Nonlinear time series analysis

Introduction

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Campus d'Excel·lència Internacional

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Presentation

- Originally from Montevideo, Uruguay
- PhD in physics (lasers, Bryn Mawr College, USA)
- Since 2004 @ Universitat Politecnica de Catalunya.
- Professor in the Physics Department, research group on Dynamics, Nonlinear Optics and Lasers.

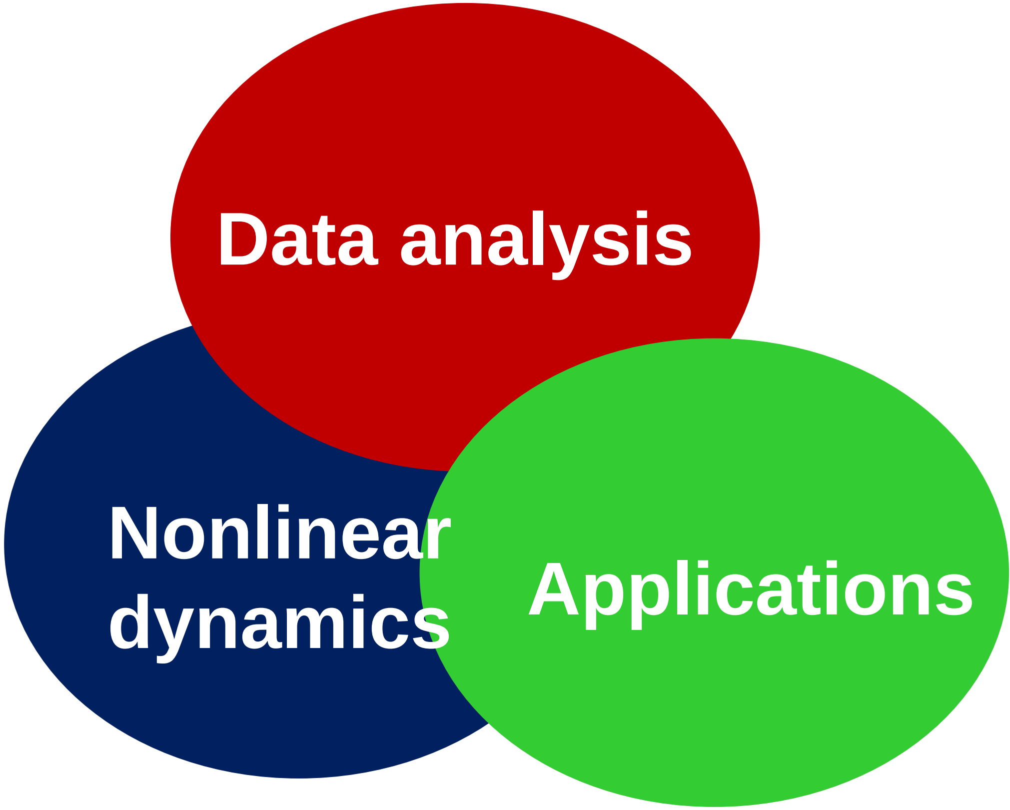
Where are we?

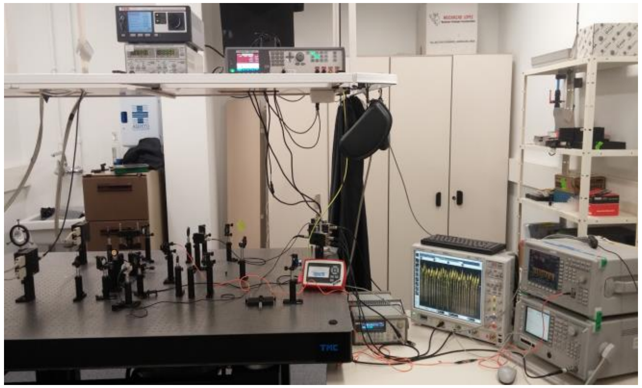
1. Barcelona
2. Castelldefels
3. Igualada
4. Manresa
5. Mataró
6. Sant Cugat del Vallès
7. Terrassa
8. Vilanova i la Geltrú

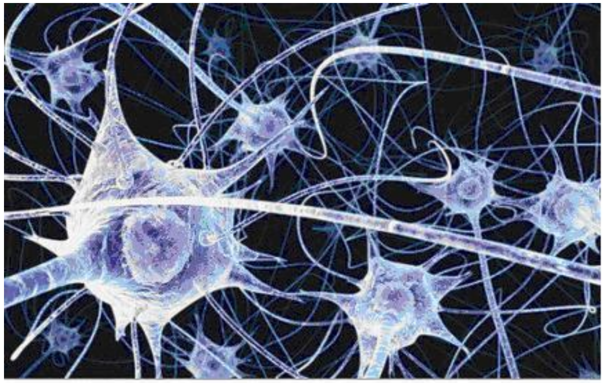
Viernes, 25 de septiembre de 2009 Diari de Terrassa
El edificio Gaia centraliza grupos científicos consolidados y emergentes.

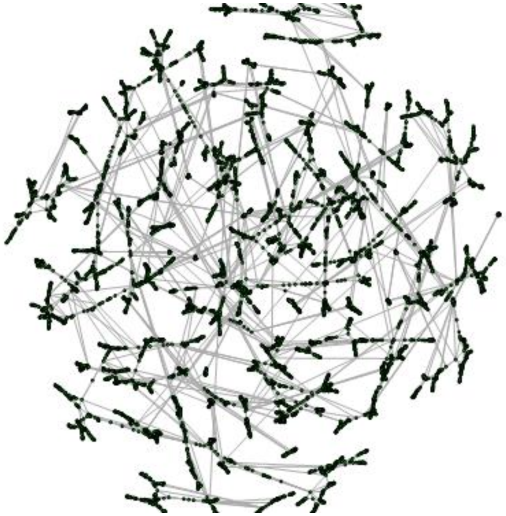
What do we study?

- Nonlinear and stochastic phenomena
  - laser dynamics
  - neuronal dynamics
  - complex networks
  - data analysis (climate, biomedical signals)



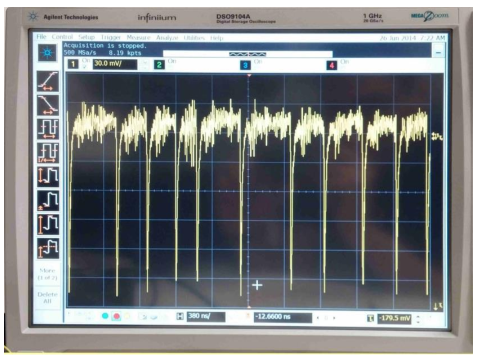




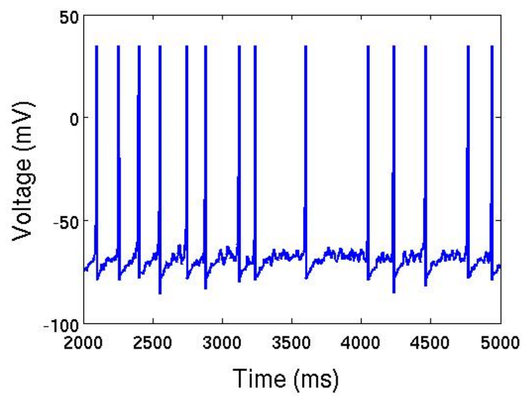


Lasers, neurons, climate, complex systems?

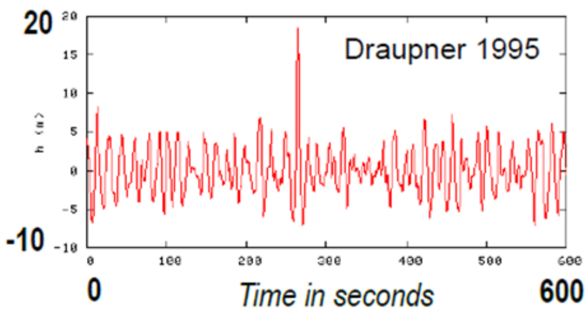
- Lasers allow us to study in a controlled way phenomena that occur in diverse complex systems.
- Laser experiments allow to generate sufficient data to test new methods of data analysis for prediction, classification, etc.



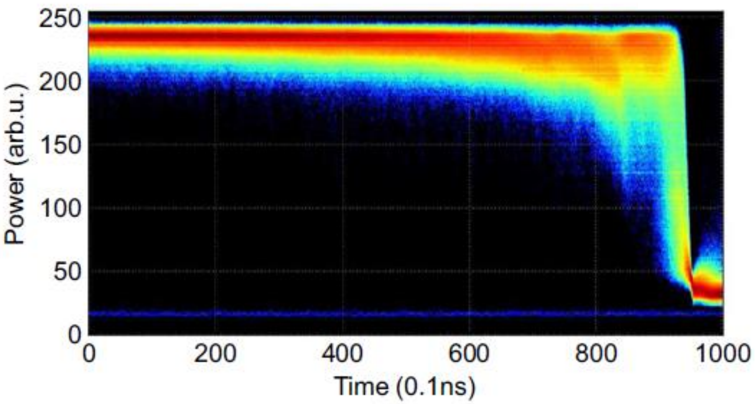
Laser & neuronal spikes



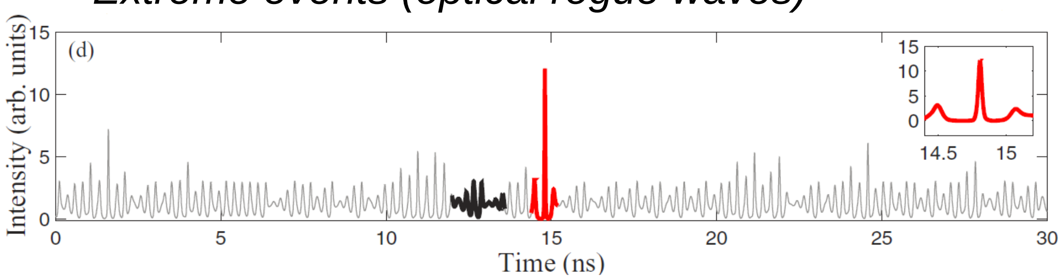
Ocean rogue wave (sea surface elevation in meters)



Abrupt switching

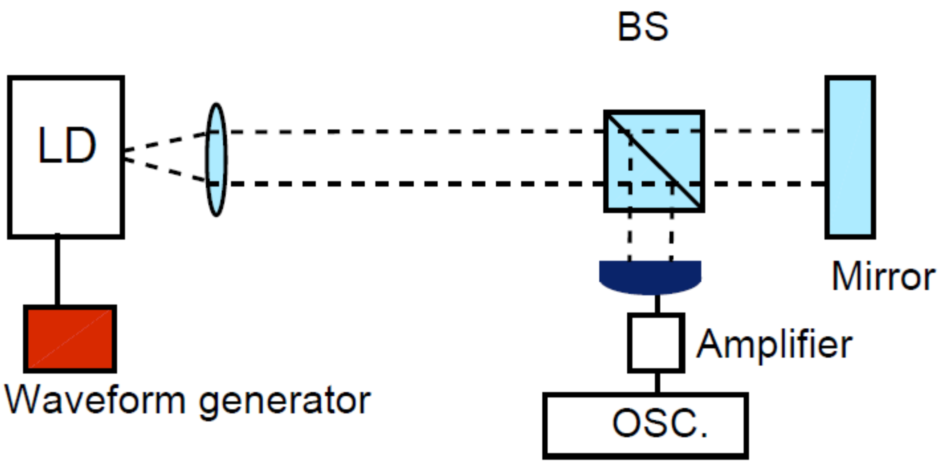


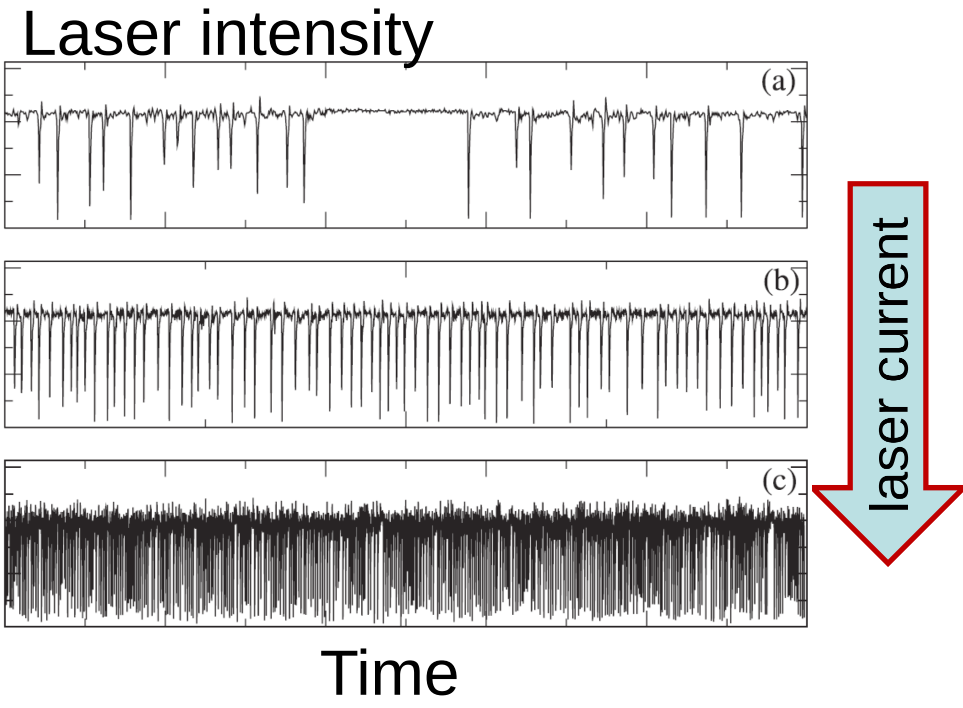
Extreme events (optical rogue waves)



