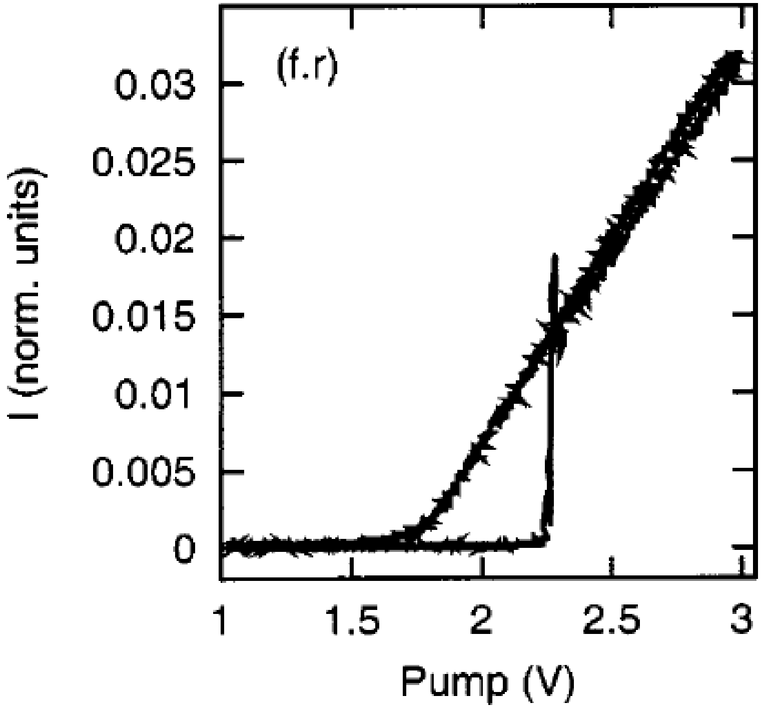
Source: Tredicce et al, Am. J. Phys., Vol. 72, No. 6, 2004

Dynamical hysteresis

Simulations



Experiments



Influence of gain saturation

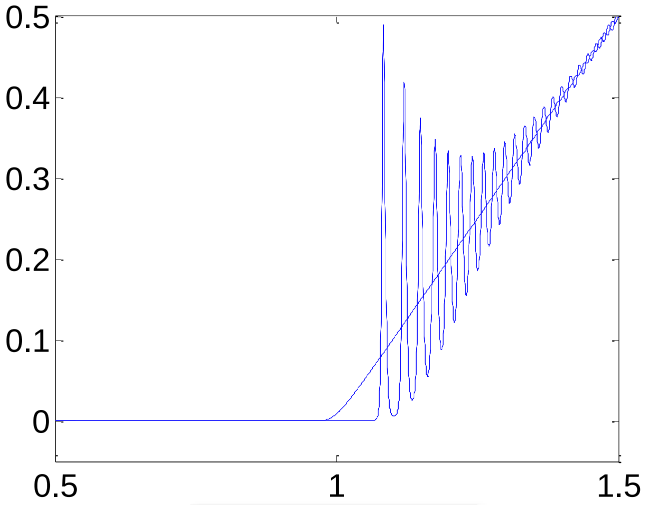
$$\frac{dS}{dt} = \frac{1}{\tau_p}(G-1)S + \frac{\beta_{sp}N}{\tau_N}$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - GS\right)$$

$$G(N,S) = \frac{N}{1+\varepsilon\, S}$$

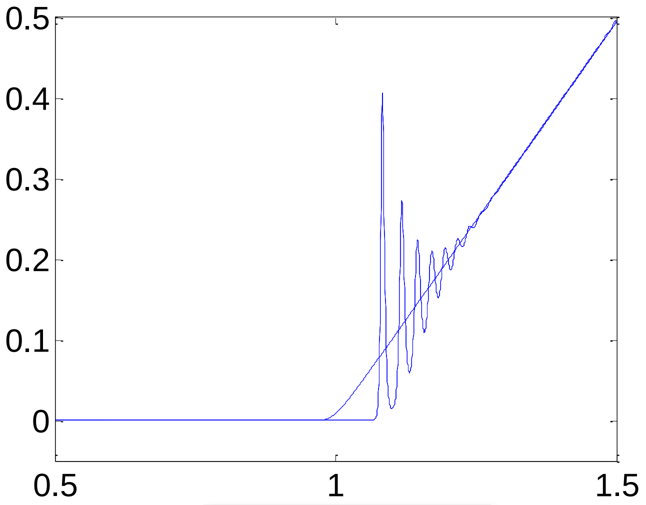
ε=0

S



Pump μ

ε=0.01



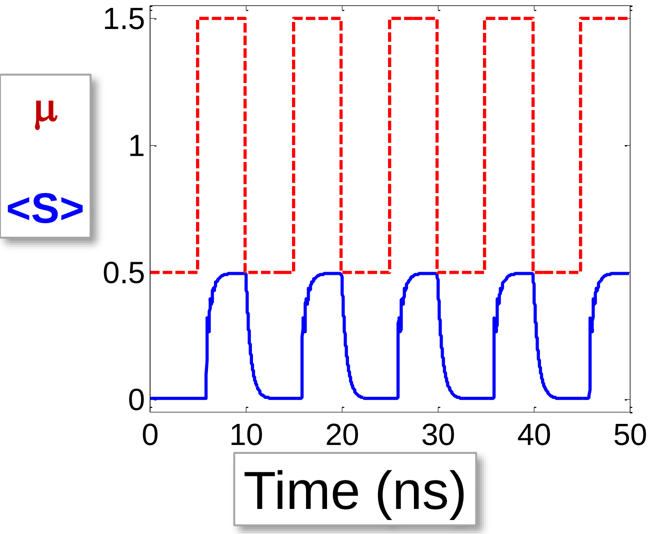
Pump μ

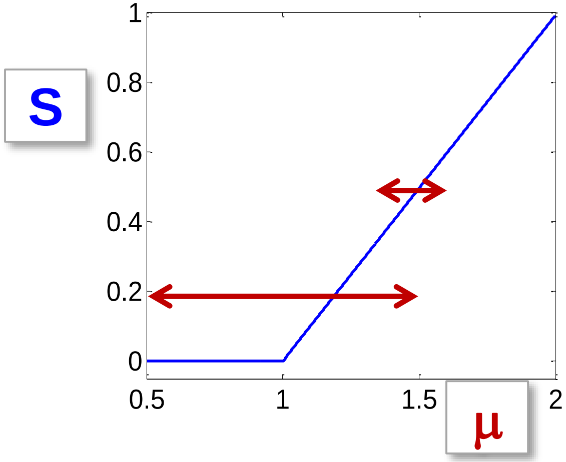
Damped relaxation oscillations

Gain saturation takes into account phenomenologically several effects (spatial and spectral hole burning, thermal effects)

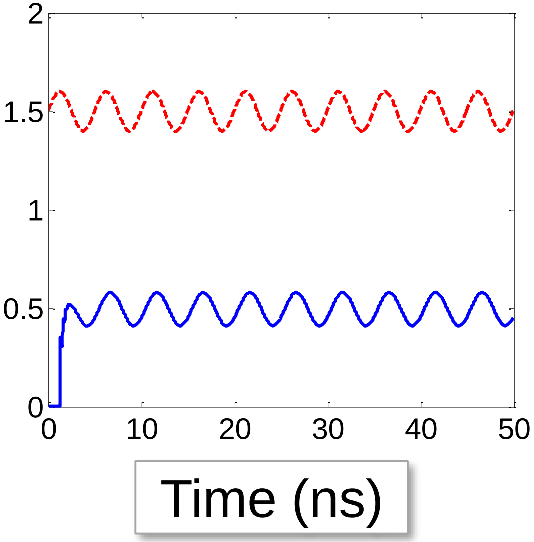
Injection current modulation

Digital





Analog



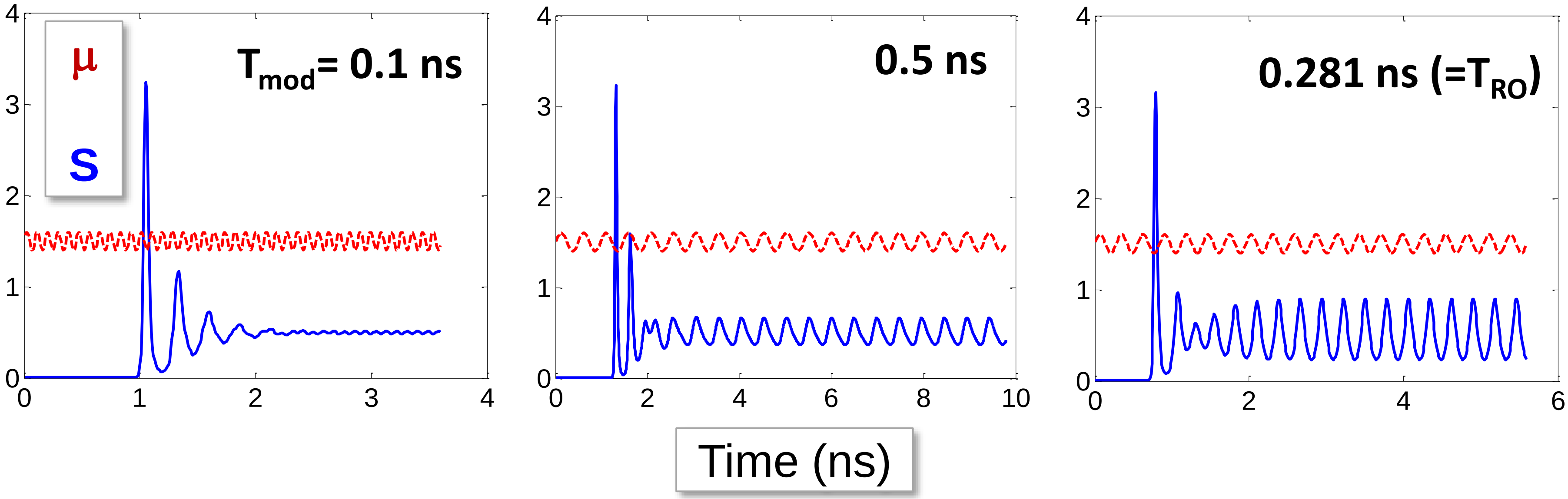
Direct modulation allows to encode information in the laser output power (“amplitude modulation”)

Weak sinusoidal modulation: influence of the modulation frequency

$\mu = \mu_{dc} + A \sin \omega_{mod} t$

$\mu_{dc} = 1.5, A=0.1$

For $\mu=1.5$: $\nu_{RO} = 3.56$ GHz

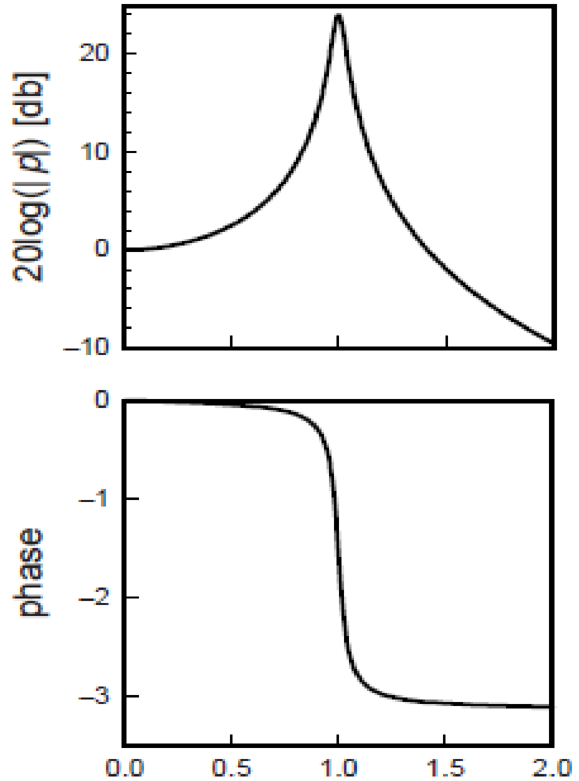


The laser intensity (**S** = photon density) is modulated at the same frequency of the pump current (**μ**), but the phase of the intensity and the current are not necessarily the same.

Small-signal modulation response

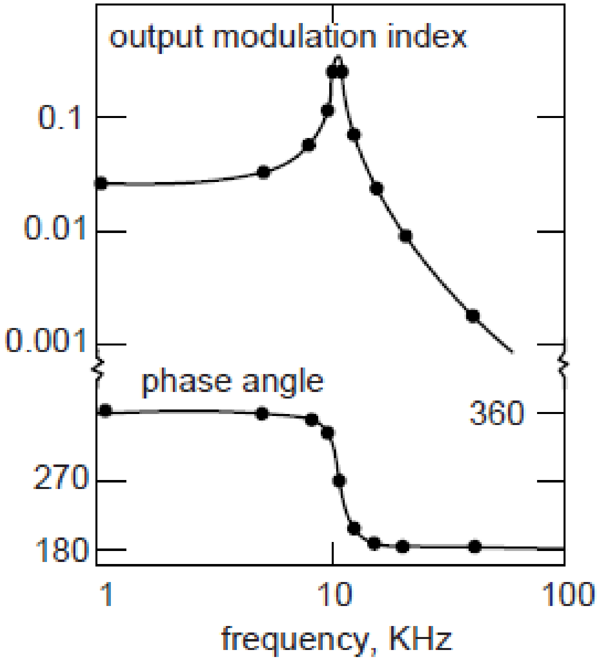
Oscillation amplitude

Analytical expressions can be calculated from the linearization of the rate equations.



$\omega_{mod}/\ \omega_{RO}$

diode-pumped Nd^{3+}:YAG laser

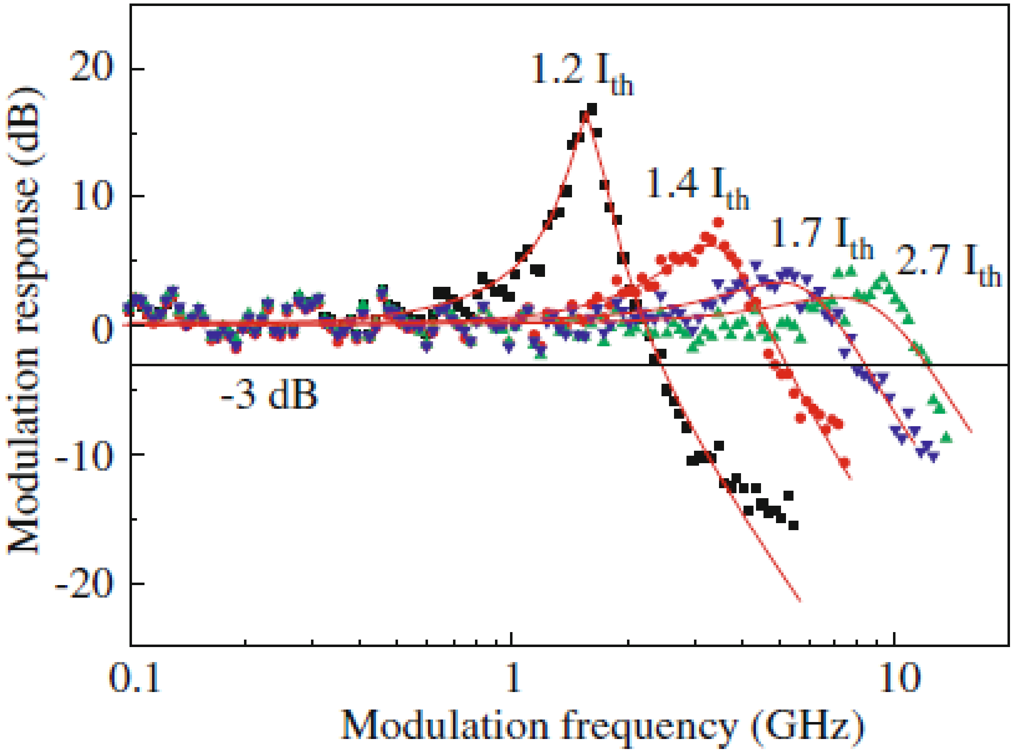


Modulation frequency (KHz)

Resonance at $\omega_{mod} = \omega_{RO}$

$$\omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}}$$

For a semiconductor laser (VCSEL)

