Semiconductor Lasers and Applications

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Outline

Part 1
1. Introduction to semiconductor lasers
2. Simplest model and dynamics with time-varying current parameter

Part 2
1. Applications of semiconductor lasers
2. More complicated models and nonlinear dynamics
Bibliography

• Saleh and Teich, *Fundamentals of photonics* (Wiley)

• J. Ohtsubo, *Semiconductor lasers: stability, instability and chaos* (Springer)


• R. Michalzik, *VCSELs* (Springer 2013)
Part 1: Introduction to SCLs

**Main goals**: acquire a basic knowledge of

- Historical development of SCLs & why they are important
- Types of SCLs & design goals
  - Cavity geometries: DFBs & DBRs; EELs & VCSELs, etc.
  - Gain medium: Bulk, QW, QDs, etc
2012: 50th anniversary of the semiconductor laser (SCL)

- First demonstration: 1962 (pulsed operation, cryogenic temperatures).
- cw RT emission: 1970
- In the 60’ & 70’: SCLs where “a solution looking for a problem”.
- The first practical application: February 1980, an optical fiber system was used to broadcast TV (Winter Olympics, Lake Placid, US).

Source: Optics & Photonics News May 2012
What are the applications of SCLs?

Impact of lasers (all types) in the US economy. Adapted from F. Schlachter & T. Baer (LaserFest 2010)
What is a diode laser?

• It is an **electrically pumped** semiconductor laser.

• As any laser, a diode laser requires a **gain medium** within an **optical cavity** (an exception to this are random lasers, which do not require a cavity).

• The semiconductor **band-gap** controls the emission **wavelength**.
After 50 years diode lasers dominate the laser market

- They enable the development of key transformation technologies with huge social impact.

Source: Laserfocusworld.com
Main applications (more in Part 2)

- Optical fiber communications
- Optical storage

No diode laser

⇒ No internet!

- But diode lasers are also widely used in printers, scanners, sensors, pumping of solid-state lasers, etc.
- A dramatic reduction of the fabrication price made possible these applications

The diode laser in a computer mouse costs about 10 US cents

Source: Laserfocusworld.com
Why are diode lasers so successful?

- The semiconductor medium has **huge gain** & do not require fragile enclosures or mirror alignment (the laser cavity is composed by the two facets of the semiconductor).

- **Low cost** fabrication because of existing semiconductor technology.

- Compared to other lasers, diode lasers are **very efficient** (nowadays 100% for the output photons with respect to the injected electrons).

- **Bright output** considering their **small size**.

- **Low threshold** current, **low energy consumption**.
Advantages for telecom applications

- Diode lasers can be **modulated at high speeds**: fast response to high-frequency information-modulated currents.

- Semiconductor materials provide a **wide range of wavelengths**. In particular, in the infrared region where silica optical fiber has minimum dispersion or transmission loss.

- Easy integration in 1D & 2D arrays.

VCSELs with diameters between 1 and 5 µm. Adapted from Saleh and Teich
Diode lasers can also provide high output power

- Diode lasers are used to pump **solid-state lasers**, such as the Nd:YAG. Laser diodes are tuned to the absorption band of the crystal providing efficient pumping.

- Also used to pump **Erbium Doped Fiber Amplifiers** (EDFAs), which allow for the amplification of signals in long distance fiber-optic links.
Laser Diode Evolution

- Scientific Discovery
- Fabrication Technologies
- Application Explosion
- Integration Age

Adapted from D. Welch, Infinera
Semiconductors

Adapted from J. Faist, course on quantum electronics, ETHZ

16/01/2014

Adapted from J. Faist, course on quantum electronics, ETHZ
Which is the difference between a 2-level system and a semiconductor?

In a 2-level system: non interacting particles & individual energy levels

A particle in an excited state decays emitting a photon

In a semiconductor: electron/hole pairs & energy bands

An electron in the CB and a hole in the VB recombine emitting a photon

Conservation of momentum: \( p_e \approx p_h \) (\( p_{\text{photon}} \approx 0 \)) \( \Rightarrow k_e \approx k_h \)

\( \Rightarrow \) Optical transitions are \textbf{vertical} in k space

Adapted from Saleh and Teich & W Coomans PhD thesis
Optical transitions in a semiconductor

**Photo-detectors**

The absorption of a photon results in the generation of an e/h pair.