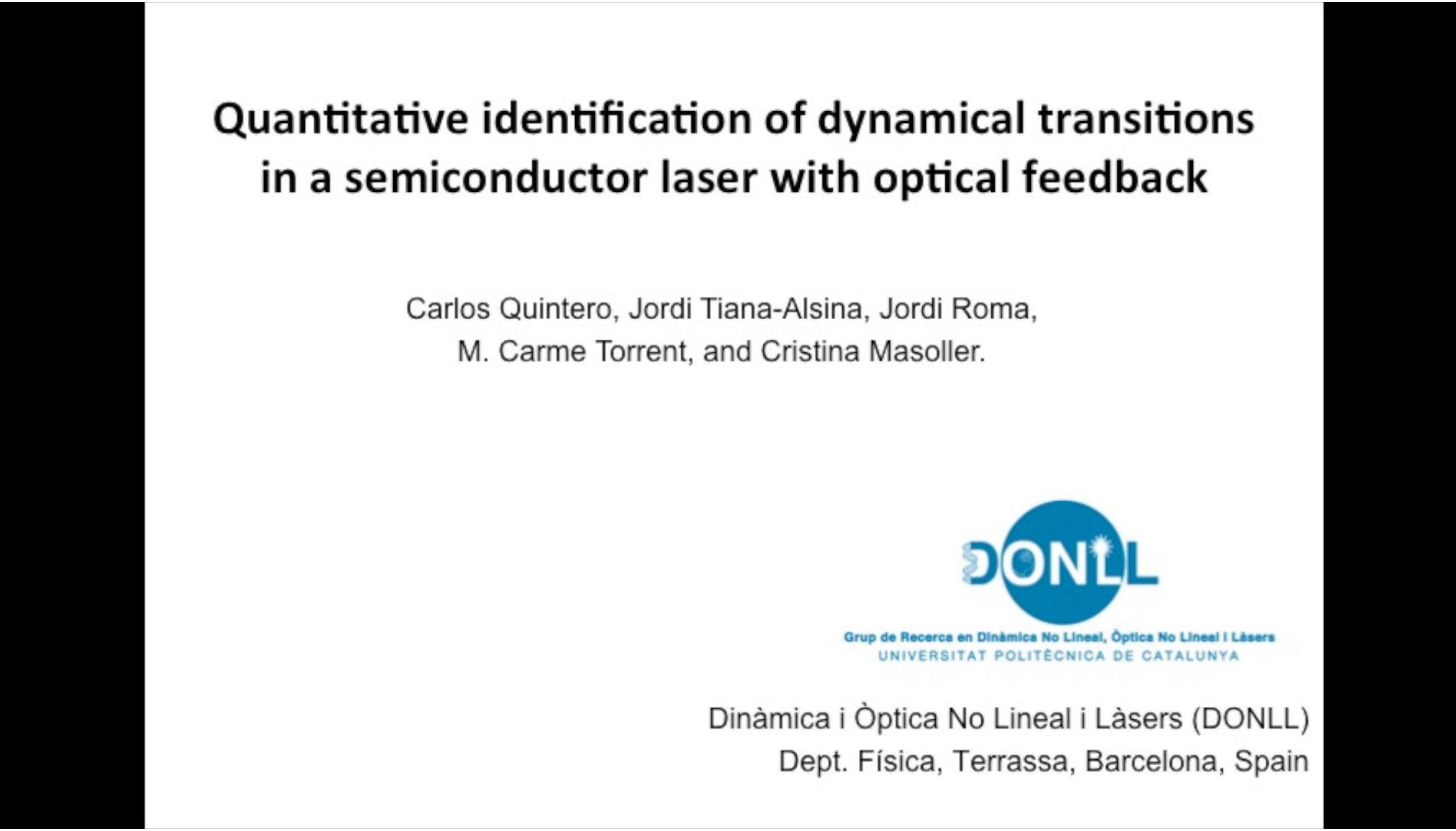
In complex systems dynamical transitions are difficult to identify and to characterize.

Example: laser with time delayed optical feedback

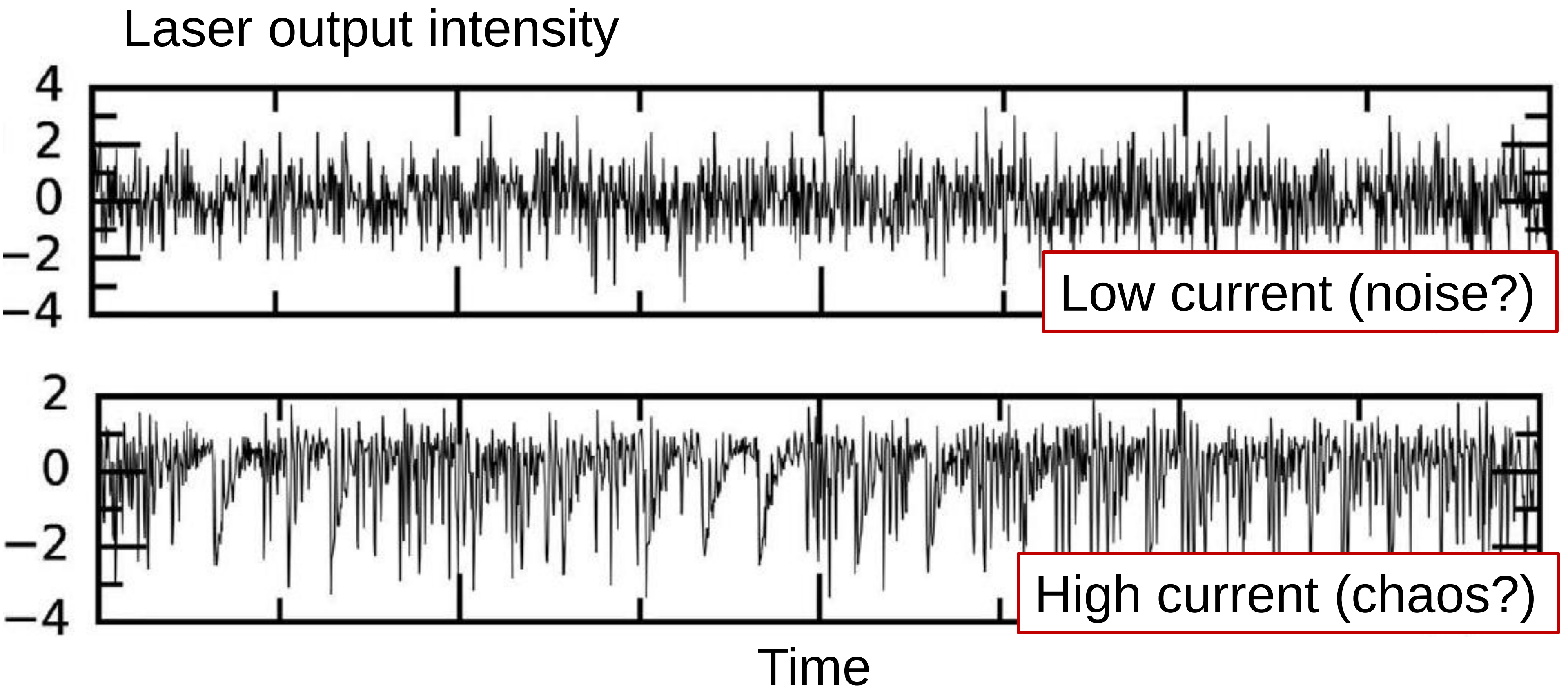




How complex optical signals emerge from noise



Video: how complex optical signals emerge from noisy fluctuations



Can differences be quantified? With what reliability?

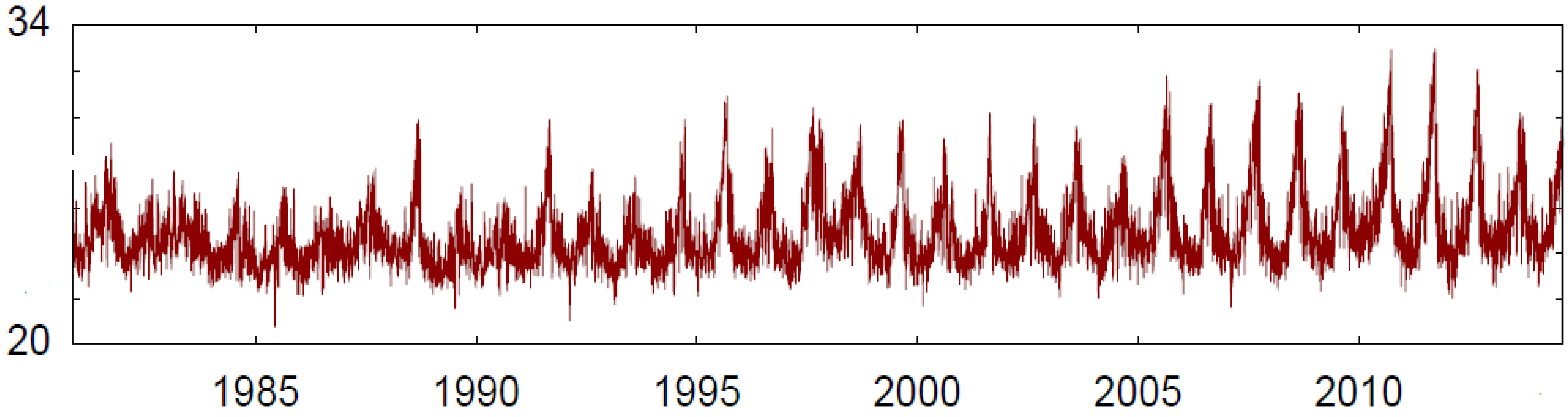
Are weather extremes becoming more frequent? more extreme?

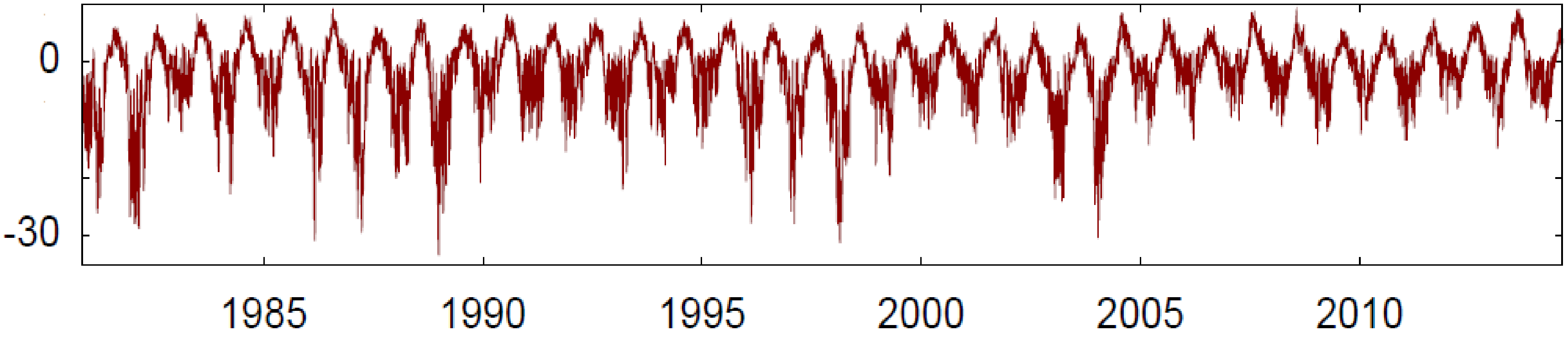
Physics Today, Sep. 2017

ECMWF

Credit: Richard Williams, North Wales, UK

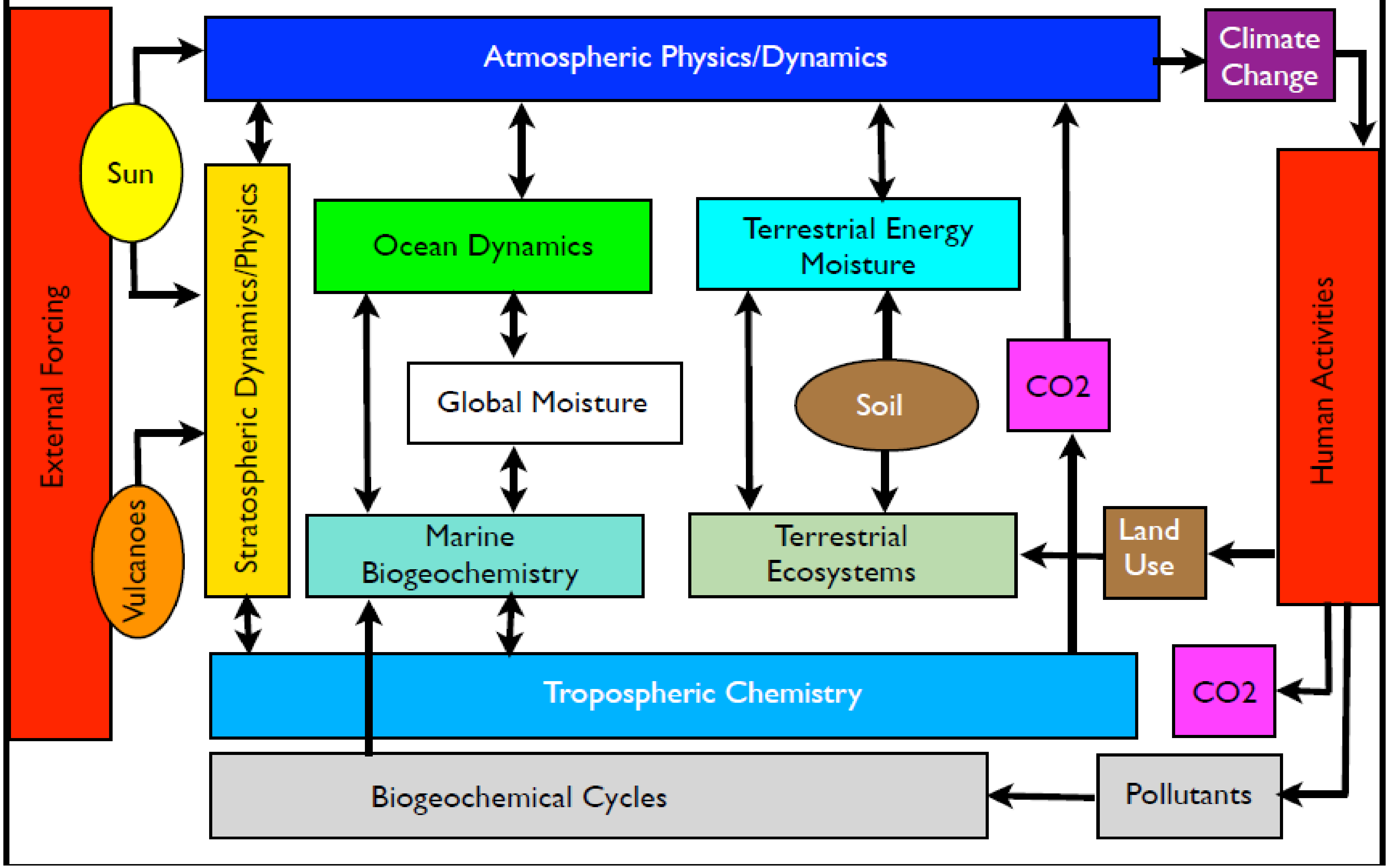
Surface air temperature in two different regions





Can changes be quantified? With what reliability?