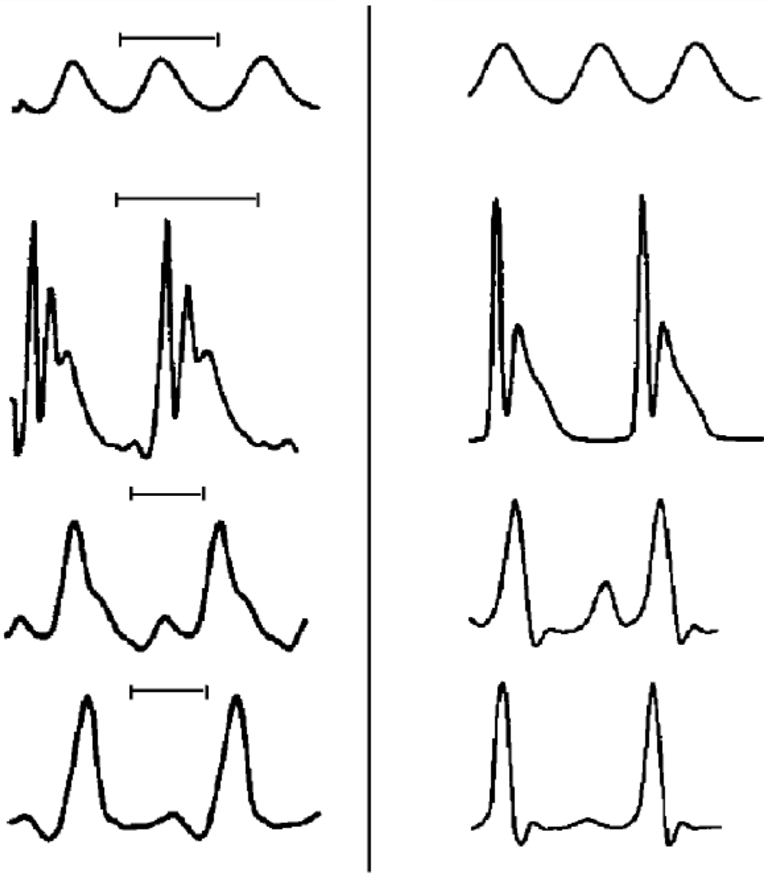


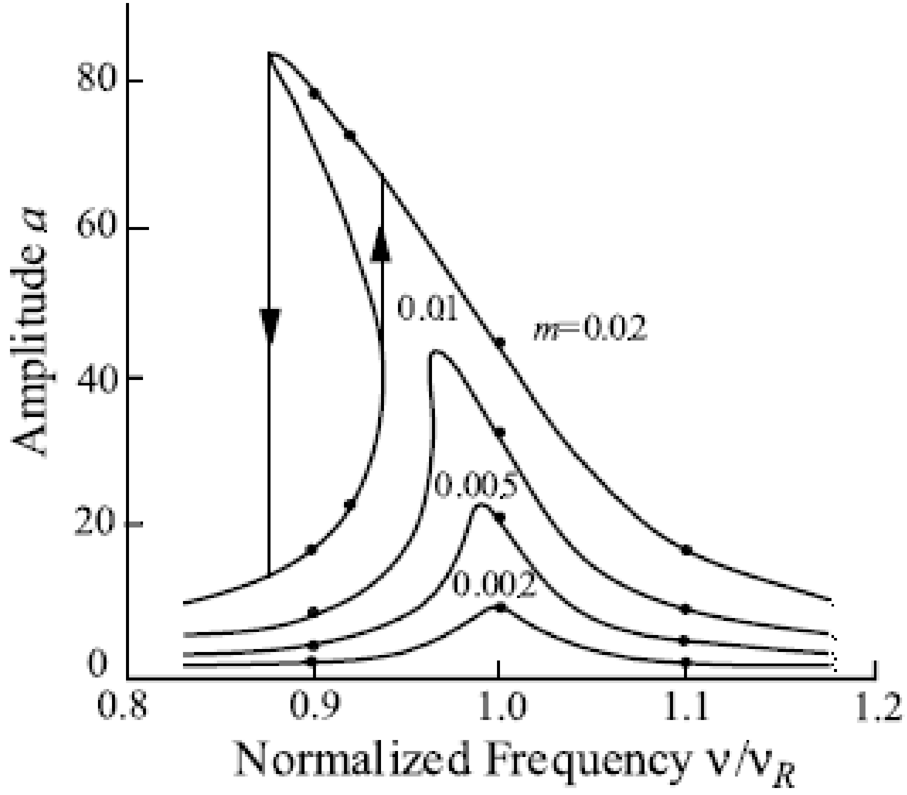
Source: R. Michalzik, VCSELs (2013)

Large-signal modulation response

Experiments

Simulations

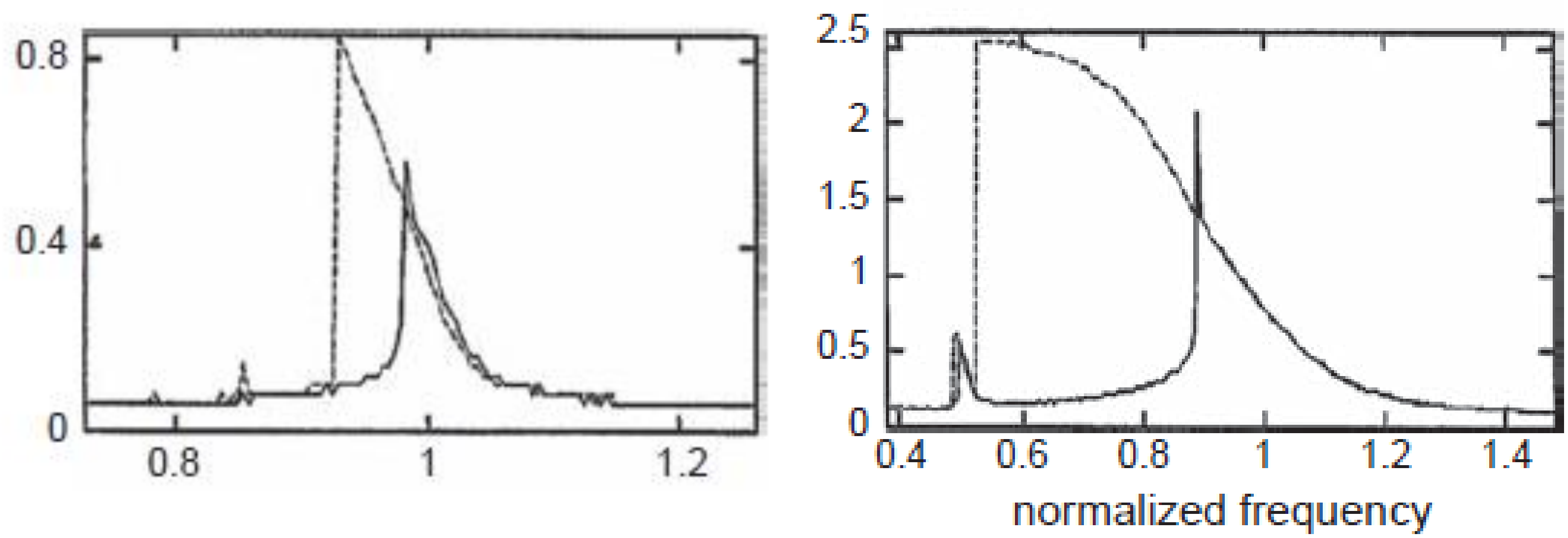




$$\mu(t) = \mu_0\left[1 + m\sin\left(\omega_{\mathrm{mod}} t\right)\right]$$

Source: J. Ohtsubo, *Semiconductor lasers*

Hysteresis induced by strong modulation



- The figures represents the maxima of the oscillations as a function of the normalized frequency near the onset of hysteresis (left), and far away from the onset of hysteresis (right).
- Hysteresis cycle obtained by slowly changing the modulation frequency forward (full line) and then backward (broken line).
- The additional smaller jump near ω_{mod}/ω_{RO} =1/2 is a signature of another resonance.
- The laser is a Nd $^{3+}$:YAG laser subject to a periodically modulated pump.

Summary

The simple rate equation model for the photon and carrier densities allows to understand the main features of the laser dynamics when the injection current varies:

- The turn on delay & relaxation oscillations
- The LI curve (static & dynamic)
- The modulation response (small and large signal response)

TF test

- Semiconductor lasers are described by two rate equations, one for the photon density and another for the carrier density.
- The laser relaxation oscillation (RO) frequency is proportional to the injected current.
- The delay in the laser turn-on is independent of the current ramp.
- The modulation response has a resonance at the RO frequency.
- Small and large amplitude modulation result in sinusoidal oscillation of the output power.
- The relative phase of the output power and input signal depends on the modulation frequency.
- In a multimode model with a parabolic gain profile, mode competition is a transient dynamics.

RATE EQUATION MODEL FOR A SINGLE-MODE COMPLEX OPTICAL FIELD

-ALPHA FACTOR, LINEWIDTH & INTENSITY NOISE

-OPTICAL PERTURBATIONS (INJECTION, FEEDBACK)

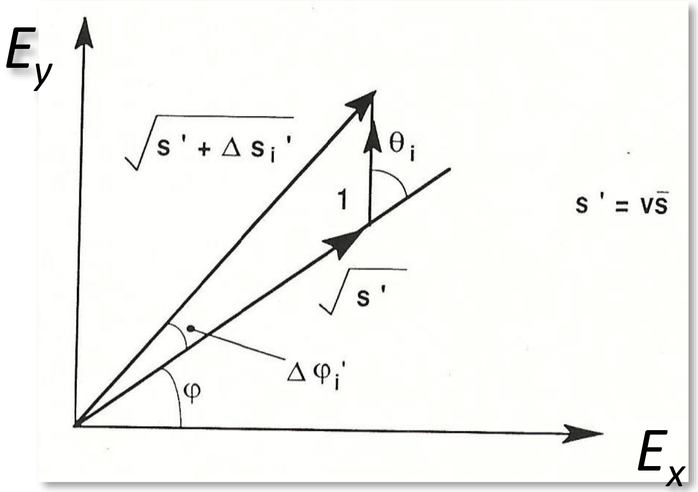
-POLARIZATION INSTABILITIES

Laser linewidth

Schematic representation of the change of **magnitude and phase** of the lasing field E due to the spontaneous emission of one photon.

$$S = |E|^2$$

$$E = E_x + iE_y$$



- The line-width of **gas and solid-state lasers** is well described by the classic Schawlow-Townes formula (Δf~1/P)

Schawlow-Townes formula

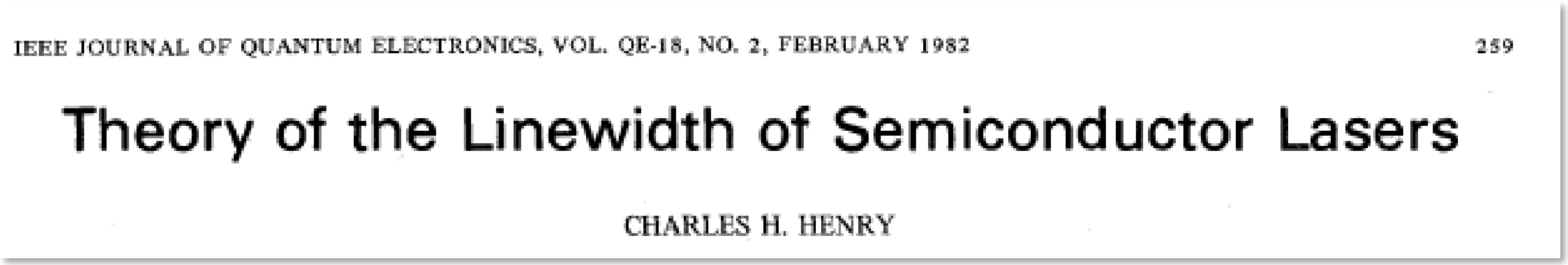
$$\Delta\nu_{ST} = \frac{\pi h\nu(\Delta\nu_c)^2}{P_{out}}$$

$\Delta\nu_c$=half-width of the cavity resonance

Physical interpretation:

- In each round-trip, some noise (spontaneous emission) is added to the circulating field
- It changes the amplitude and the phase of the field.
- Amplitude fluctuations are damped: the power returns to values close to the steady state.
- For phase fluctuations, there is no restoring force.
- Therefore, the phase undergoes a random walk, which leads to phase noise, which causes a finite line-width.
- **But the line-width of semiconductor lasers is significantly higher**.

https://www.rp-photonics.com/schawlow_townes_linewidth.html



IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. QE-18, NO. 2, FEBRUARY 1982 259

Theory of the Linewidth of Semiconductor Lasers

CHARLES H. HENRY

- In diode lasers the enhancement of the line-width is due to the dependence of the refractive index (n) on the carrier density (N)

$$\Delta S \rightarrow \Delta N \rightarrow \Delta n \rightarrow \Delta \phi$$

- Henry introduced a **phenomenological** factor (α) to account for amplitude–phase coupling.
- The **linewidth enhancement factor α** is a very important parameter of semiconductor lasers. Typically **α=2-5**

$$\Delta \nu = \left(1 + \alpha^2\right) \Delta \nu_{ST}$$

Single-mode *slowly-varying* optical field

$$\mathrm{E}(t) = E(t)e^{i\omega_0 t}$$

Photon density $S = |E|^2$

Complex field $E = E_x + iE_y$

$$\frac{dS}{dt} = \frac{1}{\tau_p}(N-1)S + \frac{\beta_{sp}N}{\tau_N} \Rightarrow \frac{dE}{dt} = \frac{1}{2\tau_p}(1+i\alpha)(N-1)E + \sqrt{\frac{\beta_{sp}N}{\tau_N}}\xi$$

α factor

Langevin stochastic term: complex, uncorrelated, Gaussian white noise

$$\xi = \xi_x + i\xi_y$$

$$k = \frac{1}{2\tau_p}, \; D = \frac{\beta_{sp}N_0}{\tau_N}$$

$$\frac{dE_x}{dt} = k(N-1)(E_x - \alpha E_y) + \sqrt{D}\xi_x$$

$$\frac{dE_y}{dt} = k(N-1)(\alpha E_x + E_y) + \sqrt{D}\xi_y$$

Derivation of the equations: Ohtsubo Cap. 2

Rate equation for optical phase

$$\frac{dE_x}{dt} = k(N-1)(E_x - \alpha E_y) + \sqrt{D}\xi_x$$

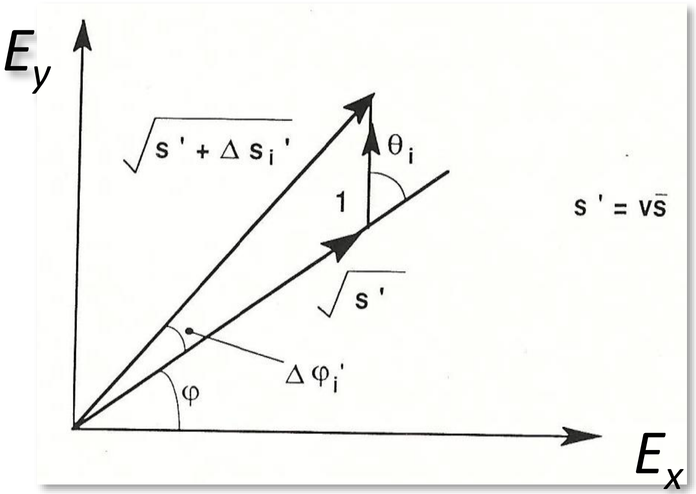
$$\frac{dE_y}{dt} = k(N-1)(\alpha E_x + E_y) + \sqrt{D}\xi_y$$

$$E = E_x + iE_y = \sqrt{S}e^{i\phi}$$

$$\frac{dS}{dt} = \frac{1}{\tau_p}\left(N-1\right)S + D + \xi_S(t)$$

$$\frac{d\phi}{dt} = \frac{\alpha}{2\tau_p}\left(N-1\right) + \xi_\phi(t)$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - GS\right)$$

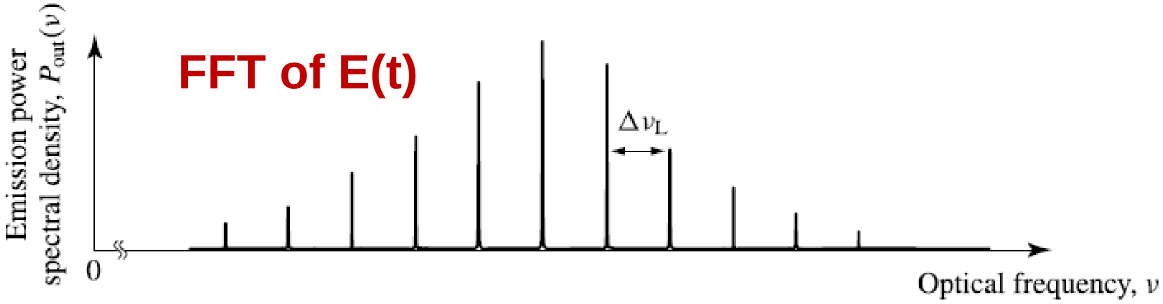


- The instantaneous frequency depends on the carrier density.
- ϕ is a “slave” variable.
- The noise sources are not independent.

