Laser Models and Dynamics

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Outline

- Models
  - Intensity (Photon) rate equation model
  - Optical field rate equation model

- Dynamics
  - Current modulation
  - Optical injection
  - Optical feedback
  - Polarization instability

(Springer, 3rd ed. 2013, ebook available in UPC library)

Laser dynamics, T. Erneux & P. Glorieux (Cambridge University Press 2010)
Learning objectives

• Acquire a basic knowledge of the simplest laser rate equation model, for the photon and carrier densities.
• Understand the relaxation oscillations and dynamics during the laser turn on.
• Understand the small and large signal modulation response.
• Perform simple numerical simulations.
• Become familiar with the single-mode equation for the complex optical field.
• Understand the effects of optical perturbation.
• Acquire a basic knowledge of multi-mode models.
Semiconductor lasers are **class B** lasers

- Governed by **two rate-equations**: one for the photons ($S$) and one for the carriers ($N$).
- Display a stable output (with only transient relaxation oscillations).
- Single-mode “conventional” EELs diode lasers are class B lasers.
- Other class B lasers are ruby, Nd:YAG, and CO2 lasers.
- Because of the $\alpha$-factor (a specific feature of diode lasers, more latter) diode lasers display complex dynamics when they are optically perturbed.

<table>
<thead>
<tr>
<th>Laser</th>
<th>$\tau_p$ (s)</th>
<th>$\tau_n$ (s)</th>
<th>$\gamma = \tau_p / \tau_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO$_2$</td>
<td>$10^{-8}$</td>
<td>$4 \times 10^{-6}$</td>
<td>$2.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>solid state (Nd$^{3+}$:YAG)</td>
<td>$10^{-6}$</td>
<td>$2.5 \times 10^{-4}$</td>
<td>$4 \times 10^{-3}$</td>
</tr>
<tr>
<td>semiconductor (GaAs)</td>
<td>$10^{-12}$</td>
<td>$10^{-9}$</td>
<td>$10^{-3}$</td>
</tr>
</tbody>
</table>

$\tau_n =$ Carrier lifetime  
$\tau_p =$ Photon lifetime
Other types of lasers

• **Class A** (Visible He-Ne lasers, Ar-ion lasers, dye lasers): governed by one rate equation for the optical field (the material variables can be adiabatically eliminated), no oscillations.

• **Class C** (infrared He-Ne lasers): governed by three rate equations (N, S, P=macroscopic atomic polarization), display sustained oscillations and even a chaotic output. No commercial applications.
Dynamics of Class C, B and A lasers

infrared He-Ne lasers | Semiconductor, ruby, Nd:YAG, CO2 lasers | Visible He-Ne lasers, Ar-ion lasers, dye lasers

Fig. 1. Sketches of a typical trajectory approaching a stable fixed point in class-C, class-B, and class-A free-running lasers.
Diode lasers: electrical to power conversion

Injected electrical current
Carrier density
Net gain: emission – absorption
Photon density
Cavity losses

Net gain:
emission – absorption
Rate equation for the carrier density $N$

\[
\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau_N} - GS
\]

- **Pump**: injection of carriers
- **Recombination of carriers**
- **Stimulated emission & absorption**

**I**: Injection current ($I/eV$ is the number of electrons injected per unit volume and per unit time).

**$\tau_N$**: Carrier lifetime.

**G (N,S)**: Net gain
Rate equation for the photon density $S$

$$\frac{dS}{dt} = GS - \frac{S}{\tau_p} + \frac{\beta_{sp} N}{\tau_N}$$

$\tau_p$ : Photon lifetime.

$G (N,S)$ : Net gain

$\beta_{sp}$ : Spontaneous emission rate
Simple model for the semiconductor gain

We assume single-mode emission at $\lambda_0$. The differential gain coefficient $a$ depends on $\lambda_0$ (multi-mode model latter).

