Lecture 2: Multivariate time series analysis and applications to climate and biomedical networks

Prof. Cristina Masoller
Universitat Politècnica de Catalunya
cristina.masoller@upc.edu
www.fisica.edu.uy/~cris

http://www.fisica.edu.uy/~cris/teaching/lecture1_masoller_winter_school_2021.pdf
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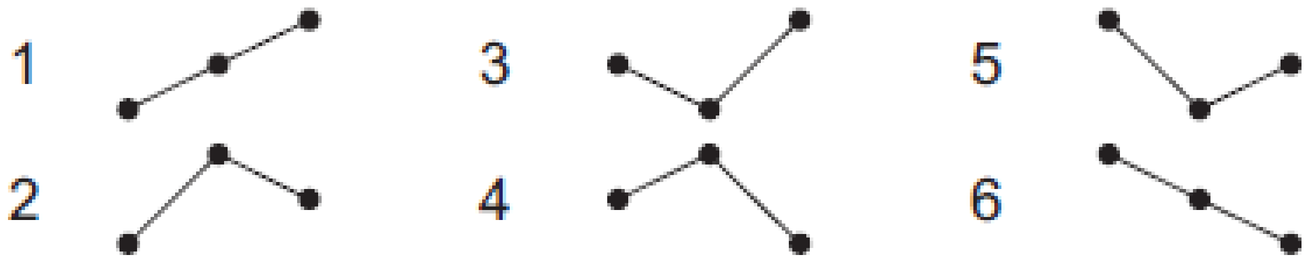
Outline

- Quick review of ordinal analysis (lecture 1)
- Permutation entropy
- Network representation of a time-series
- Spatio-temporal representation of a time-series
- Instantaneous phase and amplitude
- Bivariate time series analysis: cross-correlation and mutual information
- Multivariate time series analysis: brain functional networks, climate networks, network inference

Ordinal analysis: threshold-less method to define symbols

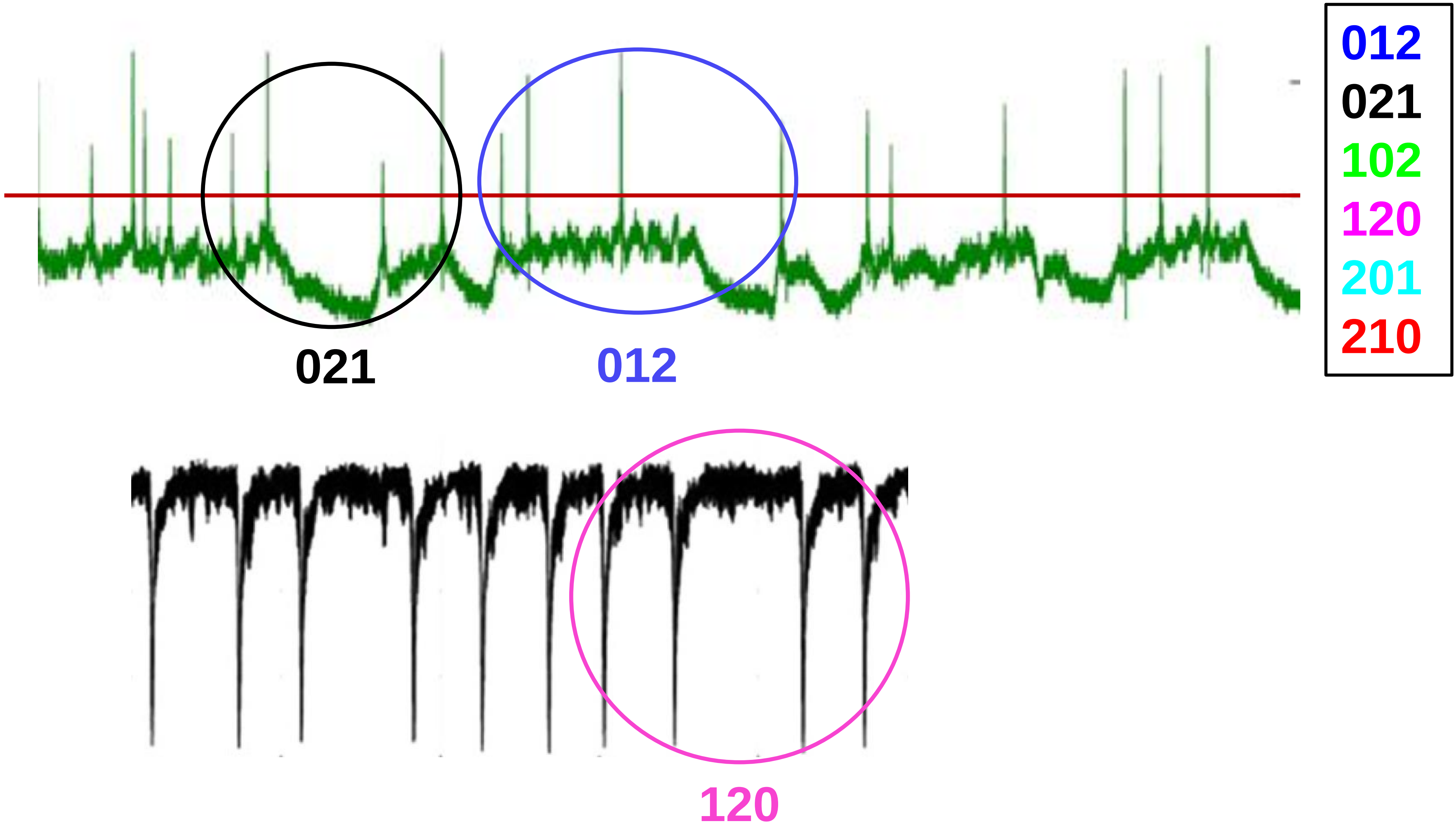
- Consider a time series $x(t)=\{\ldots x_i, x_{i+1}, x_{i+2}, \ldots\}$
- Which are the possible order relations among three consecutive data points?



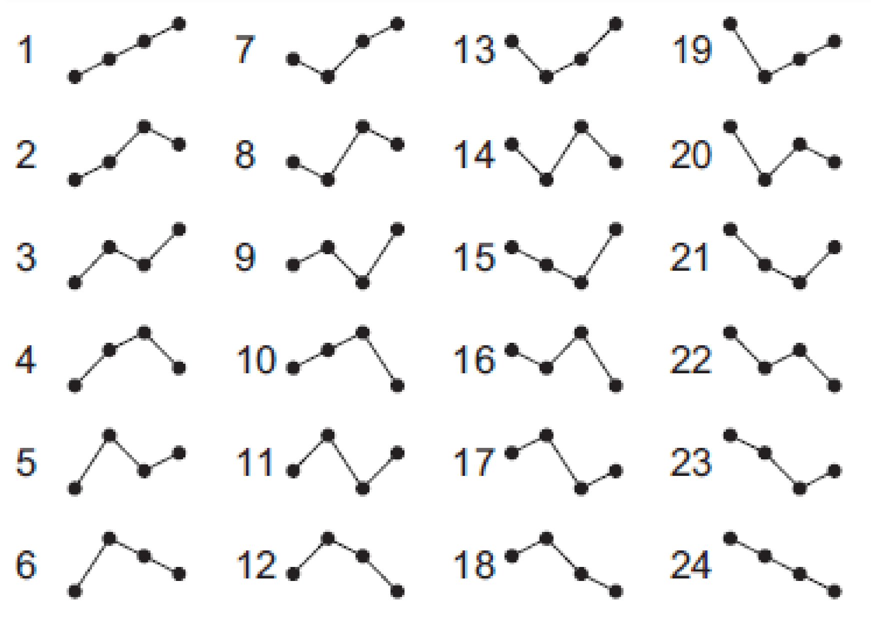
- Count how many times each "ordinal pattern" appears.
- Advantages: allows to identify temporal structures & is robust to noise.
- Drawback: information about actual data values is lost.

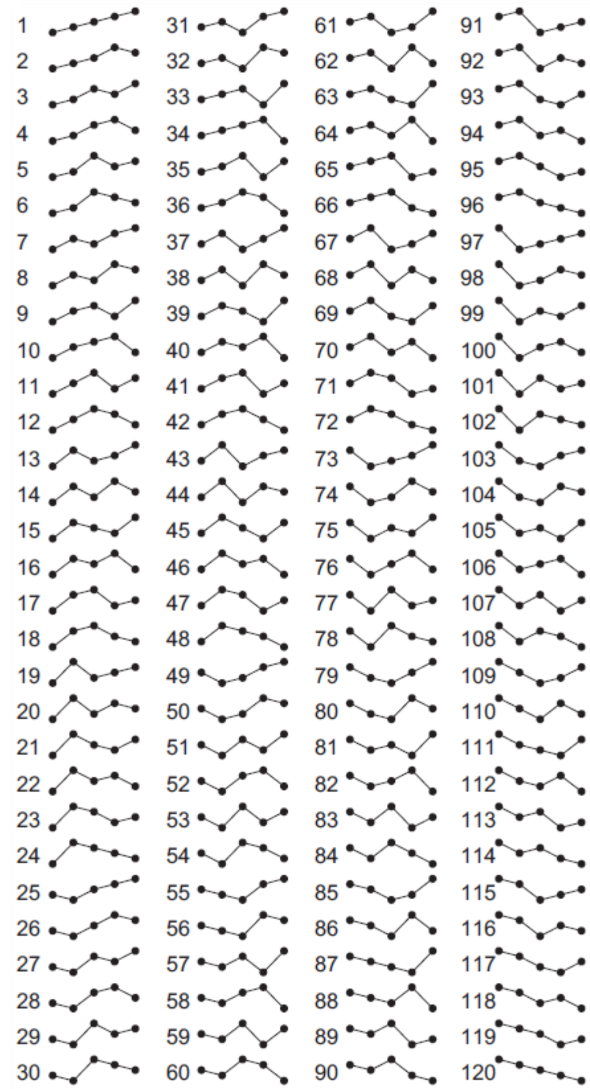
Bandt and Pompe, Phys. Rev. Lett. 88, 174102 (2002)

Analysis of D=3 patterns in spike sequences



The number of ordinal patterns increases as D!