These equations allow to understand the **optical spectrum** of a semiconductor laser

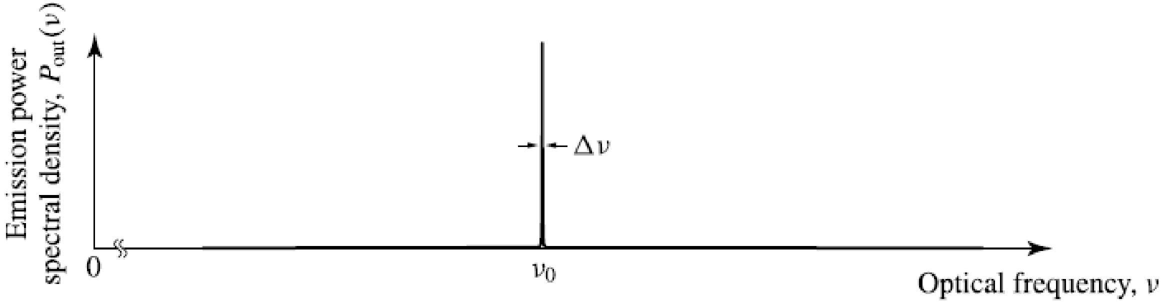
- In EELs: L= 200–500 µm ⇒ Δν=100–200 GHz. Because the gain bandwidth is 10–40 THz ⇒ 10–20 longitudinal modes.
- The line-width of each longitudinal mode depends on the alpha-factor and is of the order of 10 MHz.

Multimode optical spectra



FFT of E(t)

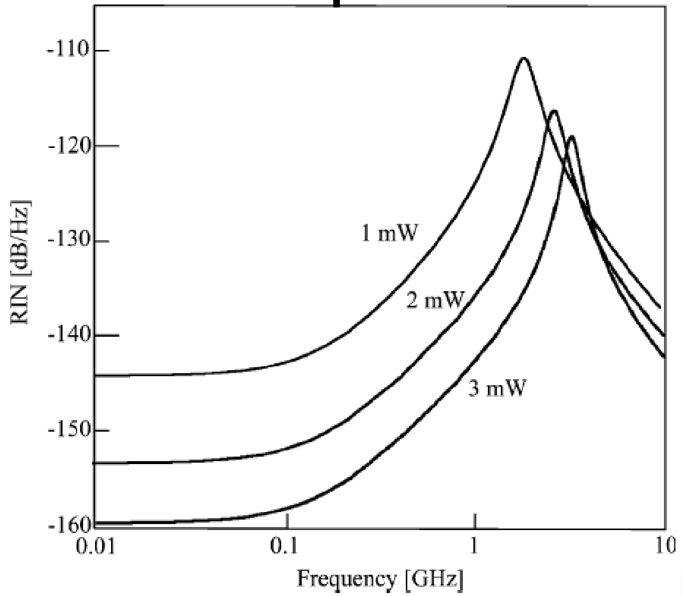
Single-mode optical spectra (Source: J. M. Liu, *Photonic devices*)



These equations also allow to understand the Relative Intensity Noise (RIN): **FFT of S(t)**

The laser output intensity is detected by a photo-detector, converted to an electric signal and sent to a RF spectrum analyzer. The RIN is a measure of the relative noise level to the average dc power.

Numerical expression

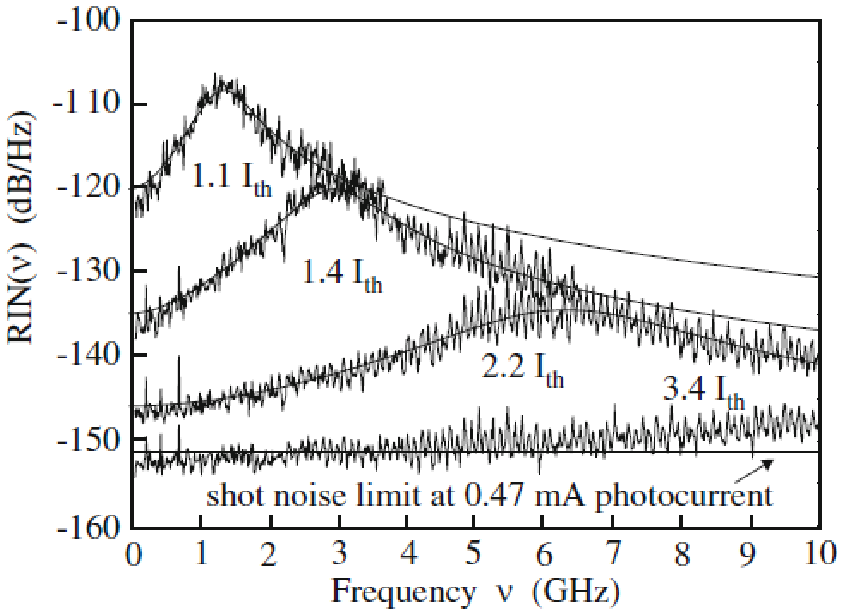


Peak at the relaxation oscillation frequency

$$\omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}}$$

Source: J. Ohtsubo, *Semiconductor lasers*

Experimentally measured (VCSEL)



Source: R. Michalzik, *VCSELs*

RATE EQUATION MODEL FOR A SINGLE-MODE COMPLEX OPTICAL FIELD

-ALPHA FACTOR, LINEWIDTH & INTENSITY NOISE

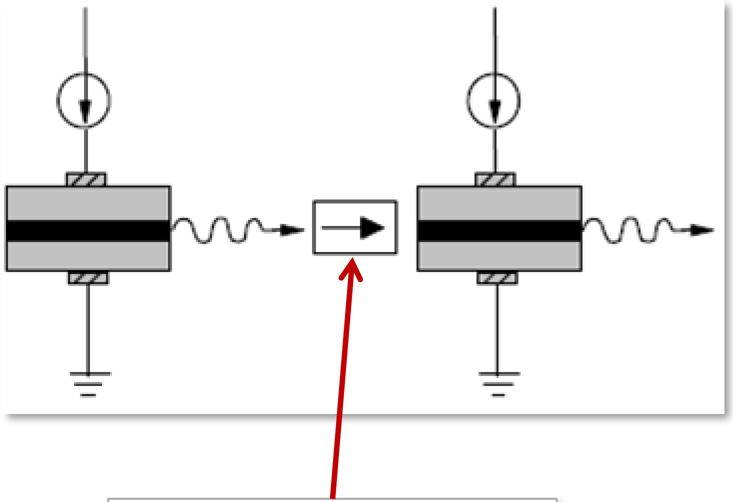
-OPTICAL PERTURBATIONS (INJECTION, FEEDBACK)

-POLARIZATION INSTABILITIES

Optical perturbations

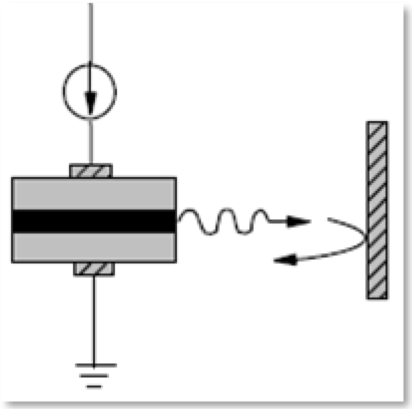
“Solitary” diode lasers display a stable output (only transient oscillations) but they can be easily perturbed by injected light and can display sustained periodic or irregular oscillations.

Optical injection

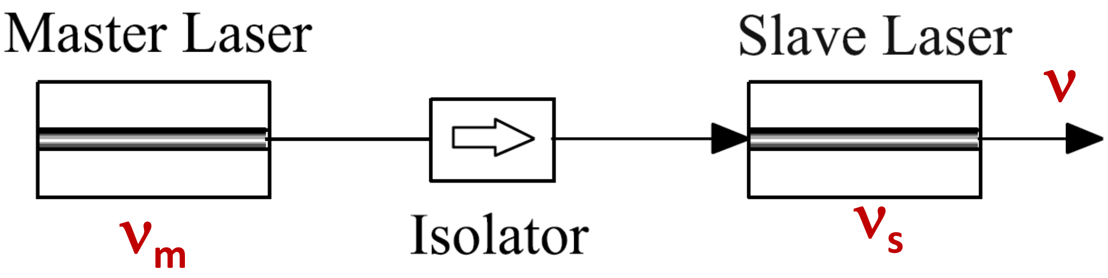


Optical isolator

Optical feedback

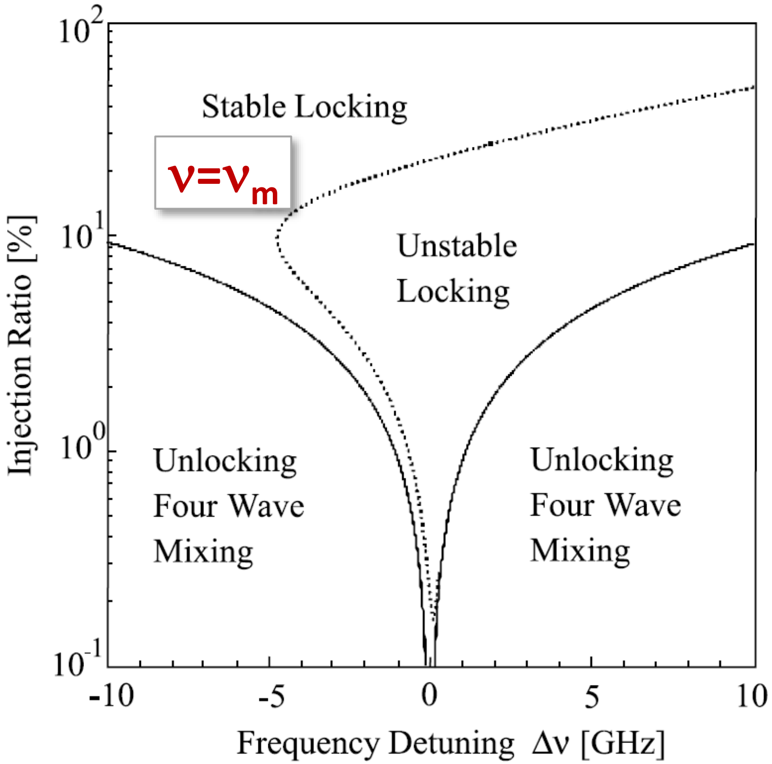


Optical Injection



Detection system (photo detector, oscilloscope, spectrum analyzer)

- Two Parameters:
  - Injection ratio
  - Frequency detuning $\Delta\nu = \nu_s - \nu_m$
- Dynamical regimes:
  - Stable locking (cw output)
  - Periodic oscillations
  - Chaos
  - Beating (no interaction)



Source: J. Ohtsubo, *Semiconductor lasers*

Model for the injected laser

Optical field $\mathrm{E}(t) = E(t) \exp(i\omega_s t)$; $E(t)$ = slowly varying amplitude

Without injection:

$$\frac{dE}{dt} = \frac{1}{2\tau_p}(1+i\alpha)(N-1)E + \sqrt{D}\xi \qquad D = \frac{\beta_{sp} N_0}{\tau_N}$$

With injection:

$$\frac{dE}{dt} = \frac{1}{2\tau_p}(1+i\alpha)(N-1)E + \underbrace{i\Delta\omega E + \sqrt{P_{\mathrm{inj}}}}_{\text{optical injection from master laser}} + \underbrace{\sqrt{D}\xi(t)}_{\text{spontaneous emission noise}}$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu\text{-}N\text{-}N|E|^2\right)$$

optical injection
from master laser

P_{inj}: injection strength

$\Delta\omega=\omega_s-\omega_m$: detuning

spontaneous
emission
noise

μ: pump current parameter

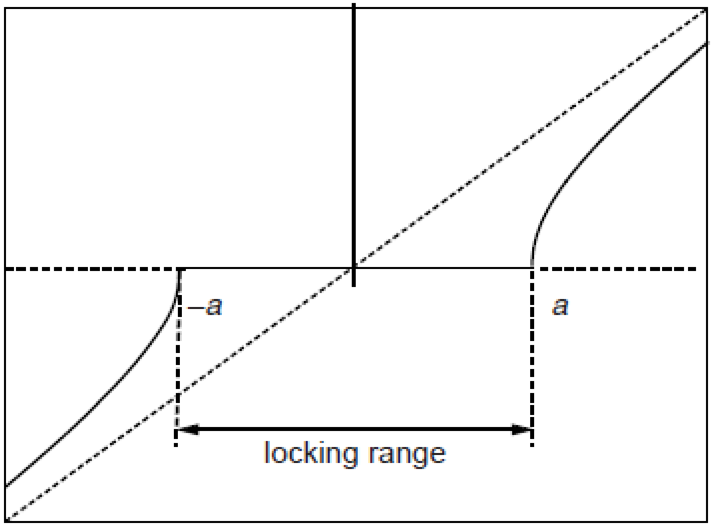
Typical parameters:

$\alpha = 3$, $\tau_p = 1$ ps,

$\tau_N = 1$ ns, $D=10^{-4}$ ns^{-1}

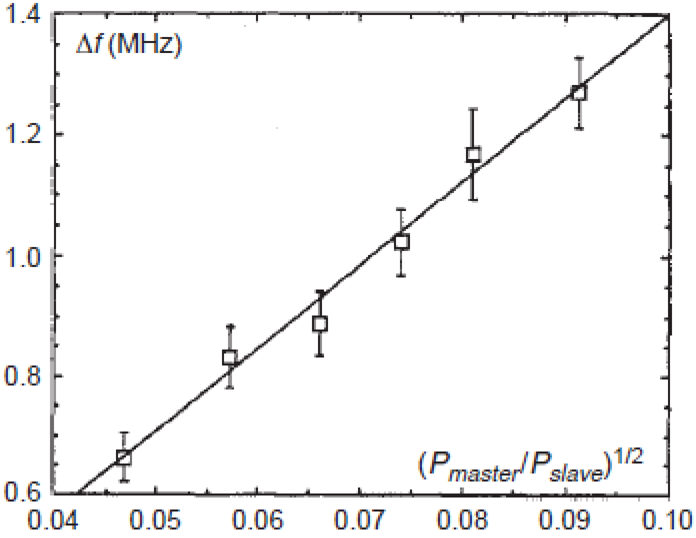
Injection locking

Beat frequency $\Omega=0$



$\Delta\nu = \nu_s - \nu_m$

Experimental verification

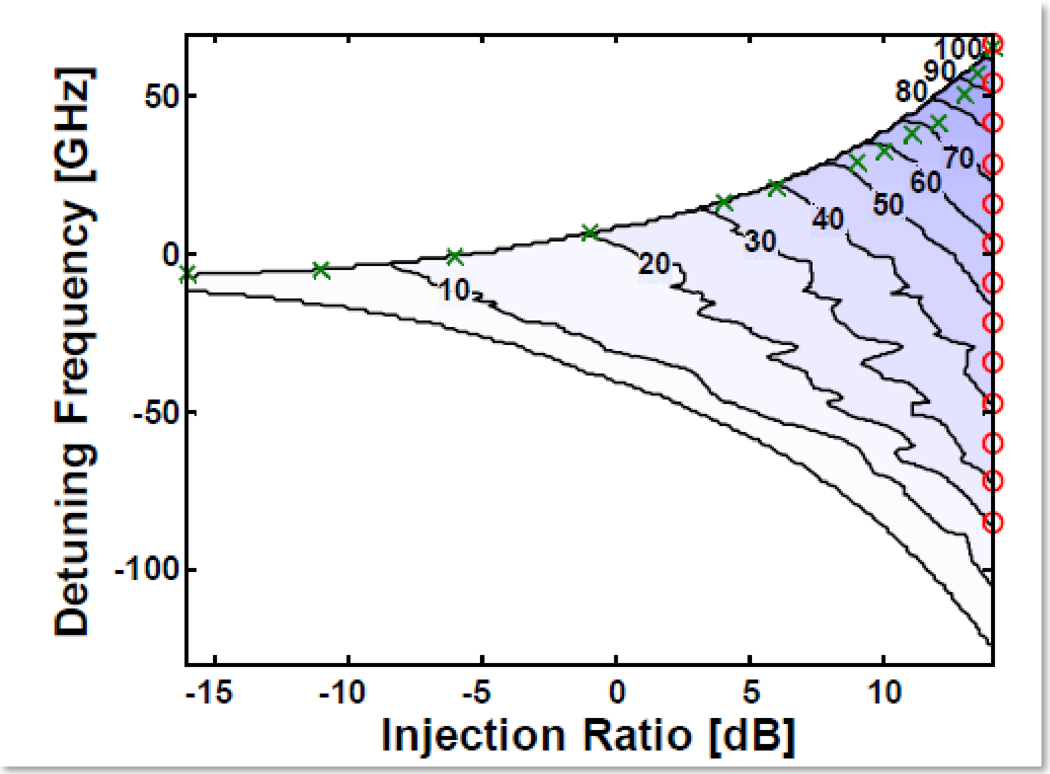


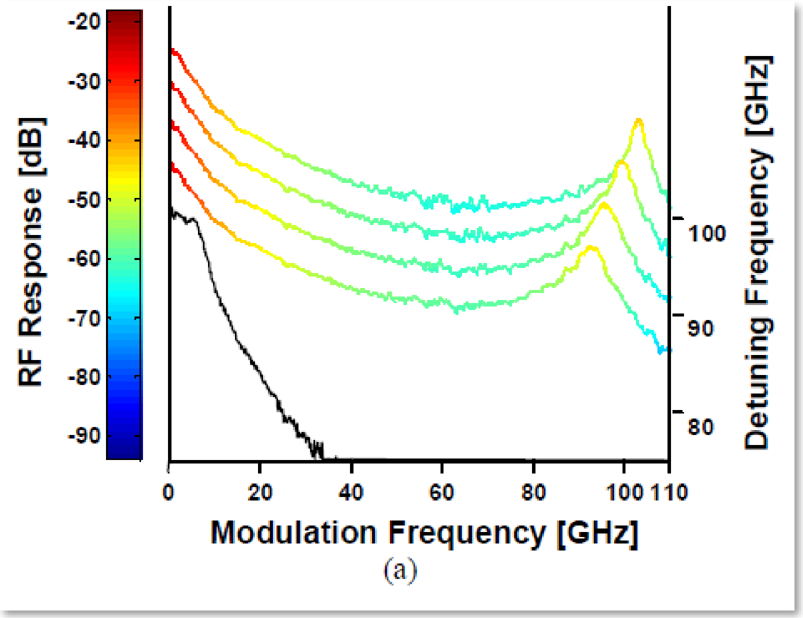
Nd $^{3+}$:YAG laser

Model prediction:

the locking range is proportional to the relative injection strength.

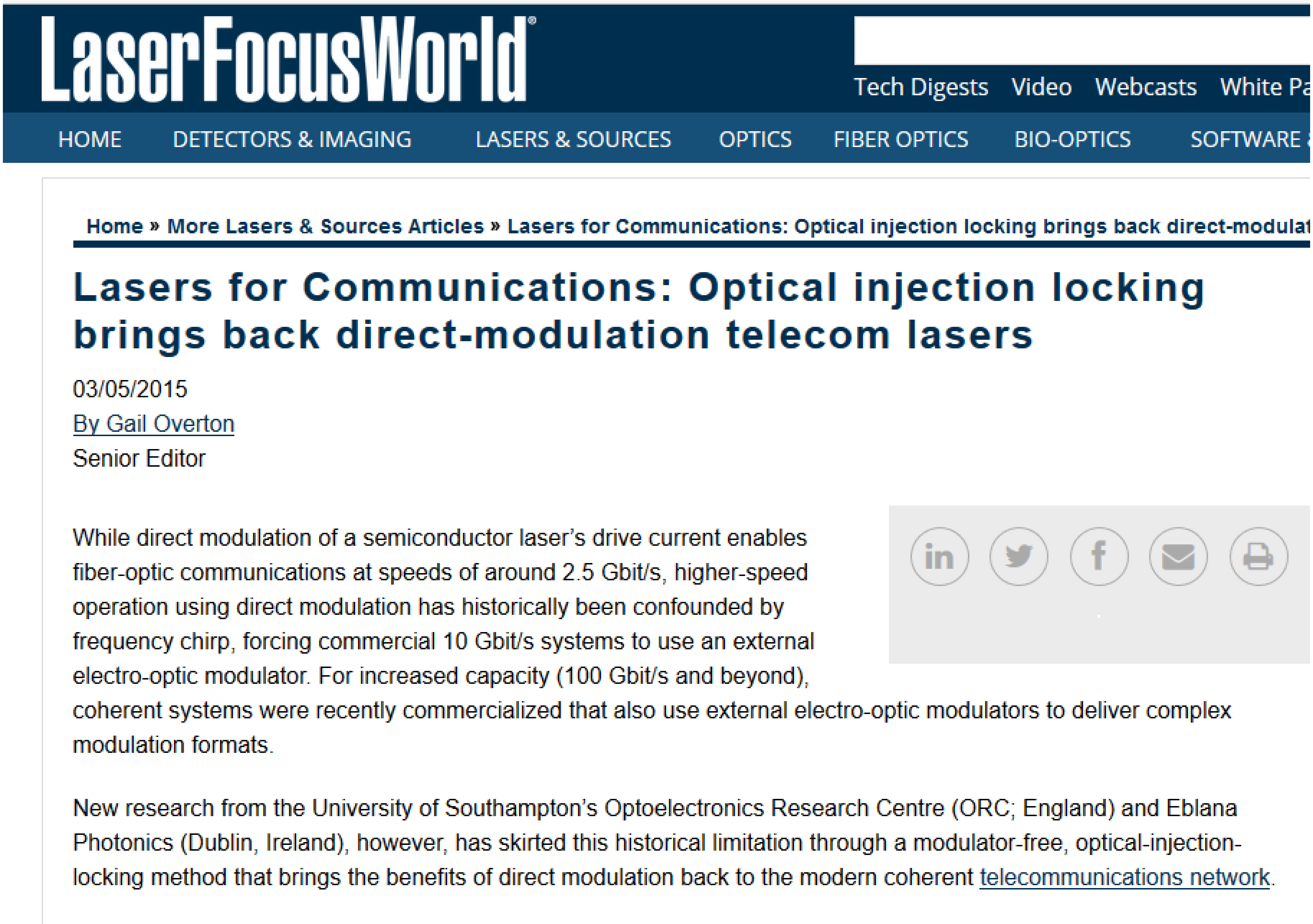
Injection locking increases the resonance frequency and the modulation bandwidth





(a)

#90115 - $15.00 USD Received 26 Nov 2007; revised 17 Mar 2008; accepted 20 Mar 2008; published 24 Apr 2008
(C) 2008 OSA 28 April 2008 / Vol. 16, No. 9 / OPTICS EXPRESS 6609



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Lasers for Communications: Optical injection locking brings back direct-modulation telecom lasers

03/05/2015

By Gail Overton

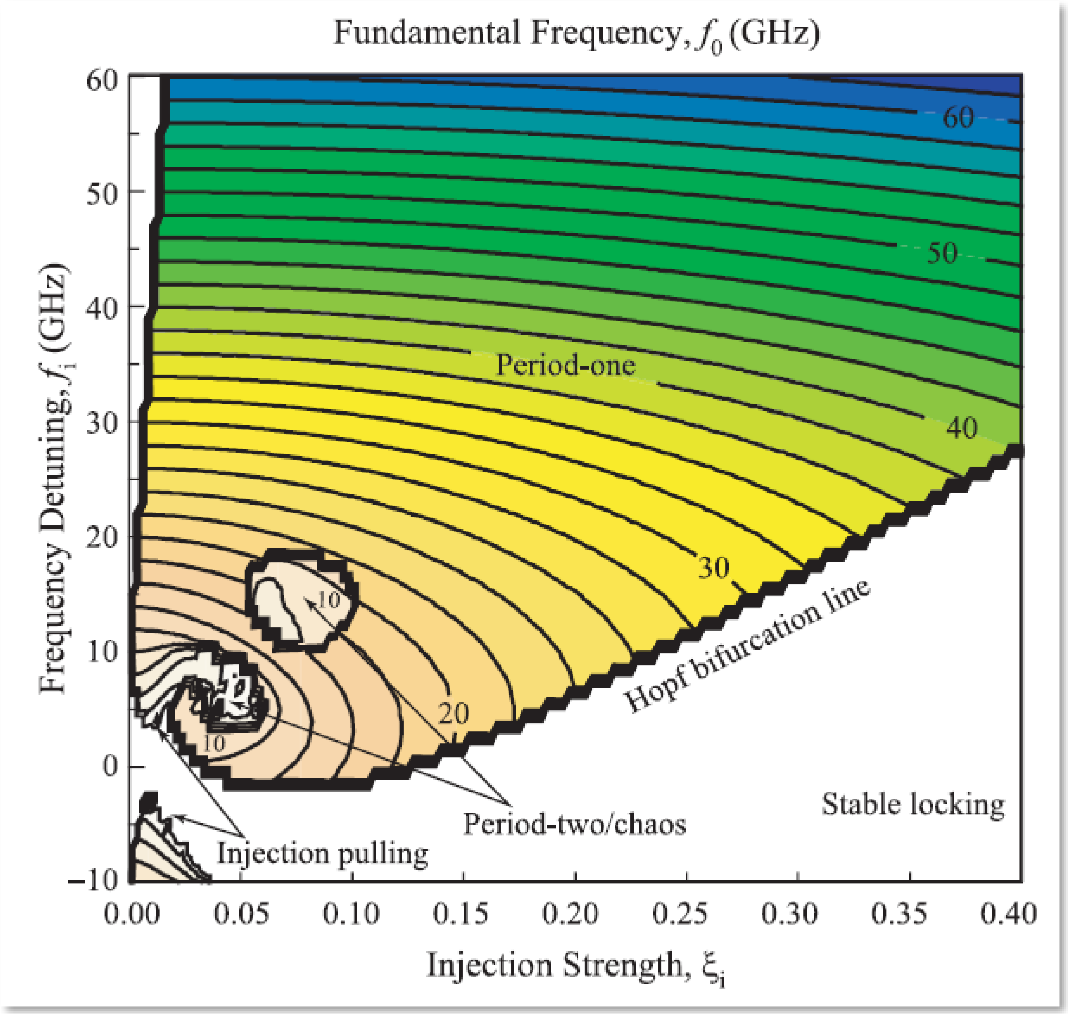
Senior Editor

While direct modulation of a semiconductor laser's drive current enables fiber-optic communications at speeds of around 2.5 Gbit/s, higher-speed operation using direct modulation has historically been confounded by frequency chirp, forcing commercial 10 Gbit/s systems to use an external electro-optic modulator. For increased capacity (100 Gbit/s and beyond), coherent systems were recently commercialized that also use external electro-optic modulators to deliver complex modulation formats.

New research from the University of Southampton's Optoelectronics Research Centre (ORC; England) and Eblana Photonics (Dublin, Ireland), however, has skirted this historical limitation through a modulator-free, optical-injection-locking method that brings the benefits of direct modulation back to the modern coherent telecommunications network.

http://www.laserfocusworld.com/articles/print/volume-51/issue-03/world-news/lasers-for-communications-optical-injection-locking-brings-back-direct-modulation-telecom-lasers.html

Outside the injection locking region: regular intensity oscillations

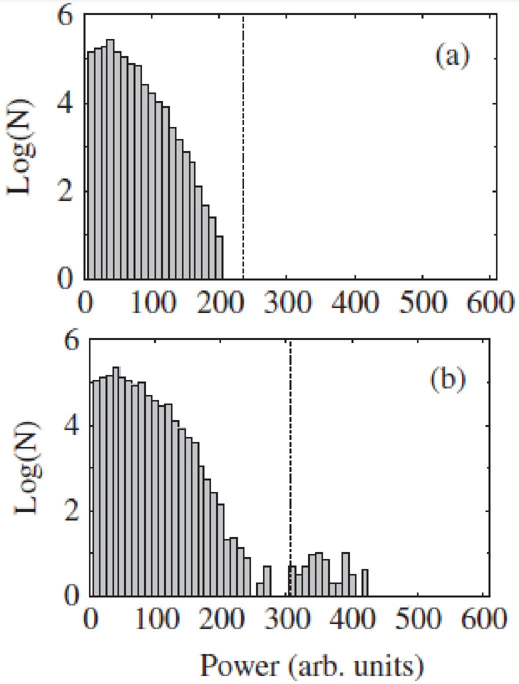


The frequency of the intensity oscillations, f_0, can be controlled by tuning the injection strength and the detuning.

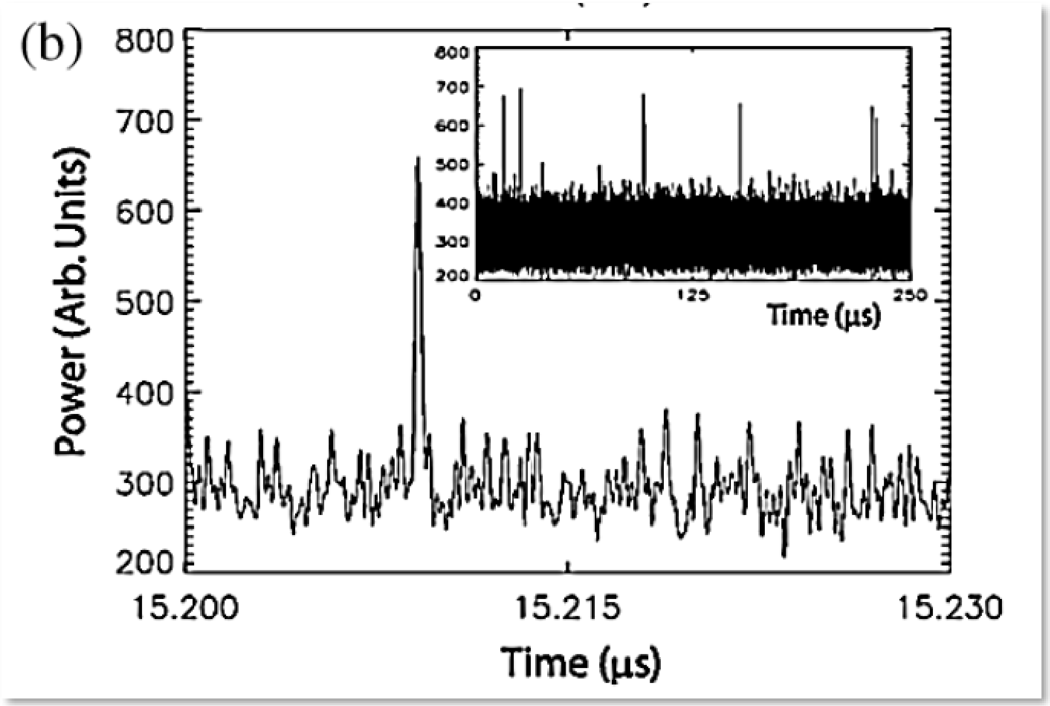
S-C Chan et al, Optics Express 15, 14921 (2007)

Also outside the locking region: ultra-high intensity pulses (“optical rogue waves”)