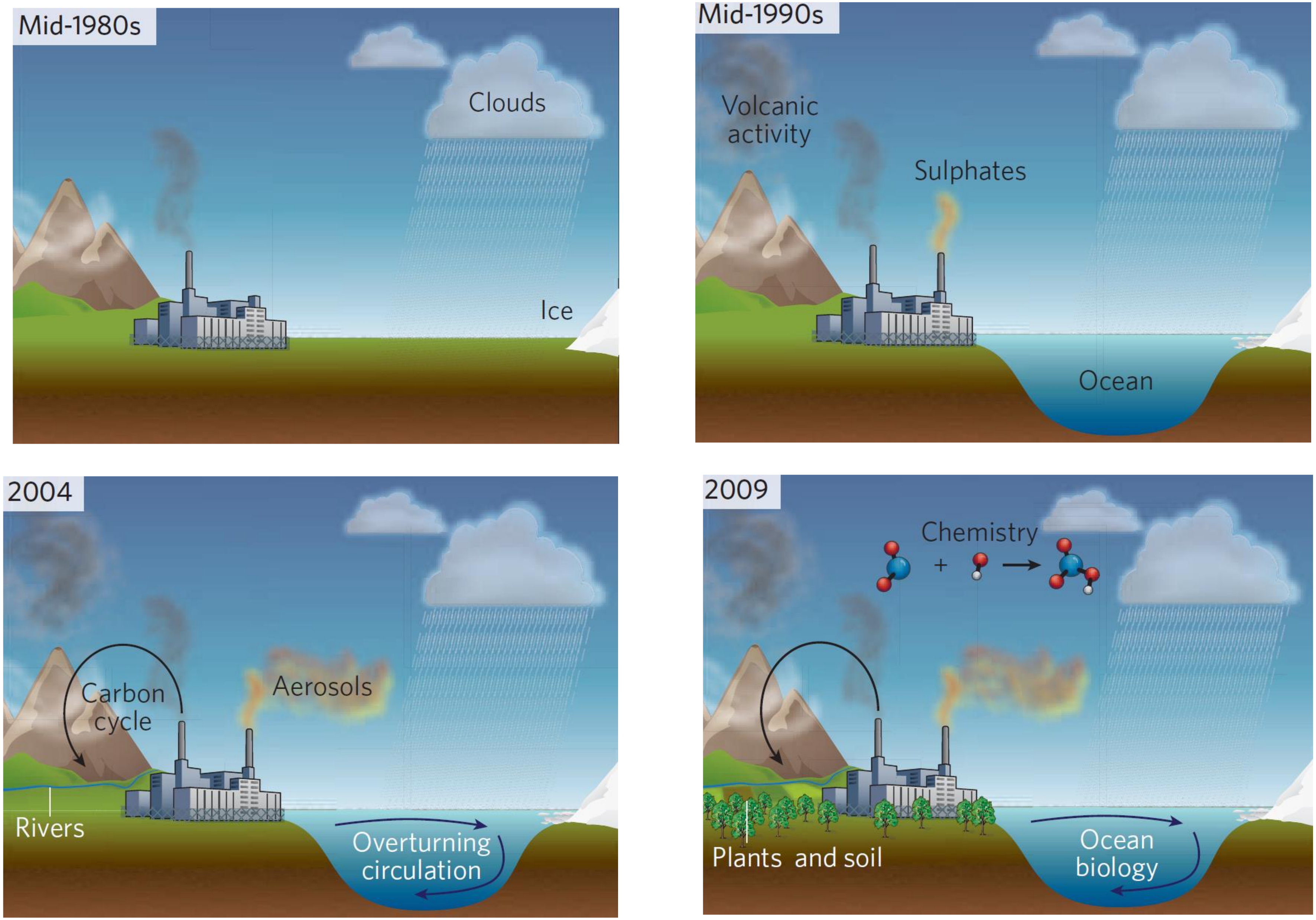
The Climate System is a "complex system"



Courtesy of Henk Dijkstra (Ultrech University)

Thanks to advances in computer science, global climate models allow for good weather forecasts

Nature, February 2010



Nature, February 2010

But global climate models are not very useful for improving our understanding

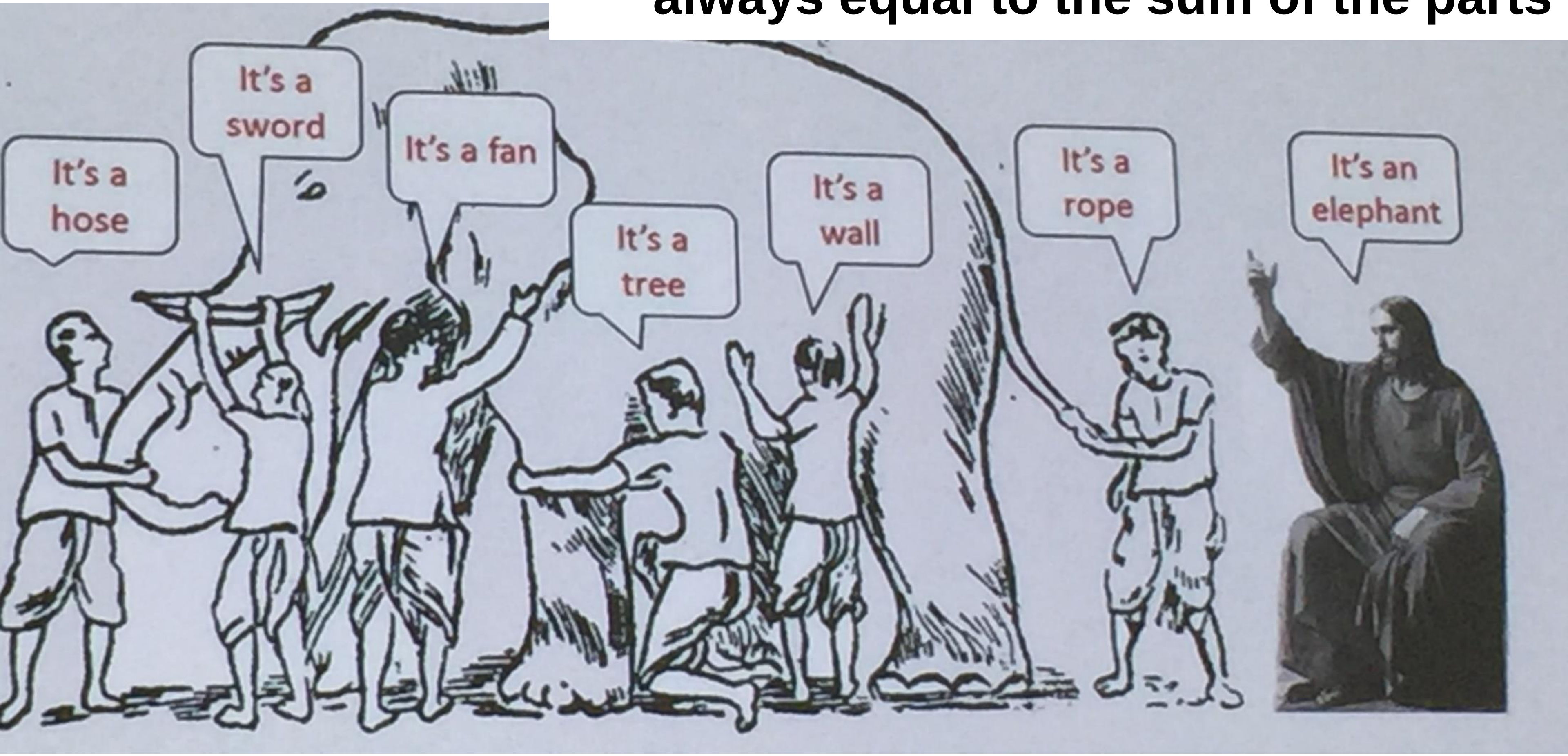
- But “over-simplified models” do not always provide useful information.

In early summer, 1996, milk production at a Wisconsin dairy farm was very low. The farmer wrote to the state university, asking help from academia. A multidisciplinary team of professors was assembled, headed by a theoretical physicist, and two weeks of intensive on-site investigation took place. A few weeks later, a physicist phoned the farmer, "I've got the answer," he said, "But it only works when you consider spherical cows in a vacuum. . . .“
Source: https://mirror.uncyc.org/wiki/Spherical_Cows

Strong need of nonlinear methods to extract reliable information from data

Why nonlinear ?

Because in nature the whole is not always equal to the sum of the parts



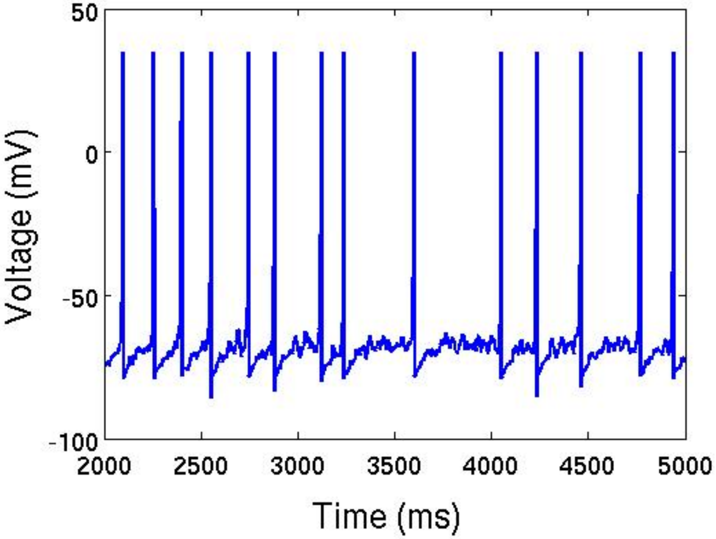
Outline

- **Introduction**
  - Historical development: from dynamical systems to complex systems
- **Univariate analysis**
  - Methods to extract information from a time series.
  - Applications.
- **Bivariate analysis**
  - Correlation, directionality and causality.
  - Applications.
- **Multivariate analysis**
  - Many time series: complex networks.
  - Network characterization and analysis.
  - Applications.

Time Series Analysis: what is this about?

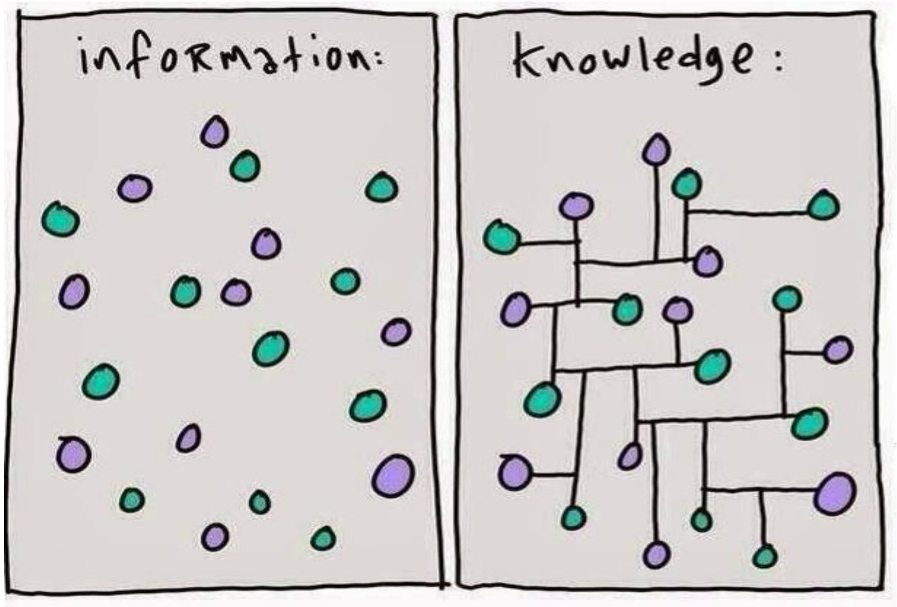
Optical spikes

Neuronal spikes



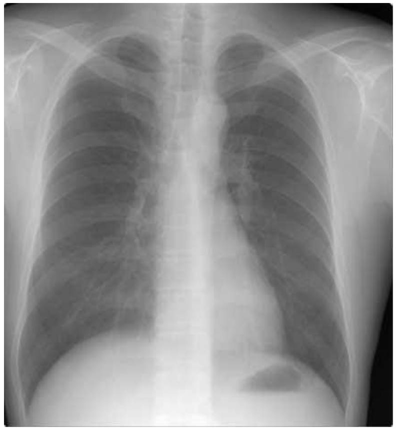
- Similar dynamical systems generate these signals?
- Ok, very different dynamical systems, but maybe similar statistical properties?
- Time series analysis finds "hidden similarities" in very different systems.

Main goal of Time Series Analysis: to extract meaningful information

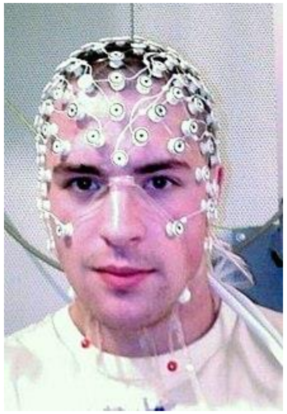


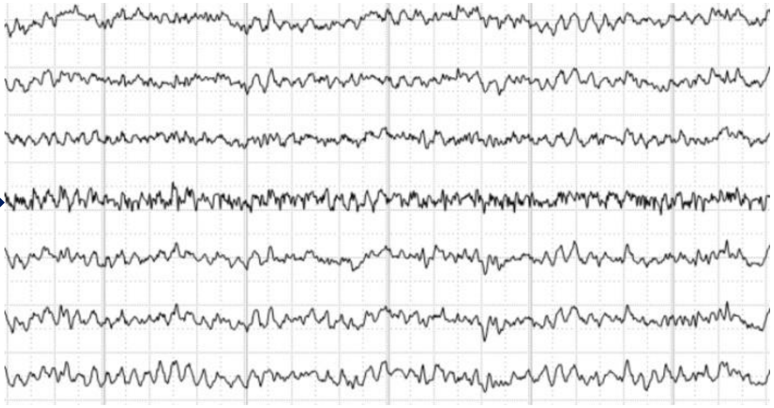
What for?

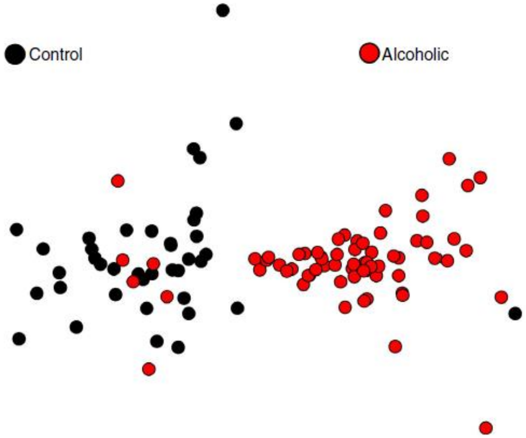
– Classification
– Prediction
– Model verification
– Parameter estimation
– Etc.