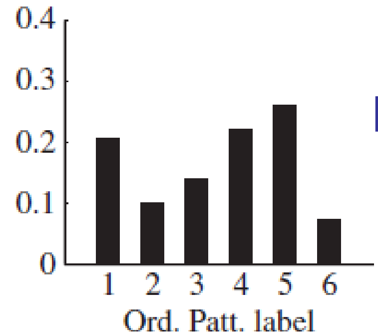
U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

Permutation entropy

- Entropy computed from ordinal probabilities.
- Number of probabilities = # of ordinal patterns (D!)

Time series

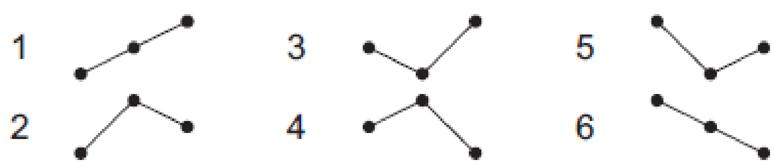
Ordinal probabilities



Permutation entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

$$\sum_{i=1}^{N} p_i = 1$$



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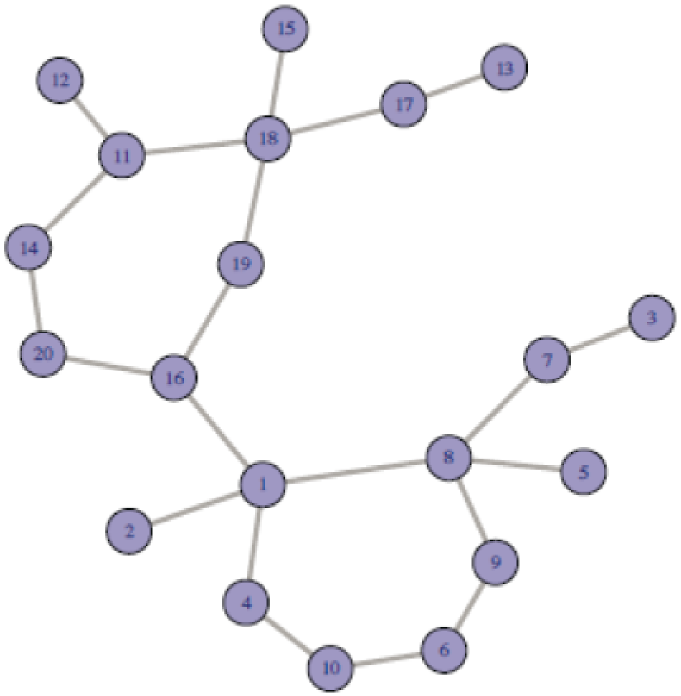
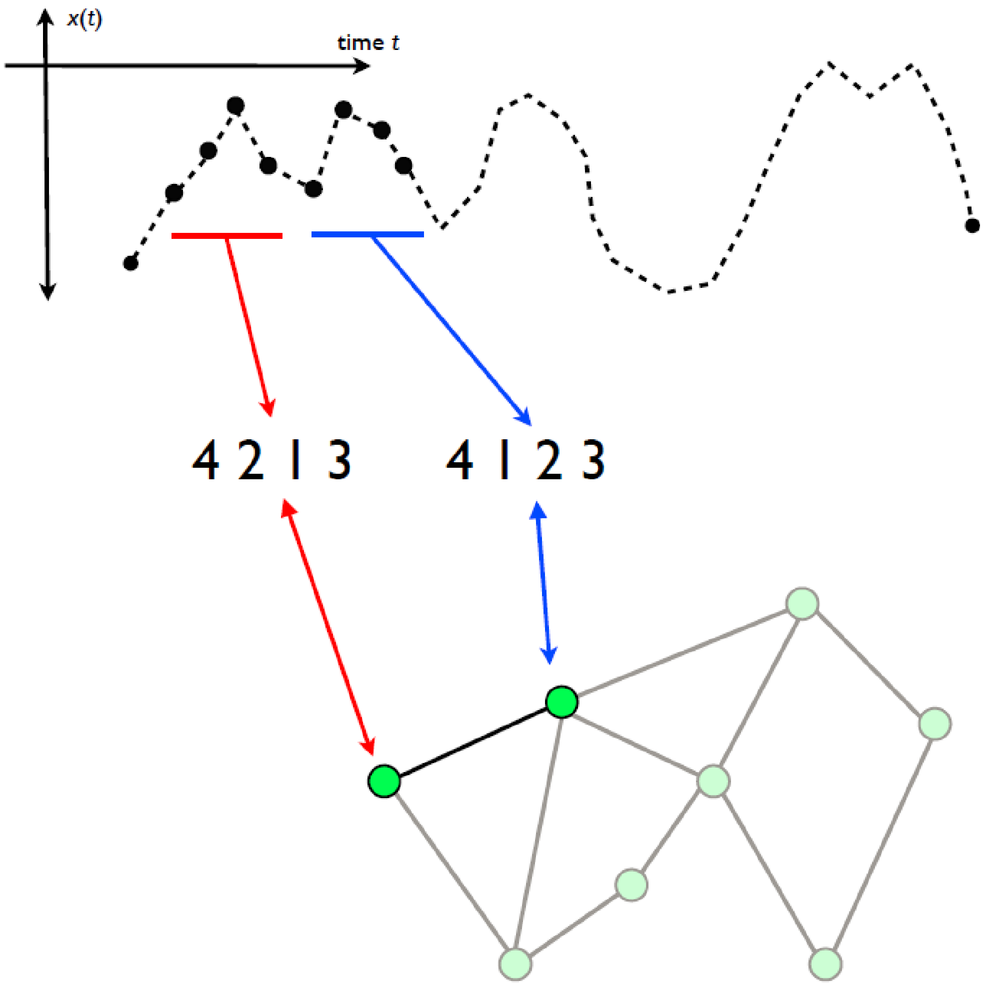
What is a network?

- A graph: a set of “nodes” connected by a set of “links”
- Nodes and links can be weighted or unweighted
- Links can be directed or undirected
- More in part 3 (multivariate time series analysis)



We use symbolic patterns as the *nodes* of the network.
And the *links*? Defined as the transition probability $\alpha \to \beta$



- In each node *i*:
  $\sum_j w_{ij}=1$
- <u>Weigh of node i</u>: the probability of pattern *i* ($\sum_i p_i=1$)

$\Rightarrow$ **Weighted and directed network**

Adapted from M. Small (The University of Western Australia)

Network-based diagnostic tools

- Entropy computed from node weights (**permutation entropy**)

$$s_p = -\sum p_i \log p_i$$

- Average node entropy (entropy of the link weights)

$$s_n = \frac{1}{M}\sum_{i=1}^{M} s_i \qquad s_i = -\sum w_{ij} \log w_{ij}$$

- Asymmetry coefficient: normalized difference of transition probabilities, P(‘01’→ ‘10’) - P(‘10’→ ’01’), etc.

$$a_c = \frac{\sum_i \sum_{j\neq i} \left| w_{ij} - w_{ji} \right|}{\sum_i \sum_{j\neq i} \left(w_{ij} + w_{ji} \right)}$$

(0 in a fully symmetric network;
1 in a fully directed network)

