Nonlinear time series analysis

Bivariate analysis

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Outline

- **Introduction**
  - Historical developments: from dynamical systems to complex systems
- **Univariate analysis**
  - Methods to extract information from a time series.
  - Applications.
- **Bivariate analysis**
  - Correlation, directionality and causality.
  - Applications.
- **Multivariate analysis**
  - Many time series: complex networks.
  - Network characterization and analysis.
  - Climate networks.

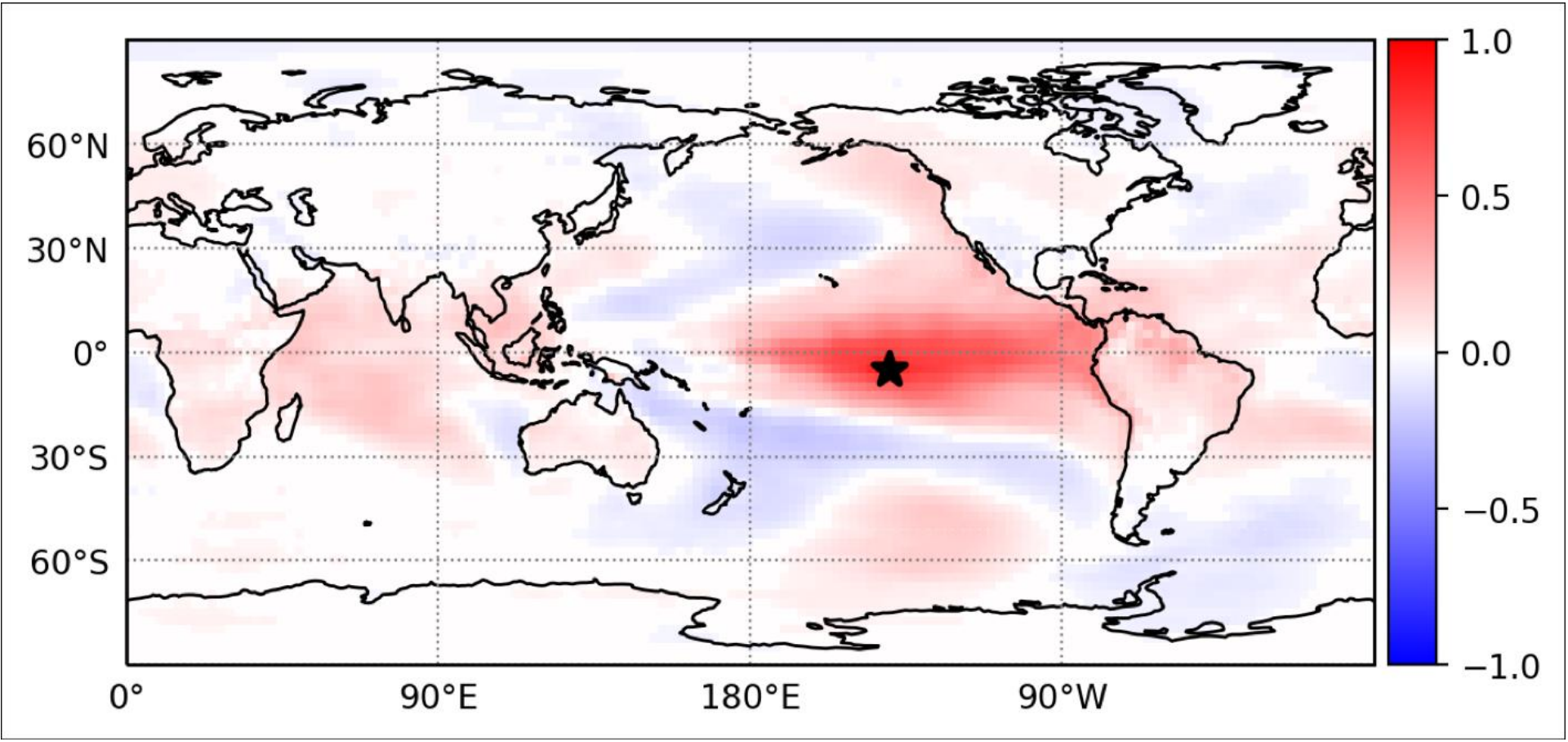
Cross-correlation of two time series *X* and *Y* of length *N*

$$C_{xy}(\tau) = \frac{1}{N-\tau}\sum_{k=1}^{N-\tau} x(k+\tau)y(k)$$

the two time series are normalized to zero-mean **μ=0** and unit variance, **σ=1**

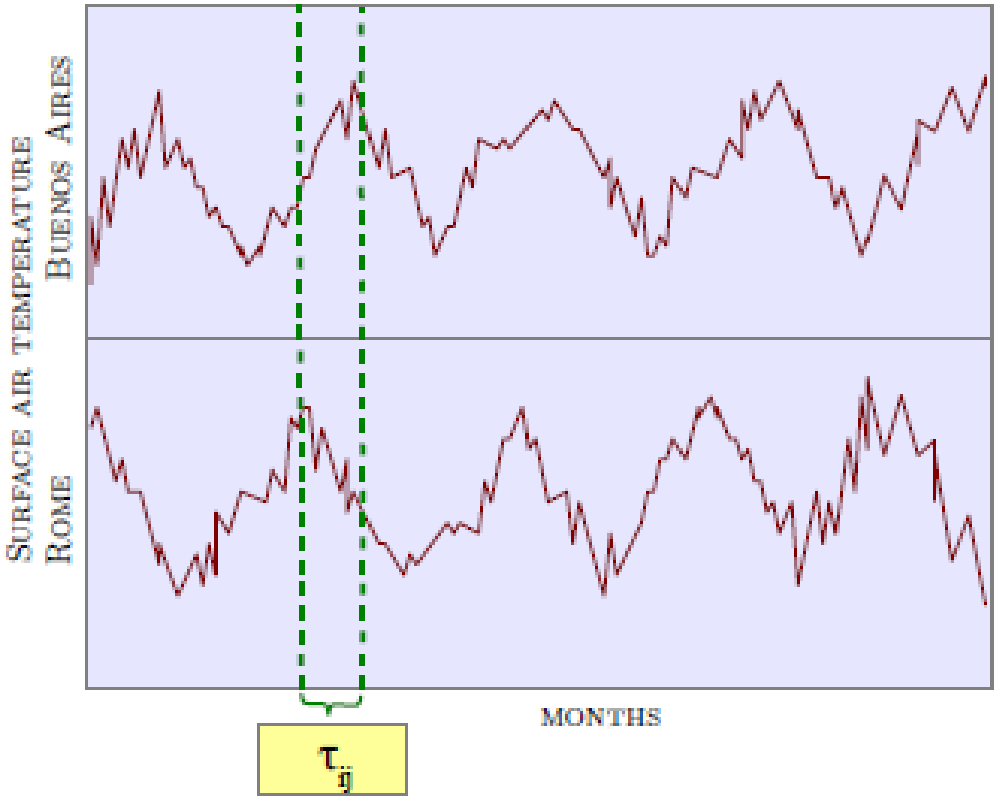
- $-1 \leq C_{X,Y} \leq 1$
- $C_{X,Y} = C_{Y,X}$
- The maximum of $C_{X,Y}(\tau)$ indicates the **lag** that renders the time series *X* and *Y* best aligned.
- Pearson coefficient: $\rho = C_{X,Y}(0)$

An example of cross correlation map: monthly surface air temperature (SAT) anomalies

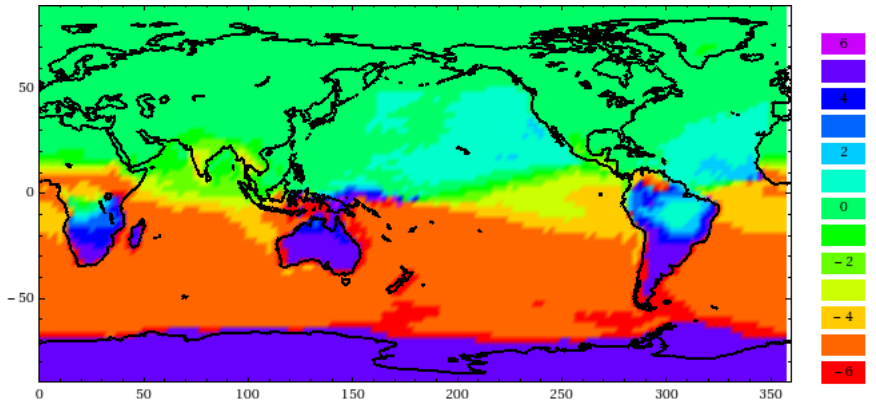


Color code represents the Pearson coefficient of $*$ and all the world

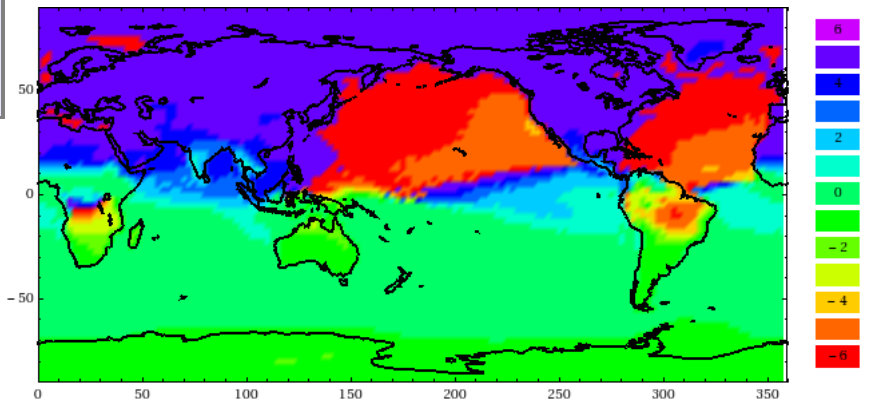
Correlation analysis of lag-times between seasonal cycles (Surface Air Temperature)