**LEDs**

The spontaneous recombination of an e/h pair results in the spontaneous emission of a photon.

**Diode lasers & amplifiers**

The stimulated recombination of an e/h pair by a photon results in the emission of an identical photon.

Adapted from Saleh and Teich
Direct and indirect semiconductors

Direct optical transitions (GaAs):
- efficient photon sources

Indirect optical transitions (Si, Ge):
- inefficient photon sources
  (but efficient photo-detectors)

Almost all the III–V semiconductors can be used to fabricate semiconductor lasers

16/01/2014

Adapted from J. Faist, course on quantum electronics, ETHZ
2-level system vs. a semiconductor

In a 2-level system:

\[ N_e \quad \text{and} \quad N_g \]

For lasing we need **population inversion** \( (N_e > N_g) \)

In a semiconductor:

Charge neutrality \( N_e \approx N_h = N \)

For lasing: \( N > N_0 \)

\[ N_0 = \text{transparency value} \]

Threshold value \( N_{th} > N_0 \)
Diode lasers: electrical to optical power conversion

Injected electrical current

Carrier density: *electrons and holes* in the active region

Photon density

In modern lasers the efficiency approaches one output photon for each injected electron
Light vs. Input (LI) current curve

- The laser efficiency in converting electrical power to optical power is determined by the slope of the LI curve, $\Delta P_0/\Delta I$.

- Another measure: overall quantum efficiency (also called the power-conversion efficiency or wall-plug efficiency): the emitted optical power, $P_0$ / the applied electrical power, $iV$.

Nonlinearity at high currents leads to saturation (shown latter).

Adapted from Saleh and Teich.
How does a diode laser work?

**Homo-structure lasers (early lasers)**

- **p doped:** extra holes
- **n doped:** extra electrons

The **diffusion** of electrons and holes creates the "**depletion layer**" (e/h are within the same region).

Adapted from Saleh and Teich
Forwards bias decreases the potential barrier; reverse bias increases the potential barrier.

\[ L_n, L_p = \text{diffusion lengths} \]
\[ \tau_n, \tau_p = \text{recombination times} \]

\[ \Rightarrow \text{The p-n junction acts as a diode} \]

Source: K. Kieu (University of Arizona)
The depletion layer is also a wave guide for the generated photons.

The electron/hole concentration in the depletion layer modifies the refracting index, creating a wave guide that helps to confine the photons.

Refractive index

Distribution of photons

Adapted from Saleh and Teich
How to improve the gain & the optical confinement?

**Hetero-structure lasers (2nd generation)**

Semiconductors with different band-gaps: improved e/h confinement

From K. Kieu (University of Arizona)
Double Hetero-structure (DH) laser diodes

- **Improved photon confinement** in the GaAs active region due to the larger index of refraction of GaAs ($n = 3.6$) compared to the $p$- and $n$- cladding layers ($n = 3.4$).

- **Improved carrier confinement** in the GaAs active region due to the smaller band gap ($E_g \approx 1.5$ eV) of the GaAs compared to the $p$- and $n$- cladding layers ($E_g \approx 1.8$ eV).

Source: Thorlabs tutorial
Drawback of DH lasers

More complicated to fabricate!

Require **strict matching** conditions between the two semiconductor layers (the **lattice constant** and the **thermal expansion coefficient**).

Adapted from Bhattacharya, *Optoelectronic devices*

Adapted from D. Welch, Infinera
The 2000 Nobel Prize in Physics

The improved photon – electron/hole confinement of double hetero-structure lasers allowed for cw RT emission, enabling the development of technologies with huge social impact.

“For basic work on information and communication technology"

“For developing semiconductor heterostructures used in high-speed opto-electronics"

Zhores I. Alferov
laffe Physico-Technical Institute, St. Petersburg Russia

Herbert Kroemer
University of California USA
Heterostructures are grown epitaxially, as lattice-matched layers of one semiconductor material over another, by

- molecular-beam epitaxy (MBE) uses molecular beams of the constituent elements in a high-vacuum environment,
- liquid-phase epitaxy (LPE) uses the cooling of a saturated solution containing the constituents in contact with the substrate (but layers are thick)
- vapor-phase epitaxy (VPE) and metal-organic chemical vapor deposition (MOCVD) use gases in a reactor.