$$G = a(N - N_0)$$

$$G = aN_0 \ln \left( \frac{N}{N_0} \right)$$

RT InGaASP laser

Differential gain coefficient

Carrier density at transparency
Threshold carrier density

Threshold condition: net gain = cavity loss

\[ G(N_{th}) = \frac{1}{\tau_p} \]

\[ G = a(N - N_0) \]

\[ \Rightarrow \frac{1}{\tau_p} = a(N_{th} - N_0) \]

\[ \frac{dS}{dt} = GS - \frac{S}{\tau_p} + \frac{\beta_{sp}N}{\tau_N} \]

\[ G - \frac{1}{\tau_p} = a(N - N_0) - a(N_{th} - N_0) = a(N - N_{th}) \]

\[ \Rightarrow \frac{dS}{dt} = a(N - N_{th})S + \frac{\beta_{sp}N}{\tau_N} \]

Fig. 1.20 Optical gain \( g(N) \) vs. carrier density for an InGaAsP strained quantum well active layer (1.55 \( \mu \text{m} \)) at 20°C. The power gain is defined by \( \Gamma G(N) = \Gamma v_g g(N) \), where \( v_g \) is the photon group velocity (\( \sim 10^{10} \text{ cm s}^{-1} \)) and \( \Gamma \) is the confinement factor (\( \sim 0.1 \)) (redrawn from Figure 3.1 of Piprek and Bowers [45]).
SIMPLEST RATE EQUATION MODEL
-STEADY-STATE SOLUTIONS & LI CURVE
-TIME-DEPENDENT SOLUTIONS WHEN THE INJECTION CURRENT VARIES
• Ordinary differential equations (spatial effects neglected)
• Additional nonlinearities from carrier recombination and gain saturation

\[
\frac{dS}{dt} = GS - \frac{S}{\tau_p} + \frac{\beta_{sp} N}{\tau_N}
\]
\[
\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau_N} - GS
\]

\[
\frac{1}{\tau_N} = A + BN + CN^2
\]
\[
G = \frac{a(N - N_0)}{1 + \varepsilon S}
\]

• These equations allow to understand the LI curve and the modulation response.
• To understand the intensity noise and the line-width (the optical spectrum), we need a stochastic equation for the complex field E (S=|E|^2) (more latter).
• Spatial effects (diffraction, carrier diffusion) and thermal effects can be included phenomenologically.
Normalized equations

- Define the dimensionless variable:

\[
\frac{dS}{dt} = \frac{1}{\tau_p} (N' - 1)S + \frac{\beta_{sp} N'}{\tau_N}
\]

\[
\frac{dN'}{dt} = \frac{1}{\tau_N} (\mu - N' - N'S)
\]

Pump current parameter: proportional to \( I/I_{th} \)

- Normalizing the equations eliminates two parameters \( (a, N_0) \)
- In the following we drop the “ ‘ ”

\[
N' = \frac{N - N_0}{N_{th} - N_0}
\]

Threshold carrier density: gain = loss

\[
a(N_{th} - N_0) = \frac{1}{\tau_p}
\]
Role of spontaneous emission

\[ \frac{dS}{dt} = \frac{1}{\tau_p} (N - 1)S + \frac{\beta_{sp} N}{\tau_N} \]

- If at \( t=0 \) there are no photons in the cavity: \( S(0) = 0 \)

- Then, without noise \( (\beta_{sp}=0) \): if \( S=0 \) at \( t=0 \) \( \Rightarrow \frac{dS}{dt}=0 \)
  \( \Rightarrow S \) remains 0 (regardless the value of \( \mu \) and \( N \)).

- Without spontaneous emission noise the laser does not turn.
Steady state solutions with $\beta_{sp}=0$

(Simple expressions if $\beta_{sp}$ is neglected)

\[
\frac{dS}{dt} = \frac{1}{\tau_p} (N - 1)S
\]

\[
\frac{dN}{dt} = \frac{1}{\tau_N} (\mu - N - NS)
\]

\[
\frac{dS}{dt} = 0 \implies \begin{cases} S = 0 \\ N = 1 \end{cases}
\]

\[
\frac{dN}{dt} = 0 \implies \begin{cases} S = 0 \implies N = \mu \\ N = 1 \implies S = \mu - 1 \end{cases}
\]

Laser off

Stable if $\mu < 1$

$S = 0$

$N = \mu$

Laser on

Stable if $\mu > 1$

$S = \mu - 1$

$N = 1$

$\mu_{th} = 1$

Above threshold the carriers are “clamped”.
Graphical representation

Above threshold the carriers are "clamped".
Experimental LI curve

This LI curve is obtained from model simulations when $\beta_{sp}$ is not neglected.