Rome



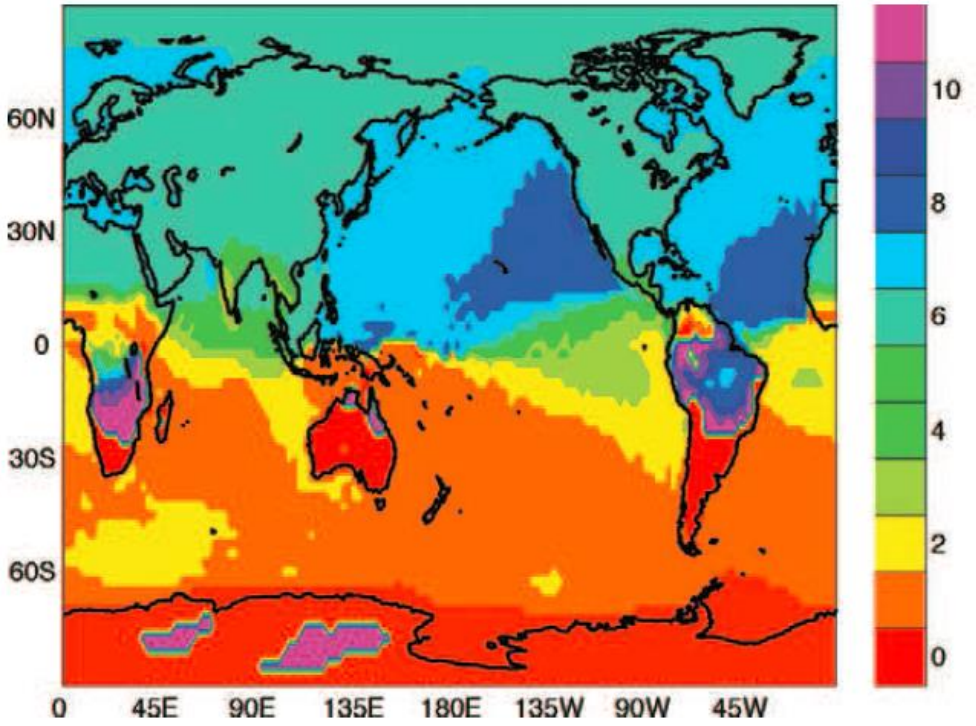
Buenos Aires



G. Tirabassi and C. Masoller,
Sci. Rep. 6:29804 (2016)

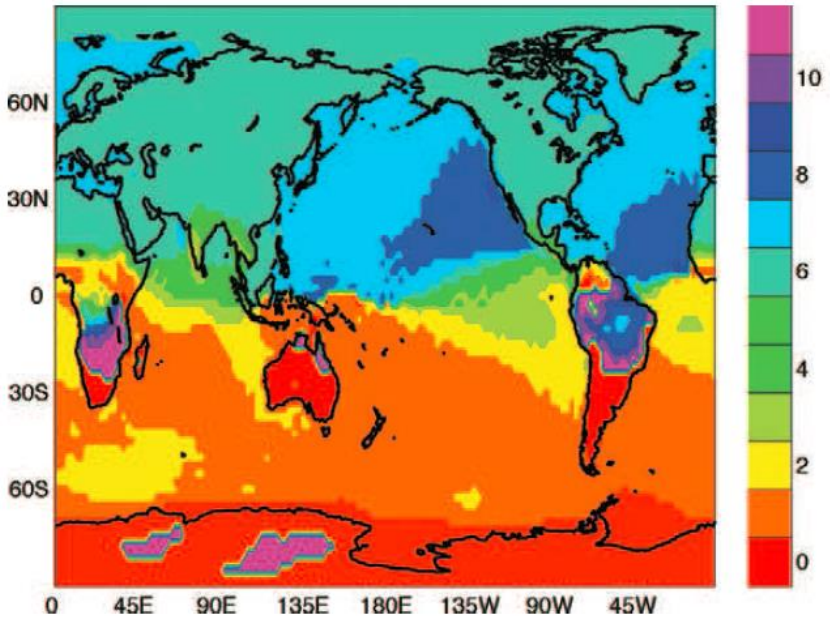
Map of lag times of raw surface air temperature series

Lag (in months) between the SAT at a reference point in Australia, and all the other time-series.

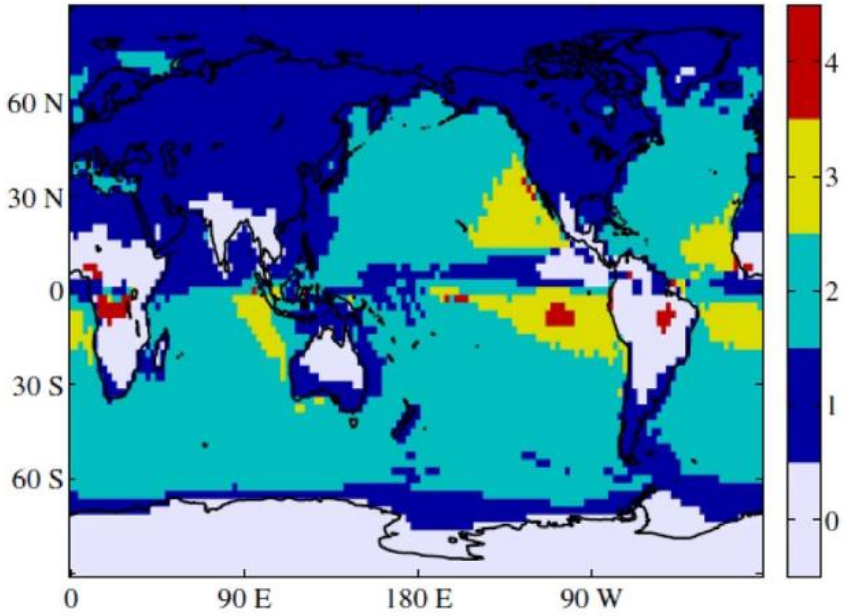


Tirabassi and Masoller EPL 102, 59003 (2013)

Lag times between SAT in different regions



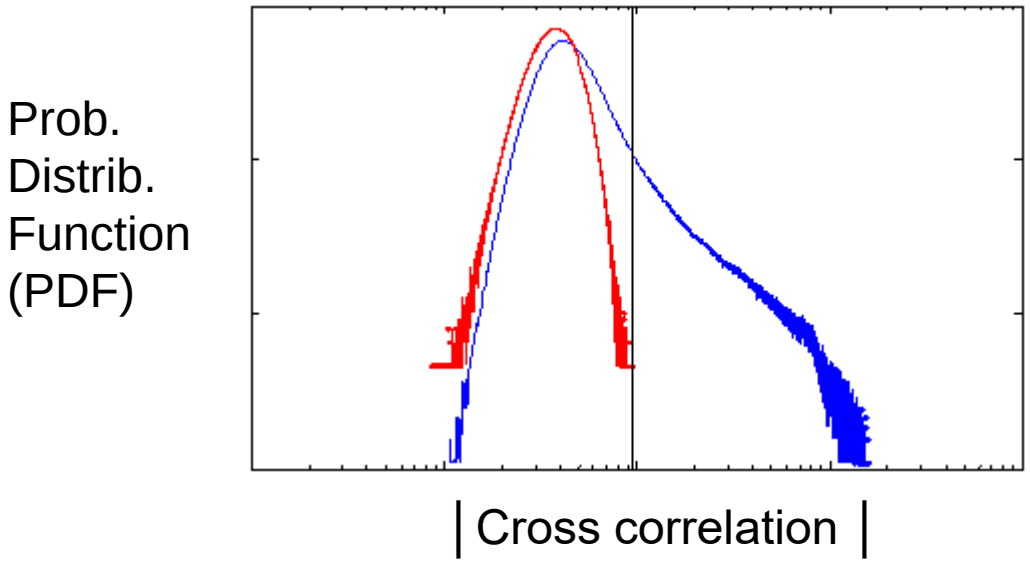
Lag times that minimize the distance between SAT and insolation in the same region



F. Arizmendi, et al.
Sci. Rep. 7, 45676 (2017).

Are these cross-correlation values significant?

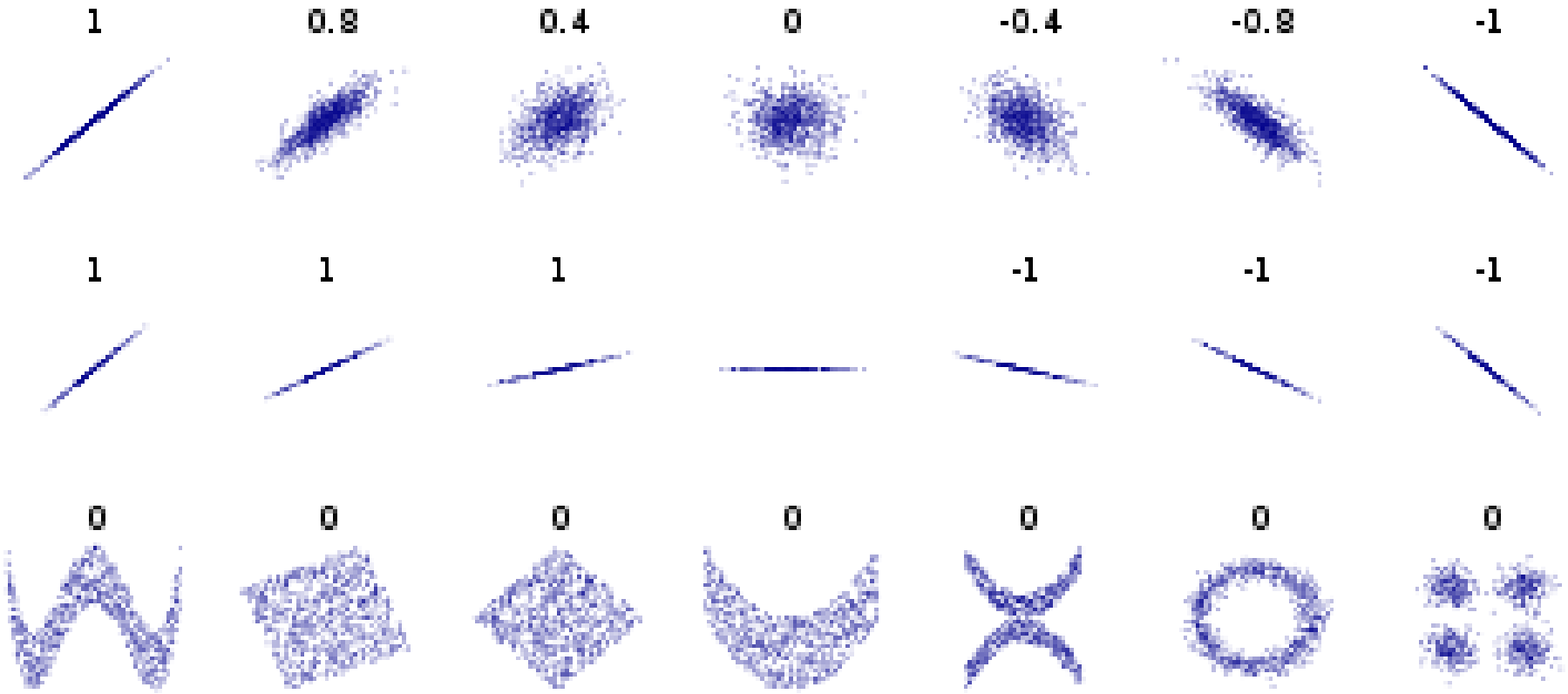
Simplest option: consider statistically significant the values that are larger than those obtained with surrogates.