The performance of early laser diodes was limited by manufacturing techniques.
The compositions and dopings of the individual layers are determined by manipulating the arrival rates of the molecules and the temperature of the substrate surface.

Individual layers can be made very thin (atomic layer accuracy)
FABRICATION STEPS FOR A SEMICONDUCTOR LASER

1- SUBSTRATE

2- EPITAXIE

3- LASER PROCESSING

4- FACETS CLEAVING

5- SINGLE CHIP PREPARATION

6- MOUNTING, BONDING

Adapted from J. Faist, ETHZ
And the final step: packaging

- Packaging allows **integrating laser diodes in devices**
  - Mechanical and optical coupling to an optical fiber
  - Temperature stabilization
  - Photodiode for monitoring of the optical power, with respect to pump current level.
  - Optical Isolation (avoid back reflections from the fiber)
- But: significantly **increases the fabrication cost**.

**Laser diode**: just the laser; **diode laser**: the complete system

A laser diode with the case cut away. The laser diode chip is the small black chip at the front; a photodiode at the back is used to control output power.

How does a diode laser work? 1) gain

**gain** = rate of stimulated emission – rate of absorption.

These rates can be calculated from first principles (Einstein theory) knowing the density of states in the CB & VB and their probability of occupancy.

In diode lasers the gain is temperature-dependent

Adapted from Saleh and Teich
Semiconductor gain $G(N, \nu, T)$

The gain spectrum of the semiconductor is broad and increases with the carrier density ($N$) and the temperature.

Adapted from Saleh and Teich

RT InGaASP laser
How does a diode laser work?

2) optical cavity

• The **simplest** cavity: Fabry-Perot (FP).
• Two parallel ends of the semiconductor are cleaved along the crystal axis, creating **mirrors** forming a FP cavity with the semiconductor as the **gain** medium.
• The laser emission is taken from the low-reflectivity front facet. High-reflectivity in the back facet reduces the cavity loss.

Longitudinal modes:

\[ \nu_m = \frac{m \ (c/n)}{(2L)} \]

- \( \nu_m \): frequency
- \( m \): mode number
- \( c \): speed of light
- \( n \): refractive index
- \( L \): cavity length

Diagram:

- Gain medium
- Low-reflectivity front facet
- High-reflectivity back facet
How many modes?

- The gain spectrum of the semiconductor medium is broad ⇒ supports many longitudinal modes.

\[ \nu_m = m \left( \frac{c}{n} \right)/(2L) \]

\[ \Delta \nu = \frac{c}{(2nL)} \]

\[ \Delta \lambda = \frac{(\lambda_0)^2}{(2nL)} \]

(free-space wavelength spacing, measured with an Optical Spectrum Analyzer)

\( n = 3.5, L = 1 \text{ mm:} \)

\[ \Delta \lambda = 0.05 \text{ nm @ 635 nm} \]

\[ \Delta \lambda = 0.3 \text{ nm @ 1550 nm} \]

Adapted from Saleh and Teich
EXAMPLE 16.3-4. **Number of Longitudinal Modes in an InGaAsP Laser.** An InGaAsP crystal \((n = 3.5)\) of length \(d = 400 \, \mu\text{m}\) has resonator modes spaced by \(\nu_F = c/2d = c_0/2nd \approx 107 \, \text{GHz}\). Near the central wavelength \(\lambda_o = 1.3 \, \mu\text{m}\), this frequency spacing corresponds to a free-space wavelength spacing \(\lambda_F\), where \(\lambda_F/\lambda_o = \nu_F/\nu\), so that \(\lambda_F = \lambda_o\nu_F/\nu = \lambda_o^2/2nd \approx 0.6 \, \text{nm}\). If the spectral width \(B = 1.2 \, \text{THz}\) (corresponding to a wavelength width of 7 nm), then approximately 11 longitudinal modes may oscillate. A

\[ L = d, \Delta\lambda = \lambda_F \]
Optical spectra of Light Emitting Diodes (LEDs)

Spontaneous emission rate:

\[ r_{sp}(\nu) \approx D_0 (\nu - E_g)^{1/2} \exp \left( -\frac{\nu - E_g}{k_B T} \right), \quad \nu \geq E_g \]

Line-width:

\[ \Delta \lambda \approx 1.45 \lambda^2 p k_B T \]

Adapted from Saleh and Teich
Comparing the LI curve of diode lasers and LEDs

Note the different scales

Adapted from Saleh and Teich
Comparing the optical spectrum
Early 1980s: moving the DH technology one step further to quantum-well lasers

QW lasers are DH lasers (DH are also referred to as “bulk” lasers) where the thickness of the active layer is narrow and the energy-momentum relation of bulk material (energy bands) does not apply.