$$\frac{dS}{dt} = a\left(N - N_{th}\right)S + \frac{\beta_{sp} N}{\tau_N}$$

Hitachi Laser Diode HL6724MG (A. Aragoneses PhD thesis 2014)
LI curve with $\beta_{sp} \neq 0$

\[
\frac{dS}{dt} = \frac{1}{\tau_p} (N-1)S + \frac{\beta_{sp} N}{\tau_N}
\]

\[
\frac{dN}{dt} = \frac{1}{\tau_N} (\mu - N - NS)
\]

\[
\gamma = \frac{\tau_p}{\tau_N} \quad t' = \frac{t}{\tau_p}
\]

\[
\frac{dS}{dt'} = (N - 1)S + bN
\]

\[
\frac{dN}{dt'} = \gamma (\mu - N - NS)
\]

Steady-state solution:

\[
S = \frac{1}{2} \left[ (\mu - 1) + \sqrt{(\mu - 1)^2 + 4b\mu} \right]
\]

- If $\mu > 1$ $S \approx \mu - 1$
- If $\mu < 1$ $S \approx 1/|\mu - 1|$
“kink” of the LI curve

- If $\mu > 1$ $S \approx \mu - 1$
- If $\mu < 1$ $S \approx 1/|\mu - 1|$

$\mu_{th} = 1$

GaAs diode laser (1982)

Model for a multi-mode laser

Gain coefficient for mode $j$:

\[ G_{n,j} = G_n \left\{ 1 - \left( \frac{j}{M} \right)^2 \right\} \]

\[
\frac{dS_j(t)}{dt} = \left[ G_{n,j} \{n(t) - n_{th,j}\} \right] S_j(t) + R_{sp}(\omega_j)
\]

\[
\frac{dn(t)}{dt} = \frac{J(t)}{ed} - \frac{n(t)}{\tau_s} - \sum_{j=-M}^{M} G_{n,j} \{n(t) - n_0\} S_j(t)
\]

Carrier density ($n$) + several photon densities (for each longitudinal mode)
SIMPLEST RATE EQUATION MODEL
-STEADY-STATE SOLUTIONS & LI CURVE
-TIME-DEPENDENT SOLUTIONS WHEN THE INJECTION CURRENT VARIES
Time variation of the injection current

\[ \frac{dN}{dt} = \frac{1}{\tau_N} (\mu - N - NS) \]

\[ \frac{dS}{dt} = \frac{1}{\tau_p} (N - 1)S + \frac{\beta_{sp} N}{\tau_N} \]

- **Step** (laser turn on): \( \mu_{\text{off}}, \mu_{\text{on}} \)

- **Triangular** (dynamic LI curve): \( \mu_{\text{min}}, \mu_{\text{max}}, T_{\text{ramp}} \)

- **Sinusoidal** (modulation response): \( \mu_{\text{dc}}, A, T_{\text{mod}} \)

Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_p )</td>
<td>1 ps</td>
</tr>
<tr>
<td>( \tau_N )</td>
<td>1 ns</td>
</tr>
<tr>
<td>( \beta_{sp} )</td>
<td>( 10^{-4} )</td>
</tr>
</tbody>
</table>
Current step: turn-on delay & relaxation oscillations

A linear stability analysis of the rate equations allows to calculate the RO frequency

\[ \omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}} \]

Source: J. Ohtsubo, *Semiconductor lasers*
Variation of the relaxation oscillation frequency with the output power

\[ \omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}} \]

Laser on: \[ S \approx \mu - 1 \]

\[ \omega_{RO} \approx \sqrt{\frac{S}{\tau_p \tau_N}} \]

Fig. 1.5 Square of the relaxation oscillation frequency \( f_R \) vs. pump power \( P \) for an erbium doped fiber laser. Adapted Figure 4 from Sola et al. [35] with

Turn-on transient of a Nd$^{3+}$:YAG laser

Note the time-scale: for diode lasers is a few ns

Phase-space representation

For the Nd$^{3+}$:YAG laser

What is $D$? \[ D = \frac{dI}{dt} / I + 1 \]
Turn-on of a multi-mode laser

Parabolic gain profile: \[ G_{n,j} = G_n \left\{ 1 - \left( \frac{j}{M} \right)^2 \right\} \]

- **Winner takes all:** after transient “mode-competition”, the mode with maximum gain coefficient wins.
- But non-transient competition has been observed.
- More advanced gain models allow explaining non-transient competition.