Problems:

- significant weak links might be hidden by noise
- because of geographical proximity, the strongest CC values are those of neighboring points

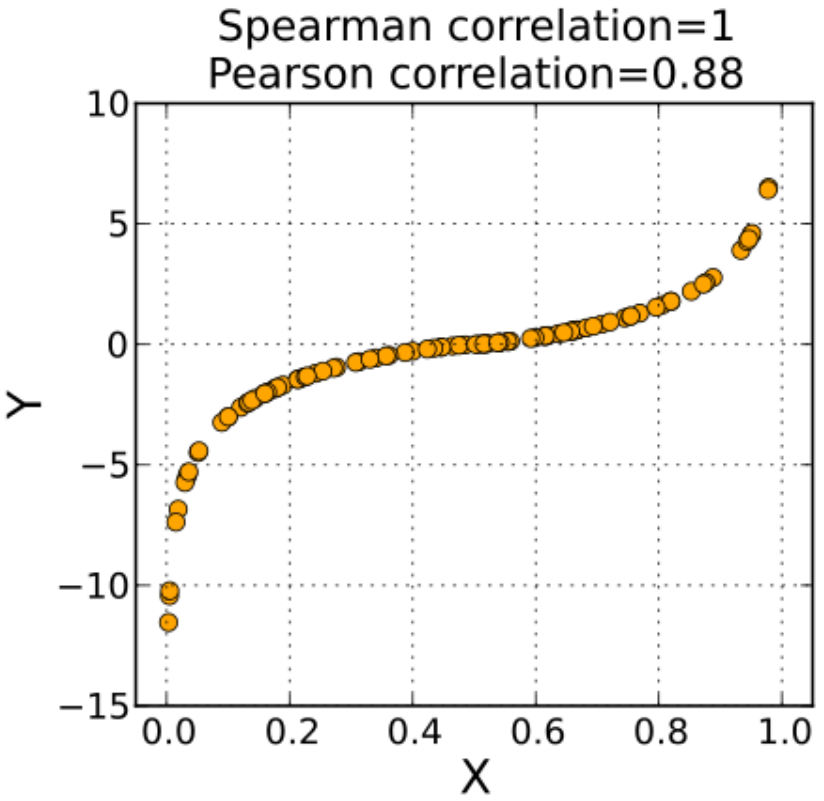
Cross-correlation analysis detects linear relationships only



Source: wikipedia

A popular nonlinear correlation measure: Spearman correlation

- It assesses how well the relationship between two variables can be described using a monotonic function.
- Spearman coefficient is the usual Pearson correlation coefficient, but applied to the **rank** variables.



- "ranking" refers to the data transformation in which numerical values are replaced by their rank when the data are sorted. For example, the ranks of (3.4, 5.1, 2.6, 7.3) are (2, 3, 1, 4).

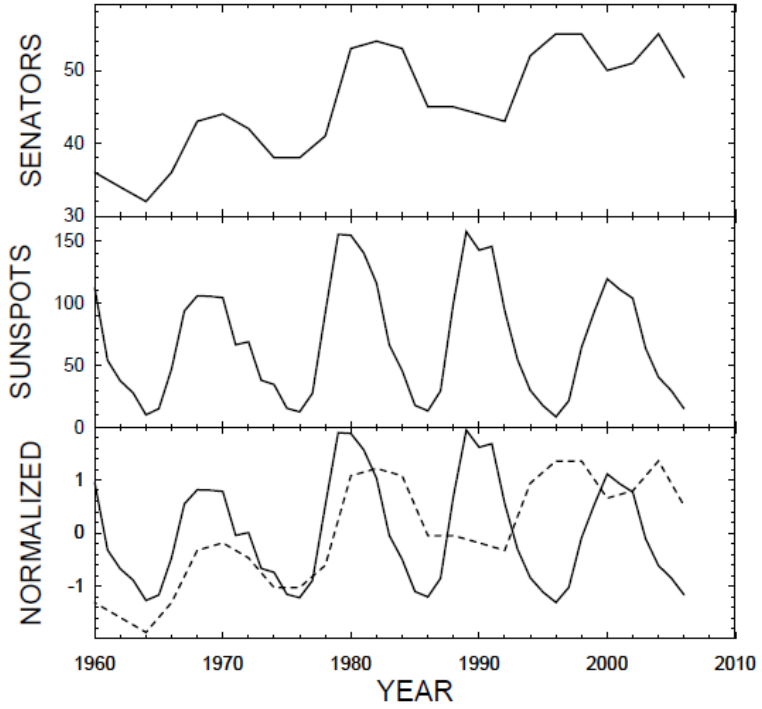
Another nonlinear correlation measure: Kendall τ coefficient

Any pair of observations (xi , yi) and (x j, yj) are said to be concordant if the ranks for both elements agree: that is, if both xi > xj and yi > yj or if both xi < xj and yi < yj; else they are said to be discordant.

$$\tau = \frac{(\text{number of concordant pairs}) - (\text{number of discordant pairs})}{n(n-1)/2}$$

Correlation is NOT causality

An illustrative example: the number of sunspots and the number of the Republicans in the U.S. Senate in the years 1960-2006.



Interval 1960 to 1986 (biannual sampling, 14 points):

C=0.52 **Is this significant?**

Null hypothesis

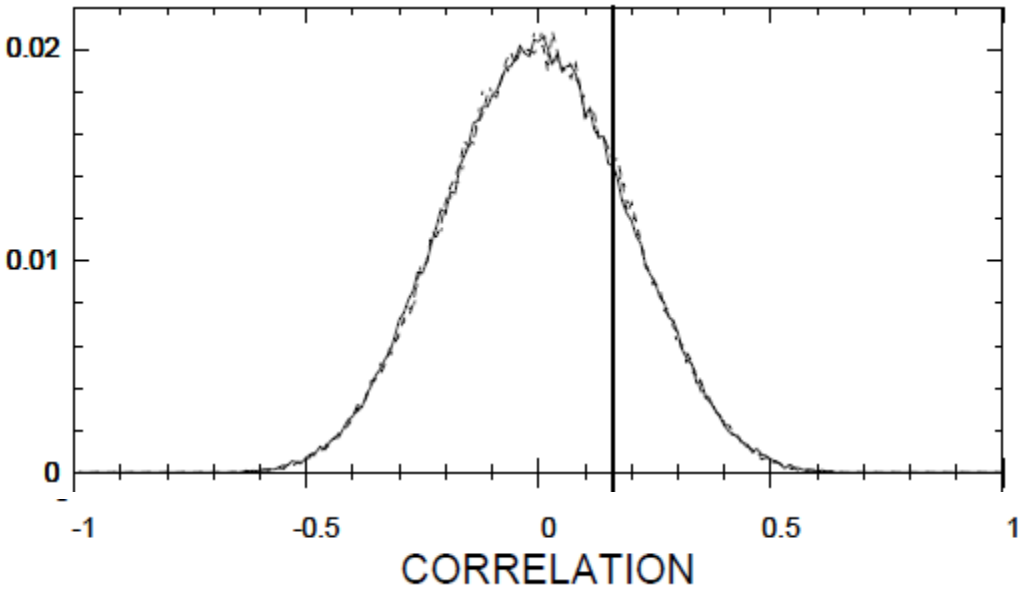
- Assuming the data sets were sampled from *independent, identically distributed* (IID) Gaussian populations and a significance level of 95%, then the significance threshold value of C is 0.458.

- Therefore, the null hypothesis (Gaussian IID) should be rejected.

- Something is wrong!

The analysis of surrogate data produces three identical distributions

- Between the number of the Republican senators in the period 1960-2006 (24 samples) with 24-sample sets randomly drawn from the Gaussian distribution (dashed);
- Between the number of the Republican senators in the period 1960-2006 (24 samples) with the 24-sample segment of the sunspot numbers randomly permutated in the temporal order (IID surrogate, dash-and-dotted)
- Two 24-sample sets randomly drawn from a Gaussian distribution (solid).



Vertical line: correlation between the number of the Republican senators and the sunspot numbers for the period 1960-2006.

What was wrong?

- The significance criterion C> 0.458 is not valid because the two datasets do not meet the *independent, identically distributed* (IID) criterion.
- IID samples: there is no relation between any x_i and x_{i+j}.
- But in both datasets there are **autocorrelations**.
- No universal table of critical values can be derived for testing the independence of serially correlated data sets.

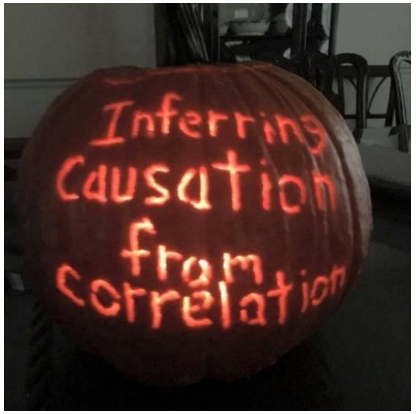
Usual solution

The significance of $C_{xy}(\tau)$ is usually checked by calculating the cross-correlation from an ensemble of signals (**surrogates**) with the same autocorrelation than the original ones but completely independent from each other.

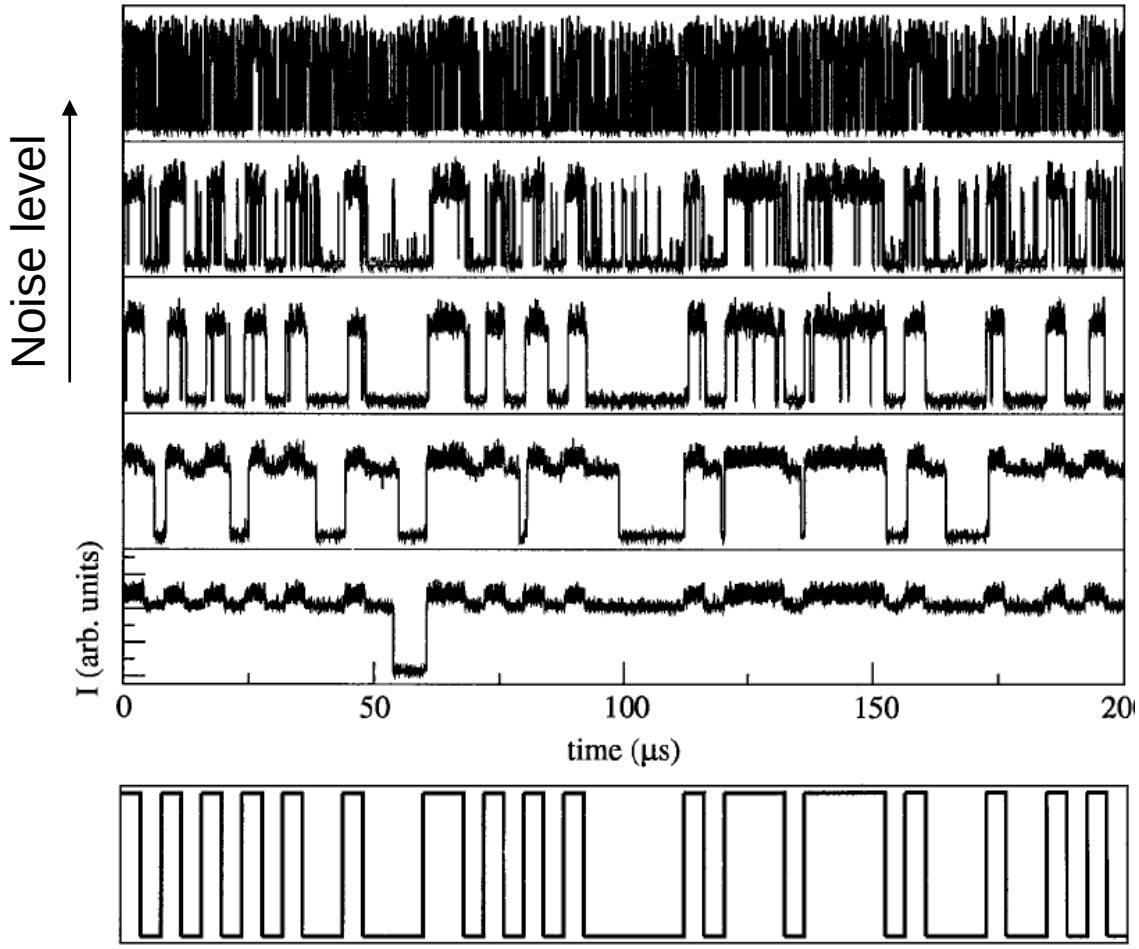
Read more: M. Palus, *From Nonlinearity to Causality: Statistical testing and inference of physical mechanisms underlying complex dynamics*. Contemporary Physics 48(6) (2007) 307-348.