Bulk:

QW:

For GaAs $\lambda_B = 50$ nm

Adapted from Saleh and Teich
In a QW laser carriers are confined in the x direction within a distance $d_1$ (the well thickness). But, in plane of the active layer (the y—z plane), they behave as in bulk semiconductor.
QW vs Bulk lasers

In QW lasers the threshold current is 4 - 5 times smaller than comparable DH lasers.

Adapted from Saleh and Teich
Multiple Quantum Well (MQW) lasers

- Alternating QW material (narrow band gap) with barrier material (high band gap).

- Advantages
  - Dramatic reduction in threshold current
  - Reduction in carrier loss
  - Reduced temperature sensitivity of threshold current

- Enable
  - Increase laser efficiency
  - Reduce thermal resistance
  - Higher output power
Novel lasers include quantum-wire, quantum-dash and quantum-dots

Adapted from Saleh and Teich
Quantum dot lasers (QDLs)

- While QW are thin layers of active material, quantum dots are (as the name suggests), dots or islands of a material surrounded by another material.

- The dots have a lower-energy bandgap than the surrounding material.

- The lasing wavelength is determined by the size and shape of the QDs.

- By controlling the size and shape of the QDs, QDLs can span a large range of wavelengths.
QDs fabrication

- Atom-like islands of 10-20 nm diameter, each one containing about $10^5$ atoms
- The size and density of QDs can be controlled by growth parameters.

InAs Quantum Dots on GaAs

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Dot density</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_{substrate} = 480 °C</td>
<td>$6 \times 10^{10} \text{ cm}^{-2}$</td>
</tr>
<tr>
<td>T_{substrate} = 510 °C</td>
<td>$2 \times 10^{10} \text{ cm}^{-2}$</td>
</tr>
<tr>
<td>T_{substrate} = 530 °C</td>
<td>$&lt; 10^9 \text{ cm}^{-2}$</td>
</tr>
</tbody>
</table>

Brighter Tutorials: http://www.ist-brighter.eu
Semiconductor lasers threshold reduction: a long way from the beginning

4 orders of magnitude
A fundamental limit for the wavelength of conventional semiconductor lasers

• In conventional semiconductor lasers, when electrons from the conduction band relax to the valence band, the energy is typically transferred to a photon.

• At longer wavelengths, depending on the band structure and temperature, this energy is often re-absorbed by another charge carrier and eventually transferred to heat.

• Thus, the emission wavelength of conventional, inter-band lasers is limited to about 3 μm.
Quantum Cascade Lasers

**Inter-band laser**

**Inter-sub-band laser**

A Conduction band

Band gap

Valence band

Interband transition

B Band gap

Intersubband transition
Quantum Cascade Laser

Jerome Faist, Federico Capasso,* Deborah L. Sivco, Carlo Sirtori, Albert L. Hutchinson, Alfred Y. Cho

A semiconductor injection laser that differs in a fundamental way from diode lasers has been demonstrated. It is built out of quantum semiconductor structures that were grown by molecular beam epitaxy and designed by band structure engineering. Electrons streaming down a potential staircase sequentially emit photons at the steps. The steps consist of coupled quantum wells in which population inversion between discrete conduction band excited states is achieved by control of tunneling. A strong narrowing of the emission spectrum, above threshold, provides direct evidence of laser action at a wavelength of 4.2 micrometers with peak powers in excess of 8 milliwatts in pulsed operation. In quantum cascade lasers, the wavelength, entirely determined by quantum confinement, can be tailored from the mid-infrared to the submillimeter wave region in the same heterostructure material.
Continuous Wave Operation of a Mid-Infrared Semiconductor Laser at Room Temperature

Mattias Beck,1* Daniel Hofstetter,1 Thierry Aellen,1 Jérôme Faist,1 Ursula Oesterle,2 Marc Illegems,2 Emilio Gini,3 Hans Melchior3