Source: J. Ohtsubo, *Semiconductor lasers*
Triangular signal: LI curve

Slow “quasi-static” current ramp (T=200 ns)
With a fast ramp: turn-on delay

Source: Tredicce et al, Am. J. Phys., Vol. 72, No. 6, 2004
Dynamical hysteresis

Simulations

Experiments

Pump $\mu$

$f(r)$

$I$ (norm. units) vs Pump (V)
Influence of gain saturation

\[
\frac{dS}{dt} = \frac{1}{\tau_p} (G - 1)S + \frac{\beta_{sp} N}{\tau_N}
\]

\[
\frac{dN}{dt} = \frac{1}{\tau_N} (\mu - N - GS)
\]

\[
G(N, S) = \frac{N}{1 + \varepsilon S}
\]

\varepsilon = 0

\varepsilon = 0.01

Gain saturation takes into account phenomenologically several effects (spatial and spectral hole burning, thermal effects)

Damped relaxation oscillations
Injection current modulation

Digital

Analog

Direct modulation allows to encode information in the laser output power ("amplitude modulation")
**Weak sinusoidal modulation: influence of the modulation frequency**

\[ \mu = \mu_{dc} + A \sin \omega_{mod} t \]

\[ \mu_{dc} = 1.5, \ A=0.1 \]

For \( \mu=1.5 \): \( \nu_{RO} = 3.56 \) GHz

The laser intensity (\( S = \text{photon density} \)) is modulated at the same frequency of the pump current (\( \mu \)), but the phase of the intensity and the current are not necessarily the same.
Small-signal modulation response

Analytical expressions can be calculated from the linearization of the rate equations.

\[ \omega_{\text{mod}} / \omega_{\text{RO}} \]

Oscillation amplitude

diode-pumped Nd\(^{3+}\):YAG laser

Modulation frequency (KHz)
Resonance at $\omega_{\text{mod}} = \omega_{\text{RO}}$

For a semiconductor laser (VCSEL)

Source: R. Michalzik, VCSELs (2013)
Large-signal modulation response

\[
\mu(t) = \mu_0 \left[ 1 + m \sin(\omega_{\text{mod}} t) \right]
\]

Source: J. Ohtsubo, *Semiconductor lasers*
Hysteresis induced by strong modulation

• The figures represent the maxima of the oscillations as a function of the normalized frequency near the onset of hysteresis (left), and far away from the onset of hysteresis (right).

• Hysteresis cycle obtained by slowly changing the modulation frequency forward (full line) and then backward (broken line).

• The additional smaller jump near $\omega_{\text{mod}}/\omega_{\text{RO}} = 1/2$ is a signature of another resonance.

• The laser is a Nd$^{3+}$:YAG laser subject to a periodically modulated pump.
Summary

The simple rate equation model for the photon and carrier densities allows to understand the main features of the laser dynamics when the injection current varies:

— The turn on delay & relaxation oscillations

— The LI curve (static & dynamic)

— The modulation response (small and large signal response)
Semiconductor lasers are described by two rate equations, one for the photon density and another for the carrier density.

The laser relaxation oscillation (RO) frequency is proportional to the injected current.

The delay in the laser turn-on is independent of the current ramp.

The modulation response has a resonance at the RO frequency.

Small and large amplitude modulation result in sinusoidal oscillation of the output power.

The relative phase of the output power and input signal depends on the modulation frequency.