Example: analysis of EEG signals allows to distinguish control from alcoholic subjects

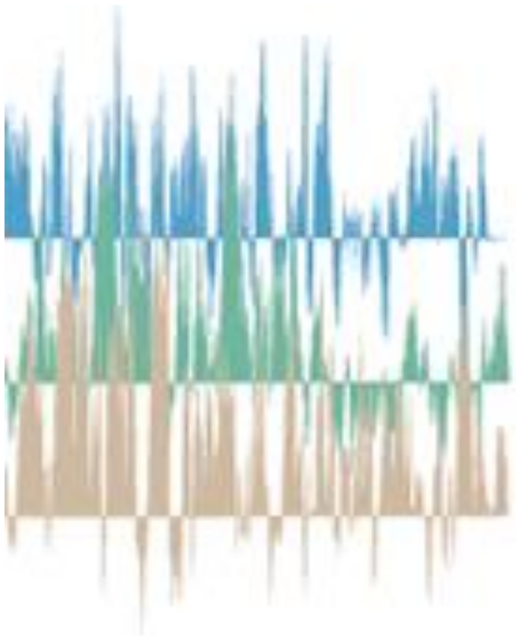




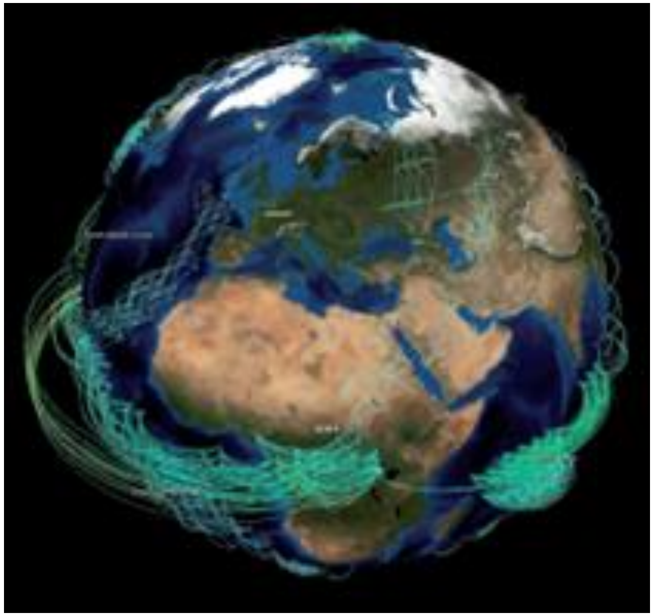


T. A. Schieber et al, Nat. Comm. 8:13928 (2017).

Example: inferring climatic interactions



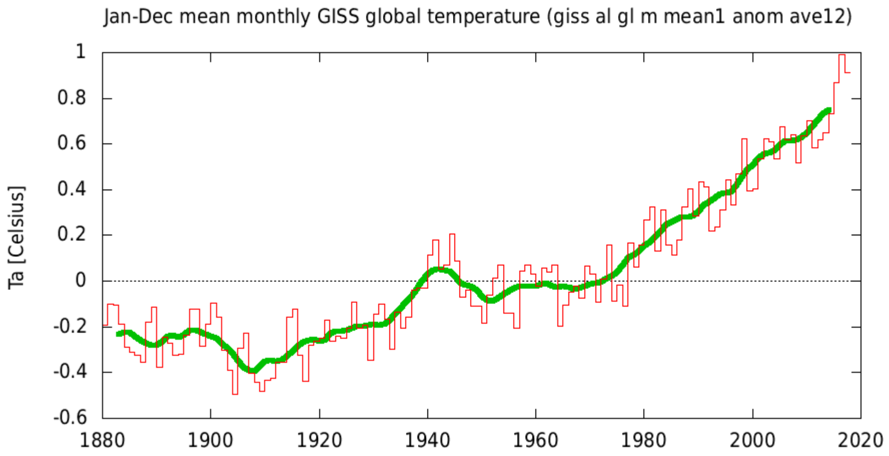
Surface Air Temperature <u>Anomalies</u> in different geographical regions



Donges et al, Chaos 2015

Methods

- Many methods have been developed to extract information from a time series (x_1, x_2, … x_N).
- The method to be used depends on the characteristics of the data
  - Length of the time series;
  - Stationarity;
  - Level of noise;
  - Temporal resolution;
  - etc.



- **Different methods provide complementary information.**

Where the data comes from?

- Modeling assumptions about the **type of dynamical system** that generates the data:
  - Stochastic or deterministic?
  - Regular or chaotic or “complex”?
  - Stationary or non-stationary? Time-varying parameters?
  - Low or high dimensional?
  - Spatial variable? Hidden variables?
  - Time delays? Etc.

- Good results depend on the *knowledge* of the system that generates the time series.

Brief historical tour, from dynamical systems to complex systems

The start of dynamical systems theory

- Mid-1600s: Ordinary differential equations (ODEs)

- **Isaac Newton**: studied planetary orbits and solved analytically the “two-body” problem (earth around the sun).

- Since then: a lot of effort for solving the “three-body” problem (earth-sun-moon) – Impossible.

Late 1800s

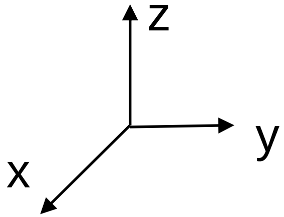
- **Henri Poincare** (French mathematician).

  Instead of asking "*which are the exact positions of planets (trajectories)*?"

  he asked: "*is the solar system* ***stable*** *for ever, or will planets eventually run away*?"

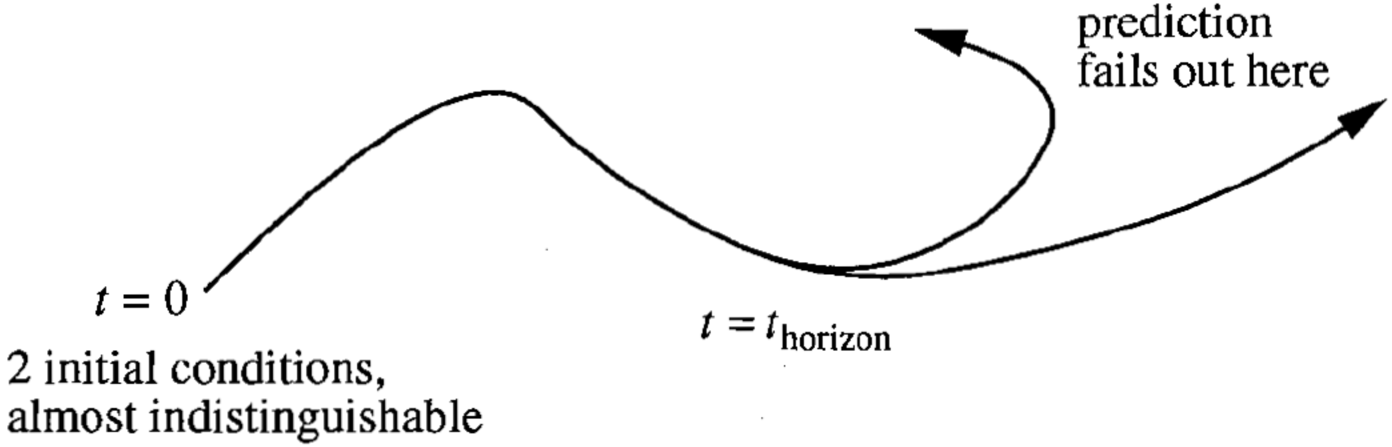
- He developed a **geometrical** approach to solve the problem.

- Introduced the concept of "phase space".



- He also had an <u>intuition</u> of the possibility of **chaos.**

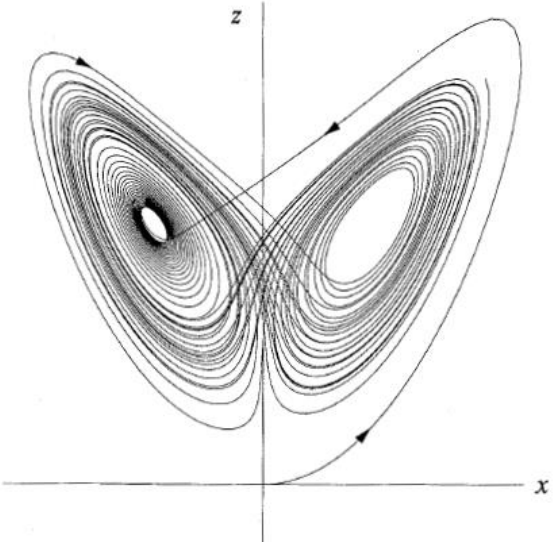
*Poincare: “The evolution of a **deterministic** system can be aperiodic, unpredictable, and strongly depends on the initial conditions”*



Deterministic system: the initial conditions fully determine the future state. **There is no randomness but the system can be unpredictable**.

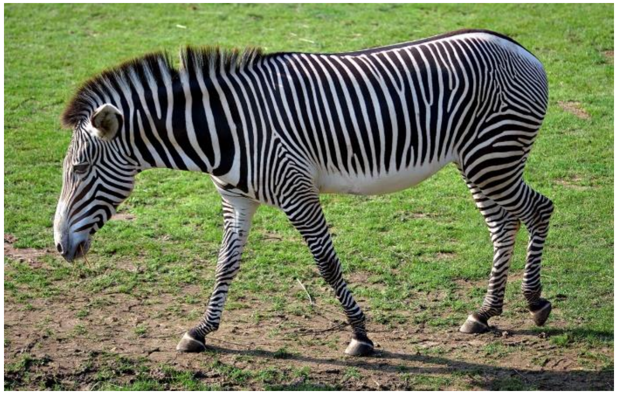
1950s: First computer simulations

- Computes allowed to experiment with equations.
- Huge advance in the field of "*Dynamical Systems*".
- 1960s: **Eduard Lorentz** (American mathematician and meteorologist at MIT): simple model of convection rolls in the atmosphere.
- **Chaotic** motion.



Order within chaos and self-organization

- **Ilya Prigogine** (Belgium, born in Moscow, Nobel Prize in Chemistry 1977)
- Thermodynamic systems far from equilibrium.
- Discovered that, in chemical systems, the interplay of (external) **input of energy** and **dissipation** can lead to “self-organized” patterns.



The study of spatio-temporal structures has uncovered striking similarities in nature

Honey bees do a spire wave to scare away predators

https://www.youtube.com/watch?v=Sp8tLPDMUyg

Rotating waves occur in the heart during ventricular fibrillation

Hurricane Maria (Wikipedia)

https://media.nature.com/original/nature-assets/nature/journal/v555/n7698/extref/nature26001-sv6.mov

Spiral vegetation patterns in high-altitude wetlands

Cristian Fernandez-Oto*, Daniel Escaff, Jaime Cisternas

Complex Systems Group, Facultad de Ingeniería y Ciencias Aplicadas, Universidad de los Andes, Av. Mon. Alvaro del Portillo, 12455 Santiago, Chile

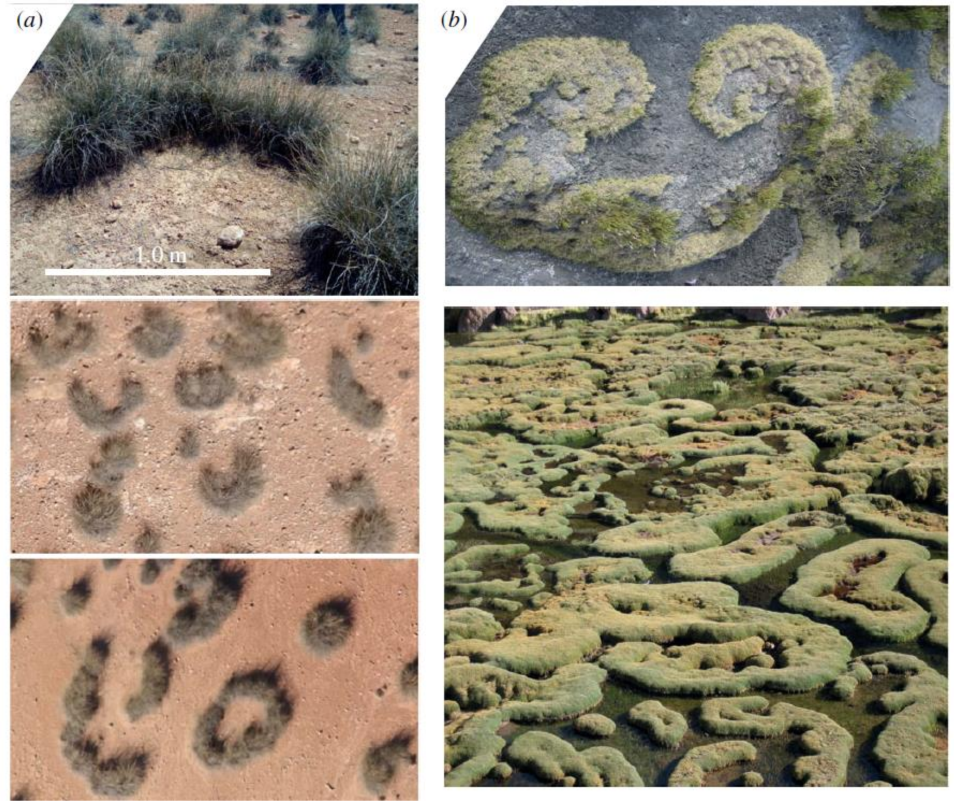
Ecological Complexity 37 (2019) 38–46

Spiral vegetation patterns in San Pedro de Atacama, Chile

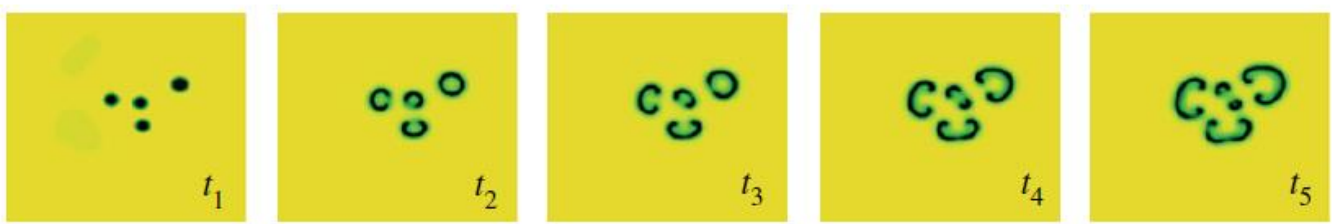
Observation and modelling of vegetation spirals and arcs in isotropic environmental conditions: dissipative structures in arid landscapes

M. Tlidi[1], M. G. Clerc[2], D. Escaff[3], P. Couteron[4], M. Messaoudi[5], M. Khaffou[5] and A. Makhoute[5]

Phil. Trans. R. Soc. A **376** 20180026 (2018)



Morocco



Model simulation showing the temporal transition from localized patterns to arcs and spirals.