Distribution of pulse amplitudes for different injection currents

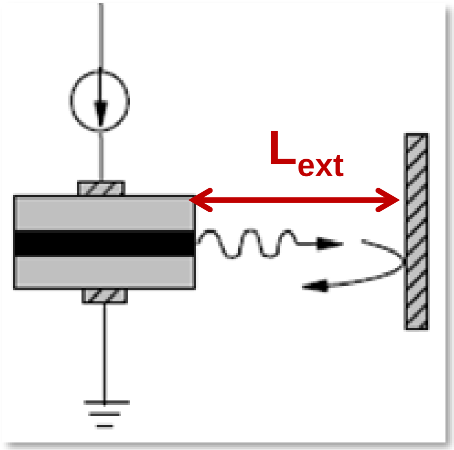


Time series of the laser intensity



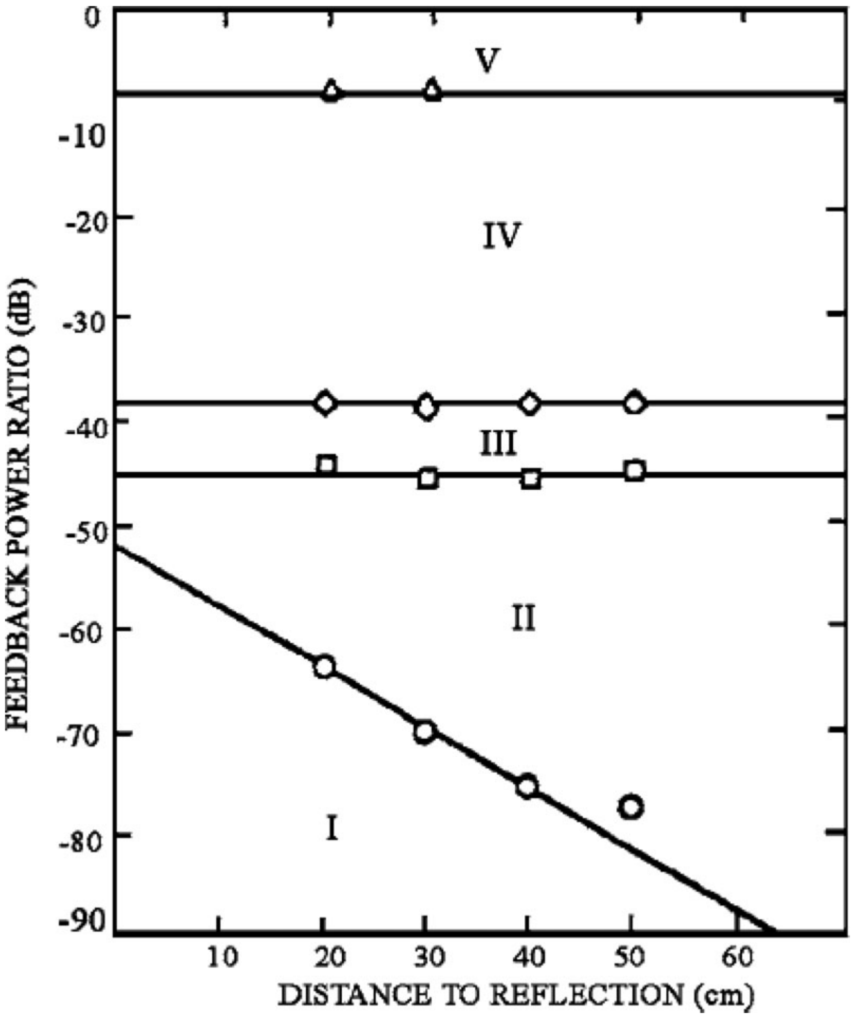
C. Bonatto et al, PRL 107, 053901 (2011),
Optics & Photonics News February 2012,
Research Highlight in *Nature Photonics* DOI:10.1038/nphoton.2011.240

Optical feedback regimes



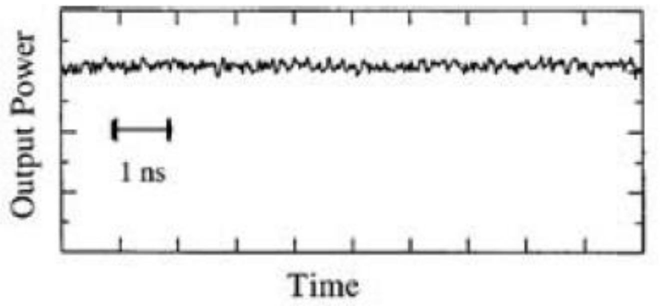
- **Regime I**: line-width narrowing/ broadening (depending on the phase of feedback),
- **Regime II**: mode-hopping,
- **Regime III**: single-mode narrow-line operation,
- **Regime IV**: coherence collapse,
- **Regime V**: single-mode operation in an extended cavity mode.

Tkach and Chraplyvy diagram

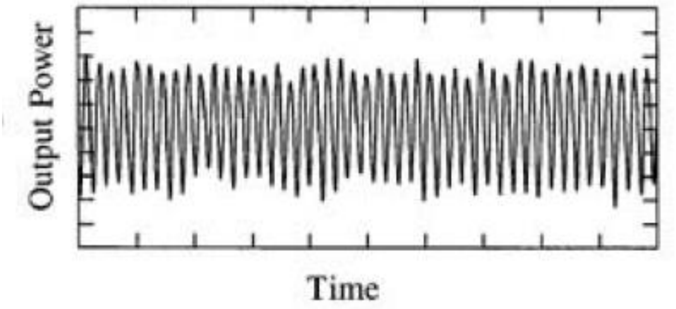


Source: S. Donati, Laser and Photonics Rev. 2012

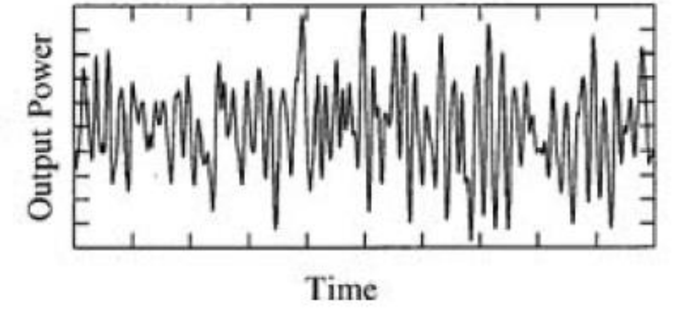
Feedback induced instabilities



negligible small feedback



periodic oscillations
with weak feedback



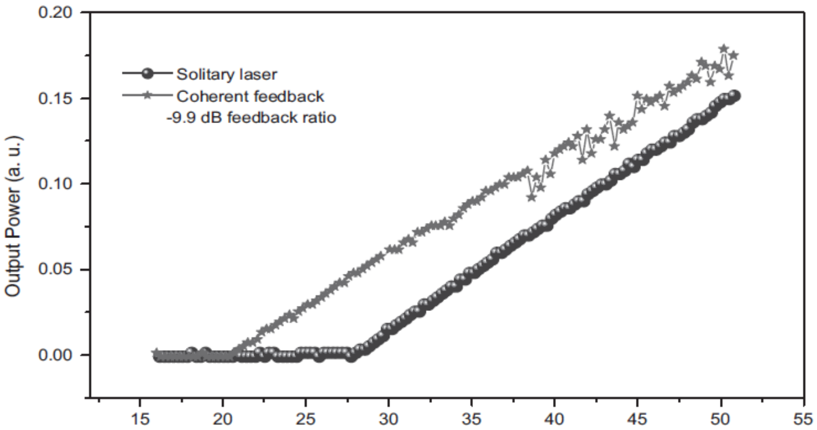
chaotic oscillations with
strong feedback
(coherence collapse,
Regime IV)

$I/I_{th} = 1.3$

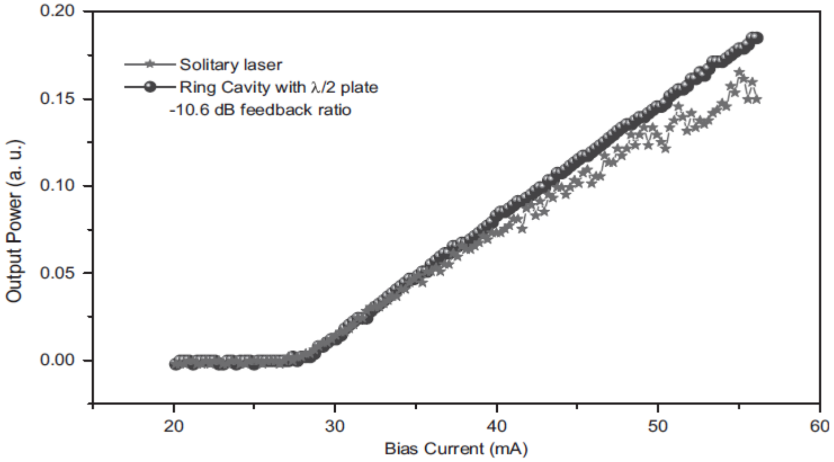
Source: J. Ohtsubo, *Semiconductor lasers*

Optical feedback effects on the LI curve

Coherent feedback



Incoherent feedback

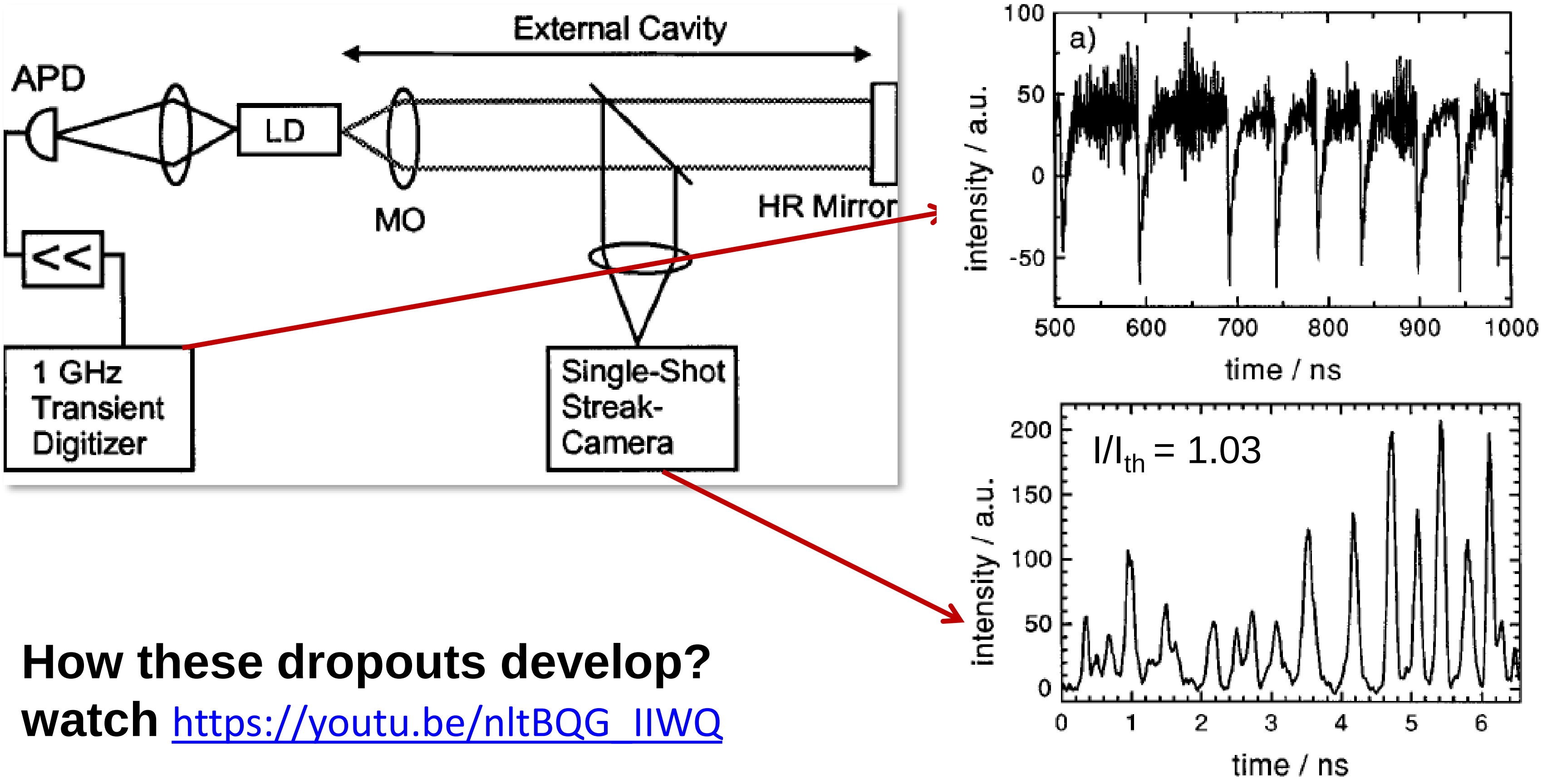


Feedback-reduced threshold: the amount of reduction quantifies the strength of the feedback.

Adapted from R. Ju et al,

IEE Proc.-Optoelectron., Vol. 153, No. 3, June 2006

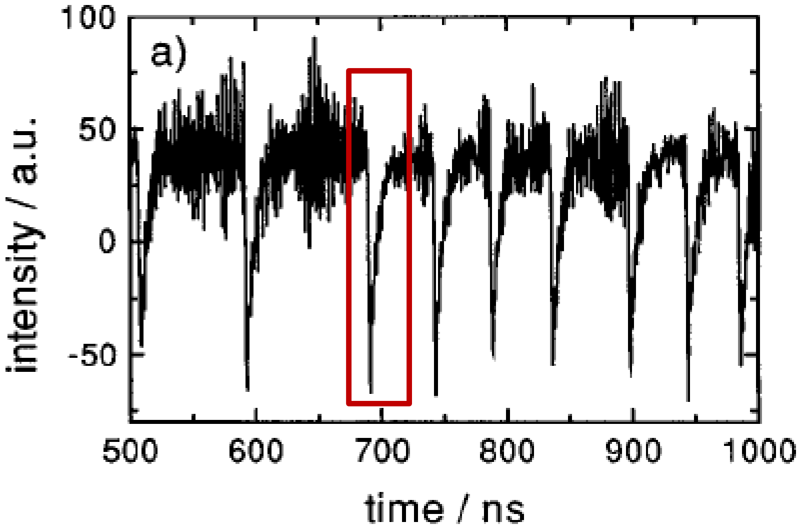
Close to the laser threshold: Low Frequency Fluctuations (LFFs)

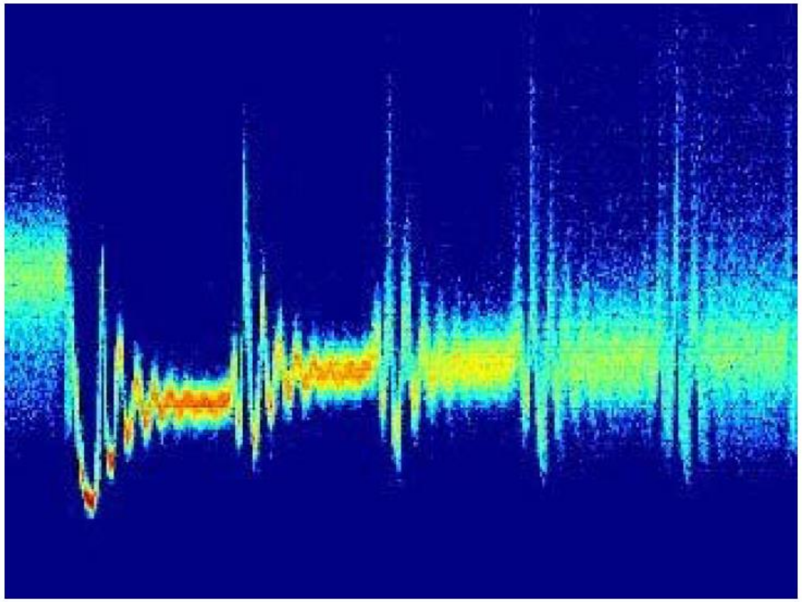


How these dropouts develop?
watch https://youtu.be/nltBQG_IIWQ

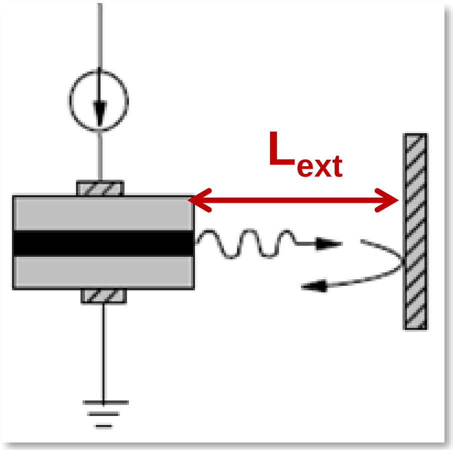
I. Fischer et al, PRL 1996

LFFs: Complex dynamics, several time-scales





Recovery after a dropout: in steps of τ with relaxation oscillations



$$\boxed{\tau = \frac{2L_{ext}}{c}}$$

If $L_{ext} = 1$ m ⇒ $\tau = 6.7$ ns

Souce: M. Sciamanna (PhD Thesis 2004)