Continuous wave operation of quantum cascade lasers is reported up to a temperature of 312 kelvin. The devices were fabricated as buried heterostructure lasers with high-reflection coatings on both laser facets, resulting in continuous wave operation with optical output power ranging from 17 milliwatts at 292 kelvin to 3 milliwatts at 312 kelvin, at an emission wavelength of 9.1 micrometers. The results demonstrate the potential of quantum cascade lasers as continuous wave mid-infrared light sources for high-resolution spectroscopy, chemical sensing applications, and free-space optical communication systems.
QCLs applications

The wavelength region accessible with quantum cascade lasers and the developing applications of this technology.

Optics and Photonics News, July/August 2008
Nowadays QCLs emitting in the mid- to long-infrared are compact and operate at RT: ideal light sources for lab-on-a-chip biosensors.

Design of fabrication of microfluidics over the facet of the quantum cascade laser integrated with plasmonic antenna. (a) The 2-mm QCL is first chipped and mounted onto a standard gold-coated c-mount that has been thinned to less than 1 mm. (b) The packaged laser is then submerged in PDMS and patterned for inlet and outlet ports. (c) Finally, the hoses are connected to the microfluidic channel, and liquid is pumped through the device.
Further improving the confinement of photons and carriers: *lateral* waveguide

**Gain guided**
(carrier induced small $\Delta n$)

**Index guided**
(build-in larger $\Delta n$)

Adapted from J. Ohtsubo
Gain + cavity determine the optical spectrum

- The number of lasing modes and their relative power depends on the type of laser, the current (I) and the temperature (T).
- It is often possible to adjust I and T for single-mode operation, but it can be achieved over a **limited** I and T range.

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**Multi longitudinal mode (gain guided)**

- $P_0 = 5\text{mW}$
- $P_0 = 3\text{mW}$
- $P_0 = 1\text{mW}$
- $P_0 = 0.5\text{mW}$

**Single longitudinal mode (index guided)**

- $P_0 = 5\text{mW}$
- $P_0 = 3\text{mW}$
- $P_0 = 1\text{mW}$
- $P_0 = 0.5\text{mW}$

---

Wavelength $\lambda$ (nm)
An example from our lab

Low pump current

High pump current

Courtesy of Andres Aragoneses,
UPC (Semiconductor laser lab, Terrassa, Spain)
Why do we need single-mode emission?

• High-data-rate optical fiber transmission requires the laser to emit single mode.

• This is because each mode travels with its own group velocity. Therefore, the optical pulses emitted by a multimode laser broaden with propagation distance, and the distinction between binary 'zero' and 'one' is gradually lost.
Can we fabricate stable single-mode lasers?

Dynamically stable?

Yes! By using a **mode-selective cavity**

- A **Bragg-Grating** (BG) mirror
  - Distributed Feedback (DFB)
  - Distributed Bragg Reflector (DBR)
  - Vertical Cavity Surface Emitting Lasers (VCSEL)

- An **External** mirror – External Cavity Laser (ECL)
Bragg-Grating (BG) mirror

- Peak reflectivity for a particular frequency (the Bragg-frequency) via coherent addition of distributed reflections.
External Cavity Laser

- With controlled feedback conditions the laser emission “locks” to one of the modes of the “compound” cavity. Additional advantages: decrease of the threshold current (reduced cavity loss) and reduced linewidth.
- Drawback: uncontrolled feedback conditions can lead to unstable (and even chaotic) output.
EELs vs VCSELS

**Edge-Emitting Laser (EEL)**

- Injection current
- Stripe contact
- Cladding layer
- Active layer ~ 10 nm
- Cladding layer
- Laser output

**Wide divergent output**

\[ L \approx 300 \ \mu m \]

The semiconductor facets serve as mirrors

Adapted from J. Mulet, PhD thesis, Universitat de les Illes Balears (2002)

**VCSEL**

- Two DBRs serve as mirrors
- \[ \Delta \lambda = (\lambda_0)^2/(2nL) \]

⇒ VCSELS emit a single-longitudinal-mode.