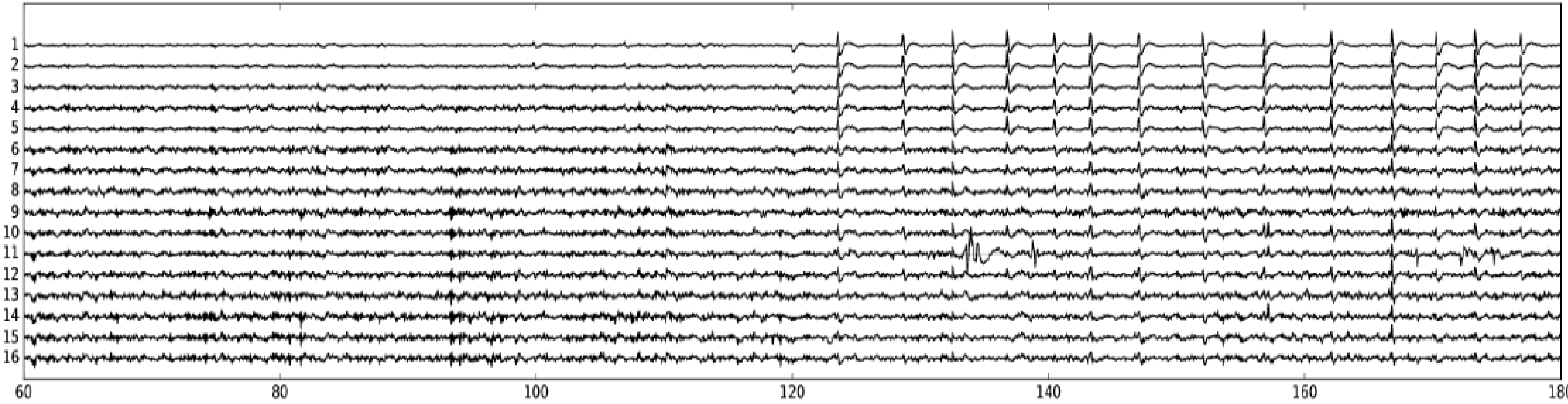
First application: distinguishing *eyes closed* and *eyes open* brain states

Analysis of two EEG datasets

	BitBrain DTS1	PhysioNet DTS2
Sampling rate(Hz)	256	160
Time task(seg)	120	60
Total points	30720	9600
Number of electrodes	16	64
Number of subjects	70	109

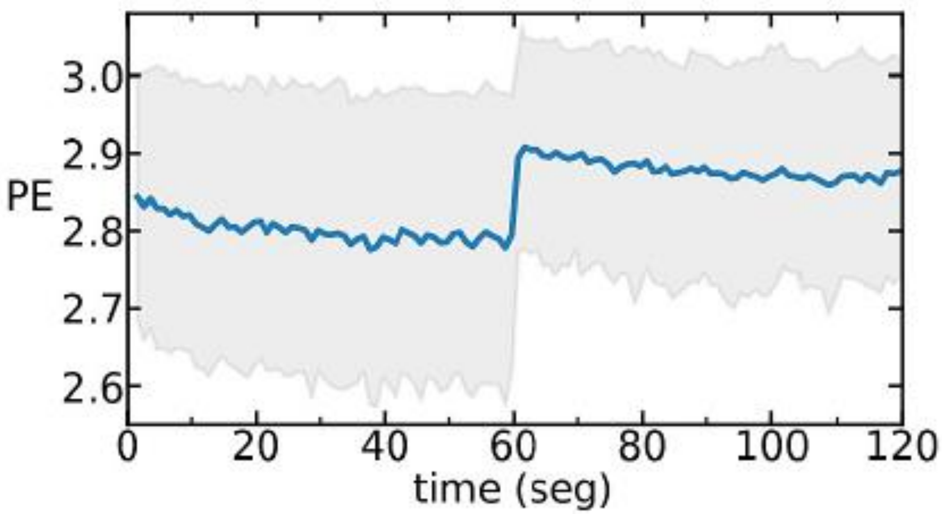
Eye closed

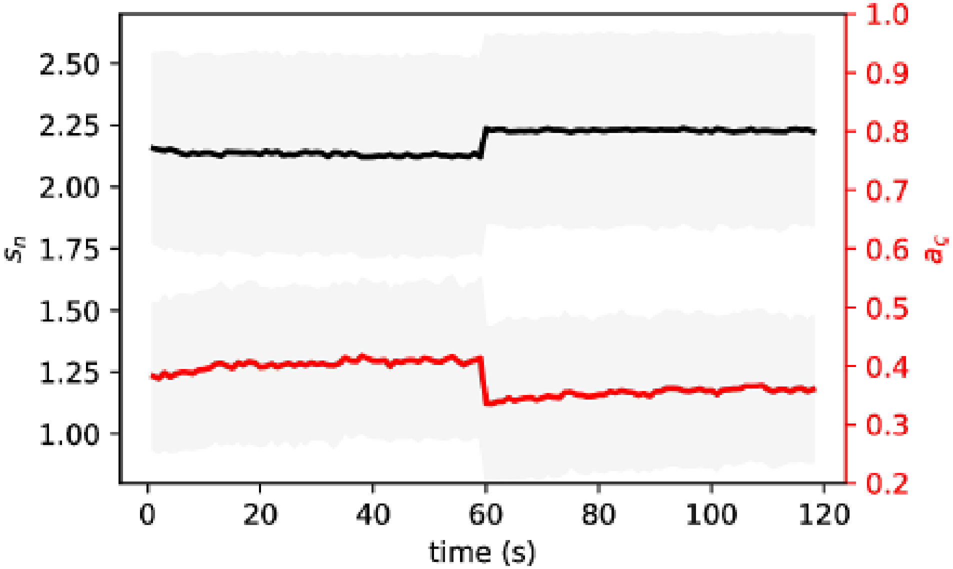
Eye open



Symbolic analysis is applied to the raw data; similar results were found with filtered data using independent component analysis.