MARINE ECOLOGY

Fairy circle landscapes under the sea

Daniel Ruiz-Reynés,[1] Damià Gomila,[1]* Tomàs Sintes,[1] Emilio Hernández-García,[1] Núria Marbà,[2] Carlos M. Duarte[3]

Ruiz-Reynés *et al.*, *Sci. Adv.* 2017; **3**: e1603262 2 August 2017

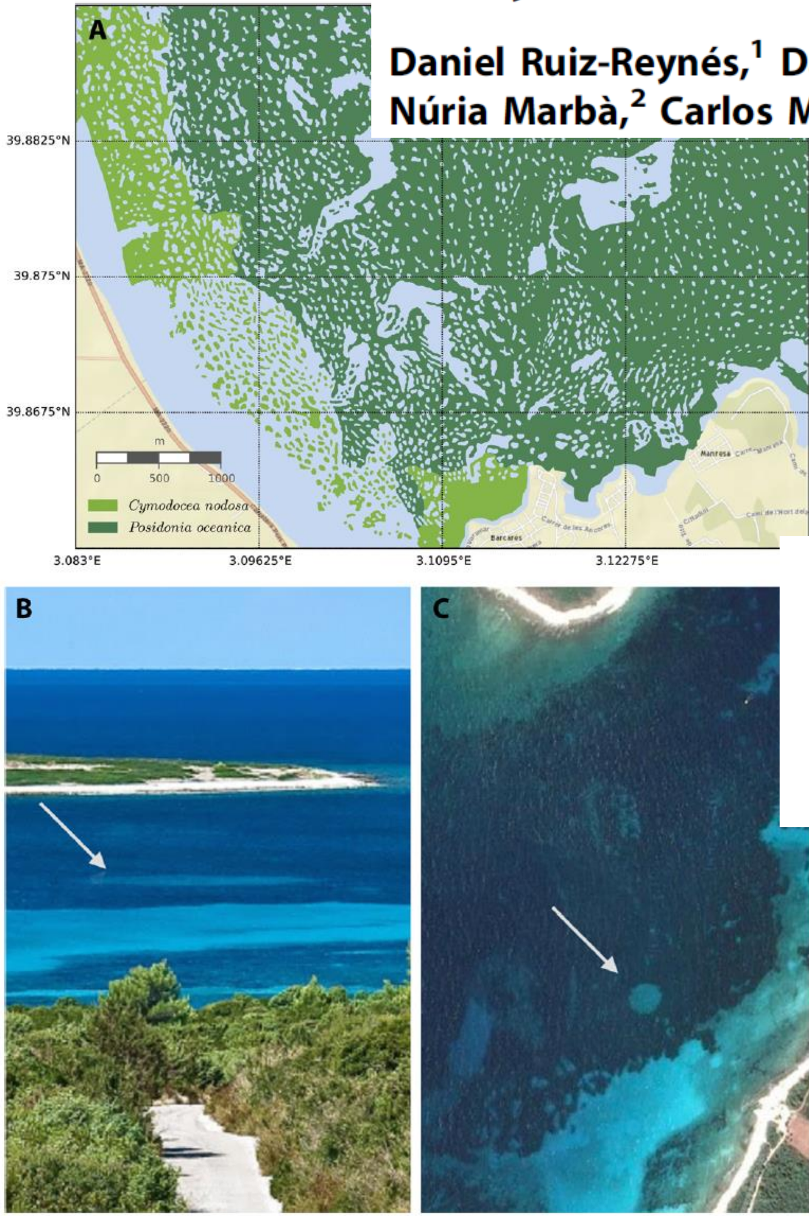
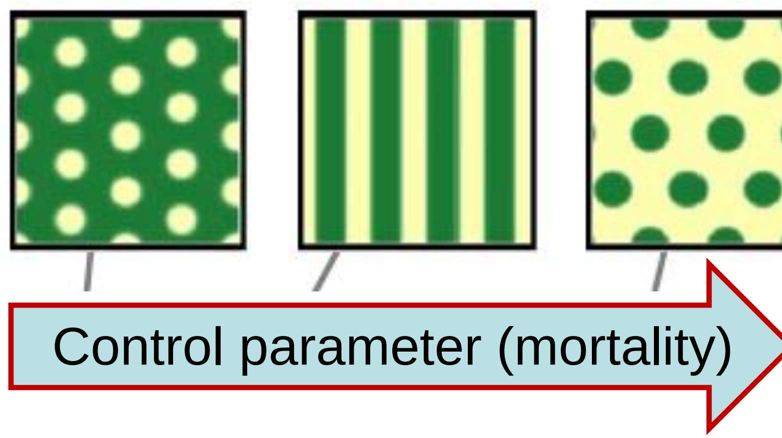
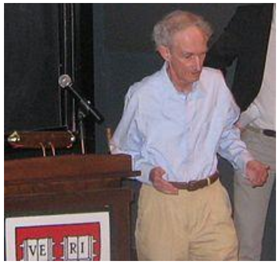


Fig. 1. Examples of fairy circles and spatial patterns in Mediterranean seagrass meadows. (**A**) Side-scan image of a seagrass meadow in Pollença bay (Mallorca



The 1970s



- **Robert May** (Australian, 1936): population biology
- "Simple mathematical models with very complicated dynamics“, *Nature* (1976).

$$x_{t+1} = f(x_t)$$

Example: $f(x) = r\,x(1-x)$

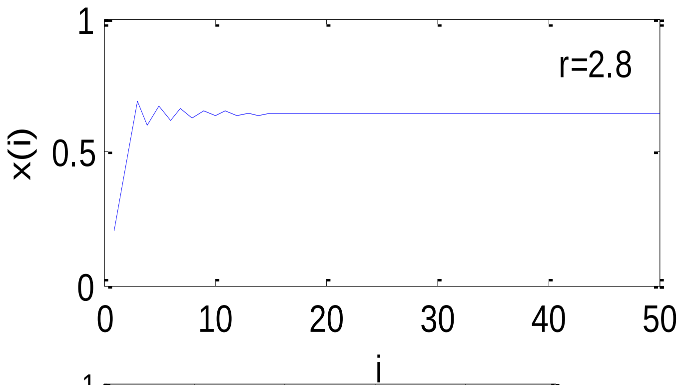
- Difference equations ("iterated maps"), even though simple and deterministic, can exhibit different types of dynamical behaviors, from **stable points**, to a bifurcating hierarchy of **stable cycles**, to **apparently random fluctuations**.

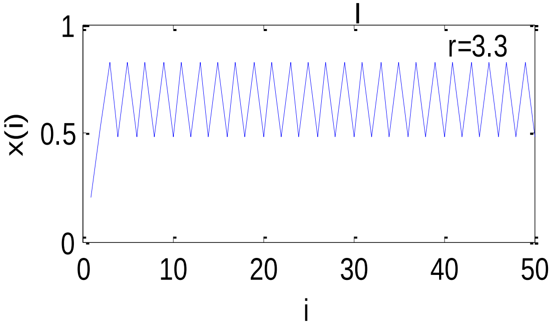
The logistic map

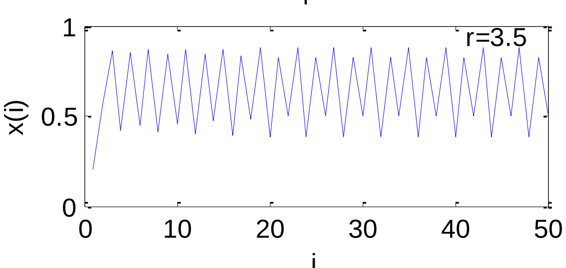
$$x(i+1) = r\, x(i)[1 - x(i)]$$

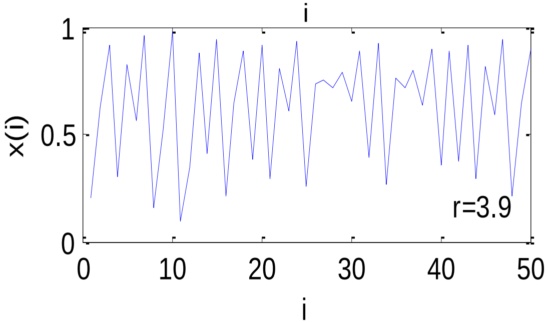
r=2.8, Initial condition: x(1) = 0.2

Transient relaxation → long-term stability



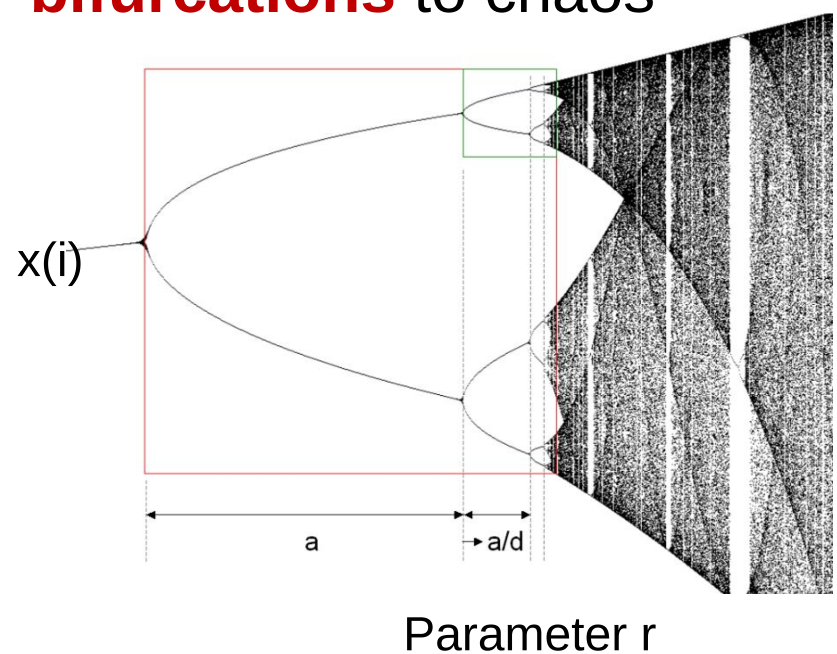






Transient dynamics → stationary oscillations (regular or irregular)

“period-doubling” **bifurcations** to chaos



Universal route to chaos

- In 1975, **Mitchell Feigenbaum** (American mathematician and physicist 1944-2019), using a small HP-65 calculator, discovered the scaling law of the bifurcation points

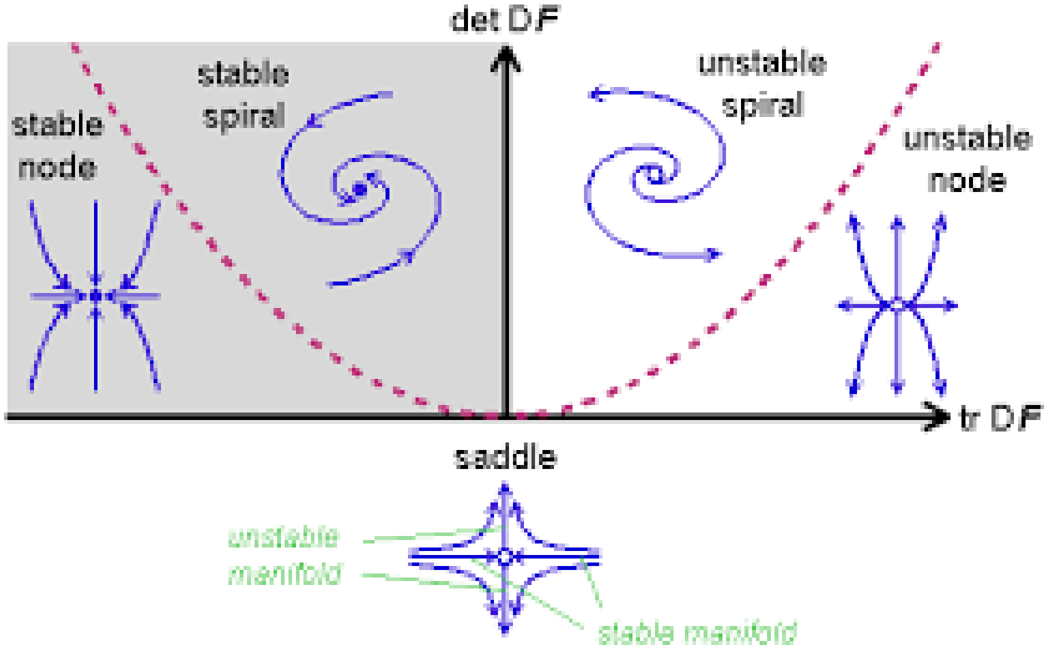
$$\lim_{n\to\infty} \frac{r_{n-1} - r_{n-2}}{r_n - r_{n-1}} = 4.6692...$$

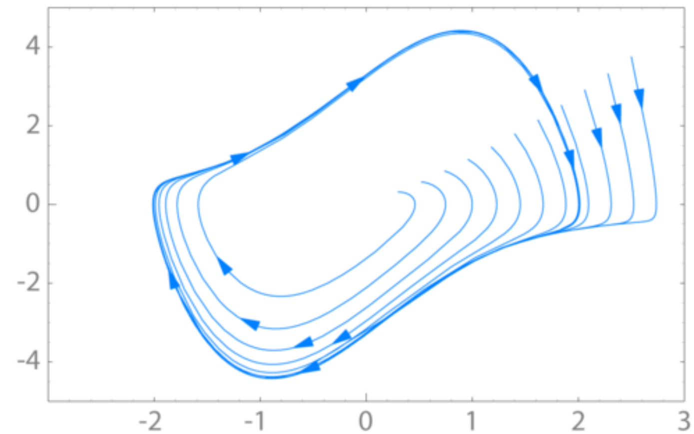
- Then, he showed that the same behavior, with the same mathematical constant, occurs within a wide class of functions, prior to the onset of chaos (**universality**).

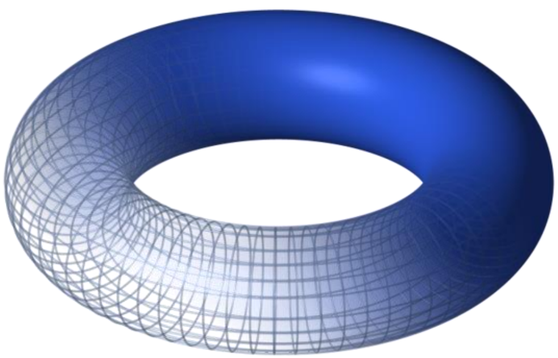
Very different systems (in chemistry, biology, physics, etc.) go to chaos in the same way, quantitatively.