In a multimode model with a parabolic gain profile, mode competition is a transient dynamics.
RATE EQUATION MODEL FOR A SINGLE-MODE COMPLEX OPTICAL FIELD

- ALPHA FACTOR, LINEWIDTH & INTENSITY NOISE
- OPTICAL PERTURBATIONS (INJECTION, FEEDBACK)
- POLARIZATION INSTABILITIES
Laser linewidth

Schematic representation of the change of magnitude and phase of the lasing field $E$ due to the spontaneous emission of one photon.

$$S = |E|^2$$

$$E = E_x + iE_y$$

- The line-width of gas and solid-state lasers is well described by the classic Schawlow-Townes formula ($\Delta f \sim 1/P$)
Schawlow-Townes formula

\[ \Delta \nu_{ST} = \frac{\pi h \nu (\Delta \nu_c)^2}{P_{out}} \]

Physical interpretation:

- In each round-trip, some noise (spontaneous emission) is added to the circulating field.
- It changes the amplitude and the phase of the field.
- Amplitude fluctuations are damped: the power returns to values close to the steady state.
- For phase fluctuations, there is no restoring force.
- Therefore, the phase undergoes a random walk, which leads to phase noise, which causes a finite line-width.

\[ \Delta \nu_c = \text{half-width of the cavity resonance} \]

But the line-width of semiconductor lasers is significantly higher.

https://www.rp-photonics.com/schawlow_townes_linewidth.html
• In diode lasers the enhancement of the line-width is due to the dependence of the refractive index (n) on the carrier density (N)

\[ \Delta S \rightarrow \Delta N \rightarrow \Delta n \rightarrow \Delta \phi \]

• Henry introduced a \textit{phenomenological} factor (\(\alpha\)) to account for amplitude–phase coupling.

• The \textbf{linewidth enhancement factor} \(\alpha\) is a very important parameter of semiconductor lasers. Typically \(\alpha = 2-5\)

\[ \Delta \nu = \left(1 + \alpha^2\right) \Delta \nu_{ST} \]
Single-mode \textit{slowly-varying} optical field

Photon density

\[ S = \left| E \right|^2 \]

Complex field

\[ E(t) = E(t)e^{i\omega_0 t} \]

\[ E = E_x + iE_y \]

\[ \frac{dS}{dt} = \frac{1}{\tau_p} (N - 1)S + \frac{\beta_{sp} N}{\tau_N} \]

\[ \frac{dE}{dt} = \frac{1}{2\tau_p} (1 + i\alpha)(N - 1)E + \sqrt{\frac{\beta_{sp} N}{\tau_N}} \xi \]

\[ \xi = \xi_x + i\xi_y \]

\[ k = \frac{1}{2\tau_p}, \quad D = \frac{\beta_{sp} N_0}{\tau_N} \]

\[ \frac{dE_x}{dt} = k(N - 1)(E_x - \alpha E_y) + \sqrt{D} \xi_x \]

\[ \frac{dE_y}{dt} = k(N - 1)(\alpha E_x + E_y) + \sqrt{D} \xi_y \]

Derivation of the equations: Ohtsubo Cap. 2

Langevin stochastic term: complex, uncorrelated, Gaussian white noise
Rate equation for optical phase

\[
\begin{align*}
\frac{dE_x}{dt} &= k(N-1)(E_x - \alpha E_y) + \sqrt{D}\xi_x \\
\frac{dE_y}{dt} &= k(N-1)(\alpha E_x + E_y) + \sqrt{D}\xi_y \\
E &= E_x + iE_y = \sqrt{S}e^{i\phi}
\end{align*}
\]

\[
\begin{align*}
\frac{dS}{dt} &= \frac{1}{\tau_p}(N-1)S + D + \xi_s(t) \\
\frac{d\phi}{dt} &= \frac{\alpha}{2\tau_p}(N-1) + \xi_\phi(t) \\
\frac{dN}{dt} &= \frac{1}{\tau_N}(\mu - N - GS)
\end{align*}
\]

- The instantaneous frequency depends on the carrier density.
- \( \phi \) is a “slave” variable.
- The noise sources are not independent.
These equations allow to understand the **optical spectrum** of a semiconductor laser

- In EELs: \( L = 200–500 \, \mu m \Rightarrow \Delta v = 100–200 \, \text{GHz} \). Because the gain bandwidth is 10–40 THz \( \Rightarrow 10–20 \) longitudinal modes.
- The line-width of each longitudinal mode depends on the alpha-factor and is of the order of 10 MHz.