“Randomization”: the entropies increase and the asymmetry coefficient decreases

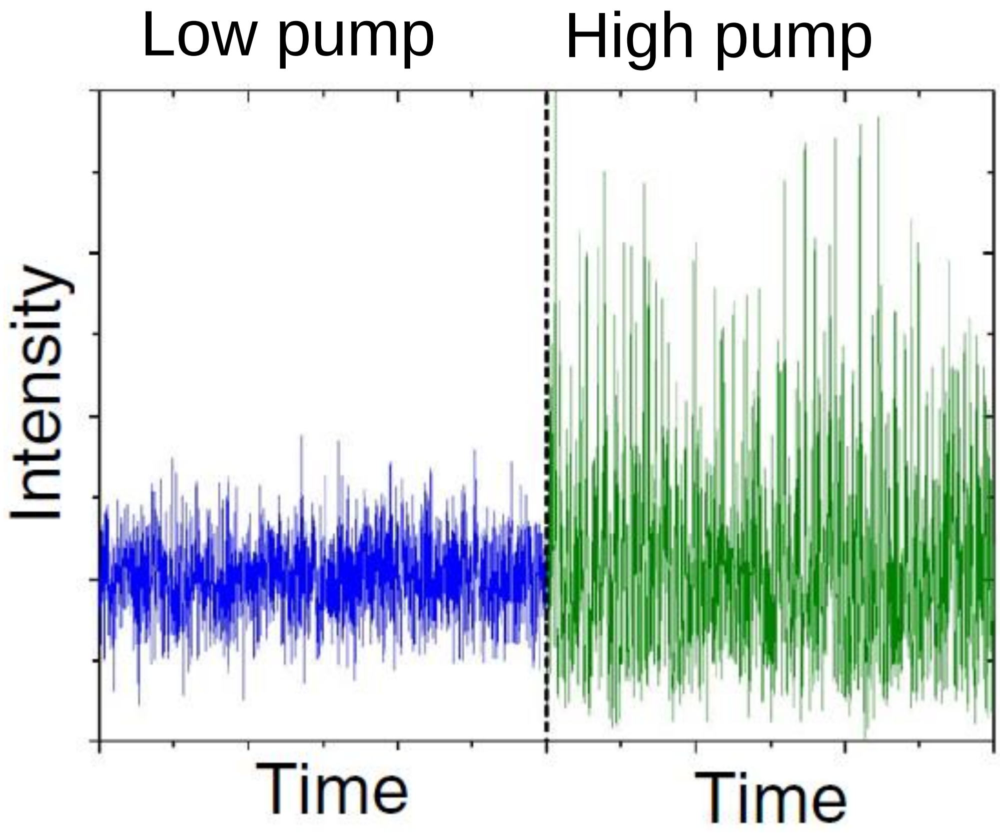




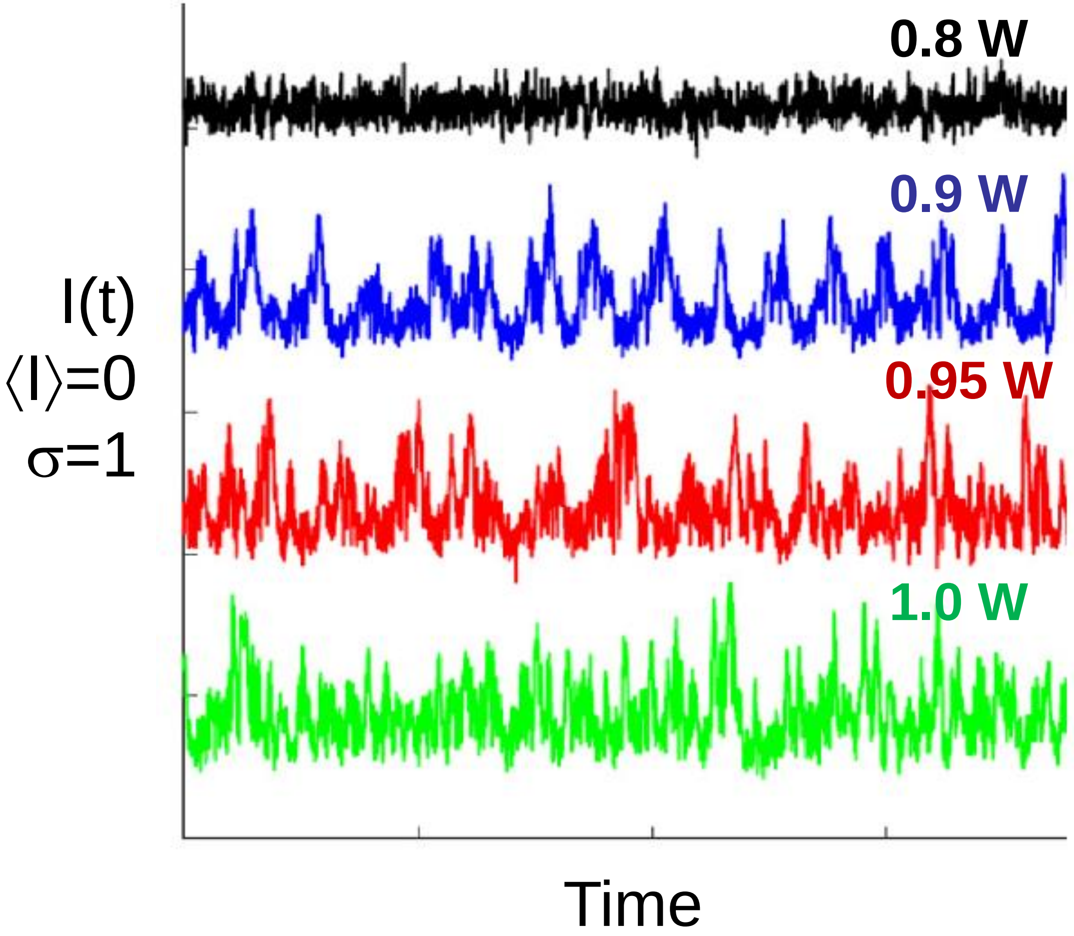
Time window = 1 s
(160 data points)

C. Quintero-Quiroz et al, “*Differentiating resting brain states using ordinal symbolic analysis*”, Chaos 28, 106307 (2018).

Second application: Laminar → Turbulence transition in a fiber laser as the pump (control parameter) increases

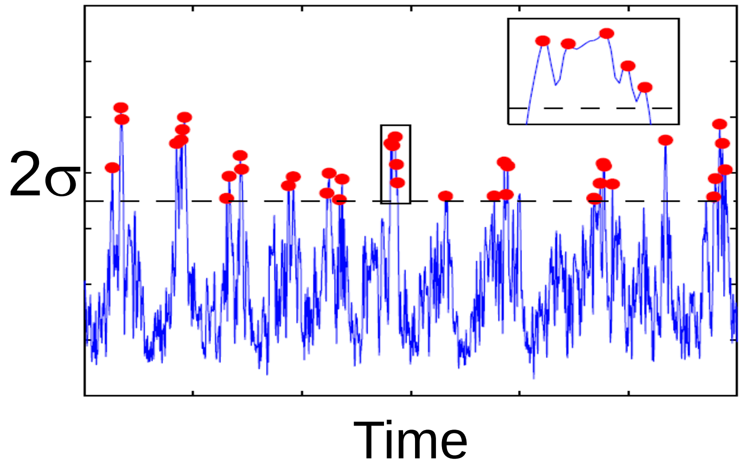


E. G. Turitsyna et al Nat. Phot. 7, 783 (2013)



Nonlinear temporal correlations?

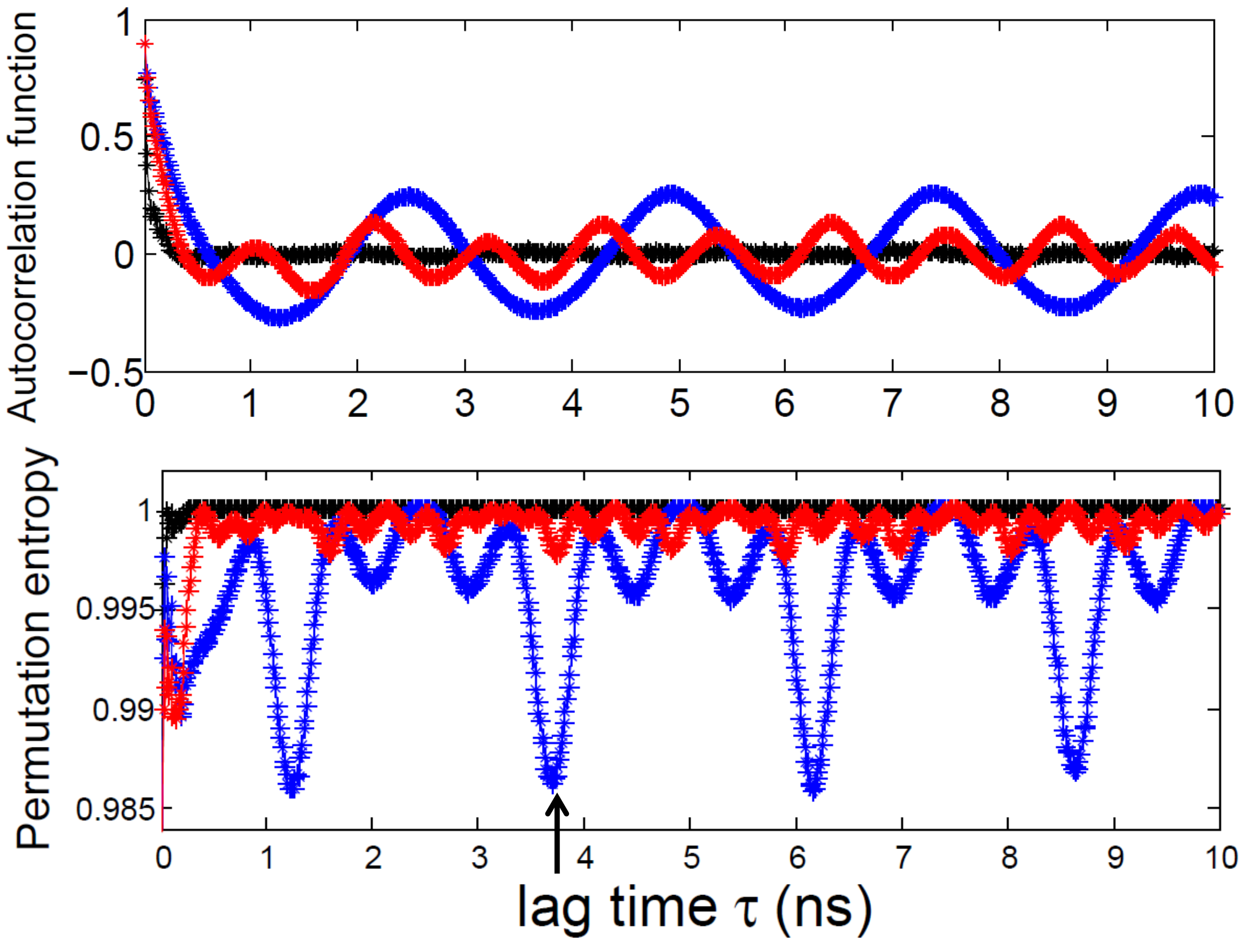
Raw and **thresholded** data



L. Carpi and C. Masoller, “*Persistence and stochastic periodicity in the intensity dynamics of a fiber laser during the transition to optical turbulence*”, Phys. Rev. A **97**, 023842 (2018).

Ordinal analysis identifies ``hidden'' periodicity

$$\{I_i, I_{i+\tau}, I_{i+2\tau}, \ldots\}$$



Autocorrelation

$$C(\tau) = \frac{\langle [I_i - \langle I \rangle][I_{i-\tau} - \langle I \rangle] \rangle}{\sigma^2}$$

Permutation entropy

$$H = -\sum_i p_i \log_2 p_i$$

Aragoneses et al, PRL (2016)