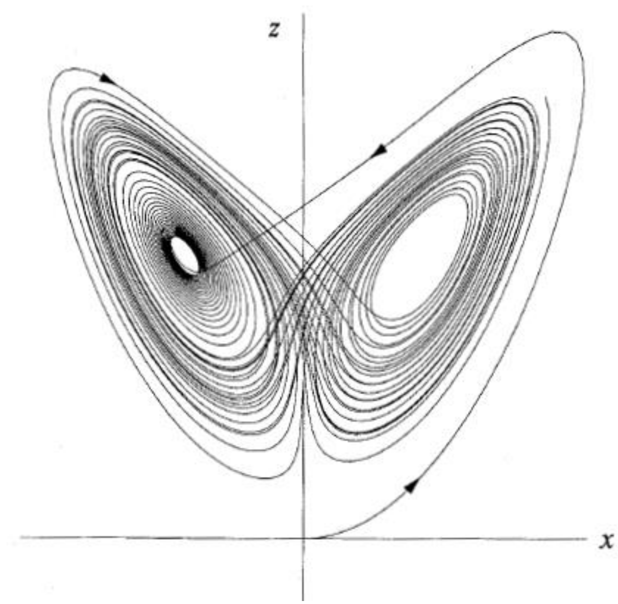
HP-65 calculator: the first magnetic card-programmable handheld calculator

Attractors: fixed points, limit cycles, torus, chaotic (strange) attractors



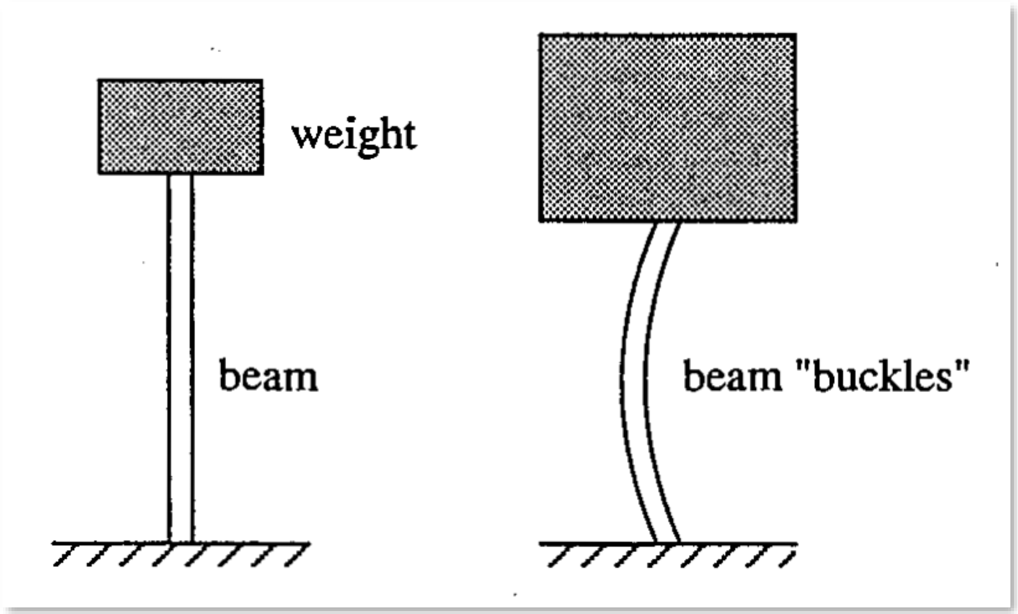






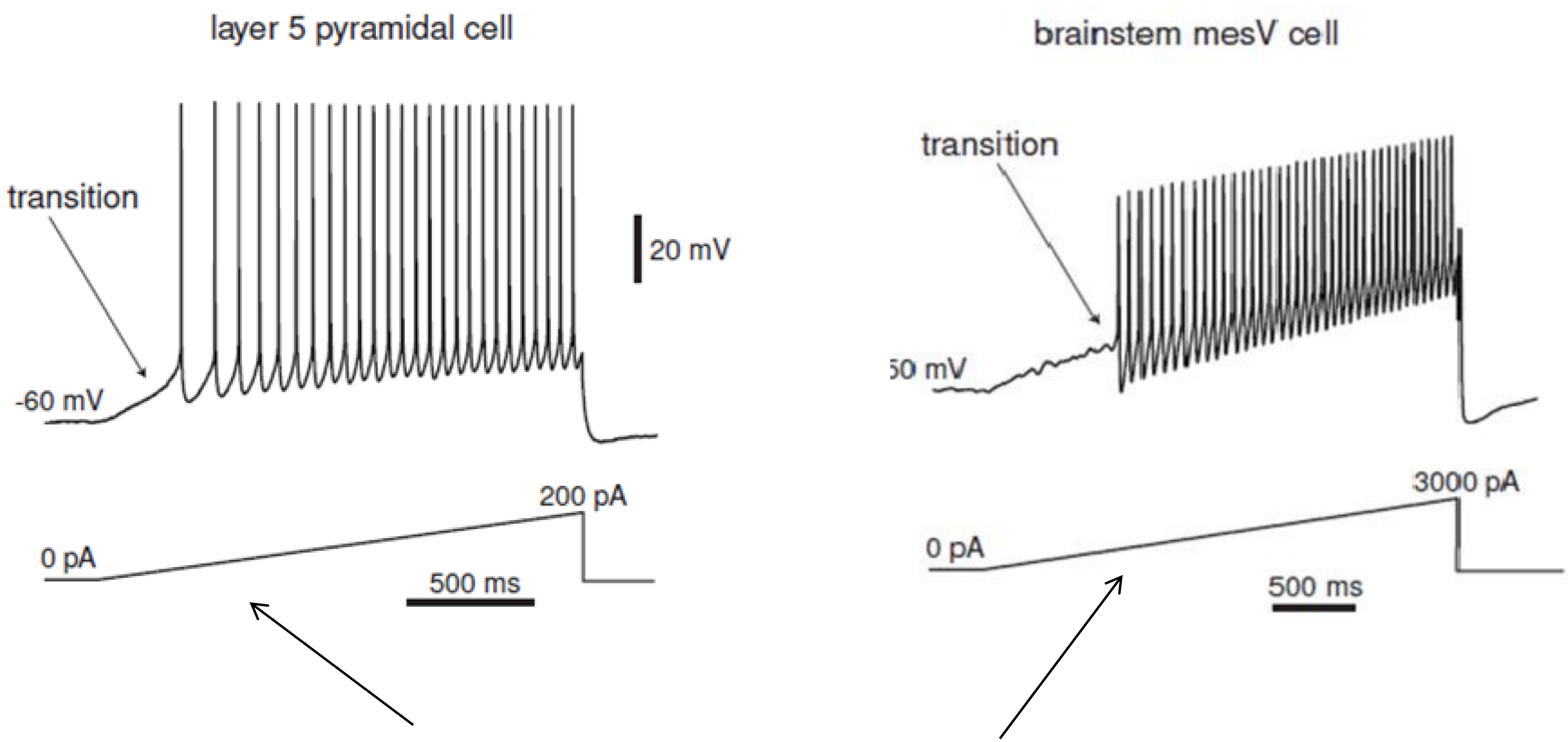
Brief introduction to bifurcations

- A qualitative change (in the structure of the phase space) when a **control parameter is varied**:
  - Attractors can be created or destroyed
  - The stability of an attractor can change
- There are many examples in physical systems, biological systems, etc.



Further reading: Strogatz, *Nonlinear dynamics and chaos*

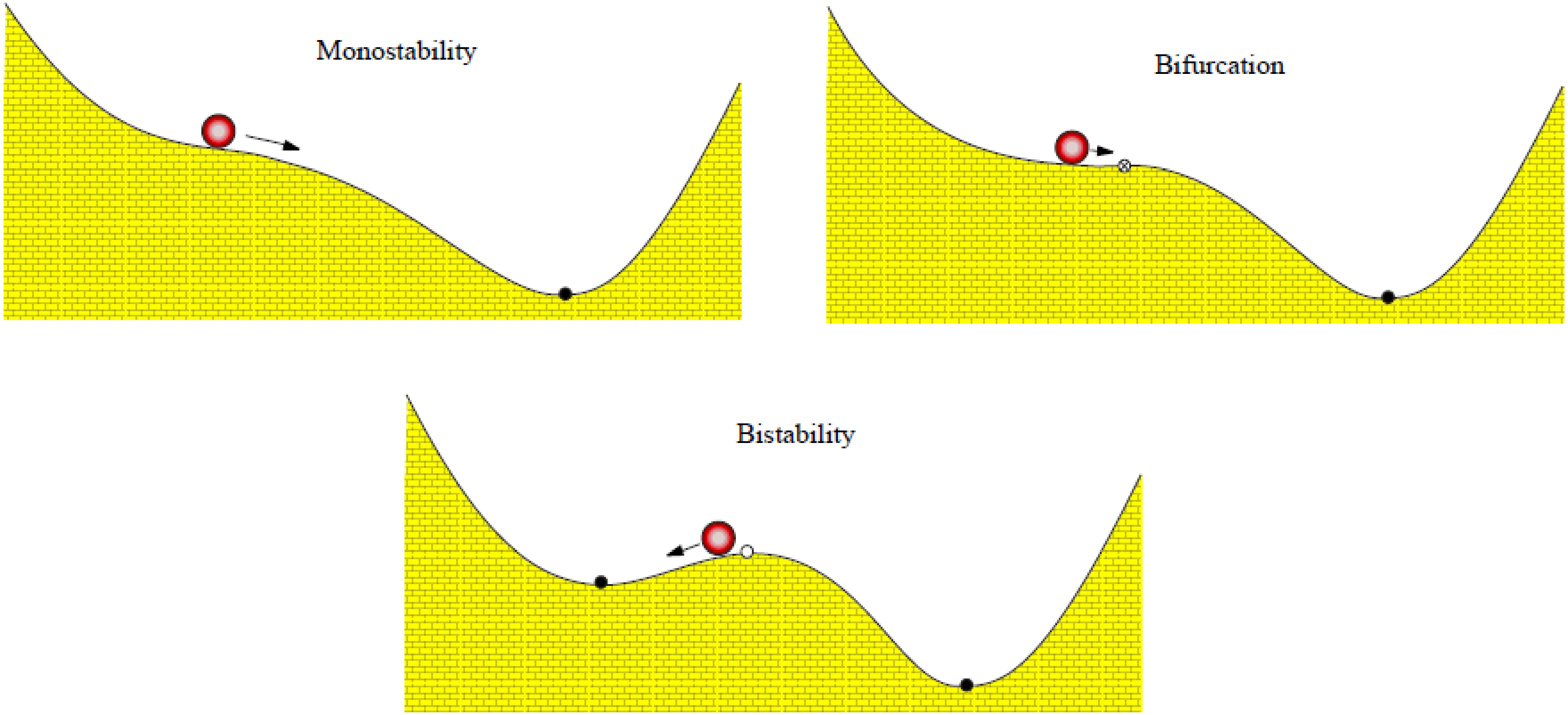
Example: neuronal spikes



Control parameter increases in time

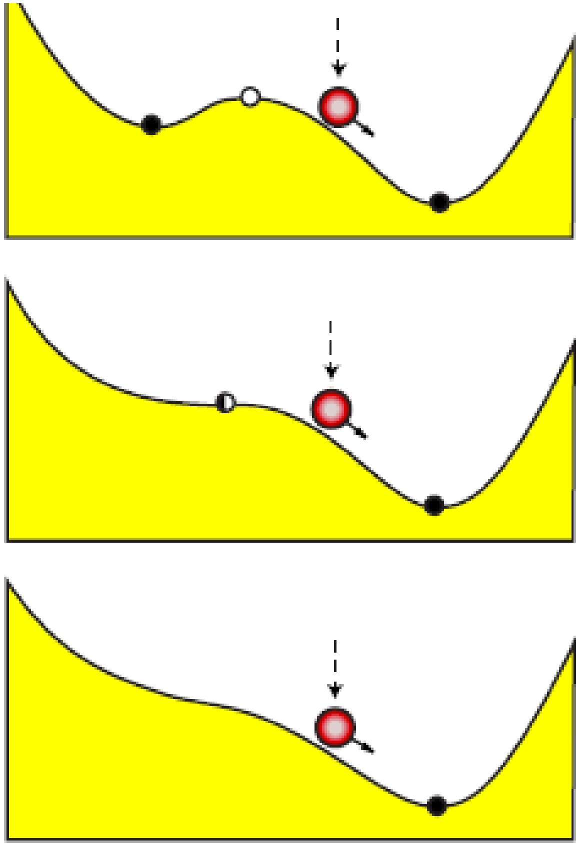
Further reading: Eugene M. Izhikevich, *Dynamical Systems in Neuroscience*

Physical interpretation of a bifurcation

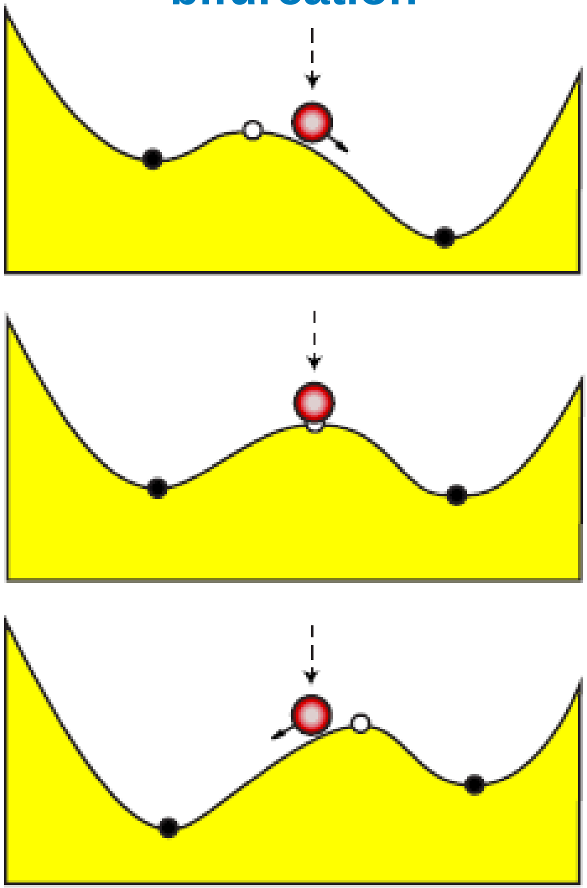


Bifurcations are not the same a qualitative change of behavior

Bifurcation but no change of behavior



Change of behavior but no bifurcation

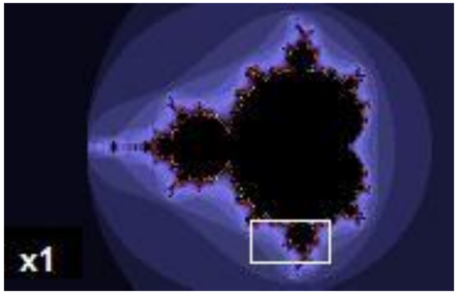


The late 1970s

- **Benoit Mandelbrot** (Polish-born, French and American mathematician 1924-2010): “self-similarity” and **fractal objects:**

  each part of the object is like the whole object but smaller.