Multimode optical spectra

Single-mode optical spectra
(Source: J. M. Liu, *Photonic devices*)
These equations also allow to understand the Relative Intensity Noise (RIN): \( \text{FFT of } S(t) \)

The laser output intensity is detected by a photo-detector, converted to an electric signal and sent to a RF spectrum analyzer. The RIN is a measure of the relative noise level to the average dc power.

\[
\omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}}
\]

Peak at the relaxation oscillation frequency

Source: J. Ohtsubo, *Semiconductor lasers*

Source: R. Michalzik, *VCSELs*
RATE EQUATION MODEL FOR A SINGLE-MODE COMPLEX OPTICAL FIELD

- ALPHA FACTOR, LINEWIDTH & INTENSITY NOISE
- OPTICAL PERTURBATIONS (INJECTION, FEEDBACK)
- POLARIZATION INSTABILITIES
Optical perturbations

“Solitary” diode lasers display a stable output (only transient oscillations) but they can be easily perturbed by injected light and can display sustained periodic or irregular oscillations.
Optical Injection

- Two Parameters:
  - Injection ratio
  - Frequency detuning $\Delta \nu = \nu_s - \nu_m$

- Dynamical regimes:
  - Stable locking (cw output)
  - Periodic oscillations
  - Chaos
  - Beating (no interaction)

Source: J. Ohtsubo, *Semiconductor lasers*
Model for the injected laser

Optical field $E(t) = E(t) \exp (i\omega_s t)$; $E(t) = $ slowly varying amplitude

Without injection:

$$\frac{dE}{dt} = \frac{1}{2\tau_p} (1 + i\alpha)(N - 1)E + \sqrt{D}\xi$$

With injection:

$$\frac{dE}{dt} = \frac{1}{2\tau_p} (1 + i\alpha)(N - 1)E + i\Delta\omega E + \sqrt{P_{\text{inj}}} + \sqrt{D}\xi(t)$$

$$\frac{dN}{dt} = \frac{1}{\tau_N} \left( \mu - N - N|E|^2 \right)$$

$\mu$: pump current parameter

optical injection from master laser

$P_{\text{inj}}$: injection strength

$\Delta\omega = \omega_s - \omega_m$: detuning

spontaneous emission noise

Typical parameters:

$\alpha = 3$, $\tau_p = 1$ ps,
$\tau_N = 1$ ns, $D = 10^{-4}$ ns$^{-1}$
Injection locking

Beat frequency $\Omega=0$

$\Delta \nu = \nu_s - \nu_m$

Model prediction: the locking range is proportional to the relative injection strength.

Experimental verification

Nd$^{3+}$:YAG laser
Injection locking increases the resonance frequency and the modulation bandwidth.
Lasers for Communications: Optical injection locking brings back direct-modulation telecom lasers

03/05/2015
By Gail Overton
Senior Editor

While direct modulation of a semiconductor laser's drive current enables fiber-optic communications at speeds of around 2.5 Gbit/s, higher-speed operation using direct modulation has historically been confounded by frequency chirp, forcing commercial 10 Gbit/s systems to use an external electro-optic modulator. For increased capacity (100 Gbit/s and beyond), coherent systems were recently commercialized that also use external electro-optic modulators to deliver complex modulation formats.

New research from the University of Southampton's Optoelectronics Research Centre (ORC; England) and Eblana Photonics (Dublin, Ireland), however, has skirted this historical limitation through a modulator-free, optical-injection-locking method that brings the benefits of direct modulation back to the modern coherent telecommunications network.
Outside the injection locking region: regular intensity oscillations

The frequency of the intensity oscillations, $f_0$, can be controlled by tuning the injection strength and the detuning.

S-C Chan et al, Optics Express 15, 14921 (2007)
Also outside the locking region: ultra-high intensity pulses ("optical rogue waves")

Distribution of pulse amplitudes for different injection currents

Time series of the laser intensity

C. Bonatto et al, PRL 107, 053901 (2011),
Optics & Photonics News February 2012,
Research Highlight in *Nature Photonics* DOI:10.1038/nphoton.2011.240
Optical feedback regimes

- **Regime I**: line-width narrowing/broadening (depending on the phase of feedback),
- **Regime II**: mode-hopping,
- **Regime III**: single-mode narrow-line operation,
- **Regime IV**: coherence collapse,
- **Regime V**: single-mode operation in an extended cavity mode.