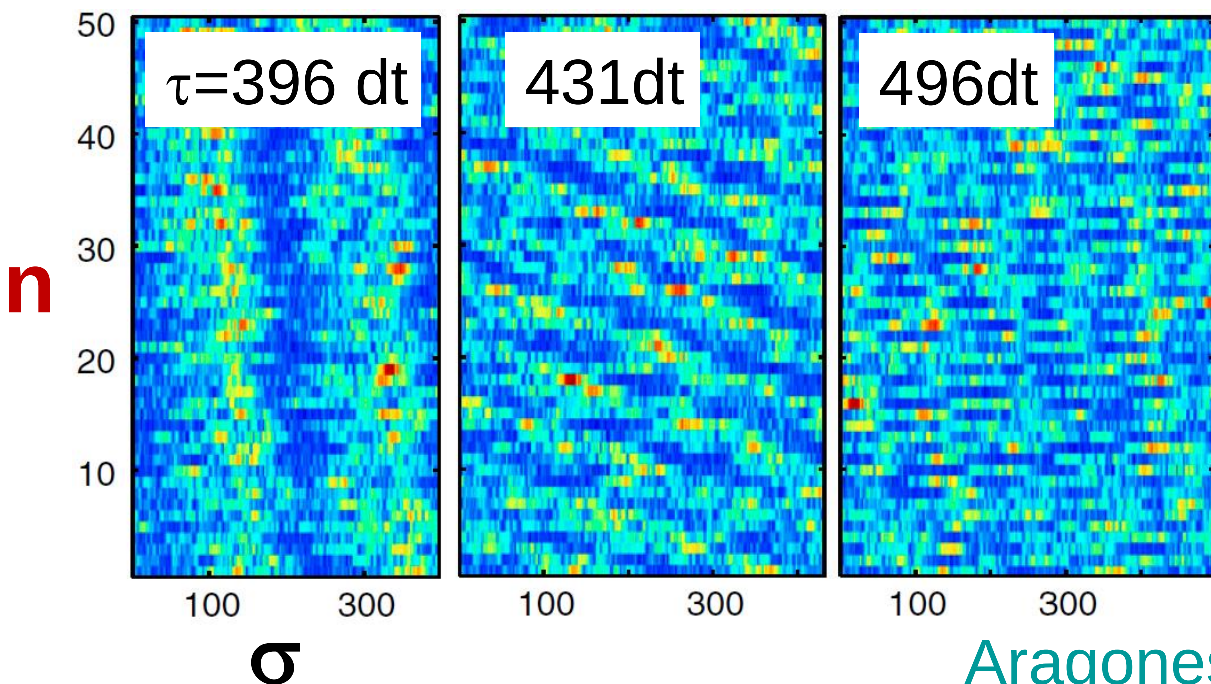
The space-time representation of the intensity time series: a convenient way to visualize the dynamics

$$\{I_1, I_2, \ldots I_\tau, I_{\tau+1}, \ldots\} \Rightarrow \begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ I_{2\tau+1} & I_{2\tau+2} & \cdots & I_{3\tau} \\ I_{\tau+1} & I_{\tau+2} & \cdots & I_{2\tau} \\ I_1 & I_2 & \cdots & I_\tau \end{bmatrix}$$

$\sigma \longrightarrow$

Color scale: I_i

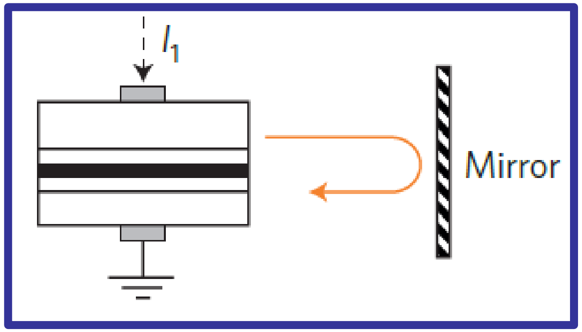
$\uparrow$ **n**



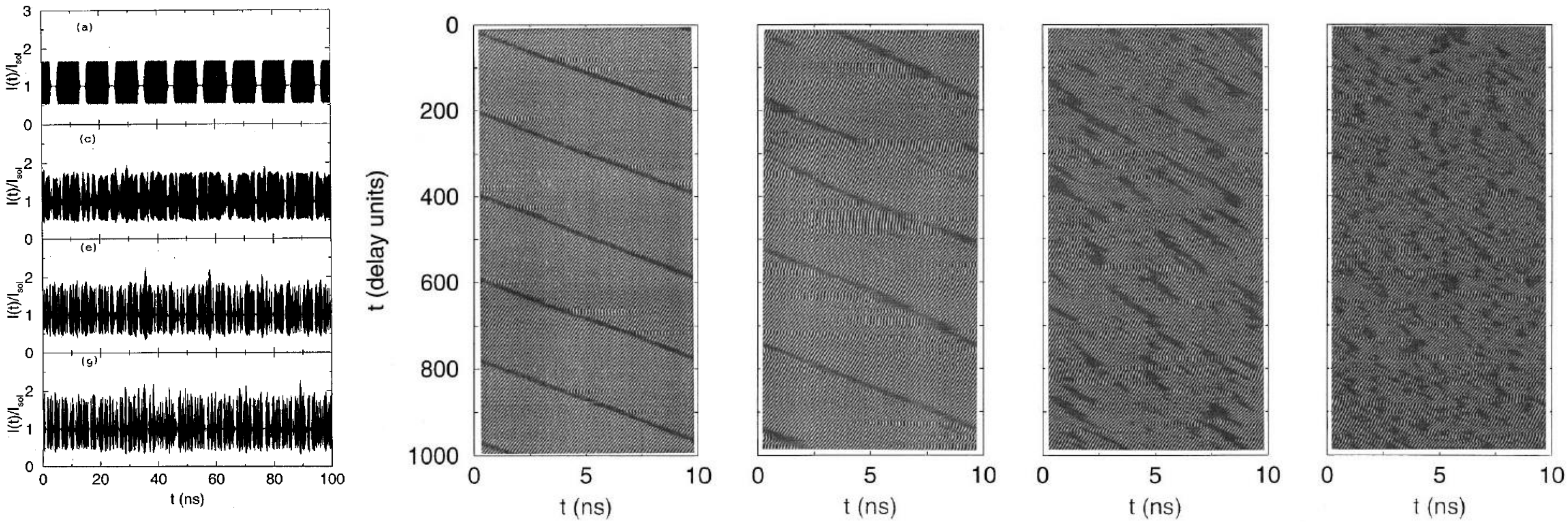
Aragoneses et al, PRL (2016)

Semiconductor laser with feedback light

- Time-delay due to propagation time (ns)
- Near threshold: stochastic spiking dynamics (quantum spontaneous emission noise).



Model simulations: increasing the feedback strength → complex dynamics



C. Masoller, Chaos 7, 455 (1997)

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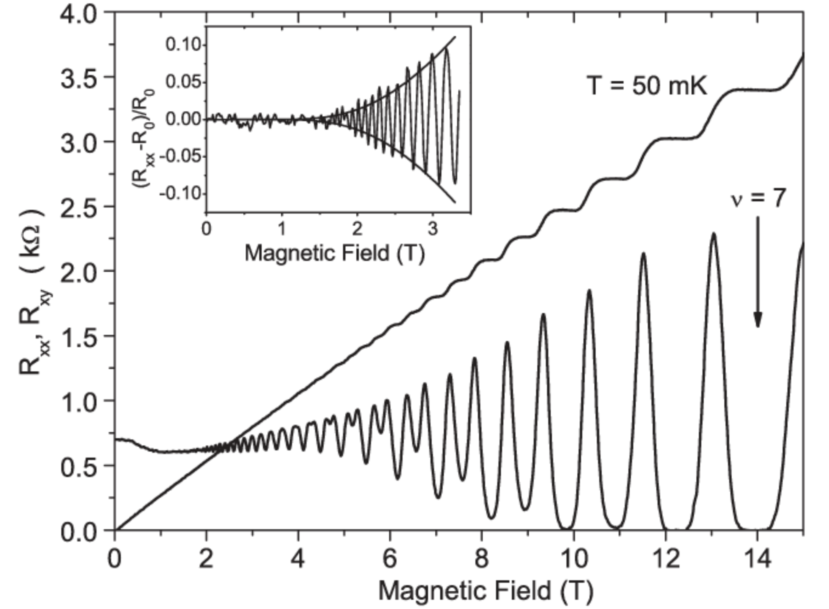
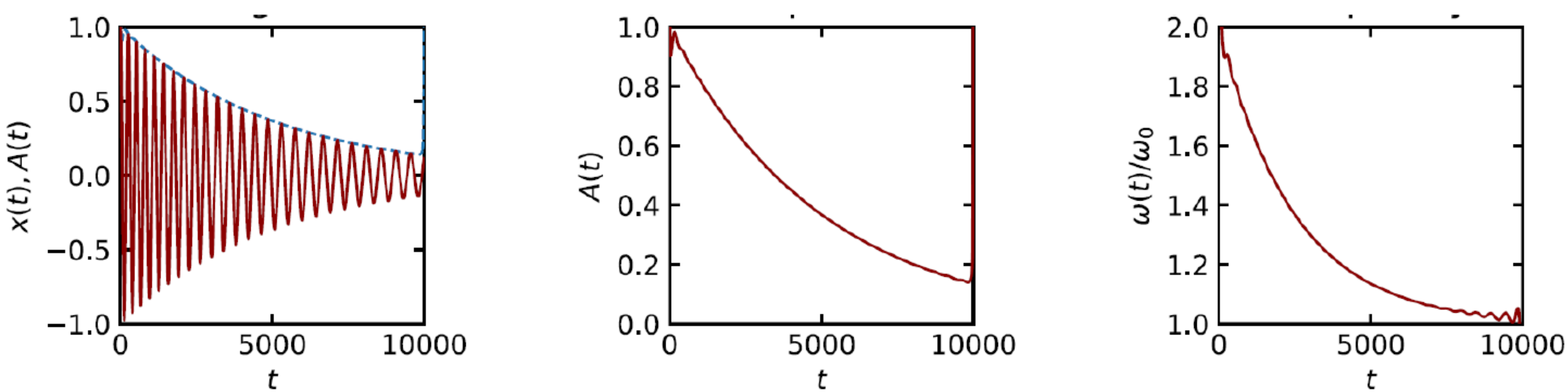
How to obtain information about instantaneous amplitude and frequency?