http://tylervigen.com/spurious-correlations

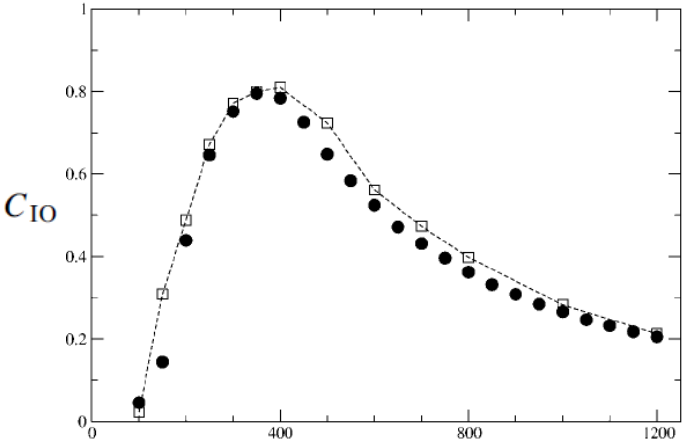
G. Lancaster et al, "*Surrogate data for hypothesis testing of physical systems*", Physics Reports 748 (2018) 1–60.



Example of application: response of a bistable system to an aperiodic signal (stochastic resonance)



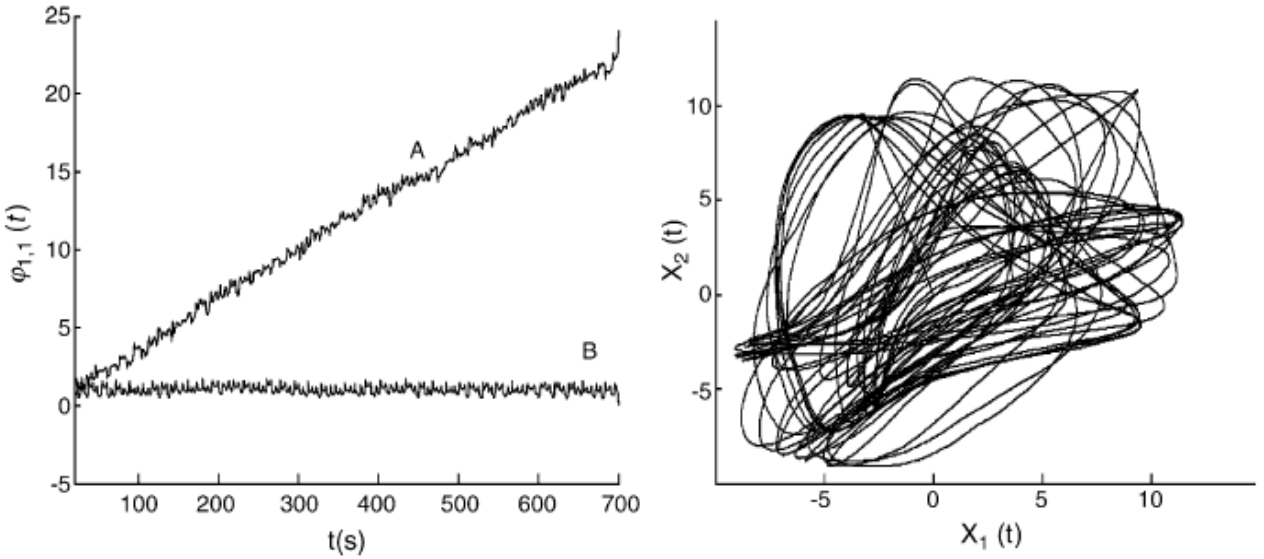
Cross-correlation between input and output signal.



Barbay et al, PRL 85, 4652 (2000)

Phase synchronization (PS)

- The phase difference between two oscillators is bounded but their amplitudes are not synchronized.



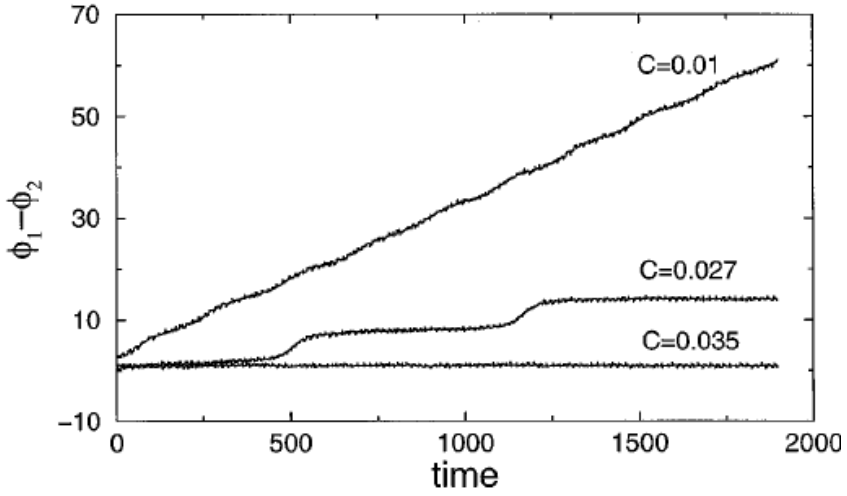
Left: Absolute phase difference between the x variables of two Rossler systems
(A)Uncoupled state: The phase difference grows and is unbounded.
(B) Strong PS: The phase difference remains constant along time.
Even in this latter case, the amplitudes remain completely uncorrelated (right).

E. Pereda et al., Progress in Neurobiology 77, 1 (2005)

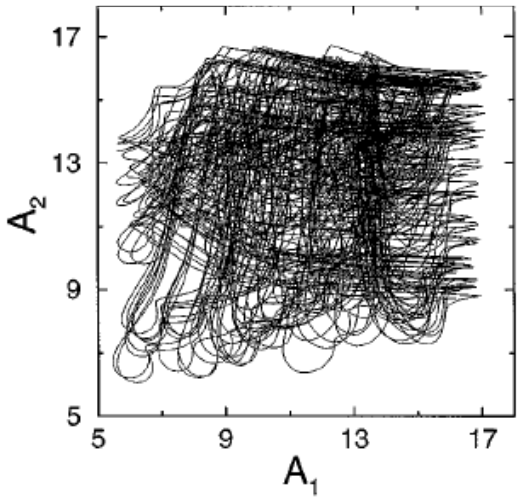
Phase synchronization: emerges as coupling increases

Mutually coupled Rossler systems

$$\dot{x}_{1,2} = -\omega_{1,2} y_{1,2} - z_{1,2} + C(x_{2,1} - x_{1,2})$$
$$\dot{y}_{1,2} = \omega_{1,2} x_{1,2} + 0.15 y_{1,2},$$
$$\dot{z}_{1,2} = 0.2 + z_{1,2}(x_{1,2} - 10).$$



C=0.035



Rosenblum et al., Phys. Rev. Lett. 76, 1804 (1996)

How to measure phase synchronization?

- Several measures have been proposed to detect PS in real (noisy) signals.

- Main Idea: If two signals are phase synchronized, the phase difference will occupy a small portion of the unit circle, while the lack of PS gives a phase difference that spreads out over the entire unit circle.

E. Pereda et al., Progress in Neurobiology 77, 1 (2005)

Nonlinear correlation measure based on information theory: the mutual Information

$$MI = \sum_{i \in x} \sum_{j \in y} p(x,y) \log \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

- *MI (x,y) = MI (y,x)*
- *p(x,y) = p(x) p(y)* $\Rightarrow$ *MI = 0*, else ***MI >0***
- *MI* can also be computed with a lag-time.
- If p(x,y) is a bivariate Gaussian distribution, then

  $MI = -1/2 \log(1-\rho^2)$

  where ρ is the cross-correlation coefficient.

MI values are systematically overestimated

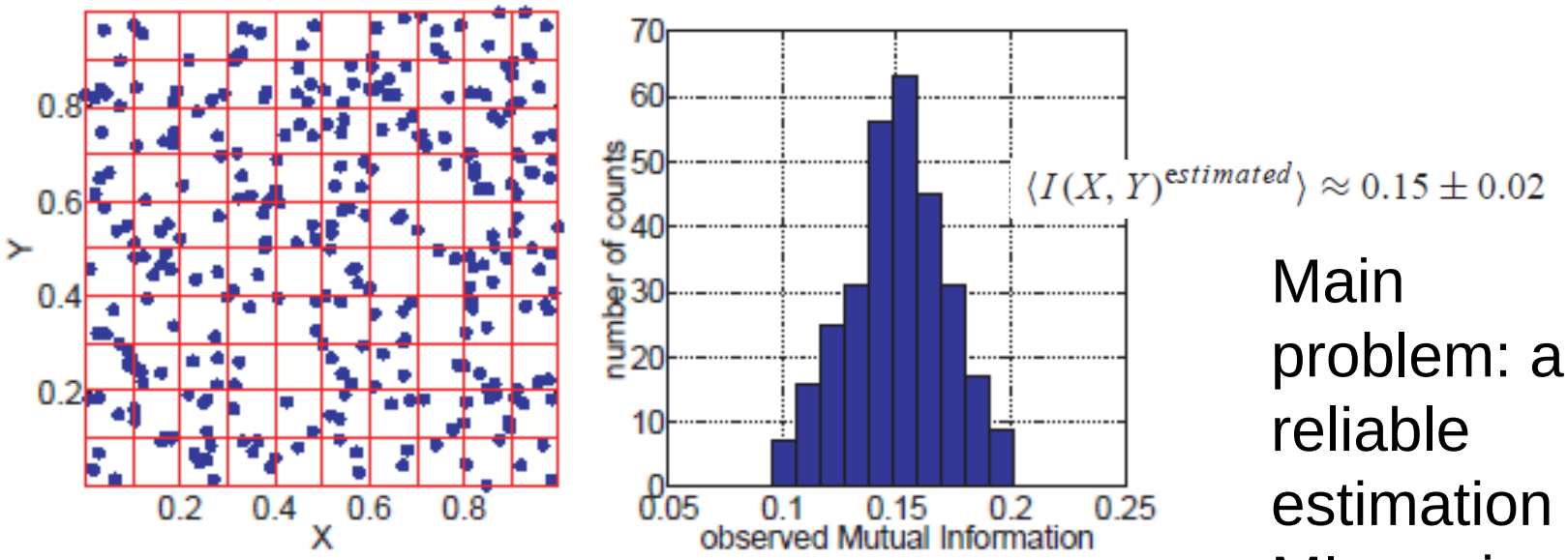


Fig. 1. Naive estimation of the mutual information for finite data. Left: The dataset consists of $N = 300$ artificially generated independent and equidistributed random numbers. The probabilities are estimated using a histogram which divides each axis into $M_X = M_y = 10$ bins. Right: The histogram of the estimated mutual information $I(X, Y)$ obtained from 300 independent realizations.