Adapted from K. Iga, JSTQE 2000
Figure 1 | Long-wavelength VCSELs. a, On a wafer. b, Individually. These have a lower power consumption than their edge-emitter counterparts at similar optical power level. In addition, their vertical-cavity design leads to better coupling to single-mode fibres (by a factor of 2–4) and enables on-wafer characterization, which greatly reduces manufacturing costs.
How does a VCSEL work?

The small cavity length requires highly-reflective DBRs, which are doped to facilitate the injection of electrons/holes.

Blue indicates n-type material and red indicates p-type.

Fig. 6. Schematic structure of a typical VCSEL using GaAs/AlAs DBR and selective-oxidation technique. Various materials emitting wide wavelength ranges can be utilized as the active layer.

Adapted from K. Iga, JLT 2008
Adapted from A. Larsson, JSTQE 2011
Spatial lateral/transverse modes

Solutions of the Helmholtz equation

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) E(x, y) + (k^2 n^2(r) - \beta^2) E(x, y) = 0
\]

Edge-Emitting Lasers:

- The circular profile allows easy coupling to an optical fiber.
- But single-transverse mode emission limited to few mW.

VCSELs:

Adapted from A. Larsson, JSTQE 2011

Adapted from Saleh and Teich
Another type of cavity geometry: Ring

- Two contra propagating “whispering-gallery” modes

The devices were fabricated in standard double-quantum-well (DQW) GaAs–AlGaAs material grown at Sheffield University by MOCVD. The active region has a GaAs double quantum well sandwiched between two $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$ waveguide regions, each 250 nm thick. The p-type and n-type cladding layers consist of 1.0- and 1.5-$\mu$m-thick $\text{Al}_{0.4}\text{Ga}_{0.6}\text{As}$, respectively. Highly p-doped GaAs 100-nm-thick is used as a contact layer. A transverse section of the waveguide is depicted in Fig. 1(b).

Adapted from Sorel et al, JQE 2003
Thermal properties of laser diodes: 1) variation of the center wavelength

Single-mode laser

\[ P_0 = 3 \text{mW} \]

Multimode: Mode hopping

\[ P_0 = 7 \text{mW} \]
2) thermal effects in the LI curve

Source: Laser Focus World, sept. 2013
Why?

• The semiconductor is not a two level system.
• **Temperature affects the band-gap** of the semiconductor material, which determines the energy (and wavelength) of the emitted photons.
• With increasing temperature (**Joule heating**) \( E_g \) decreases and the emission frequency shifts to lower frequencies.
Band-gap energy and refractive index

A variation of the gain (due to a variation of N or T) causes a change in the refractive index, $n$, of the semiconductor (via the Kramer-Kronig relation), which results in a change of the optical cavity length, $L$, and thus, in a change of the resonance frequencies of the FP cavity.

**GaAs**

$$E_g = 1.5216 - \frac{5.405 \times 10^{-4} T^2}{T + 204} \text{ (eV)}$$

At 300 K:

$$n^2 = 8.950 + \frac{2.054 \lambda^2}{\lambda^2 - 0.390}$$

$$\frac{1}{n} \frac{dn}{dT} = 4.5 \times 10^{-5} \text{ K}^{-1}$$

**InP**

$$E_g = 1.4206 - \frac{4.906 \times 10^{-4} T^2}{T + 327}$$

$$n^2 = 7.255 + \frac{2.316 \lambda^2}{\lambda^2 - 0.3922}$$

$$\frac{1}{n} \frac{dn}{dT} = 2.7 \times 10^{-5} \text{ K}^{-1}$$

Adapted from J. M. Liu, Photonic devices
Summary of diode laser design goals

- To optimize carrier injection properties
- To optimize optical confinement
- To minimize optical loss and heating
- To obtain maximum gain at a given injection power
- To obtain a high-quality spatial profile and spectral purity
Outline

Part 1

1. Introduction to semiconductor lasers (SCLs)
2. Simplest rate-equation model and dynamics with time-varying current parameter
Main goals

Acquire a basic knowledge of

• The simplest rate equation model, the normalized equations, the meaning of the parameters and the steady-state solution.