Shubnikov de Haas oscillations in a quantum spin system



$$x(t) = e^{-\alpha t}\cos[\omega_0(1 + e^{-2\alpha t})]$$

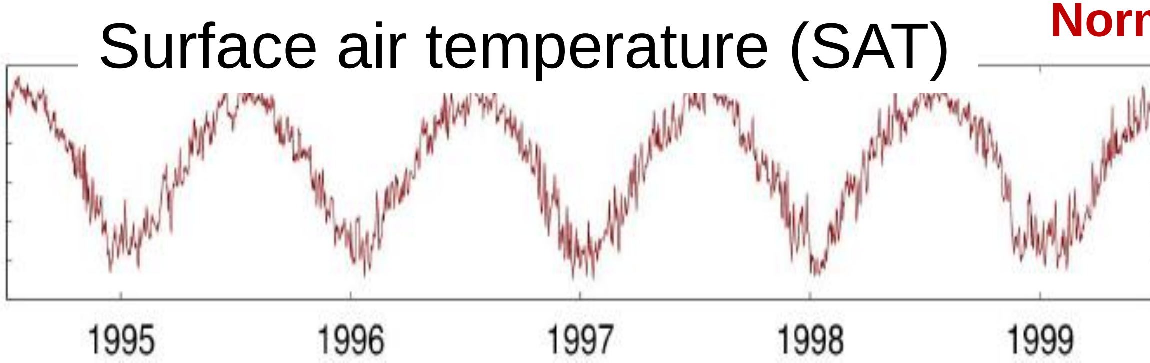


Source: Semantic scholar; D. Zappala et al, Chaos 30, 011103 (2020).

Third example

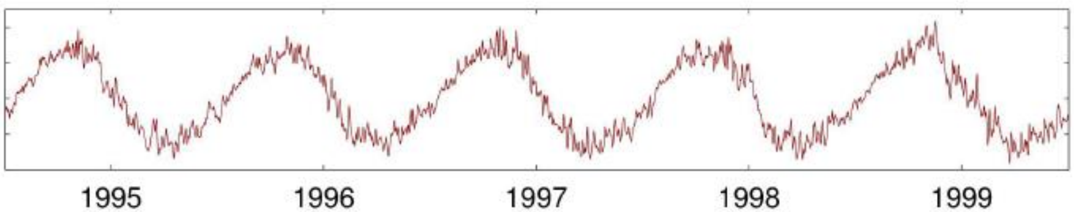
Normalization: $\mu=0$, $\sigma=1$

x

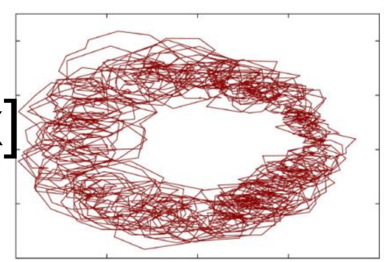


- HT[sin(ωt)]=cos(ωt)

HT[x]



y=HT[x]



x

$$a(t) = \sqrt{[x(t)]^2 + [y(t)]^2}$$

$$\varphi(t) = \arctan[y(t)/x(t)]$$

Zappala, Barreiro and Masoller, Entropy (2016)

Hilbert transform

For a real time series x(t) defines an *analytic signal*

$$\zeta(t) = x(t) + iy(t) = a(t)e^{i\varphi(t)}$$

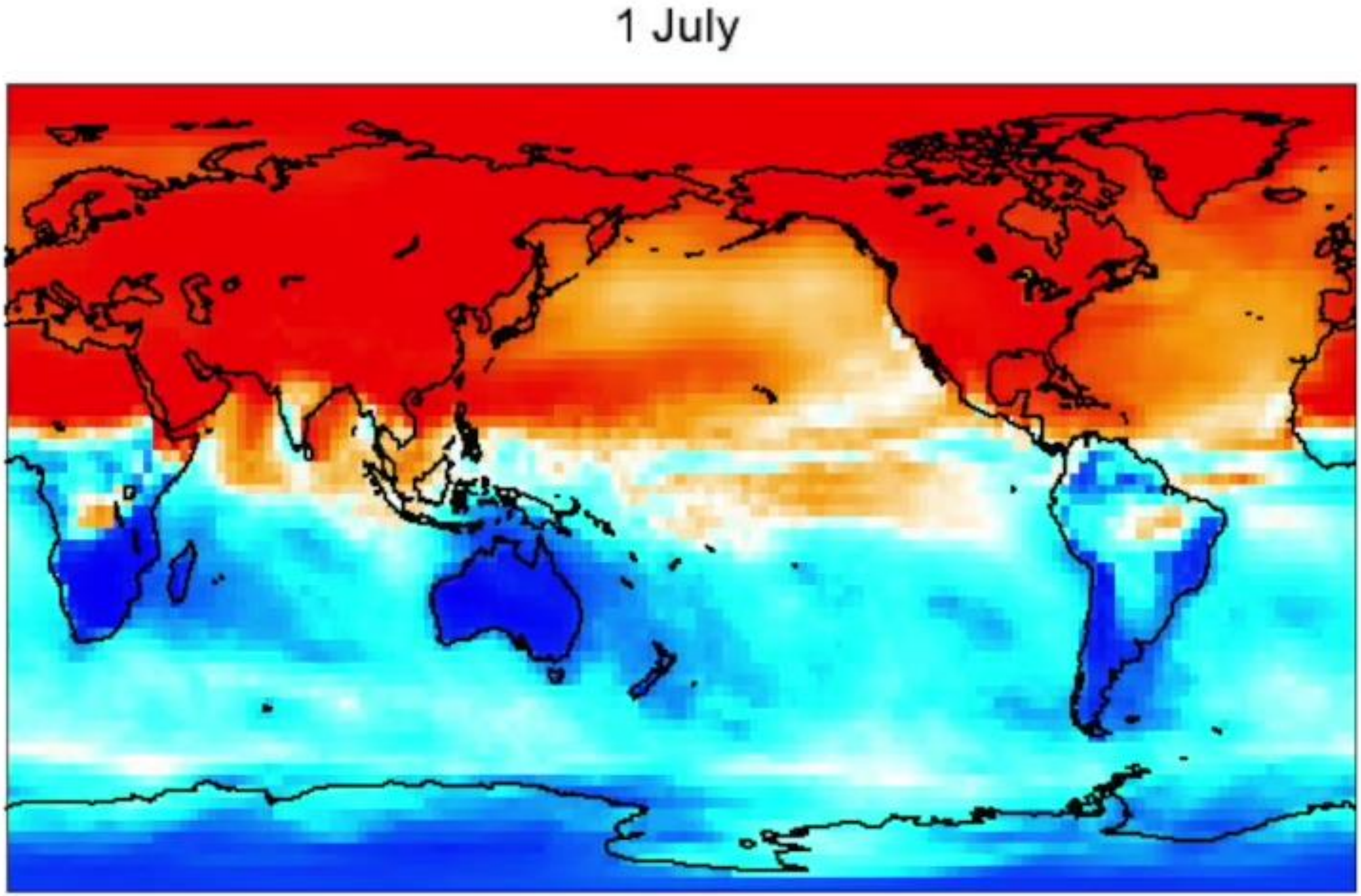
$$y(t) = H[x(t)] = \pi^{-1}\text{P.V.}\int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau$$

<u>A word of warning</u>:
Although formally *a(t)* and φ*(t)* can be defined for any *x(t)*, **they have a clear physical meaning only if *x(t)* is a narrow-band oscillatory signal**: in that case, the ***a(t)*** coincides with the envelope of *x(t)* and the **instantaneous frequency, $\omega(t)=d\varphi/dt$**, coincides with the dominant frequency in the power spectrum.