- Because of his access to IBM's computers, Mandelbrot was one of the first to use **computer graphics** to create and display fractal geometric images.



Fractal objects

- Are characterized by a “fractal” dimension that measures roughness.

Broccoli
D=2.66

Human lung
D=2.97

Coastline of Ireland
D=1.22

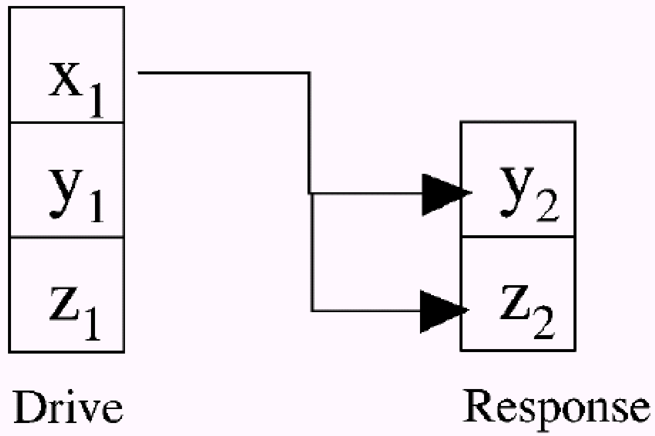
A lot of research is focused on detecting fractal objects underlying real-world signals.

Video: http://www.ted.com/talks/benoit_mandelbrot_fractals_the_art_of_roughness#t-149180

The 1990s: synchronization of chaotic systems

Pecora and Carroll, PRL 1990

Unidirectional coupling of two chaotic systems: one variable, ‘*x*’, of the response system is **replaced** by the same variable of the drive system.



$$t \to \infty \quad |y_2 - y_1| \to 0, \ |z_2 - z_1| \to 0$$

First observation of synchronization: mutual *entrainment* of pendulum clocks

In mid-1600s **Christiaan Huygens** (Dutch mathematician) noticed that two pendulum clocks mounted on a common board synchronized with their pendulums swinging in opposite directions (in-phase also possible).

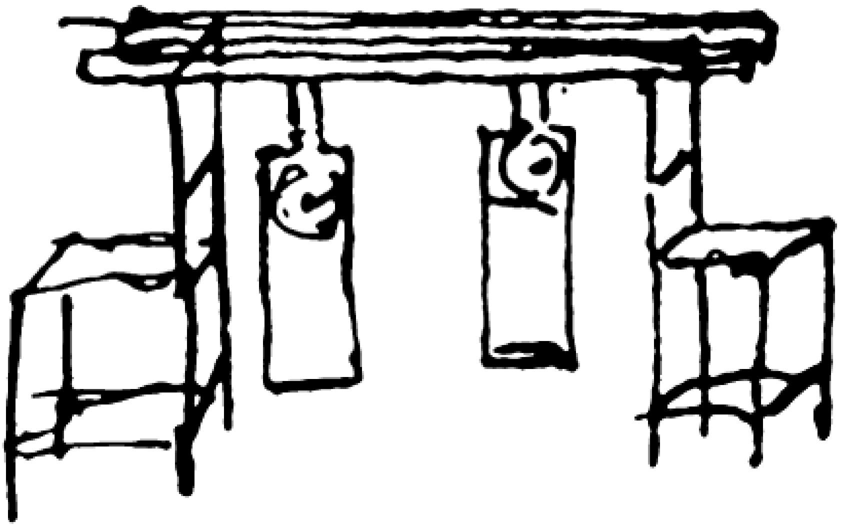
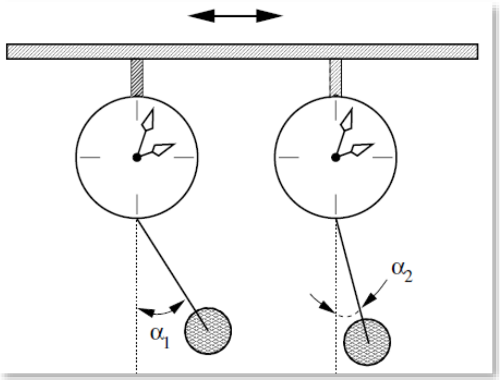


Figure 1.2. Original drawing of Christiaan Huygens illustrating his experiments with two pendulum clocks placed on a common support.



http://www.youtube.com/watch?v=izy4a5erom8

Different types of synchronization

$$dx_1 / dt = F(x_1)$$

$$dx_2 / dt = F(x_2) + \alpha E(x_1 - x_2)$$

- Complete: $x_1(t) = x_2(t)$ (identical systems)
- Phase: the phases of the oscillations synchronize, but the amplitudes are not.
- Lag: $x_1(t+\tau) = x_2(t)$
- Generalized: $x_2(t) = f(x_1(t))$ (*f* can depend on the strength of the coupling)

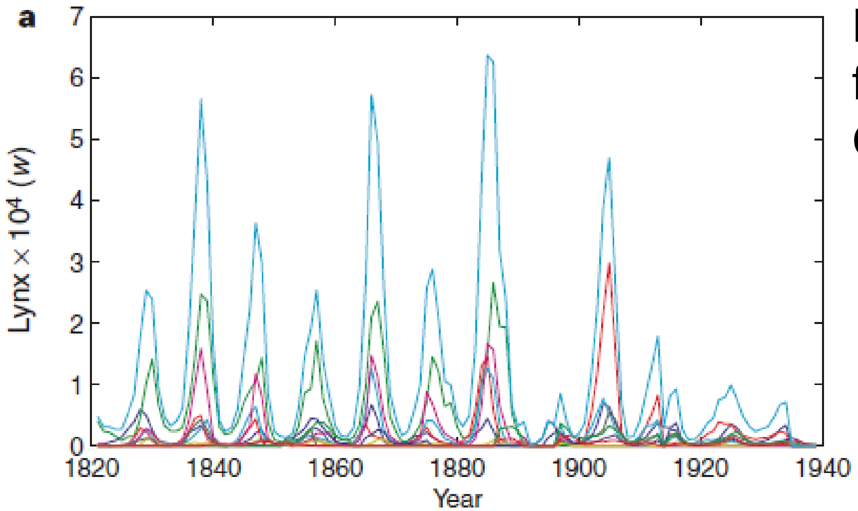
A lot of research is focused on detecting synchronization in real-world signals.

Complex dynamics and phase synchronization in spatially extended ecological systems

NATURE | VOL 399 | 27 MAY 1999

Bernd Blasius, Amit Huppert & Lewi Stone

The Porter Super-Center for Ecological and Environmental Studies & Department of Zoology, Tel Aviv University, Ramat Aviv, Tel Aviv 69978, Israel



Lynx abundances from six regions in Canada

Lynx populations oscillate regularly and periodically in phase, but with irregular and chaotic peaks in abundance.

Synchronization of a large number of coupled oscillators

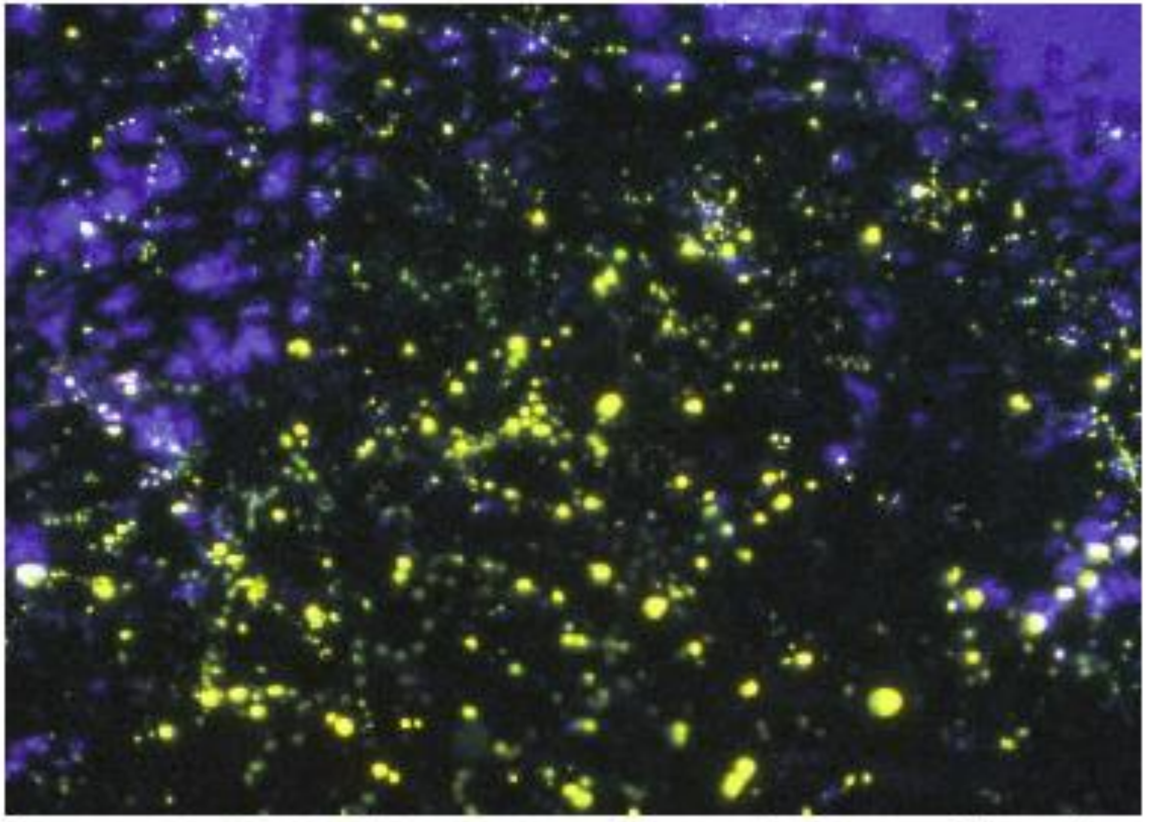
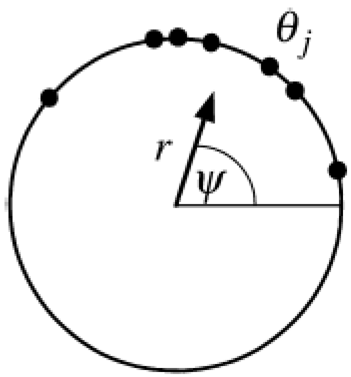


Figure 1 | Fireflies, fireflies burning bright. In the forests of the night, certain species of firefly flash in perfect synchrony — here *Pteroptyx malaccae* in a mangrove apple tree in Malaysia. Kaka *et al.*[2] and Mancoff *et al.*[3] show that the same principle can be applied to oscillators at the nanoscale.

Kuramoto model

(Japanese physicist, 1975)

Model of **all-to-all** coupled **phase oscillators**.



$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) + \xi_i, \qquad i = 1 \ldots N$$

K = coupling strength, ξ_i = stochastic term (noise)

Describes the emergence of collective behavior

How to quantify?

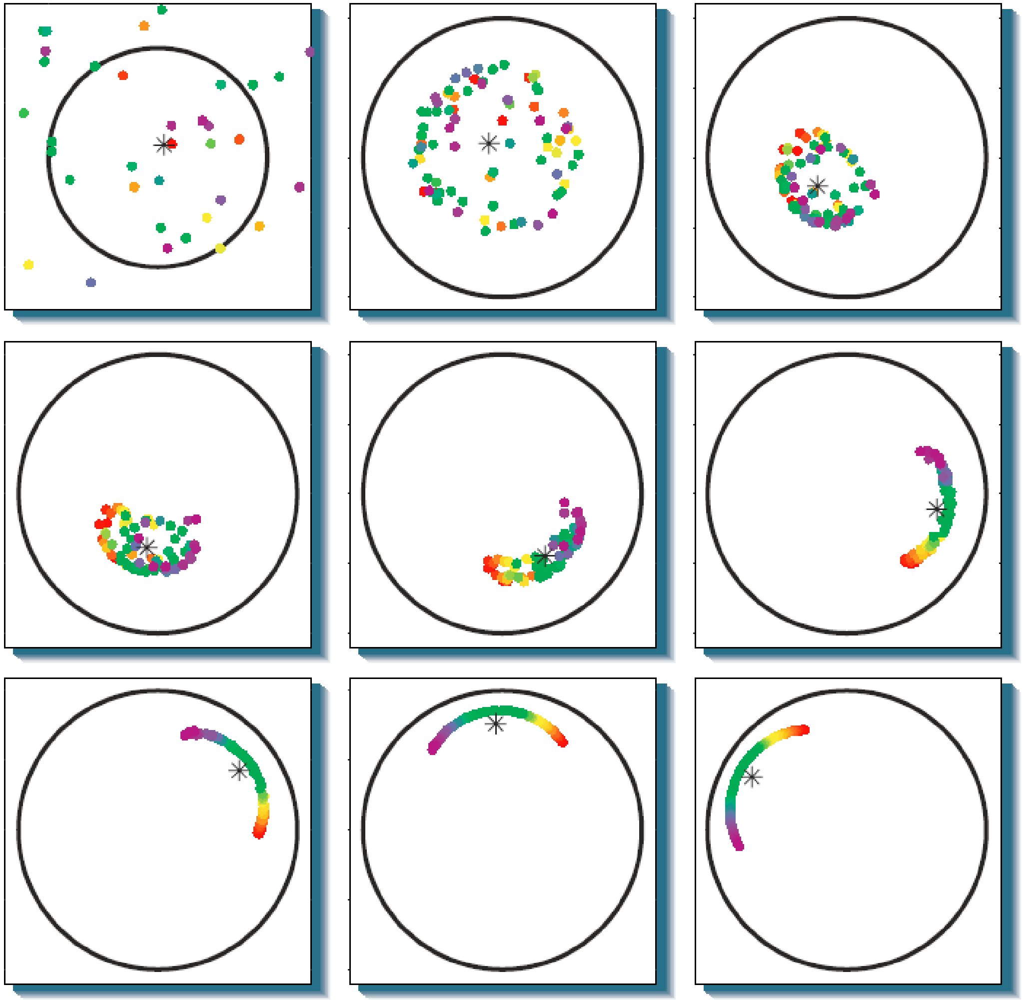
With the **order parameter**:

$$re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

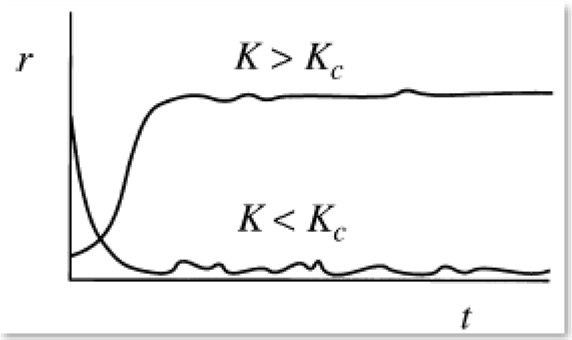
r =0 incoherent state (oscillators scattered in the unit circle)