• Perform simulations of deterministic dynamics with time-varying current parameter
  – **Turn on**: delay & relaxation oscillations
  – **LI curve**: dynamical hysteresis
  – Response to **current modulation**: understand the modulation bandwidth
Diode laser turn on (‘gain switching’): delay and relaxation oscillations

A simple model allows simulating the laser output intensity with time-varying injection current

Class B lasers only stable emission or periodic oscillations (more latter)

From T. Heil, PhD thesis (Darmstadt 2001)
Rate equation for the photon density $S$

$$\frac{dS}{dt} = GS - \frac{S}{\tau_p} + \frac{\beta_{sp} N}{\tau_N}$$

- **Stimulated emission & absorption**
- **Cavity losses**
- **Spontaneous emission**

$\tau_p$: **Photon lifetime**. The optical cavity is a photon-reservoir where photons have a finite life-time before escaping. Typically $\tau_p$ is in the range of a few pico-seconds. $1/\tau_p = k$ is the cavity loss.

$G (N,S)$: **Gain** (stimulated emission – absorption)

$\beta_{sp}$: **Spontaneous emission rate**
Rate equation for the carrier density $N$

\[
\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau_N} - GS
\]

$\tau_N$ : Carrier lifetime. In the active region carriers (electron/hole pairs) are lost due to radiative recombination (spontaneous emission) and nonradiative recombination. Typically $\tau_N$ is in the range of a few nano-seconds.

$I$ : Injection current ($I/eV$ is the number of electrons injected per unit volume and per unit time).

$G (N,S)$ : Gain
The simplest expression for the semiconductor gain

\[ G = a(N - N_0) \]

\[ G = aN_0 \ln\left(\frac{N}{N_0}\right) \]

Differential gain coefficient \(a\)

Carrier density at transparency \(N_0\)

We will assume single-mode emission at \(\lambda_0\). The differential gain coefficient \(a\) depends on \(\lambda_0\).
Nonlinear coupled equations

\[
\frac{dS}{dt} = GS - \frac{S}{\tau_p} + \frac{\beta_{sp} N}{\tau_N} \quad \frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau_N} - GS
\]

- Ordinary differential equations (spatial effects neglected!)
- Additional nonlinearities from carrier re-combination and gain saturation

\[
\frac{1}{\tau_N} = \frac{1}{\tau_{nr}} + BN + CN^2 \quad G = \frac{a(N - N_0)}{1 + \varepsilon S}
\]

- These equations allow simulating the LI curve and the laser modulation response.
- To understand the intensity noise and the laser line-width (the optical spectrum), we need a stochastic equation for the \textit{complex} field \( E \) \((S=|E|^2)\).
Normalized equations

- Define the a-dimensional variable:

\[
\frac{dS}{dt} = \frac{1}{\tau_p} (N' - 1)S + \frac{\beta_{sp} N'}{\tau_N}
\]

\[
\frac{dN'}{dt} = \frac{1}{\tau_N} (\mu - N' - N'S)
\]

Pump current parameter: proportional to \( I/I_{th} \)

- Normalizing the equations eliminates two parameters (a, \( N_0 \))
- In the following I will drop the “ ’ ”
Initial conditions

\[
\frac{dS}{dt} = \frac{1}{\tau_p} (N - 1)S + \frac{\beta_{sp} N}{\tau_N}
\]

- If at \( t=0 \) there are no photons in the cavity: \( S(0) = 0 \)

- Then, without noise (\( \beta_{sp}=0 \)): if \( S=0 \) at \( t=0 \) \( \Rightarrow \) \( dS/dt=0 \)
  \( \Rightarrow \) \( S \) remains 0 (regardless the value of \( \mu \) and \( N \)).

- Without spontaneous emission noise the laser does not turn!
Steady state solutions

(Simple expressions if $\beta_{sp}$ is neglected)

\[
\frac{dS}{dt} = \frac{1}{\tau_p} (N - 1)S
\]

\[
\frac{dN}{dt} = \frac{1}{\tau_N} (\mu - N - NS)
\]

\[
\frac{dS}{dt} = 0 \Rightarrow \begin{cases} S = 0 \\ N = 1 \end{cases}
\]

\[
\frac{dN}{dt} = 0 \Rightarrow \begin{cases} S = 0 \rightarrow N = \mu \\ N = 1 \rightarrow S = \mu - 1 \end{cases}
\]