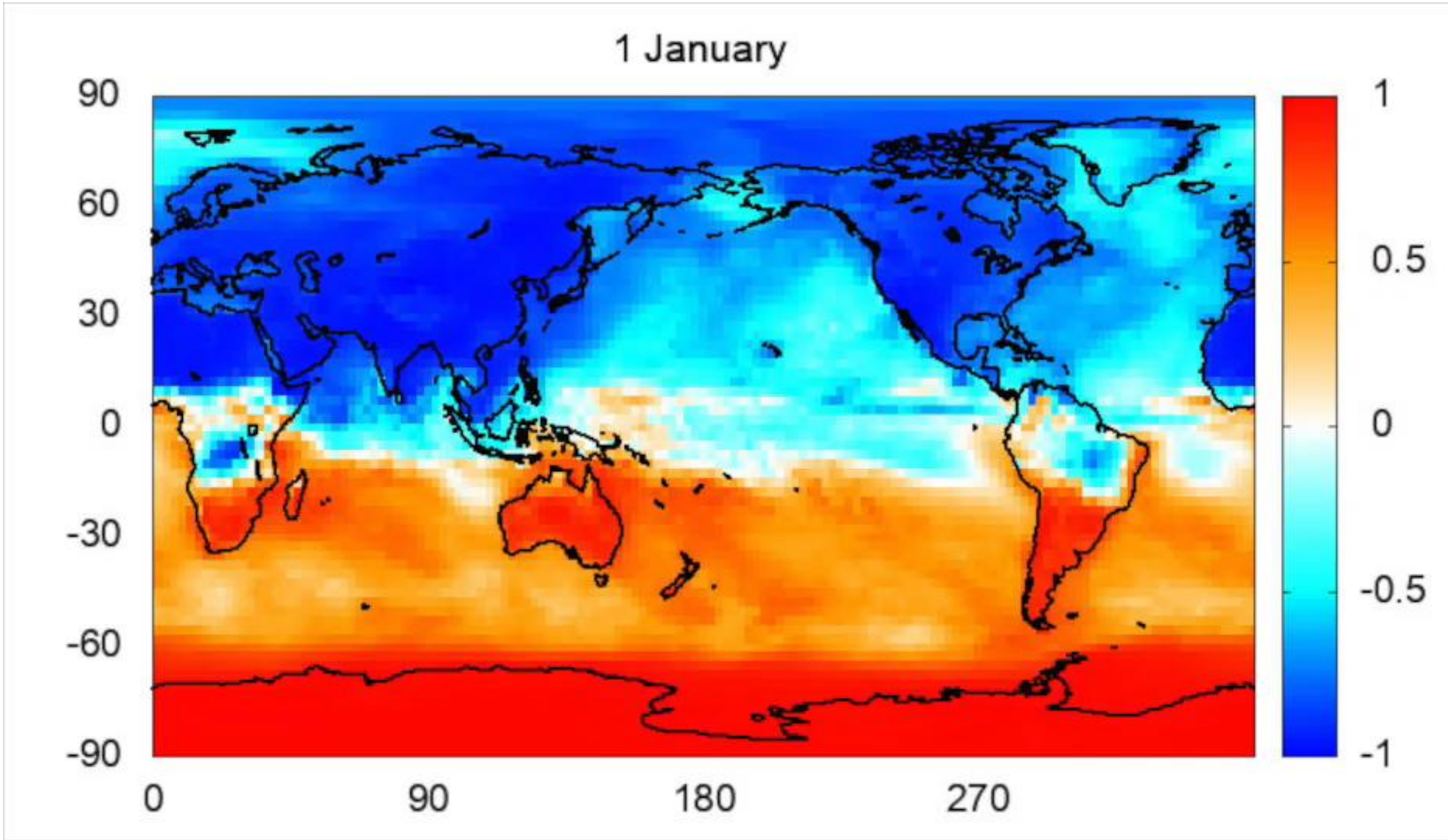
Application to climate data

- Can we use the Hilbert amplitude, phase, frequency, to :
  - Identify and quantify regional climate change?
  - Investigate synchronization in climate data?
- Problem: climate time series are not narrow-band.
- Usual solution (e.g. brain signals): isolate a narrow frequency band.
- However, the Hilbert transform applied to Surface Air Temperature time series yields meaningful insights.

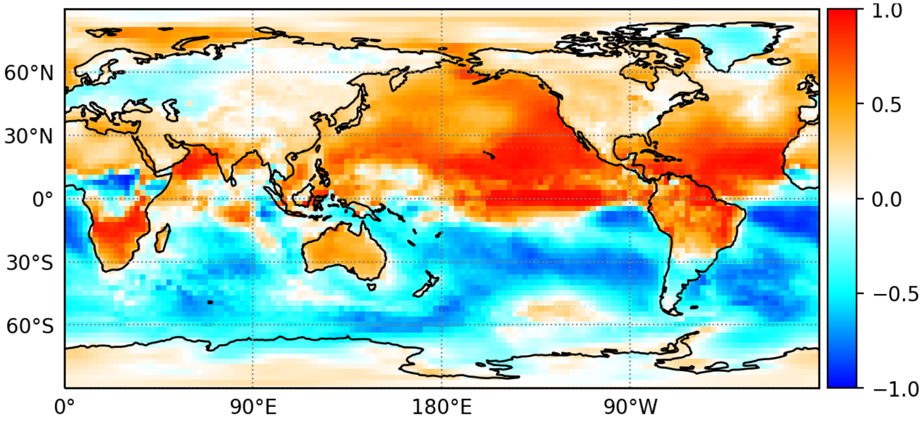
Cosine of Hilbert phase



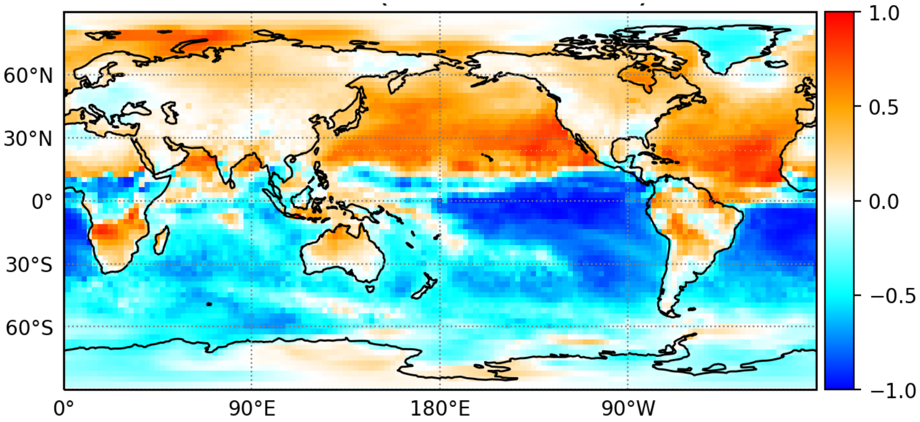
How the seasons evolve?
Temporal evolution of the cosine of the Hilbert phase



Cosine of Hilbert phase during a El Niño period (October 2015)



Cosine of Hilbert phase during a La Niña period (October 2011)



Changes in Hilbert amplitude and frequency detect inter-decadal variations in surface air temperature (SAT)

The data:

- Spatial resolution $2.5^0 \times 2.5^0 \Rightarrow$ 10226 time series
- Daily resolution 1979 – 2016 $\Rightarrow$ 13700 data points

Where does the data come from?

- European Centre for Medium-Range Weather Forecasts (ECMWF, ERA-Interim).
- Freely available.

"Features" extracted from each SAT time series

- Time averaged amplitude, $\langle a \rangle$
- Time averaged frequency, $\langle \omega \rangle$
- Standard deviations, σ_a, σ_ω

Relative decadal variations

$$\Delta a = \langle a \rangle_{2016-2007} - \langle a \rangle_{1988-1979}$$

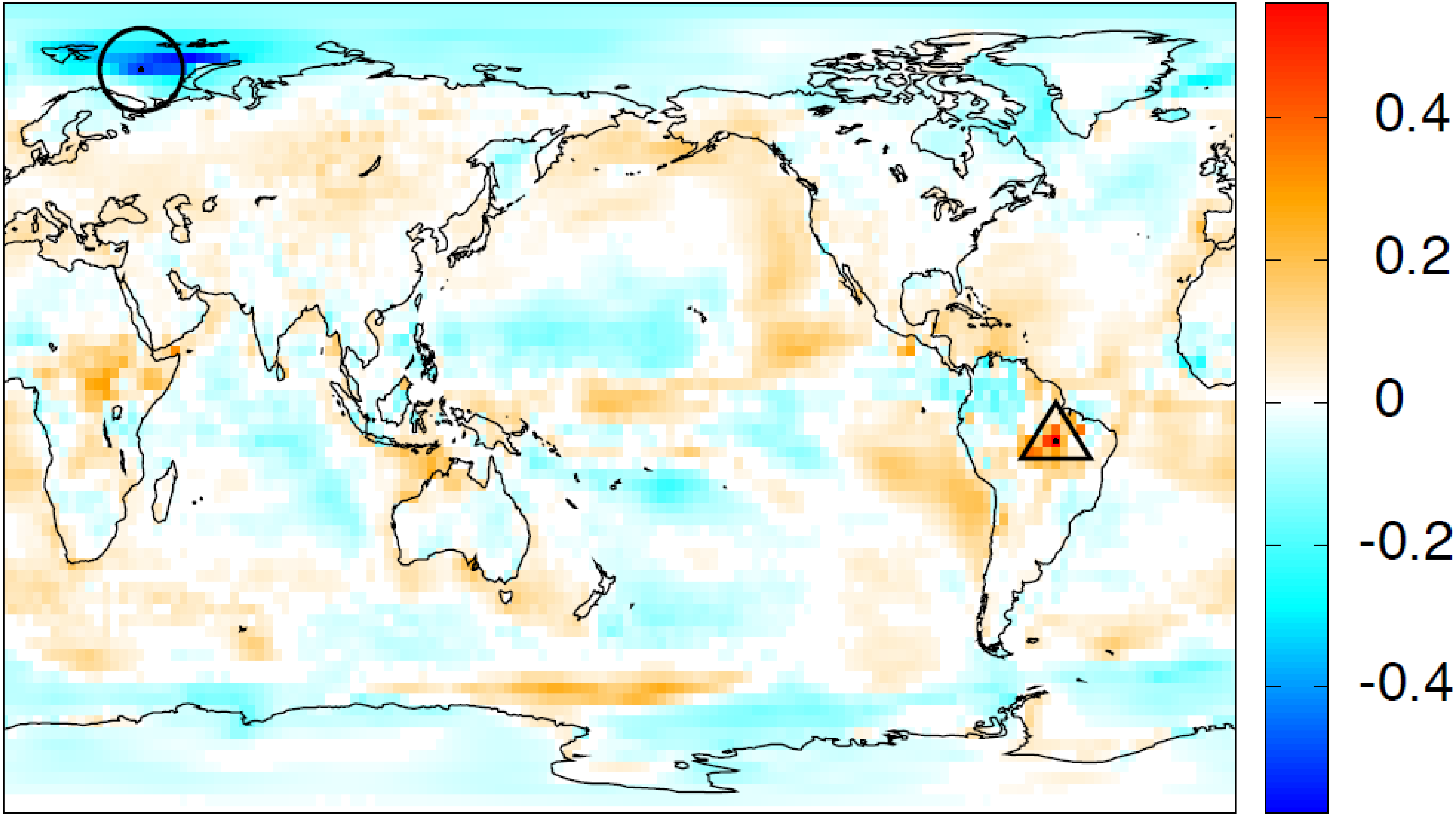
$$\frac{\Delta a}{\langle a \rangle_{2016-1979}}$$