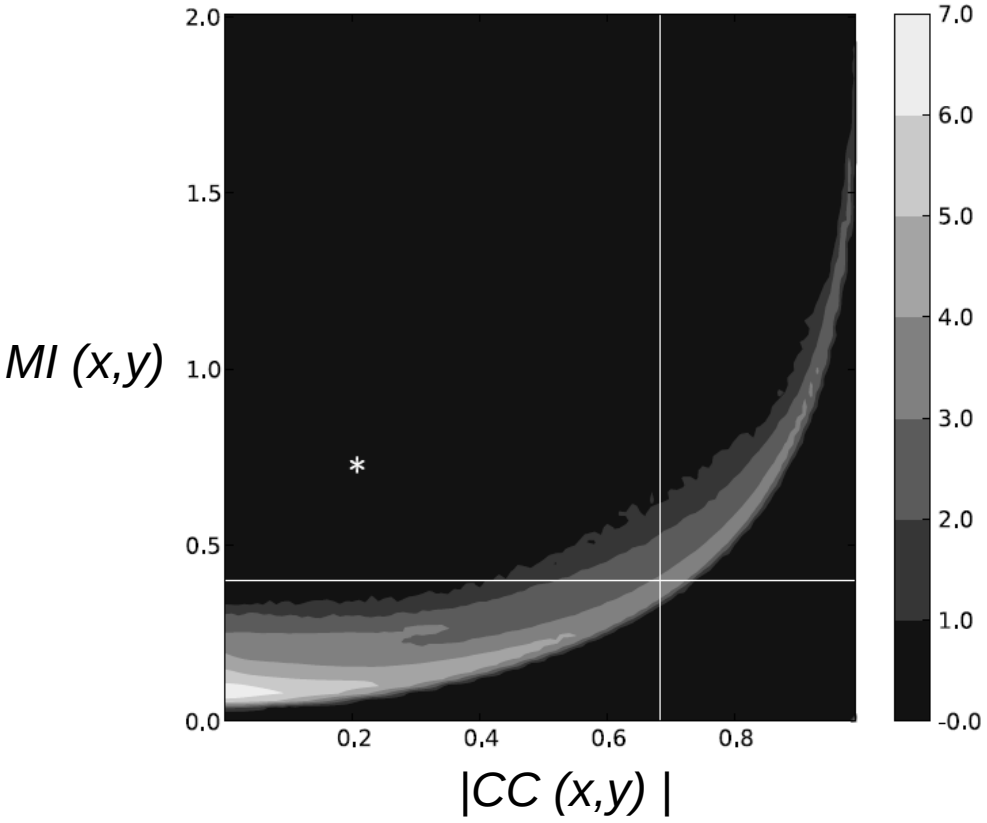
Main problem: a reliable estimation of MI requires a large amount of data

Which is the relation between |CC| and MI?

R. Steuer et al, Bioinformatics 18, suppl 2, S231 (2002).

Relation between cross-correlation and mutual information

- Depends on the data.
- Here computed from 6816 x 6816 SAT anomaly series.



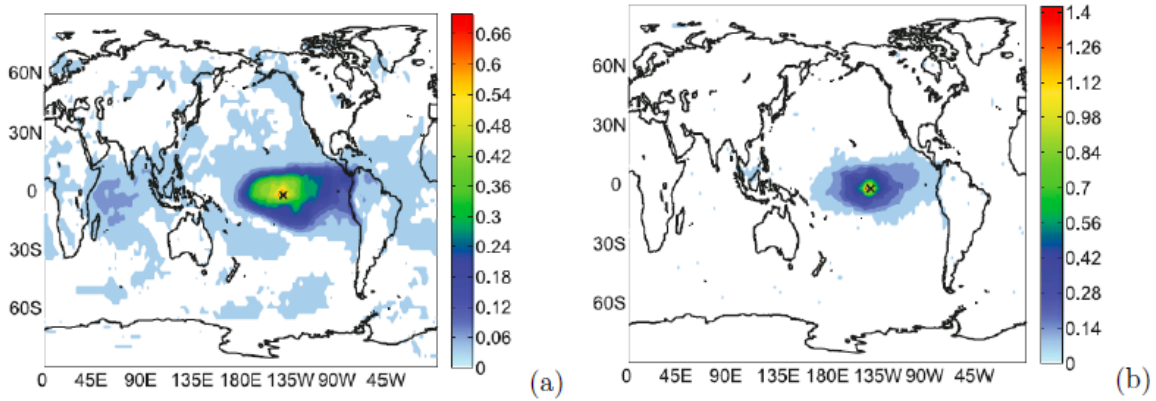
2D histogram; the color represents the number of elements in each bin in log scale

Donges et al, Eur. Phys. J. Special Topics 174, 157 (2009)

Mutual information maps

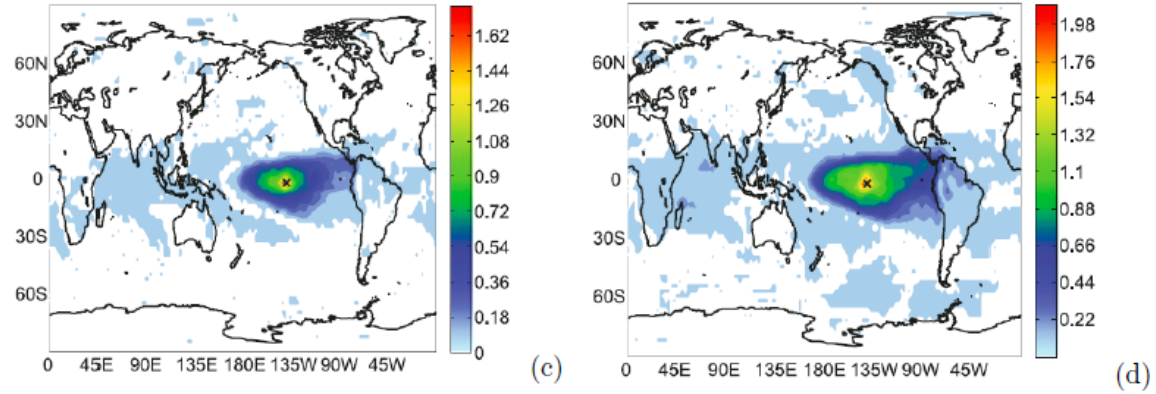
- MI between SAT anomalies time-series at a reference point located in El Niño, and all the other time-series.

Histograms



3 months ordinal patterns

Inter-annual ordinal patterns

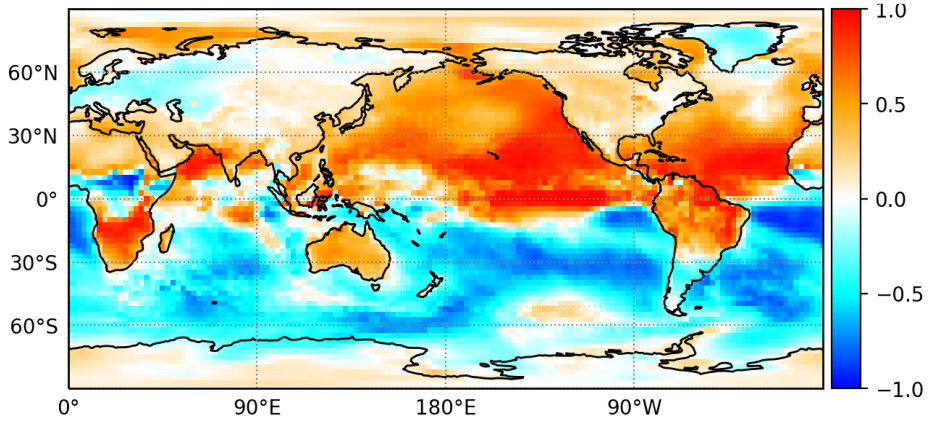


3 years ordinal patterns

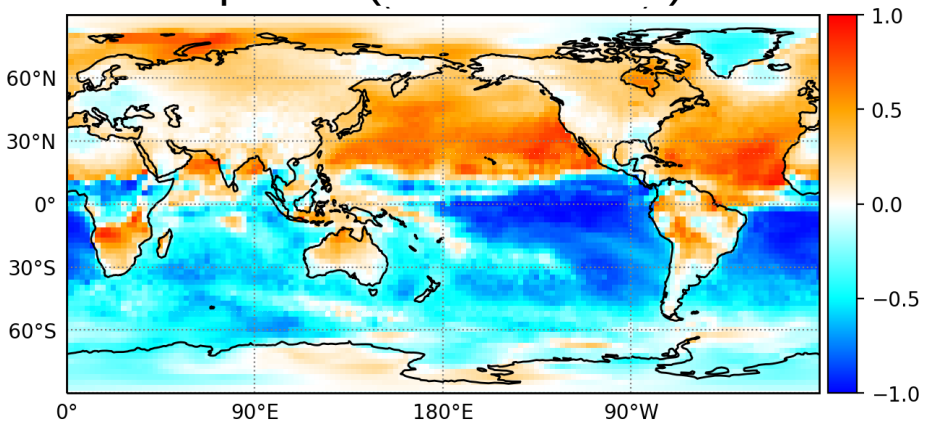
Ordinal analysis separates the times-scales of the interactions

Deza, Barreiro and Masoller, Eur. Phys. J. ST 222, 511 (2013)

Cosine of Hilbert phase in an El Niño period (October 2015)

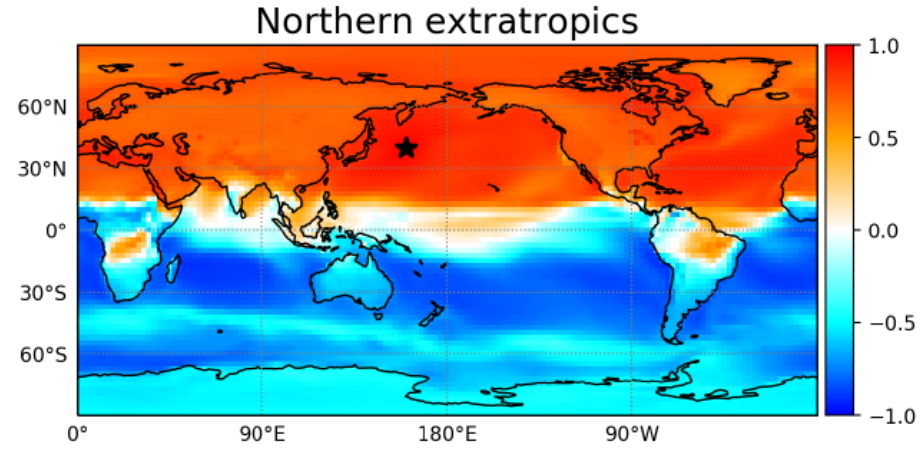


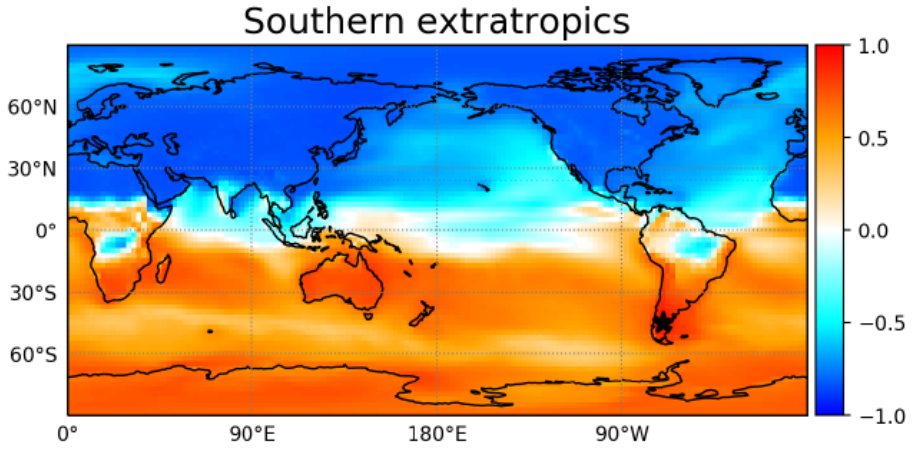
Cosine of Hilbert phase in a La Niña period (October 2011)