Laser off

S=0
N=\mu

Stable if $\mu < 1$

Laser on

S=\mu-1
N=1

Stable if $\mu > 1$

The carrier density is “clamped” above threshold

$\mu_{th} = 1$
Graphical representation

Photon density $S$

Carrier density $N$

Pump current, $\mu$

- off: $S=0$
- on: $S=\mu-1$

- off: $N=\mu$
- on: $N=1$

$\mu_{th} = 1$
Dynamics with time-varying pump current

\[
\frac{dN}{dt} = \frac{1}{\tau_N} (\mu - N - NS)
\]

\[
\frac{dS}{dt} = \frac{1}{\tau_p} (N - 1)S + \frac{\beta_{sp} N}{\tau_N}
\]

- **Step** (laser turn on): \(\mu_{\text{off}}, \mu_{\text{on}}\)
- **Triangular** signal (LI curve): \(\mu_{\text{min}}, \mu_{\text{max}}, T\)
- **Sinusoidal** signal (modulation response): \(\mu_{\text{dc}}, A, T_{\text{mod}}\)

Parameter values:

| \(\tau_p\) | 1 ps |
| \(\tau_N\) | 1 ns |
| \(\beta_{sp}\) | \(10^{-4}\) |
Current step: turn-on delay & relaxation oscillations

A linear stability analysis of the rate equations allows calculating the RO frequency

\[ \omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}} \]

Adapted from J. Ohtsubo
Triangular signal: LI curve

Slow “quasi-static” current ramp (T=200 ns)

Time (ns)

Pump parameter, $\mu$

$S$

$\mu$

$S$
But with a “fast” ramp: turn on delay (dynamical hysteresis)

T = 20 ns

Tredicce et al, Am. J. Phys., Vol. 72, No. 6, June 2004
The laser threshold: a delayed dynamical bifurcation

Simulations vs. Experiments

Tredicce et al, Am. J. Phys., Vol. 72, No. 6, June 2004
Relaxation oscillations: influence of gain saturation

\[
\frac{dS}{dt} = \frac{1}{\tau_p} (G - 1)S + \frac{\beta_{sp} N}{\tau_N}
\]

\[
\frac{dN}{dt} = \frac{1}{\tau_N} (\mu - N - GS)
\]

\[
G(N, S) = \frac{N}{1 + \epsilon S}
\]

The gain saturation coefficient \( \epsilon \) takes into account phenomenologically several effects (e.g., spatial and spectral hole burning)
Effect of asymmetric current modulation

In a solid-state diode-pumped laser pumped, on average, below the threshold.

Fig. 1. Time evolution of the laser intensity (up) and of the pump power (down) for symmetry parameter $\alpha$ equal to 1%, 50%, and 99%. (a) Experiments, (b) simulations.

Adapted from Glorieux et al Opt. Lett. 2006
Generation of optical pulses in VCSELs below the static threshold using asymmetric current modulation

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Black dashed: laser pump current
Blue and Red: laser intensity (x & y polarizations)
Why current modulation is important?

Optical waves can be modulated in Amplitude, Phase and in Frequency in order to carry information.

- **Optical Amplitude**: Amplitude Shift Keying (ASK)
- **Optical Phase**: Phase Shift Keying (PSK)
- **Optical Frequency**: Frequency Shift Keying (FSK)

Adapted from H. Jäckel, ETHZ
Digital vs analog current modulation

Digital

Time (ns)

Analog

Time (ns)
**Weak** sinusoidal modulation: influence of the modulation frequency

\[ \mu = \mu_{dc} + A \sin \omega_{mod} t \]

\[ \mu_{dc} = 1.5, A=0.1 \]

For \( \mu = 1.5 \): \( \nu_{RO} = 3.56 \text{ GHz} \)

The laser intensity \( (S = \text{photon density}) \) is modulated at the same frequency of the pump current \( (\mu) \), but the phase of the intensity and the current are not necessarily the same.
Modulation response: resonance at $\nu_{mod} = \nu_{RO}$

The modulation response can be analytically calculated by linearising the rate equations.

Adapted from A. Larsson, JSTQE 2011

Adapted from J. Ohtsubo
Large-signal modulation response

Experiments

Simulations

Adapted from J. Ohtsubo
Summary

A simple rate equation model for the photon and carrier densities allows understanding the main features of the laser dynamics with time-varying injection current:

- The turn on delay & relaxation oscillations
- The LI curve (static & dynamic)
- The modulation response (small and large signal)
THANK YOU FOR YOUR ATTENTION!

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