Feedback induced instabilities

negligible small feedback

periodic oscillations with weak feedback

chaotic oscillations with strong feedback (coherence collapse, Regime IV)

Source: J. Ohtsubo, *Semiconductor lasers*
Optical feedback effects on the LI curve

Coherent feedback

Incoherent feedback

Feedback-reduced threshold: the amount of reduction quantifies the strength of the feedback.

Adapted from R. Ju et al,

Close to the laser threshold: Low Frequency Fluctuations (LFFs)

How these dropouts develop?
watch https://youtu.be/nltBQG_IIWQ

I. Fischer et al, PRL 1996
LFFs: Complex dynamics, several time-scales

Recovery after a dropout: in steps of $\tau$ with relaxation oscillations

$$\tau = \frac{2L_{\text{ext}}}{C}$$

If $L_{\text{ext}} = 1 \text{ m} \Rightarrow \tau = 6.7 \text{ ns}$

Single-mode Lang and Kobayashi Model

Optical field \( E(t) = E(t) \exp (i\omega_0 t) \); \( E(t) = \) slowly varying amplitude

**Solitary laser**

\[
\frac{dE}{dt} = k(1 + i\alpha)(N - 1)E + \sqrt{D}\xi
\]

\( k = \frac{1}{2\tau_p} \)

\( D = \frac{\beta_{sp}}{\tau_N} \)

**With optical feedback**

\[
\frac{dE}{dt} = k(1 + i\alpha)(G - 1)E + \eta E(t - \tau)e^{-i\omega_0 \tau} + \sqrt{D}\xi
\]

\[
\frac{dN}{dt} = \frac{1}{\tau_N} \left( \mu - N - G|E|^2 \right)
\]

\[\tau = \frac{2L_{ext}}{c}\]

\( \eta = \) feedback strength

\( \tau = \) feedback delay time

\( \mu = \) pump current

*(control parameters)*

External cavity modes (ECMs)

- \( E(t) = E(t) \exp(i\omega_0 t) \)
  
  \[ E(t) = E_0 \exp[i(\omega-\omega_0)t] \; ; \; N(t) = N \]

\( \Rightarrow \) **Monocromatic solutions**: \( E(t) = E_0 \exp (i \omega t) \)
- Stable modes (constructive interference)
- Unstable models (destructive interference)

\[ \omega_0 \tau = \omega \tau + C \sin(\omega \tau + \arctan \alpha) \]

\[ C = \eta \tau \sqrt{1 + \alpha^2} \]

\[ N = 1 - \frac{\eta}{k} \cos(\omega \tau) \]

\[ |E_0|^2 = \frac{\mu - N}{N} \]

- The number of ECMs increases with:
  - The feedback strength
  - The length of the external cavity
Good agreement model-experiments

- With a “fast” detector: pulses; with a “slow” detector: dropouts

**Experiments**

**Stochastic simulations**

**Deterministic simulations**

I. Fischer et al, PRL 1996
Physics and applications of laser diode chaos

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This Review Article provides an overview of chaos in laser diodes by surveying experimental achievements in the area and explaining the theory behind the phenomenon. The fundamental physics underpinning laser diode chaos and also the opportunities for harnessing it for potential applications are discussed. The availability and ease of operation of laser diodes, in a wide range of configurations, make them a convenient testbed for exploring basic aspects of nonlinear and chaotic dynamics. It also makes them attractive for practical tasks, such as chaos-based secure communications and random number generation. Avenues for future research and development of chaotic laser diodes are also identified.
An example of an application of feedback-induced chaos: random bit generation

After processing the signal, arbitrarily long sequences can be generated at the 12.5-Gbit/s rate.

Source: I. Kanter et al, Nature Photonics 2010 & OPN
Figure 5 | The state-of-the-art of random number generation using chaos from a laser diode. Realizations differ by either the system under investigation, the post-processing method, the number of bits and/or the sampling rate. Each point corresponds one row, or multiple rows, in Table 1.
Semiconductor laser linewidth reduction by six orders of magnitude via delayed optical feedback

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Summary: optical feedback effects

- Coherent feedback reduces the lasing threshold.

- Feedback effects depend on the feedback strength, the feedback delay time and feedback phase.