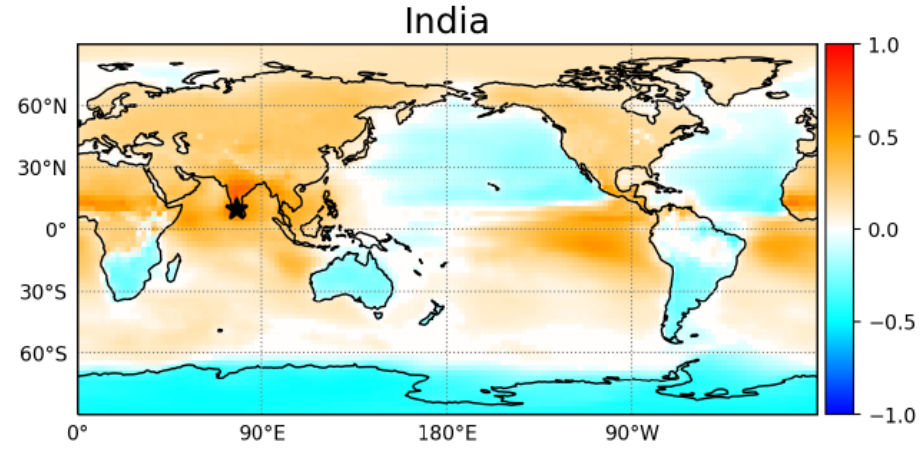


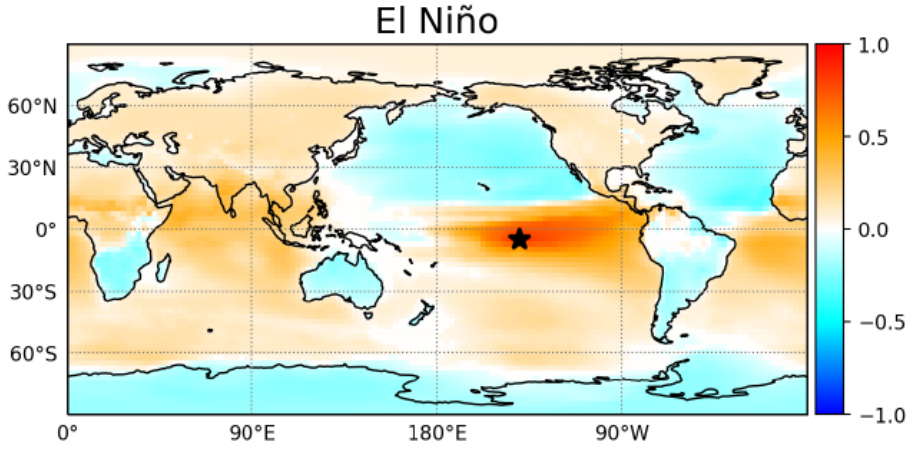
What connectivity patterns we infer using Hilbert analysis?

Cross-correlation of cosine of Hilbert phase



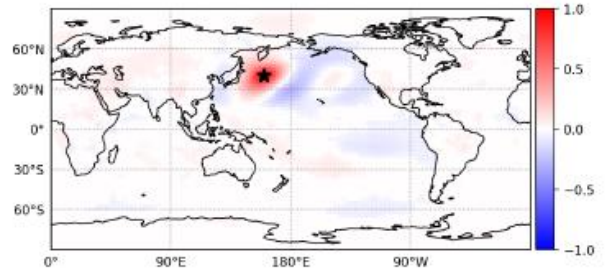


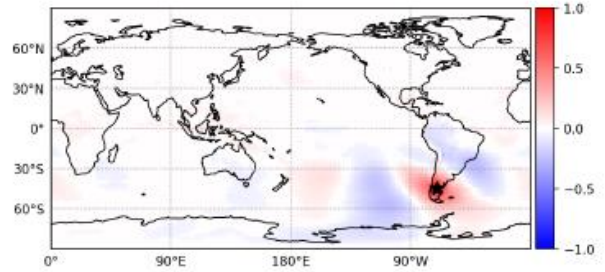




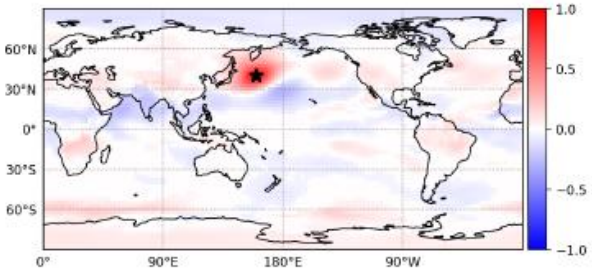
Cross-correlations in the extra-tropics

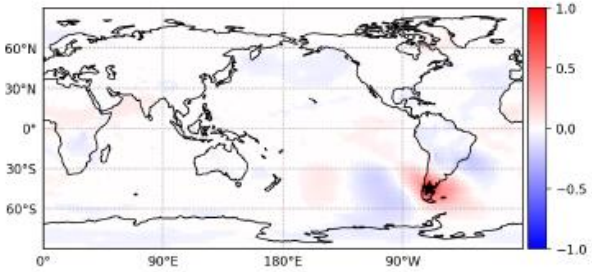
SAT Anomalies



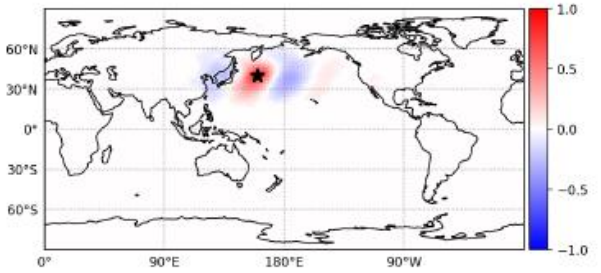


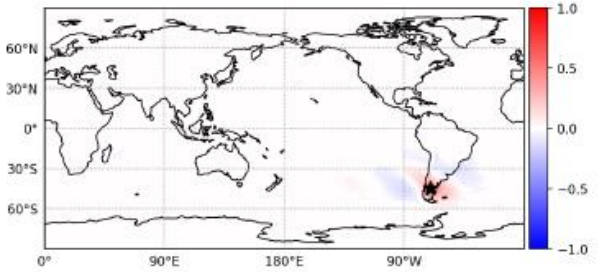
Hilbert amplitude





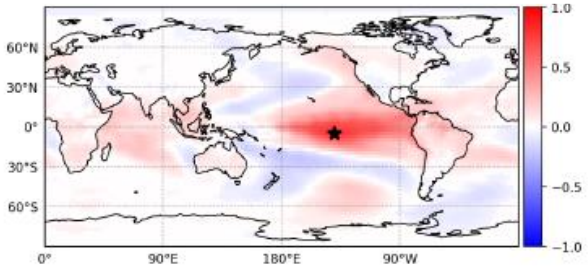
Hilbert frequency

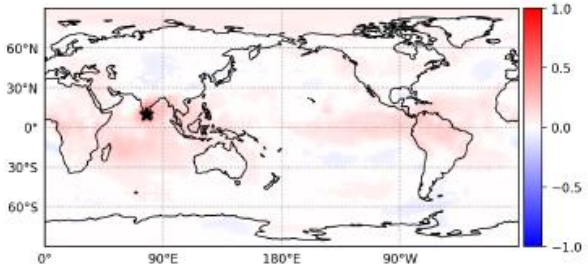




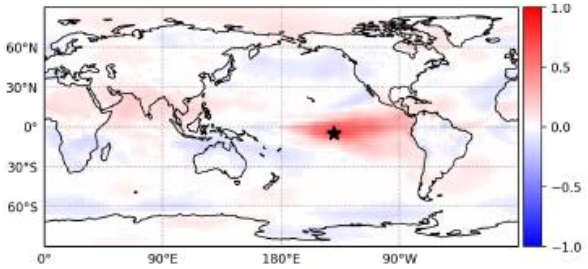
Cross-correlations in the tropics

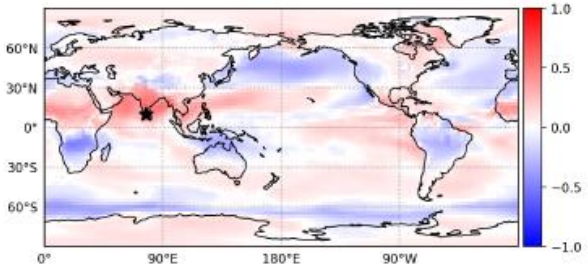
SAT Anomalies



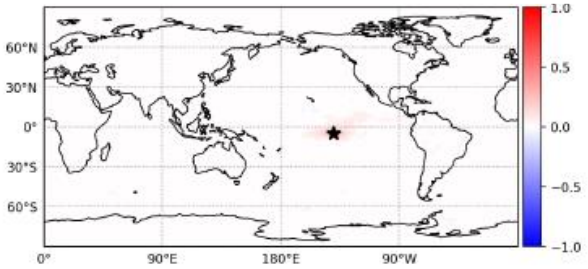


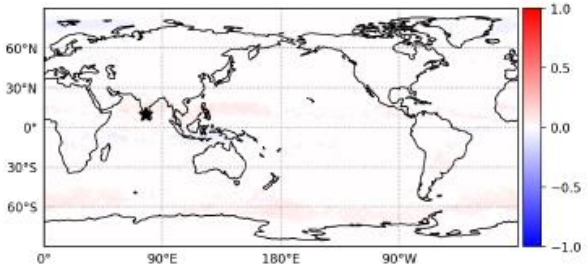
Hilbert amplitude





Hilbert frequency





Directionality of information transfer?