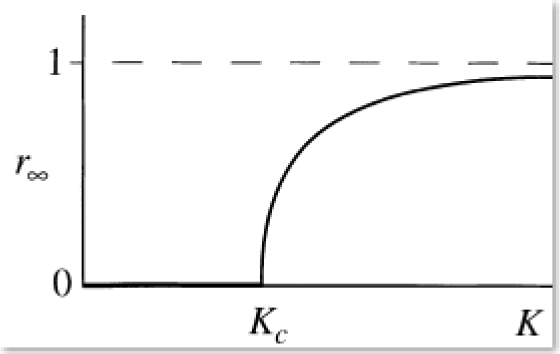
r =1 all oscillators are in phase ($\theta_i = \theta_j \; \forall \; i,j$)

Synchronization transition as the coupling strength increases



Strogatz and others, late 90’





Strogatz, Nature 2001

Video: https://www.ted.com/talks/steven_strogatz_on_sync

The synchronization transition can be explosive

Rossler oscillators

$$\dot{x}_i = -\alpha_i\left[\Gamma\left(x_i - d\sum_{j=1}^{N} a_{ij}(x_j - x_i)\right) + \beta y_i + \lambda z_i\right],$$

$$\dot{y}_i = -\alpha_i(-x_i + \nu y_i), \qquad \dot{z}_i = -\alpha_i[-g(x_i) + z_i],$$

$$\phi_i(t) = \arctan[y_i(t)/x_i(t)] \qquad S = \langle | \frac{1}{N} \sum_{j=1}^{N} e^{i\phi_j(t)} | \rangle_t$$

Simulations

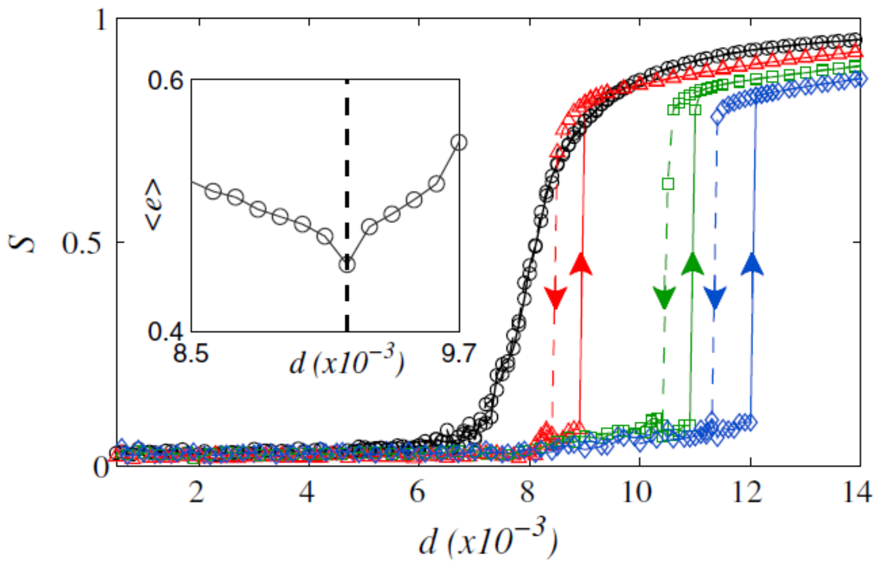
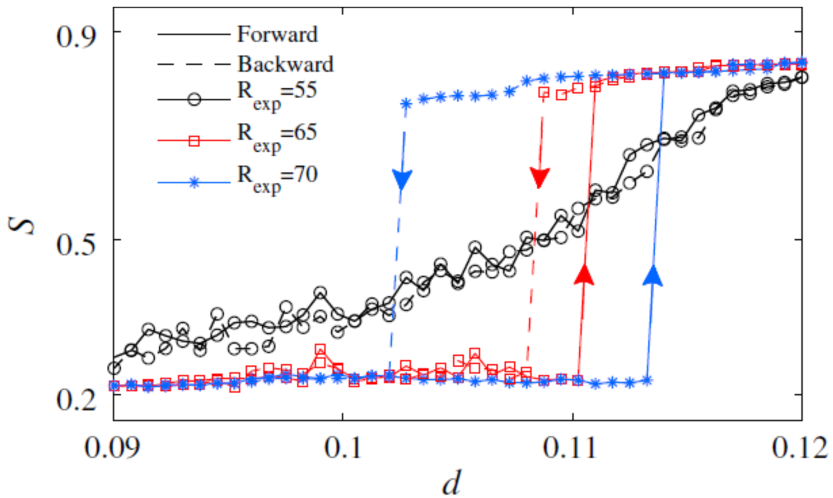


FIG. 1 (color online). Phase synchronization degree S as a function of the coupling strength d for different SF networks of size $N = 1000$, and average degree $\langle k \rangle = 6$. The networks are

Experiments (chaotic circuits)



I. Leyva et al, PRL 108, 168702 (2012)

End of 90’s - present

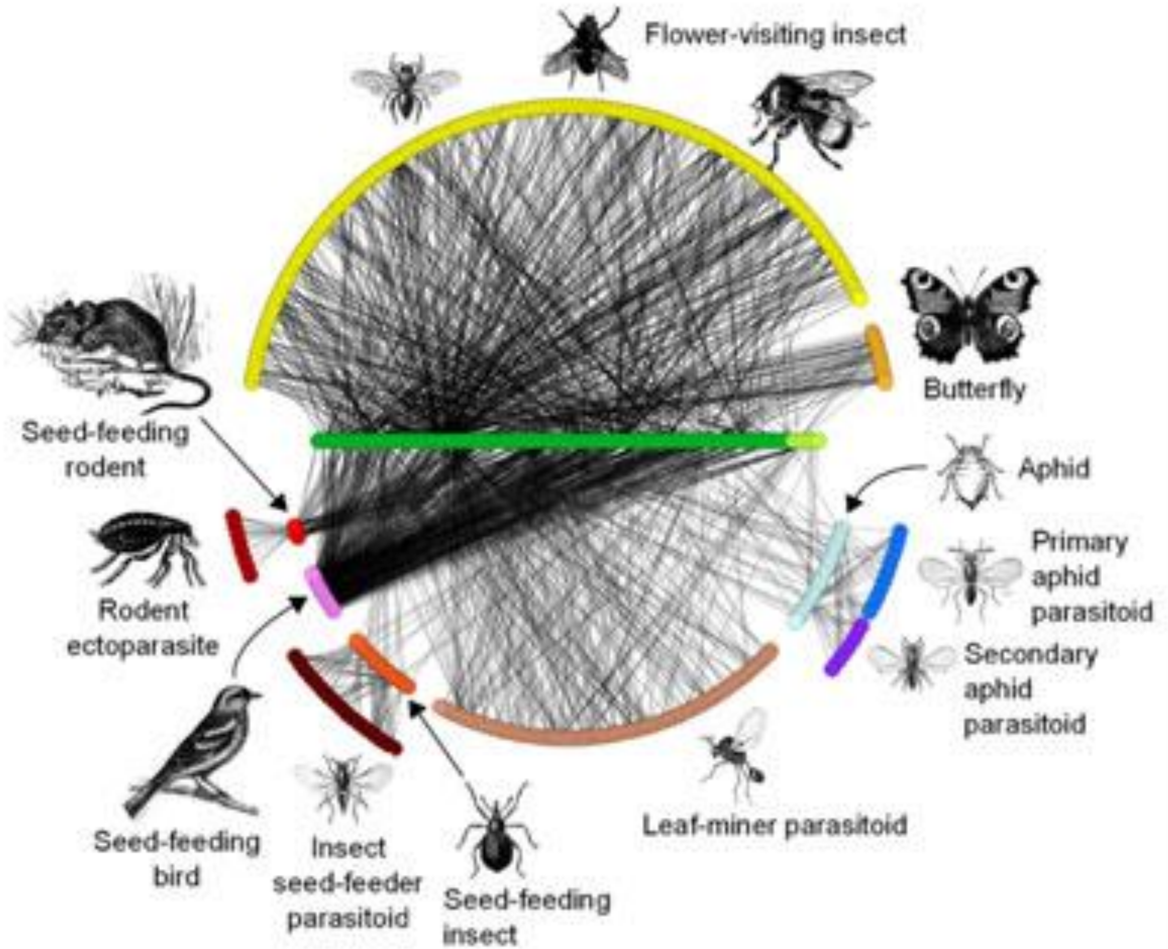
- Interest moves from <u>chaotic systems</u> to <u>complex systems</u> (small vs. very large number of variables).

- Networks (or graphs) of interconnected systems

- **Complexity science**: dynamics of emergent properties
  - Epidemics
  - Rumor spreading
  - Transport networks
  - Financial crises
  - Brain diseases
  - Etc.

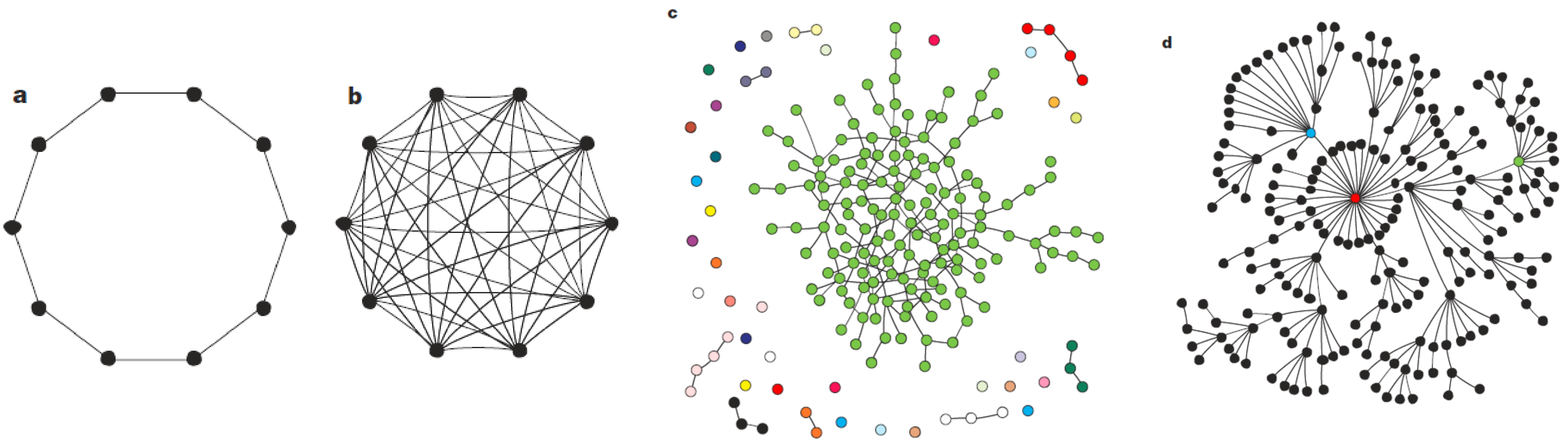
Networks in ecology

- species (nodes) are connected by pairwise interactions (links)



Network science

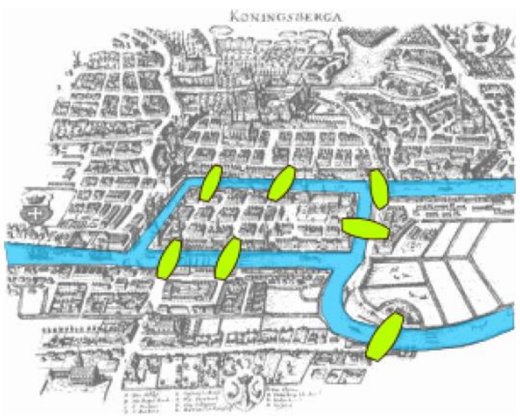
The challenge: to understand how the network **structure** and the **dynamics** (of individual units) determine the collective behavior.



Source: Strogatz
Nature 2001

The start of Graph Theory:
The Seven Bridges of Königsberg (Prussia, now Russia)

- The problem was to devise a walk through the city that would cross each of those bridges once and only once.



Source: Wikipedia

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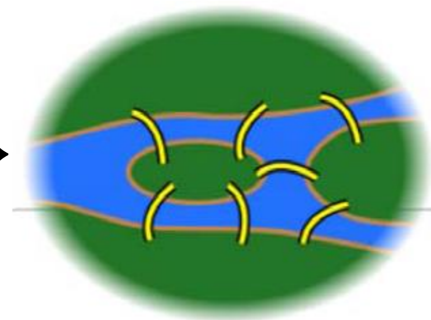

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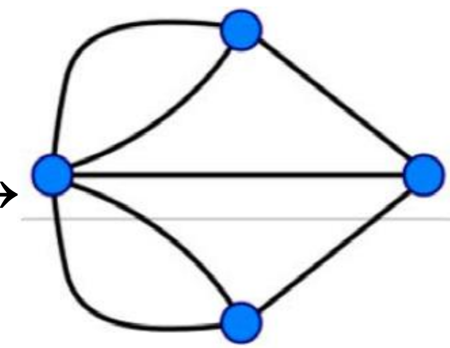

- By considering the number of odd/even links of each “node”, **Leonhard Euler** (Swiss mathematician) demonstrated in 1736 that is impossible.

Summary

- Dynamical systems allow to
  - understand low-dimensional systems,
  - uncover patterns and “order within chaos”,
  - characterize attractors, uncover universal features
- Synchronization: emergent behavior of interacting dynamical systems.
- Complexity and network science: emerging phenomena in large sets of interacting units.
- Time series analysis develops tools to characterize complex signals.
- Is an interdisciplinary research field with many applications.

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http://www.fisica.edu.uy/~cris/