- Weak feedback introduces “external cavity modes”: stable or unstable solutions of the rate equations.
RATE EQUATION MODEL FOR A SINGLE-MODE COMPLEX OPTICAL FIELD

- ALPHA FACTOR, LINEWIDTH & INTENSITY NOISE
- OPTICAL PERTURBATIONS (INJECTION, FEEDBACK)
- POLARIZATION INSTABILITIES
EELs: linearly (TE) polarized output

Polarization switching can be induced by polarization-rotated feedback.
In some VCSELs: polarization switching (PS)

- **Circular** cavity geometry: two linear orthogonal modes (x, y).
- Often there is a polarization switching when the pump current is increased.
- Also **hysteresis**: the PS points for increasing and for decreasing current are different.
- The total output power increases/decreases monotonically with pump.

Source: Hong and Shore, Bangor University, Wales, UK
Polarization-resolved LI curve

Current-driven PS

Stochastic PS

Stochastic PS

- Anti-correlated fluctuations of the two polarizations.

- Bistability + noise induced switching
Current-driven PS

- Type 1: from the Y (low freq) → X (high freq) polarization
- Type 2: from the X (high freq) → Y (low freq) polarization

Several models have been proposed to explain these PS

Thermal shift of the gain curve

When the pump current increases $\Rightarrow$ Joule heating $\Rightarrow$ different thermal shift of the gain curve and of the cavity modes

Birefringence: the polarizations have different optical frequencies

It explains Y (low freq) $\rightarrow$ X (high freq) Type I PS only

VCSEL spin-flip model

Assumes a four-level system in which e/h with **spin down (up) recombine to right (left) circularly polarized** photons:

$$\frac{dE}{dt} = \kappa (1 + i\alpha)(N - 1)E - (\gamma_a + i\gamma_p)E_D + D\xi$$

**dichroism**

**birefringence**

$$\frac{dN}{dt} = -\gamma_N(N - \mu) - \gamma_j(N - N_D) - 2\gamma_N N |E|^2$$

**Carrier recombination**

**Pump: carrier injection**

**spin-flip rate**

**Stimulated recombination**

$$E_x = (E_+ + E_-)/\sqrt{2}$$

$$E_y = -i(E_+ - E_-)/\sqrt{2}$$

The SFM model can explain both: 

\[ Y \rightarrow X \text{ and } X \rightarrow Y \] PSs

- \( \gamma_a < 0 \) and small birefringence: \( X \rightarrow Y \)
- \( \gamma_a > 0 \) and large birefringence: \( Y \rightarrow X \)

The model also explains the stochastic PS.

From M. S. Torre et al, PRA 74, 043808 (2006)
Transverse effects and polarization

When the first-order transverse mode starts lasing, it is, in general, orthogonally polarized to the fundamental transverse mode.
Partial differential equations allow to understand the interplay of polarization and transverse effects.

Fig. 4.4 Mode-resolved power-current characteristics for a 7 μm oxide-confined 850 nm VCSEL without (a), and with (b) effects of carriers and temperature accounted for.

Larsson and Gustavsson, Chapter 4 in VCSELs Editor: Michalzik (2013)
Take home message

- Optical perturbations (injection from other laser, optical self-feedback) can be useful for a number of applications.
  For appropriated parameters:
    - Optical injection induces “injection locking”: the slave laser emits at the same frequency as the master laser.
    - Optical injection increases the relaxation oscillation frequency (and thus, the laser modulation bandwidth).
    - Optical injection can induce regular oscillations.
    - Optical feedback can induce single-mode emission and reduces the laser line width.

- However, both, feedback and injection can generate a chaotic output intensity oscillations

- Due to their circular cavity geometry VCSELs can display a complex interplay of transverse modes and polarization modes.
In semiconductor laser models, the alpha factor takes into account phenomenologically the change in the refractive index induced by the variation of the carrier density.

In the injection locking regime the laser emits its natural wavelength but with larger output power.

Strong optical feedback can be used for achieving single mode emission.

The external cavity modes are coexisting monochromatic steady state solutions, the emitted wavelength depends on the feedback parameters.

In VCSELs the polarization switching can be due to thermal effects.

In VCSELs a PS always occurs when the pump current is increased, and is accompanied by a change in the transverse optical mode.
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