Conditional mutual information (CMI) and transfer entropy (TE)

- CMI measures the amount of information shared between two time series *i(t)* and *j(t)*, given the effect of a third time series, *k(t)*, over *j(t)*.

$$M_I(i;j|k) = \sum_{m,n,l} p_{ijk}(m,n,l) \log \frac{p_k(l) p_{ijk}(m,n,l)}{p_{ik}(m,l) p_{jk}(n,l)}$$

- Transfer entropy = CMI with the third time series, *k(t)*, replaced by the *past* of *i(t)* or *j(t)*.

$$\mathrm{TE}_{ij}(\tau) \equiv M_I(i;j|i_\tau) \qquad \mathrm{TE}_{ji}(\tau) \equiv M_I(j;i|j_\tau)$$

Directionality index

- τ: *time-scale* of information transfer
- *DI*: *net direction* of information transfer
- $DI_{ij} > 0 \rightarrow i$ drives j.

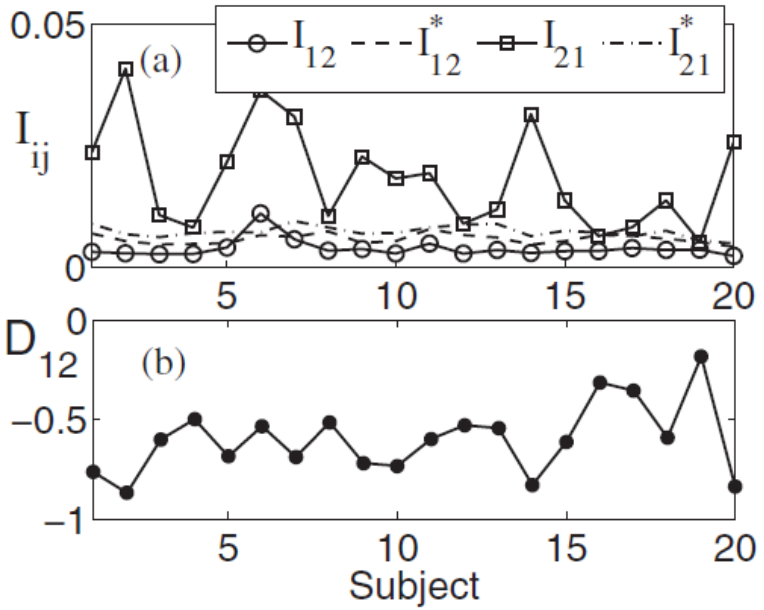
$$\mathrm{DI}_{ij}(\tau) = \frac{\mathrm{TE}_{ij}(\tau) - \mathrm{TE}_{ji}(\tau)}{\mathrm{TE}_{ij}(\tau) + \mathrm{TE}_{ji}(\tau)}$$

Problem: $x \rightarrow i$, $x \rightarrow j$ $\quad i \leftrightarrow j$??

Application to cardiorespiratory data measured from 20 healthy subjects:
(a) TEs (dashed lines: surrogate data)
(b) D_{12} (1 = heart; 2 = respiration).

$D_{12} < 0 \rightarrow$ respiration is drives cardiac activity.

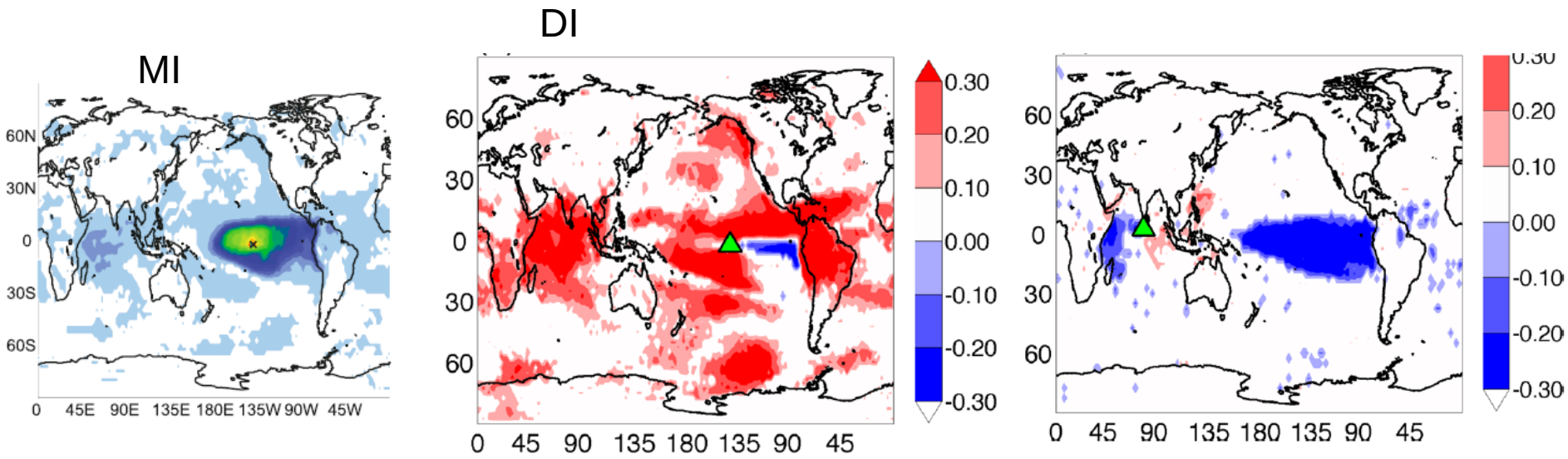
TEs were computed from ordinal probabilities and averaged over a short range of lags to decrease fluctuations.



A. Bahraminasab et al., PRL 100, 084101 (2008)

Application to climate data

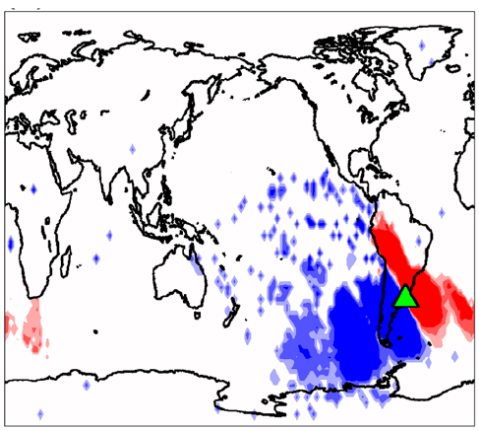
DI computed from daily SAT anomalies, PDFs estimated from histograms of values.
MI and DI are *both significant* (>3σ, bootstrap surrogates), τ=30 days.

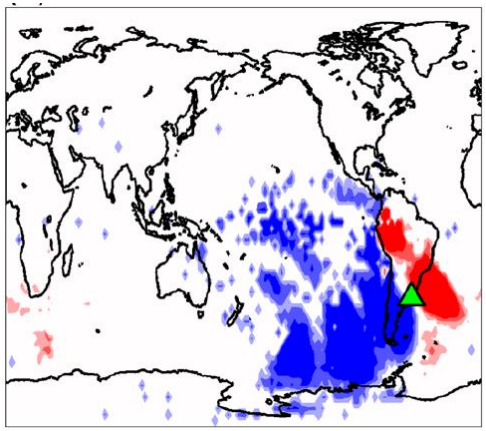


J. I. Deza, M. Barreiro, and C. Masoller, "*Assessing the direction of climate interactions by means of complex networks and information theoretic tools*", Chaos 25, 033105 (2015).

Influence of the time-scale of information transfer

τ=1 day



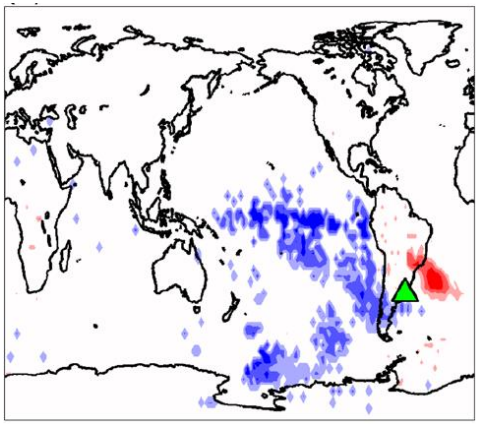


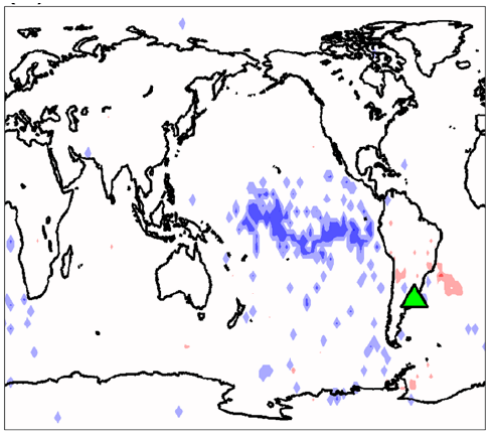
τ=3 days

Video SH

Video NH

τ=7 days





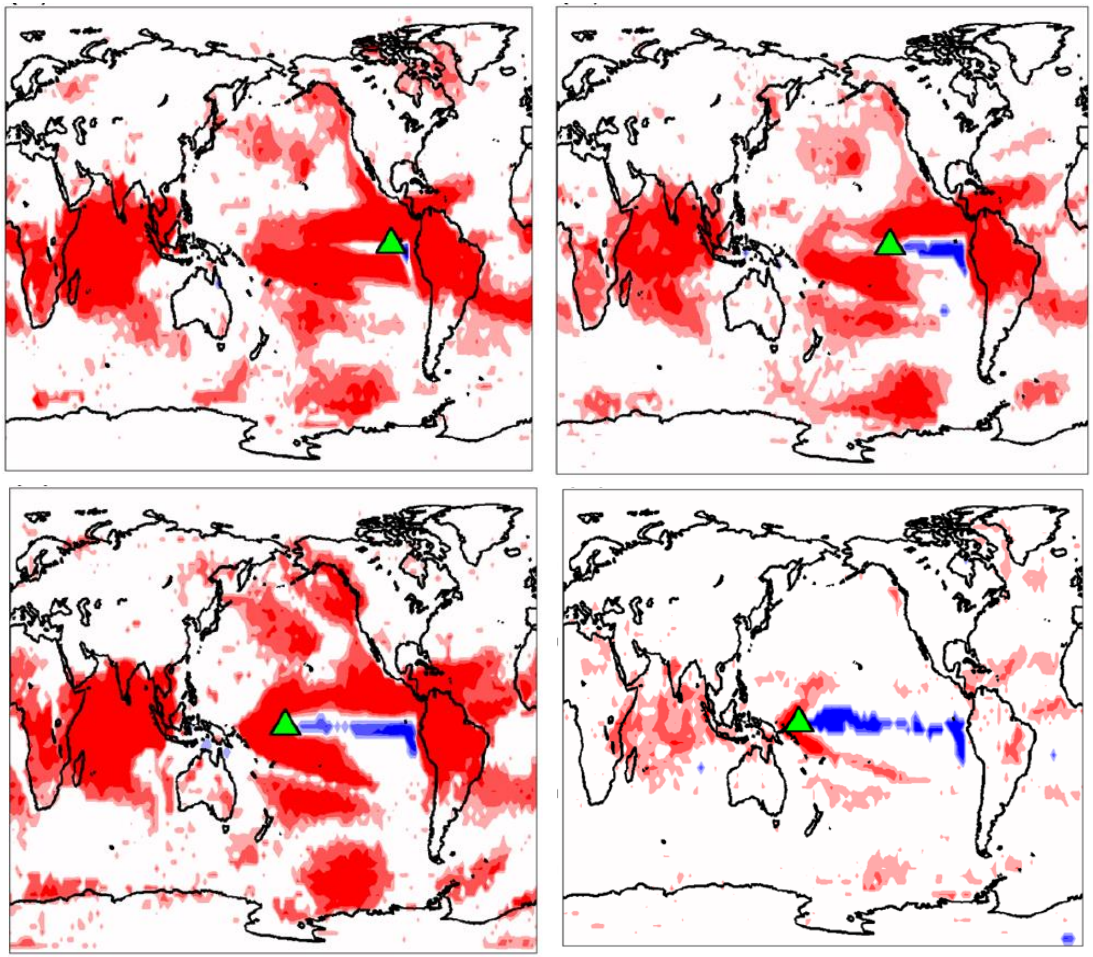
τ=30 days

Link directionality reveals wave trains propagating from west to east.