Single-mode Lang and Kobayashi Model

Optical field $\mathrm{E}(t) = E(t)\exp(i\omega_0 t)$; $E(t)$ = slowly varying amplitude

Solitary laser

$$\frac{dE}{dt} = k(1+i\alpha)(N-1)E + \sqrt{D}\xi \qquad k = \frac{1}{2\tau_p} \qquad D = \frac{\beta_{sp}}{\tau_N}$$

With optical feedback

$$\frac{dE}{dt} = k(1+i\alpha)(G-1)E + \underbrace{\eta\, E(t-\tau)e^{-i\omega_0\tau}}_{\text{feedback}} + \underbrace{\sqrt{D}\xi}_{\text{noise}}$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - G|E|^2\right)$$

$$\tau = \frac{2L_{ext}}{c}$$

η = feedback strength

τ = feedback delay time

μ = pump current

(control parameters)

R. Lang and K. Kobayashi, IEEE J. Quantum Electron. 16, 347 (1980)

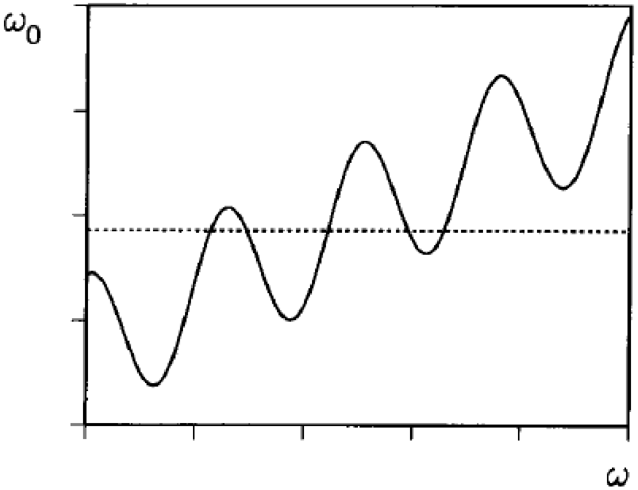
External cavity modes (ECMs)

- $\mathrm{E}(t) = E(t) \exp(i\omega_0 t)$

  $E(t) = E_0 \exp[i(\omega-\omega_0)t]$; $N(t) = N$

$\Rightarrow$ Monocromatic solutions: $\mathrm{E}(t) = E_0 \exp(i\,\omega\, t)$

  - Stable modes (constructive interference)
  - Unstable models (destructive interference)



$$\boxed{\omega_0\tau = \omega\tau + C\sin(\omega\tau + \arctan\alpha)}$$

$$C = \eta\tau\sqrt{1+\alpha^2}$$

$$N = 1 - \frac{\eta}{k}\cos(\omega\tau)$$

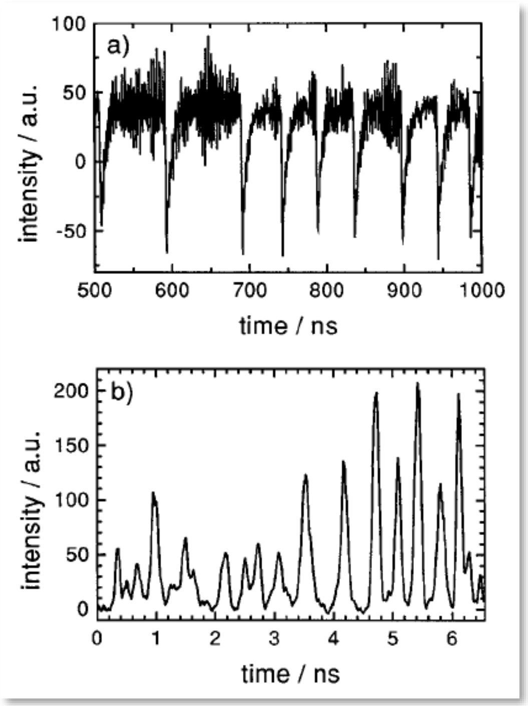
$$|E_0|^2 = \frac{\mu - N}{N}$$

- The number of ECMs increases with:
  - The feedback strength
  - The length of the external cavity

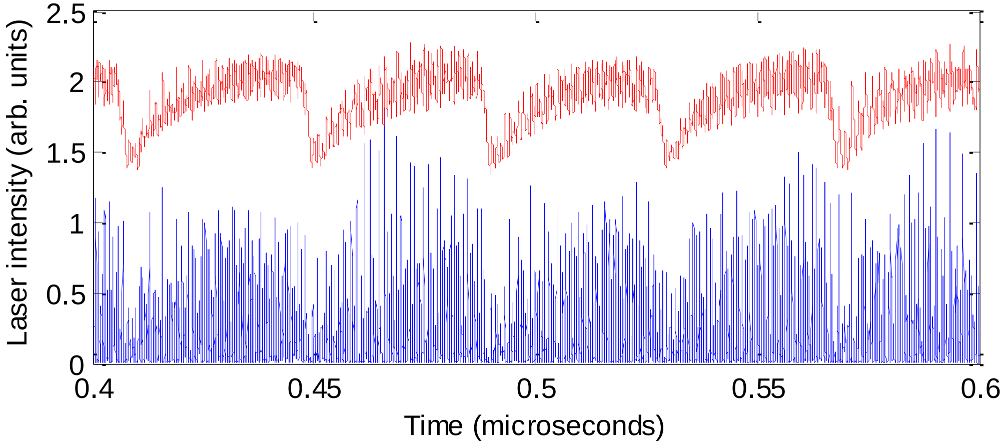
Good agreement model-experiments

- With a “fast” detector: pulses; with a “slow” detector: dropouts

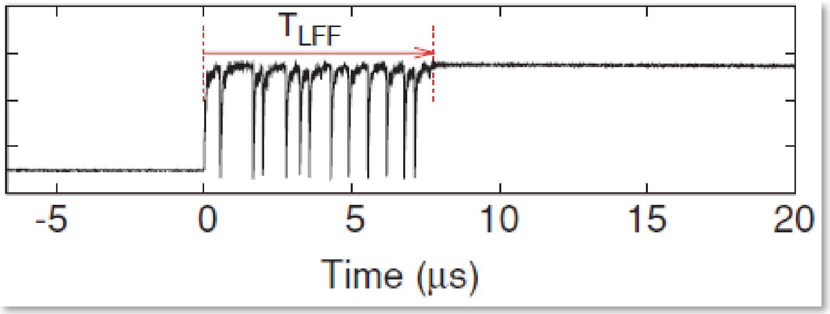
Experiments



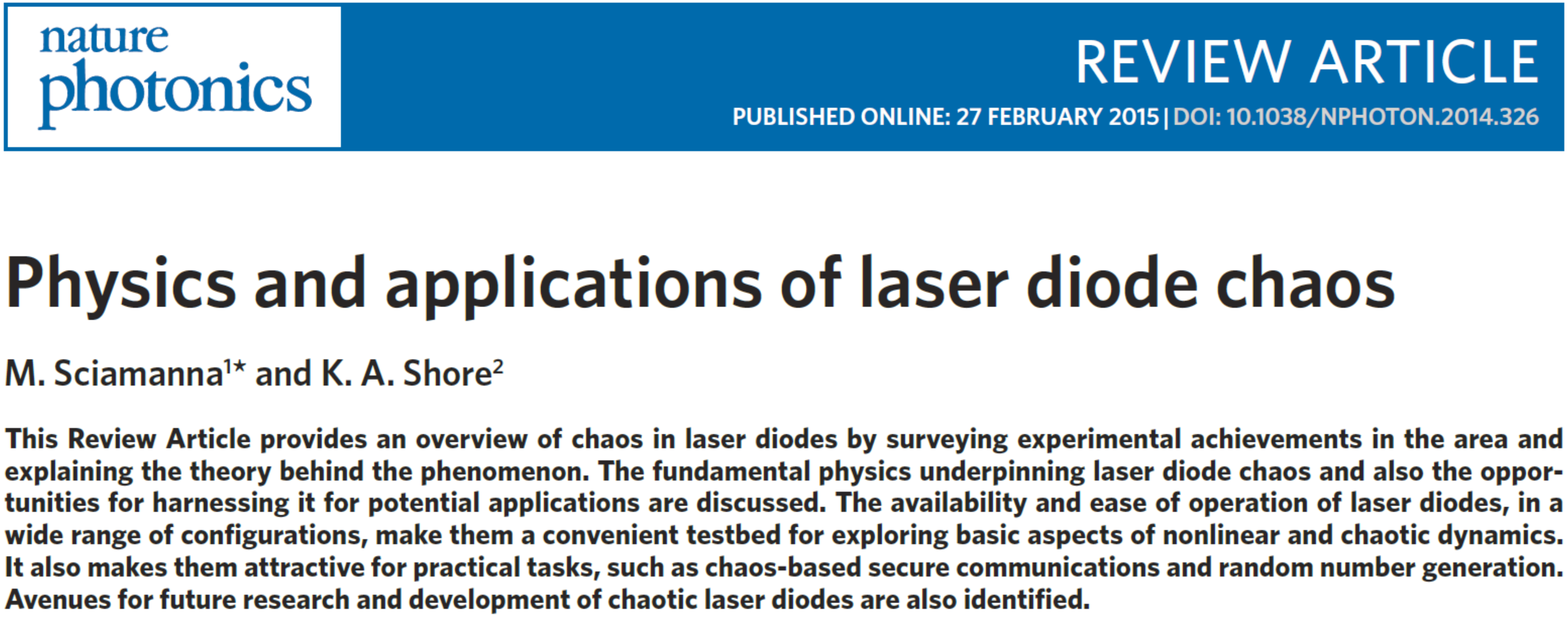
Stochastic simulations



Deterministic simulations



I. Fischer et al, PRL 1996
A. Torcini et al, Phys. Rev. A 74, 063801 (2006)
J. Zamora-Munt et al, Phys Rev A 81, 033820 (2010)



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REVIEW ARTICLE

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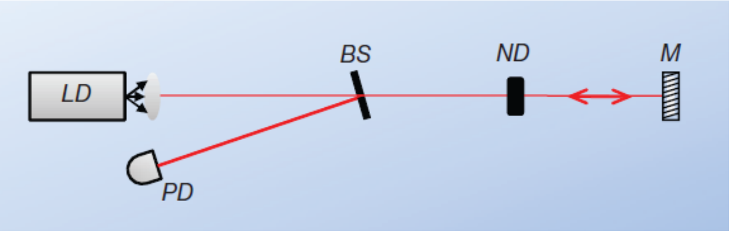
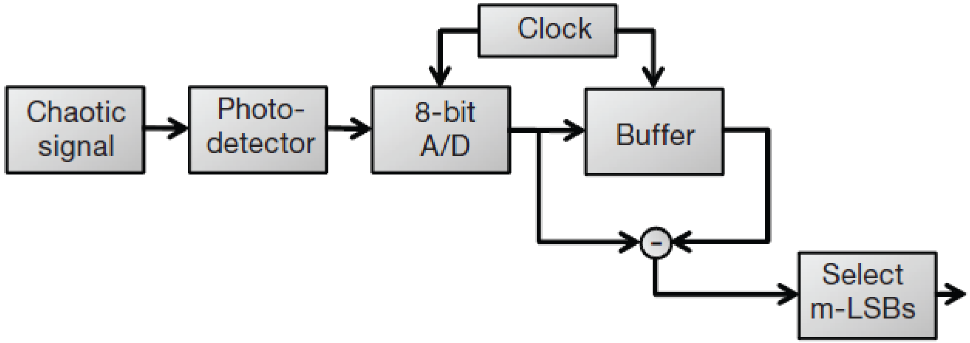
Physics and applications of laser diode chaos

M. Sciamanna[1]* and K. A. Shore[2]

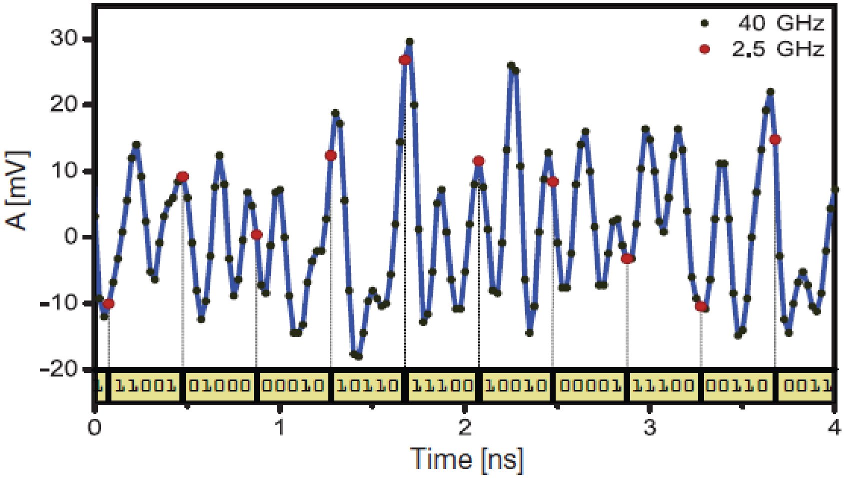
This Review Article provides an overview of chaos in laser diodes by surveying experimental achievements in the area and explaining the theory behind the phenomenon. The fundamental physics underpinning laser diode chaos and also the opportunities for harnessing it for potential applications are discussed. The availability and ease of operation of laser diodes, in a wide range of configurations, make them a convenient testbed for exploring basic aspects of nonlinear and chaotic dynamics. It also makes them attractive for practical tasks, such as chaos-based secure communications and random number generation. Avenues for future research and development of chaotic laser diodes are also identified.

An example of an application of feedback-induced chaos: random bit generation





After processing the signal, arbitrarily long sequences can be generated at the 12.5-Gbit/s rate.



Source: I. Kanter et al, Nature Photonics 2010 & OPN

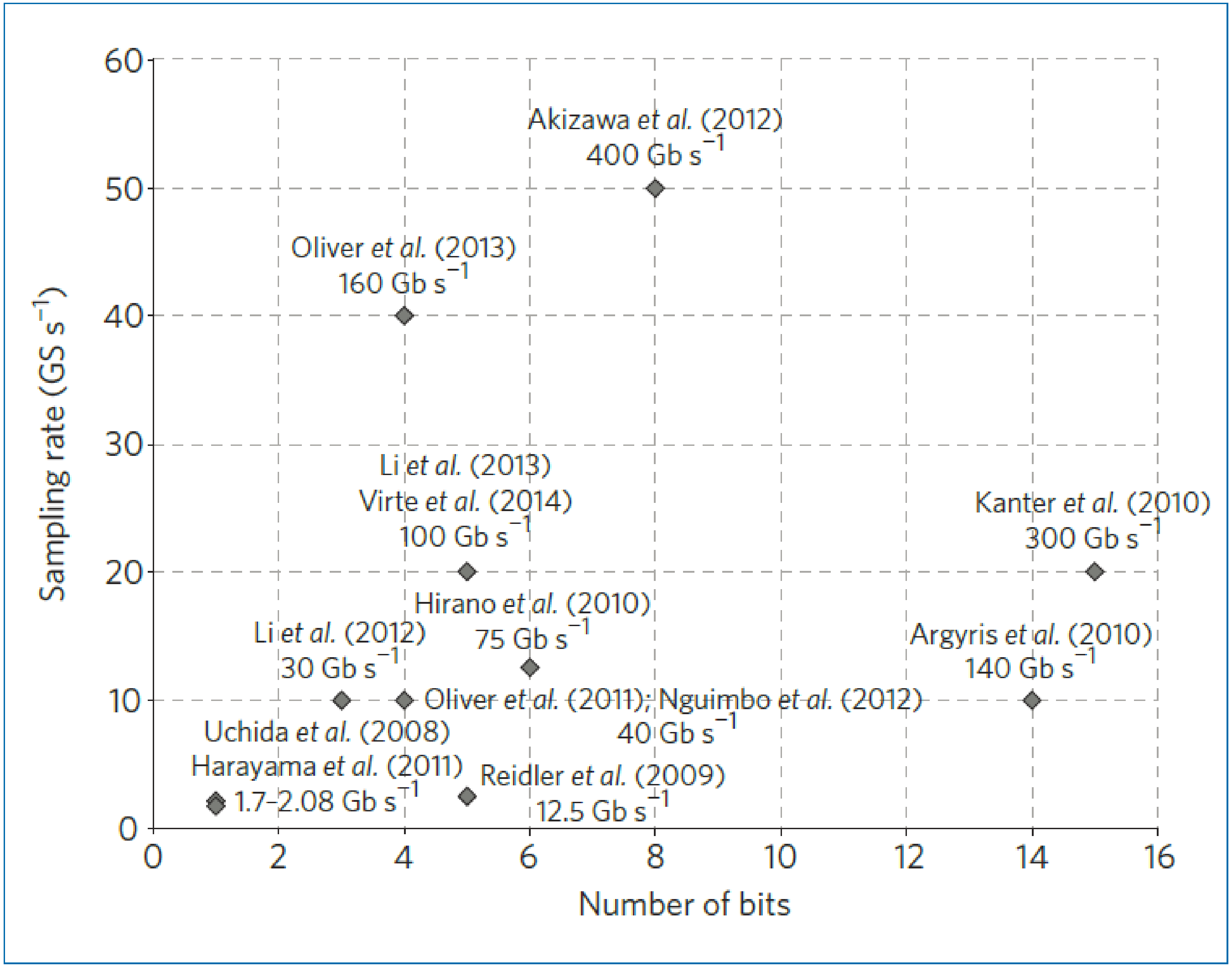


Figure 5 | The state-of-the-art of random number generation using chaos from a laser diode. Realizations differ by either the system under investigation, the post-processing method, the number of bits and/or the sampling rate. Each point corresponds one row, or multiple rows, in Table 1.

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Optics Letters

Semiconductor laser linewidth reduction by six orders of magnitude via delayed optical feedback

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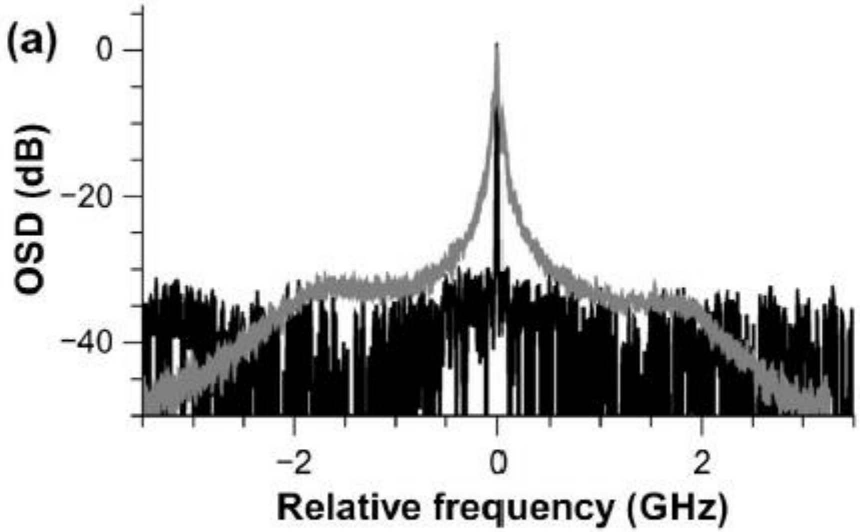
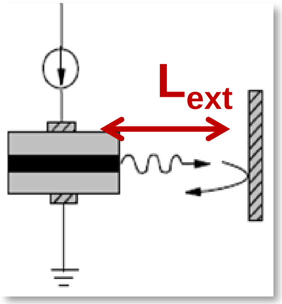


Summary: optical feedback effects



- Coherent feedback reduces the lasing threshold.

- Feedback effects depend on the feedback strength, the feedback delay time and feedback phase.

- Weak feedback introduces “external cavity modes”: stable or unstable solutions of the rate equations.

RATE EQUATION MODEL FOR A SINGLE-MODE COMPLEX OPTICAL FIELD

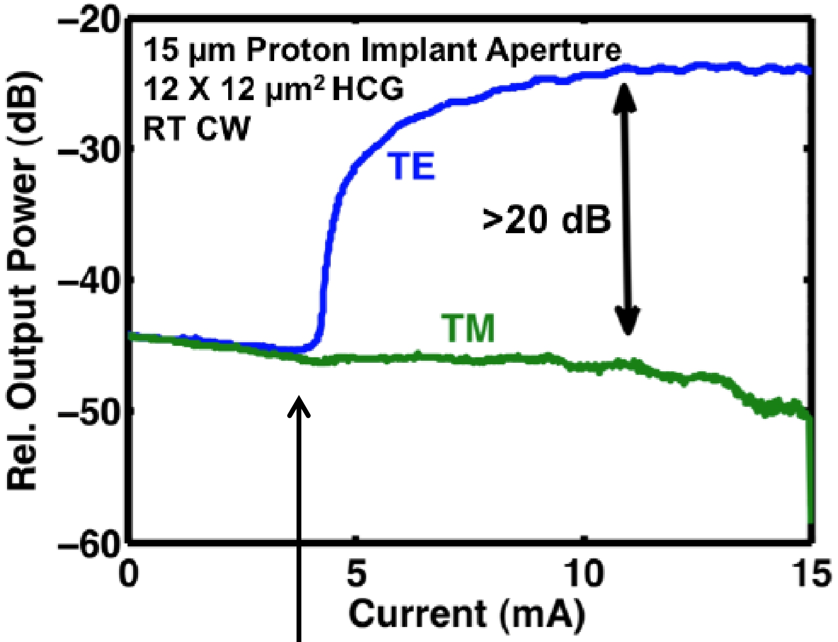
-ALPHA FACTOR, LINEWIDTH & INTENSITY NOISE

-OPTICAL PERTURBATIONS (INJECTION, FEEDBACK)

-POLARIZATION INSTABILITIES

EELs: linearly (TE) polarized output

Log scale

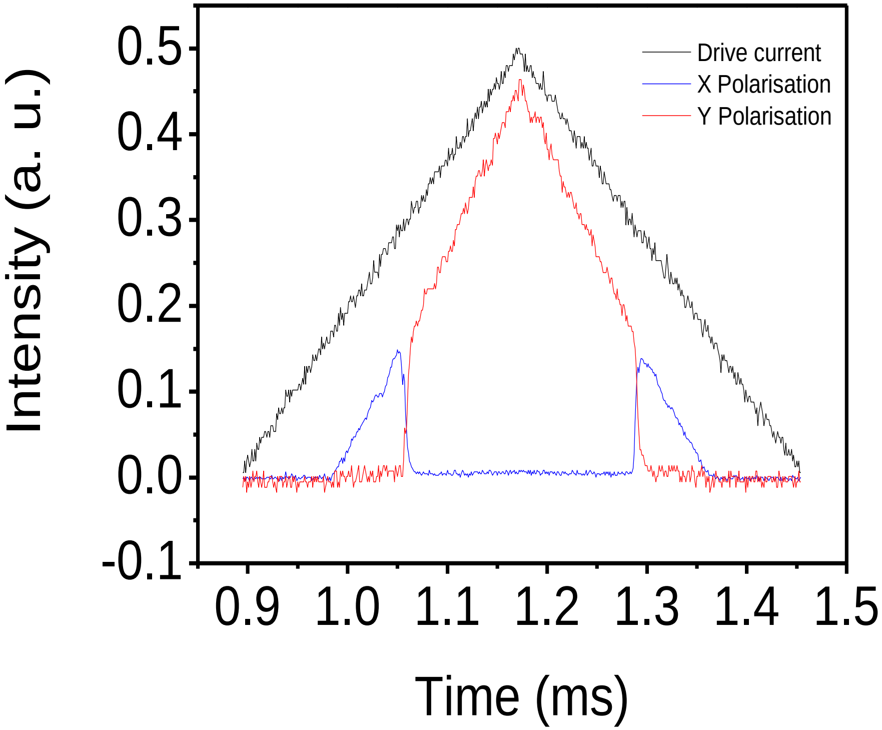


threshold current

Polarization switching can be induced by polarization-rotated feedback.

In some VCSELs: polarization switching (PS)

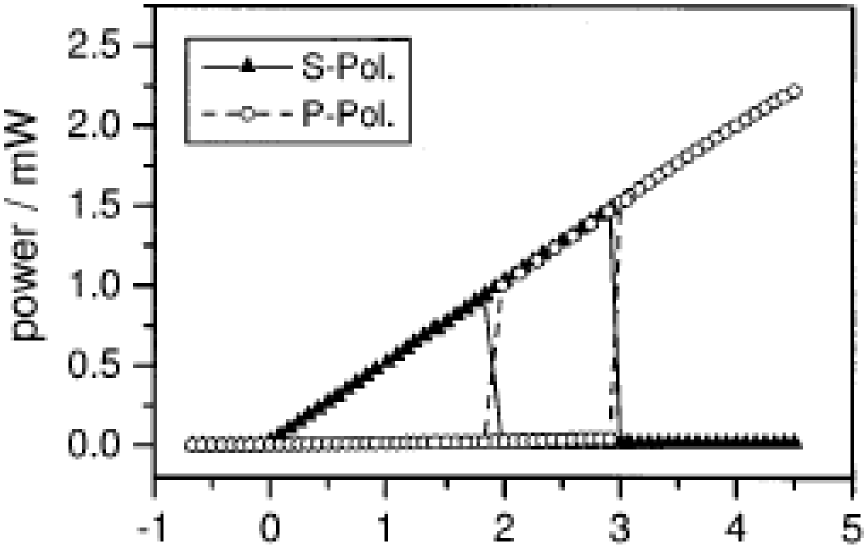
- **Circular** cavity geometry: two **linear orthogonal** modes (x, y).
- Often there is a **polarization switching** when the pump current is increased.
- Also **hysteresis**: the PS points for increasing and for decreasing current are different.
- The total output power increases/decreases monotonically with pump.



Source: Hong and Shore, Bangor University, Wales, UK

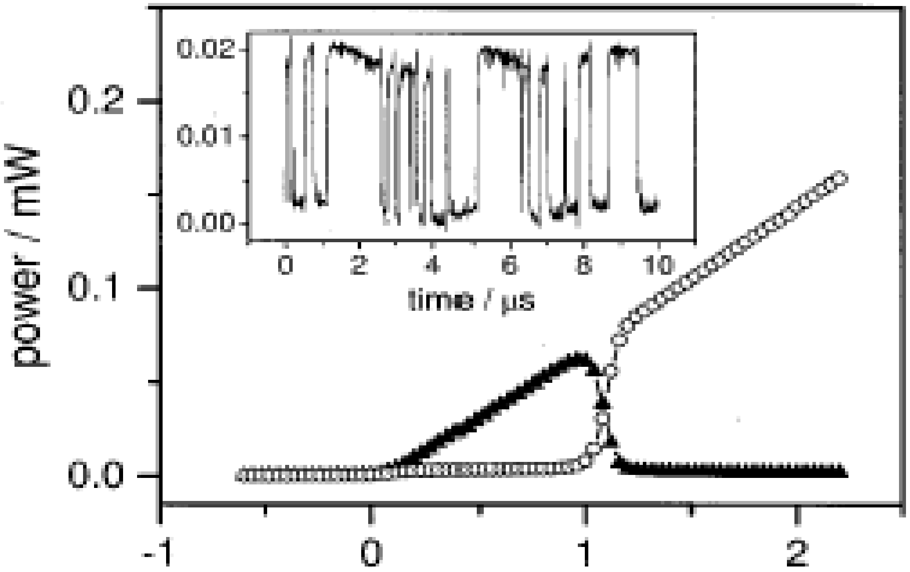
Polarization-resolved LI curve

Current-driven PS



Pump current, I/I_{th} -1

Stochastic PS

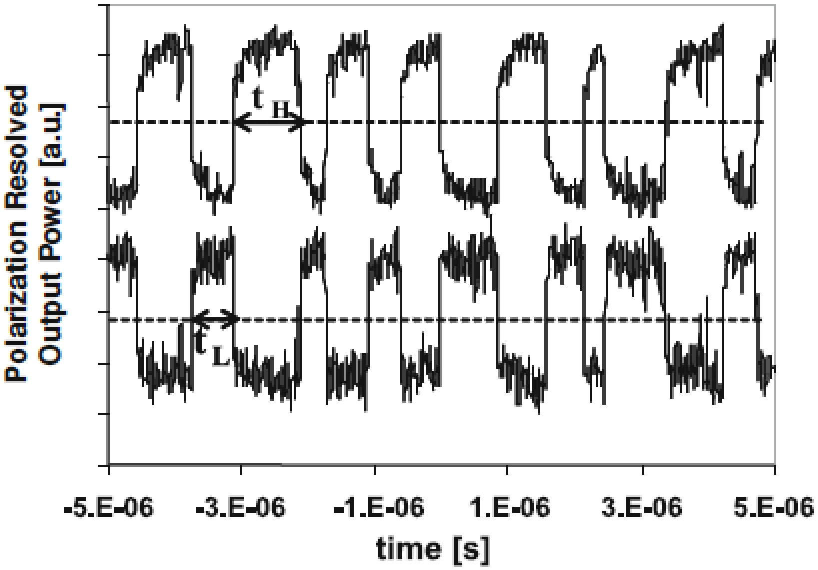


Pump current, I/I_{th} -1

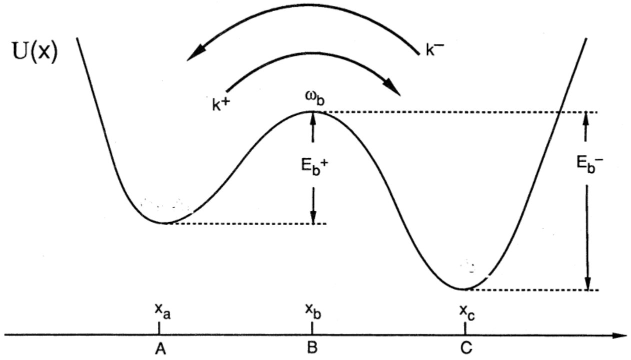
Souce: J. Kaiser et al, JOSAB 19, 672 (2002)

Stochastic PS

- Anti-correlated fluctuations of the two polarizations.

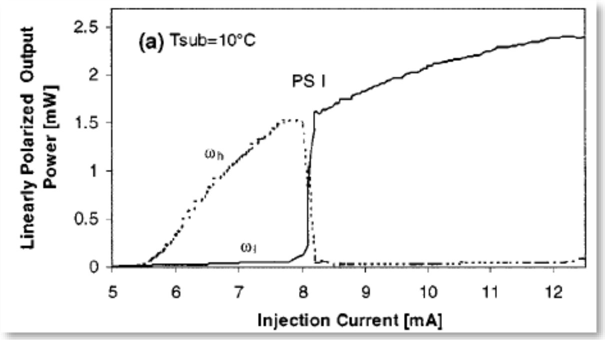


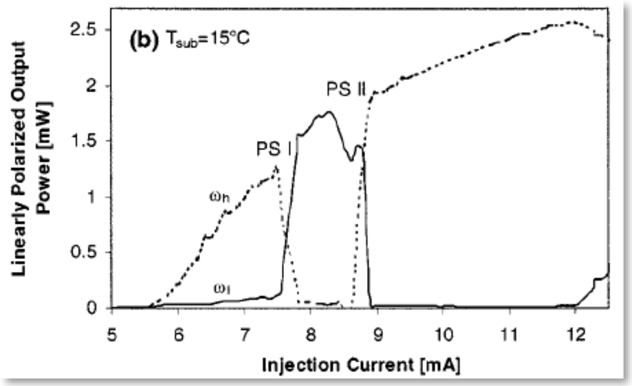
- Bistability + noise induced switching

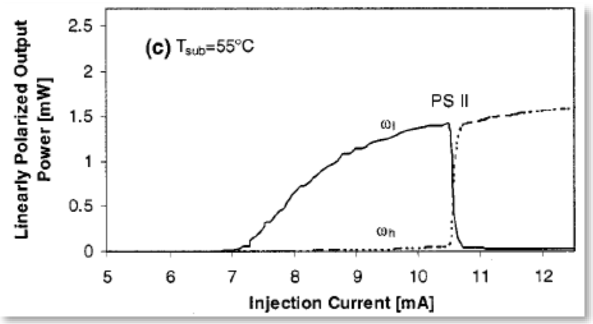


Current-driven PS

- Type 1: from the **Y** (low freq)→ **X** (high freq) polarization
- Type 2: from the **X** (high freq) → **Y** (low freq) polarization