Videos in El Niño, El Labrador and Rio de la Plata, when τ increases from 1 to 180 days

Deza, Barreiro and Masoller, Chaos 25, 033105 (2015)

Link directionality in El Niño area (τ=30 days)



Deza, Barreiro and Masoller, Chaos 25, 033105 (2015)

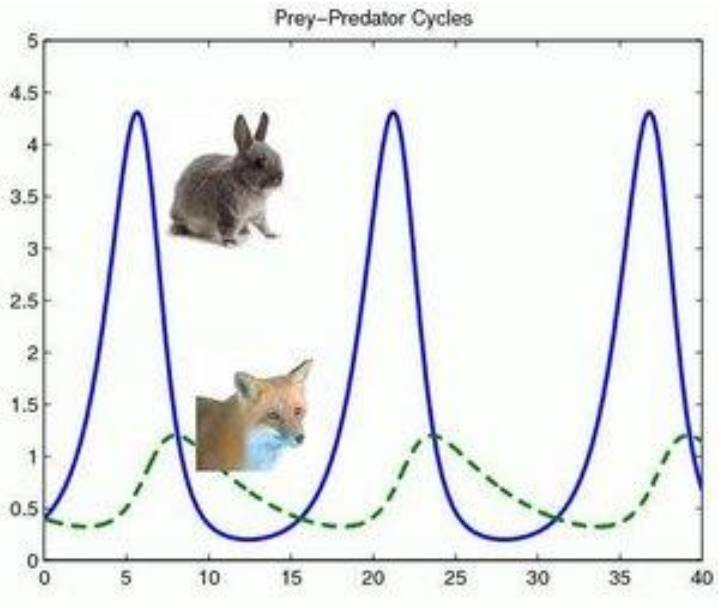
Video

Causality?

Main idea

- A time series *X* is Granger causal to a time series *Y* (*X*→*Y*) if the information given by *X* allows for a more precise prediction of *Y*.

- Example: in the predator - prey system, information about variations in the predator population can reveal properties of the prey population.



Granger causality

- Method: model *Y* as a AR(d) processes forced by *X* with residual noise ε

$$Y_t = \sum_{i=1}^{d} a_i Y_{t-i} + \sum_{i=1}^{d} b_i X_{t-i} + \epsilon_t$$

- Test the hypothesis $b \neq 0$ against the null hypothesis $b=0$.

  To do this

  - Fit vectors *a* and *b* with a linear regression and compute the variance of the residual: $\sigma^2_{\text{coupled}}$
  - Repeat with $b=0$ and compute: $\sigma^2_{\text{uncoupled}}$

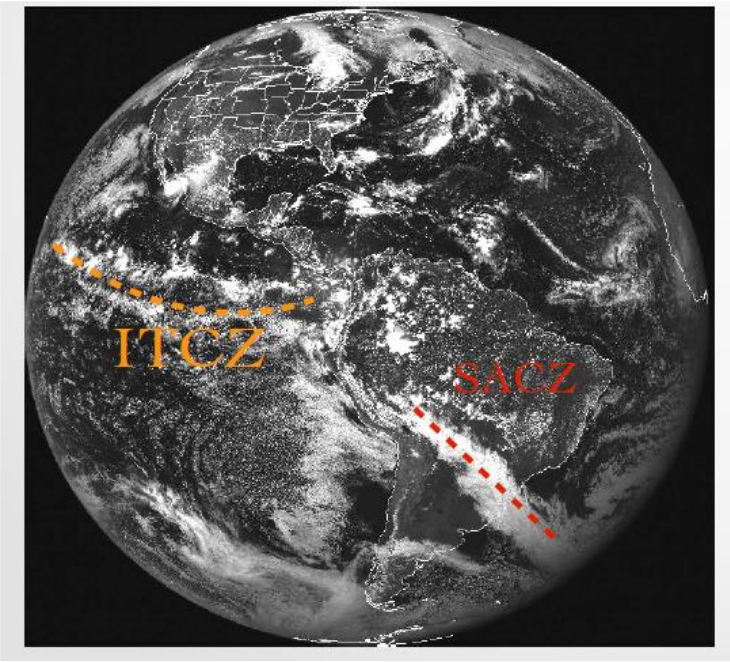
Granger causality estimator

$$GCE = \frac{\sigma^2_{\text{uncoupled}} - \sigma^2_{\text{coupled}}}{\sigma^2_{\text{uncoupled}}}$$

- If *GCE*>0 the information given by *X* allowed for a more precise prediction of *Y*.
- Problems:
  - how to select the dimension *d*?
  - how to test the statistical significance of the GCE value?

Read more: G. Tirabassi, C. Masoller and M. Barreiro, "*A study of the air-sea interaction in the South Atlantic Convergence Zone through Granger Causality*", Int. J. of Climatology, 35, 3440 (2015)

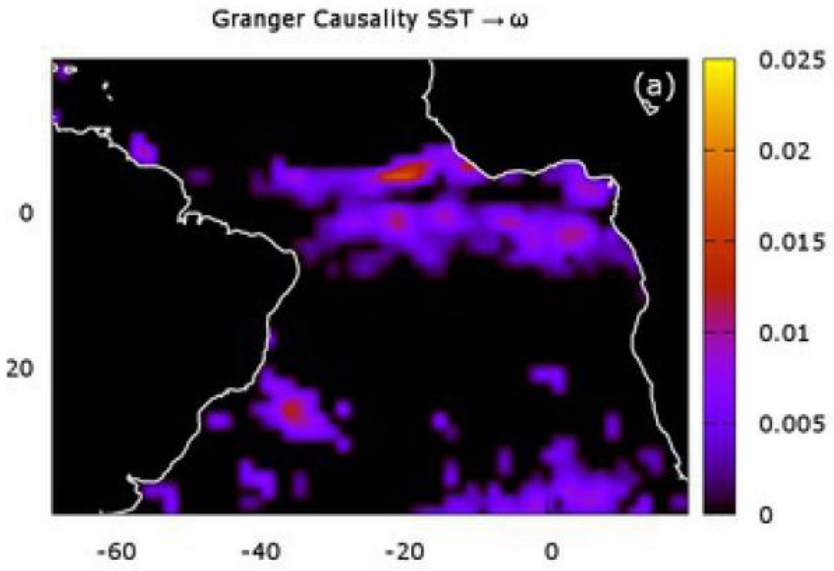
Other methods to detect causality: G. Tirabassi, L. Sommerlade and C. Masoller, "*Inferring directed climatic interactions with renormalized partial directed coherence and directed partial correlation*", Chaos 27, 035815 (2017)



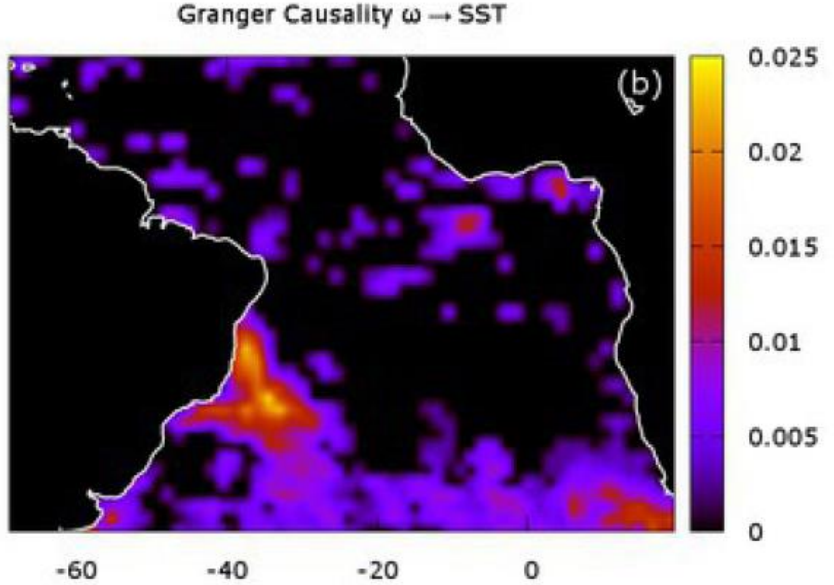
Application to climate data: rain-ocean interaction in the South Atlantic Convergence Zone

Data: two time series at the same geographical location.
SST = Surface sea temperature
ω = vertical wind velocity at 500 hPa (precipitation proxy)

Local ocean $\rightarrow$ wind



Local wind $\rightarrow$ ocean



The color code represents GCE values (only values significant at 99% confidence)

Ocean forces the atmosphere in the tropics and the subtropical waters of Brazil.

The atmosphere also forces a localized region of the ocean in front of Brazil.

G. Tirabassi et al, Int. J. of Climatology, 35, 3440 (2015)

How to find “synchronized events” in two time series?

Measures of event synchronization

- Define “events” in each time series. m_x, m_y are the number of events in each time series.
- Count $c^\tau(x|y)$ = number of times an event appears in x shortly after an event appears in y. Analogous for $c^\tau(y|x)$.
- Measures:

$$Q_\tau = \frac{c^\tau(y|x) + c^\tau(x|y)}{\sqrt{m_x m_y}} \qquad q_\tau = \frac{c^\tau(y|x) - c^\tau(x|y)}{\sqrt{m_x m_y}}$$

- $Q_\tau = 1$ if and only if the events of the signals are fully synchronized.
- $q_\tau = 1$ if the events in x always precede those in y.
- Many applications. Further reading: Quian Quiroga et al, PRE 66, 041904 (2002).

Take home messages

- Cross-correlation: detects linear interdependencies.
- Mutual information: detects nonlinear interdependencies.
- The MI computed from the probabilities of ordinal patterns allows to select the time-scale of the analysis.
- The directionality index detects the net direction of the information flow.
- Granger causality can “disentangle” mutual interactions.

References

- M. Palus, Contemporary Physics 48, 307(2007)

- M. Barreiro, et. al, Chaos 21, 013101 (2011)
- Deza, Barreiro and Masoller, Eur. Phys. J. ST 222, 511 (2013)
- Tirabassi and Masoller, EPL 102, 59003 (2013)
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