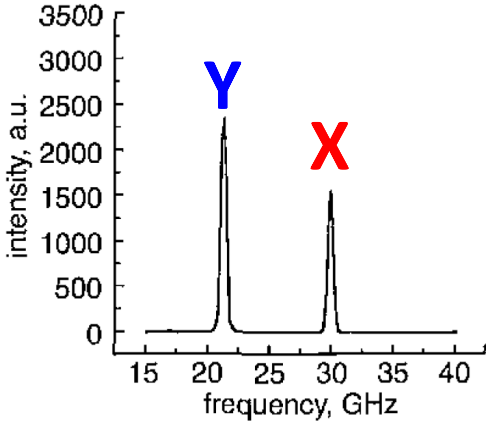


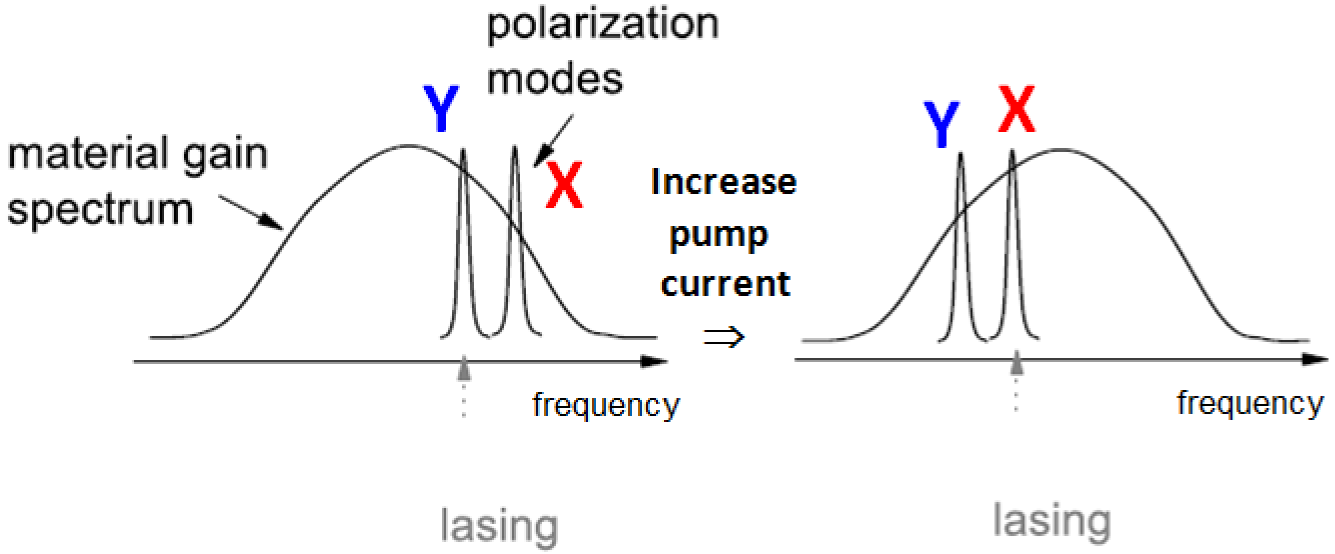


- Several models have been proposed to explain these PS

Source: B. Ryvkin et al, J. Opt. Soc. Am. B 1997

Thermal shift of the gain curve





Birefringence: the polarizations have **different optical frequencies**

When the pump current increases ⇒ Joule heating ⇒ **different thermal shift** of the **gain** curve and of the **cavity modes**

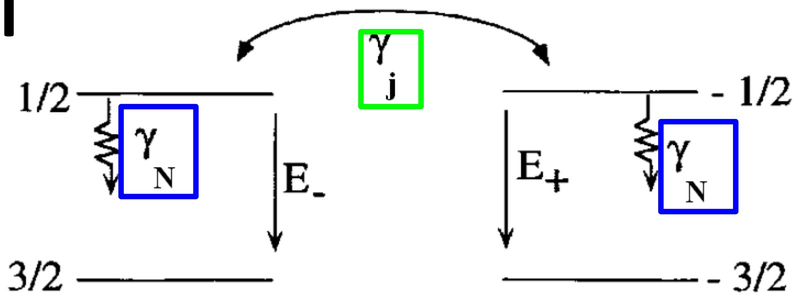
It explains **Y** (low freq)→ **X** (high freq) Type I PS only

Source: Y. Hong et al, Elec. Lett. (2000)

Source: M. Sciamanna PhD Thesis (2004)

VCSEL spin-flip model

Assumes a four-level system in which e/h with **spin down (up)** recombine to **right (left) circularly polarized** photons:



$$\frac{dE_\pm}{dt} = \kappa(1+i\alpha)(N_\pm - 1)E_\pm - (\gamma_a + i\gamma_p)E_\mp + D\xi_\pm$$

dichroism

birefringence

$$E_x = (E_+ + E_-)/\sqrt{2}$$

$$E_y = -i(E_+ - E_-)/\sqrt{2}$$

$$\frac{dN_\pm}{dt} = -\boxed{\gamma_N}(N_\pm - \mu) - \boxed{\gamma_j}(N_\pm - N_\mp) - 2\gamma_N \; N_\pm \,|E_\pm|^2$$

Carrier recombination

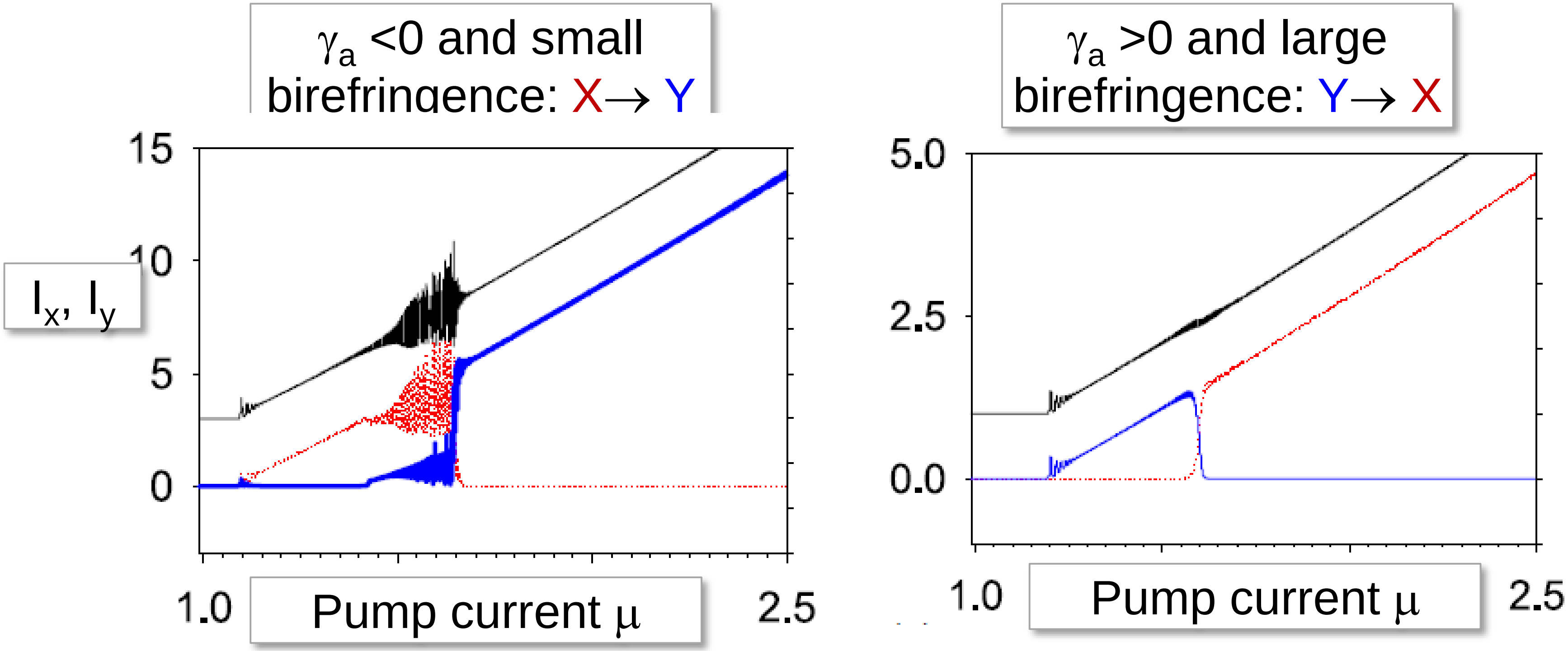
Pump: carrier injection

spin-flip rate

Stimulated recombination

Martin Regalado et al, JQE 33, 765 (1997).

The SFM model can explain both: Y → X and X→Y PSs



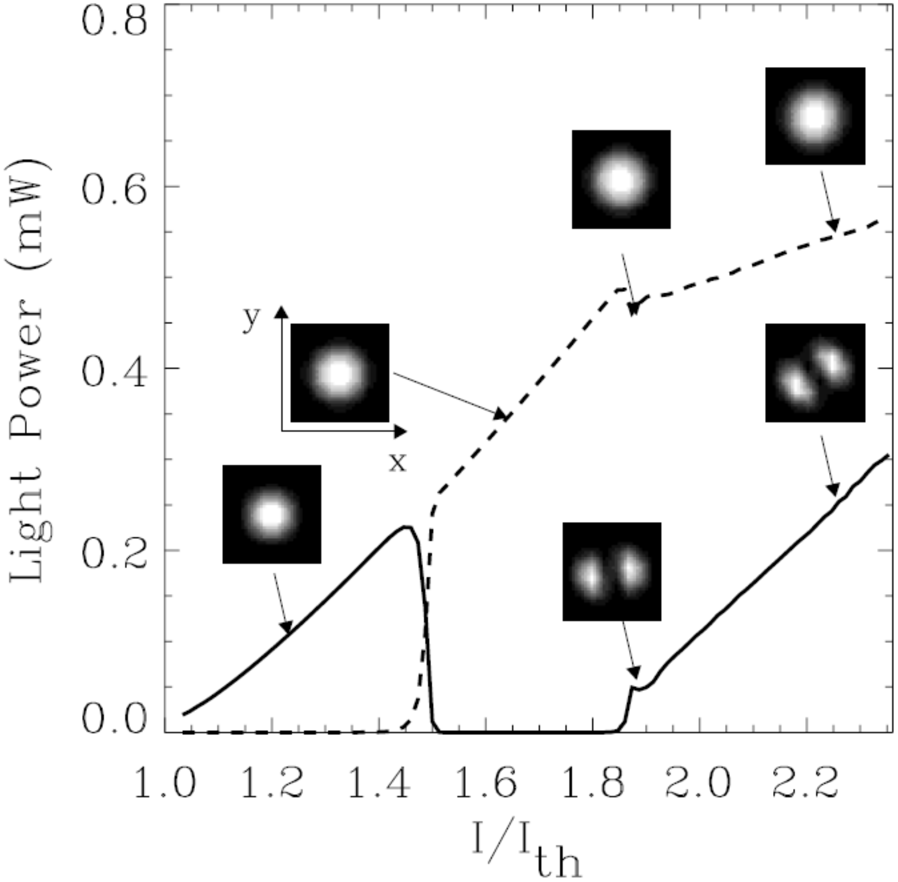
γ_a <0 and small birefringence: X→ Y

γ_a >0 and large birefringence: Y→ X

- The model also explains the stochastic PS.

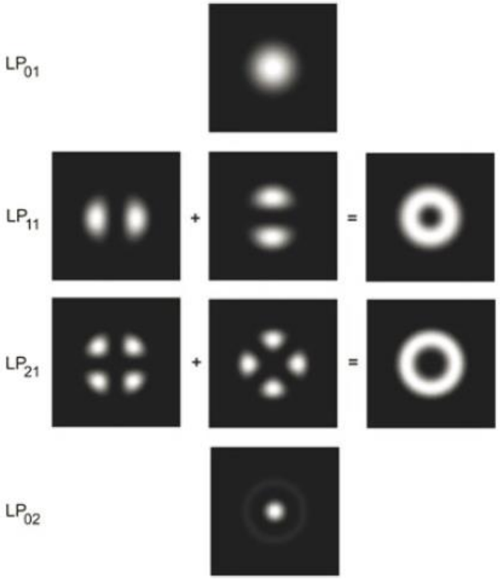
From M. S. Torre et al, PRA 74, 043808 (2006)

Transverse effects and polarization



When the first-order transverse mode starts lasing, it is, in general, orthogonally polarized to the fundamental transverse mode.

Partial differential equations allow to understand the interplay of polarization and transverse effects



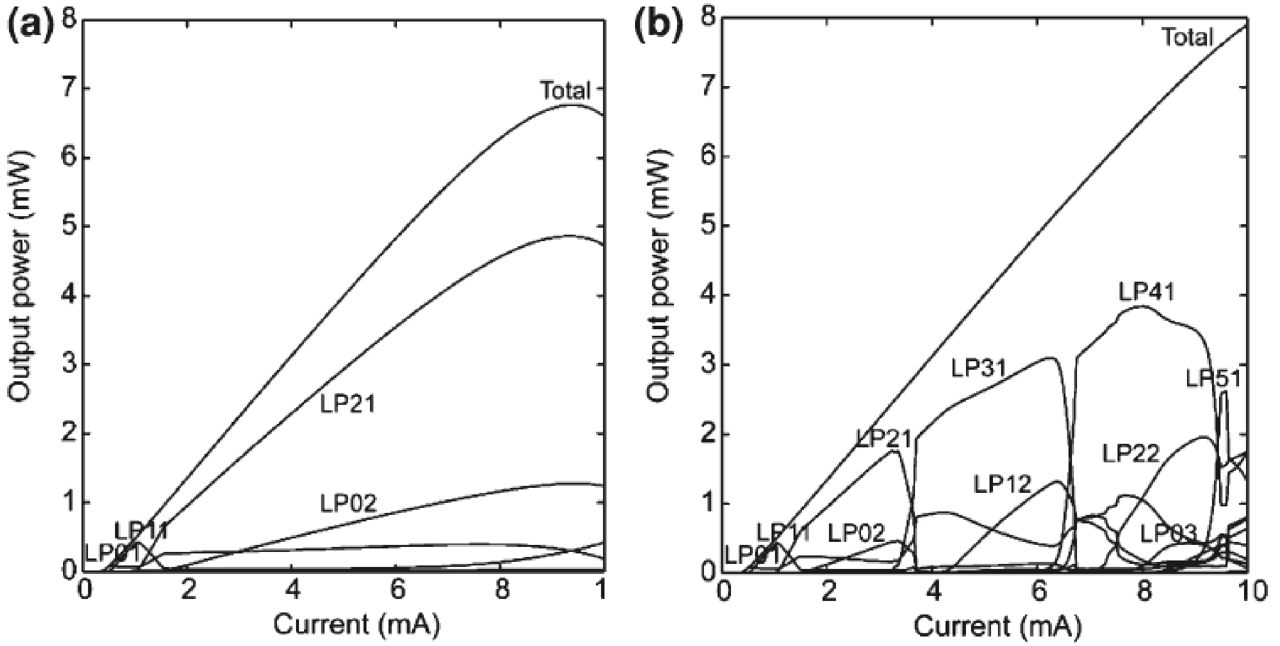


Fig. 4.4 Mode-resolved power-current characteristics for a 7 µm oxide-confined 850 nm VCSEL without (**a**), and with (**b**) effects of carriers and temperature accounted for

Larsson and Gustavsson, Chapter 4 in VCSELs Editor: Michalzik (2013)

Take home message

- Optical perturbations (injection from other laser, optical self-feedback) can be useful for a number of applications.

  For appropriated parameters:
  - Optical injection induces “injection locking”: the slave laser emits at the same frequency as the master laser.
  - Optical injection increases the relaxation oscillation frequency (and thus, the laser modulation bandwidth).
  - Optical injection can induce regular oscillations.
  - Optical feedback can induce single-mode emission and reduces the laser line width.
- However, both, feedback and injection can generate a chaotic output intensity oscillations
- Due to their circular cavity geometry VCSELs can display a complex interplay of transverse modes and polarization modes.

VF test

- In semiconductor laser models, the alpha factor takes into account phenomenologically the change in the refractive index induced by the variation of the carrier density.
- In the injection locking regime the laser emits its natural wavelength but with larger output power.
- Strong optical feedback can be used for achieving single mode emission.
- The external cavity modes are coexisting monochromatic steady state solutions, the emitted wavelength depends on the feedback parameters.
- In VCSELs the polarization switching can be due to thermal effects.
- In VCSELs a PS always occurs when the pump current is increased, and is accompanied by a change in the transverse optical mode.

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