Relative variation is considered significant if:

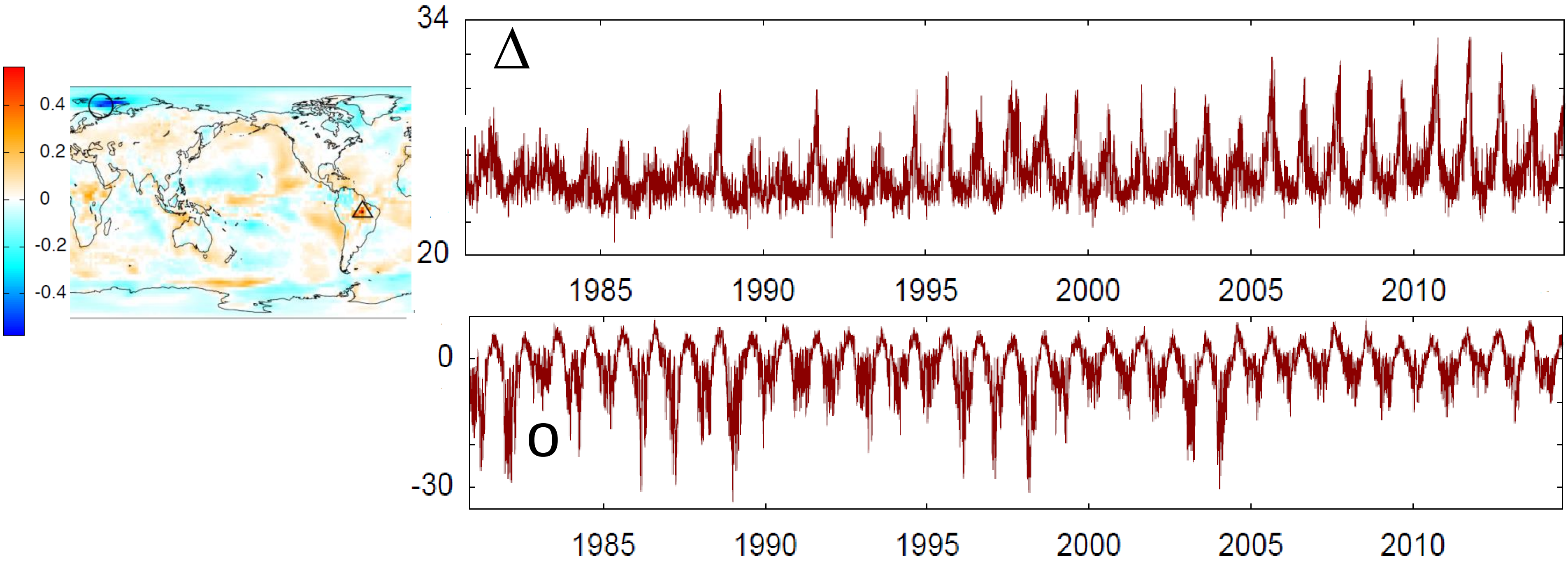
$$\frac{\Delta a}{\langle a \rangle} \geq \langle . \rangle_s + 2\sigma_s \quad \text{or} \quad \frac{\Delta a}{\langle a \rangle} \leq \langle . \rangle_s - 2\sigma_s$$

100 “block” surrogates

D. A. Zappala, M. Barreiro and C. Masoller, “*Quantifying changes in spatial patterns of surface air temperature dynamics over several decades*”, Earth Syst. Dynam. **9**, 383 (2018)


0.4
0.2
0
-0.2
-0.4

Relative variation of average Hilbert amplitude uncovers regions where the amplitude of the seasonal cycle increased or decreased



- **Decrease of precipitation**: the solar radiation that is not used for evaporation is used to heat the ground.
- **Melting of sea ice**: during winter the air temperature is mitigated by the sea and tends to be more moderated.

D. A. Zappala et al., Earth Syst. Dynam. **9**, 383 (2018)

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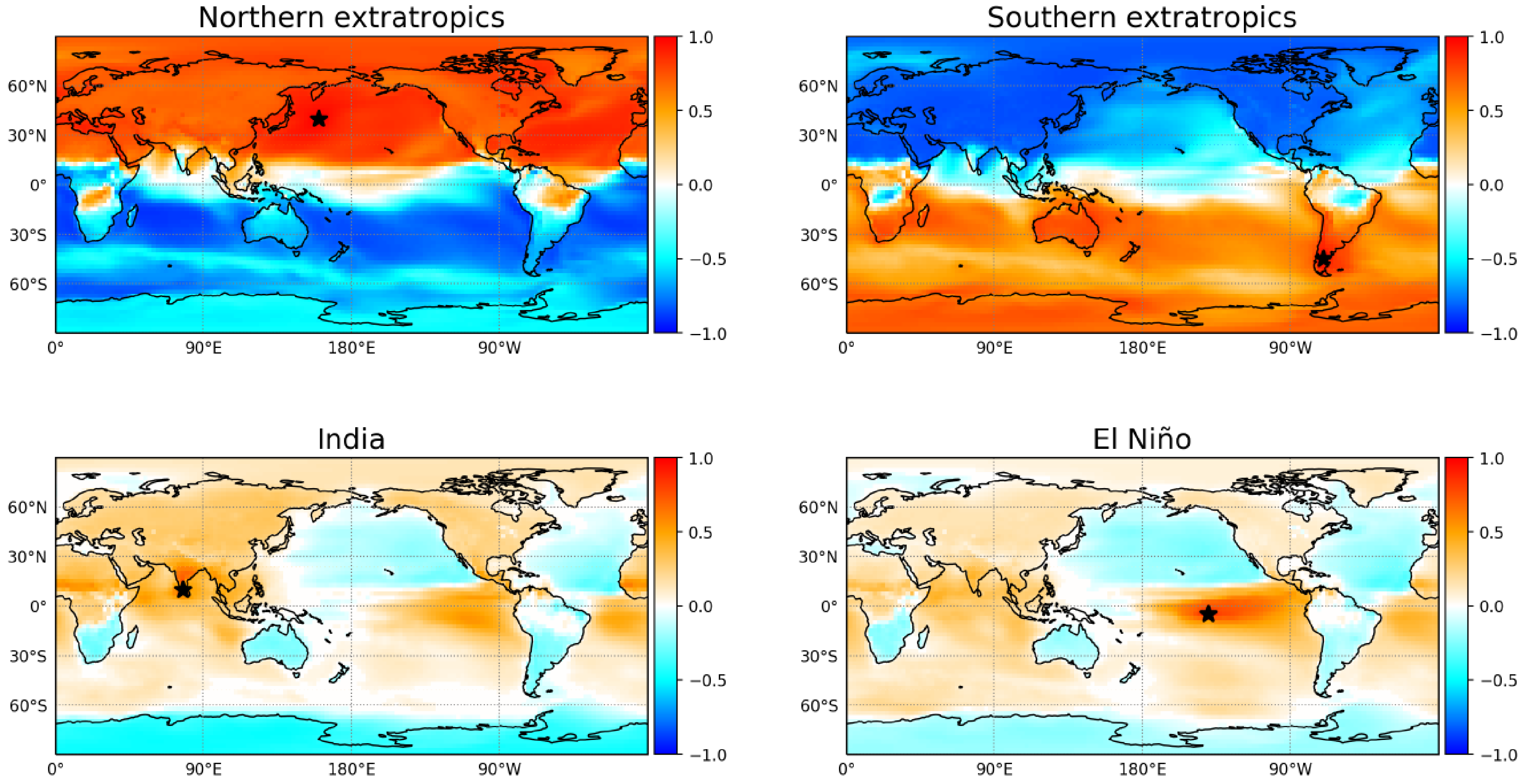
Cross-correlation of two time series *X* and *Y* of length *N*

$$C_{xy}(\tau) = \frac{1}{N-\tau} \sum_{k=1}^{N-\tau} x(k+\tau) y(k)$$

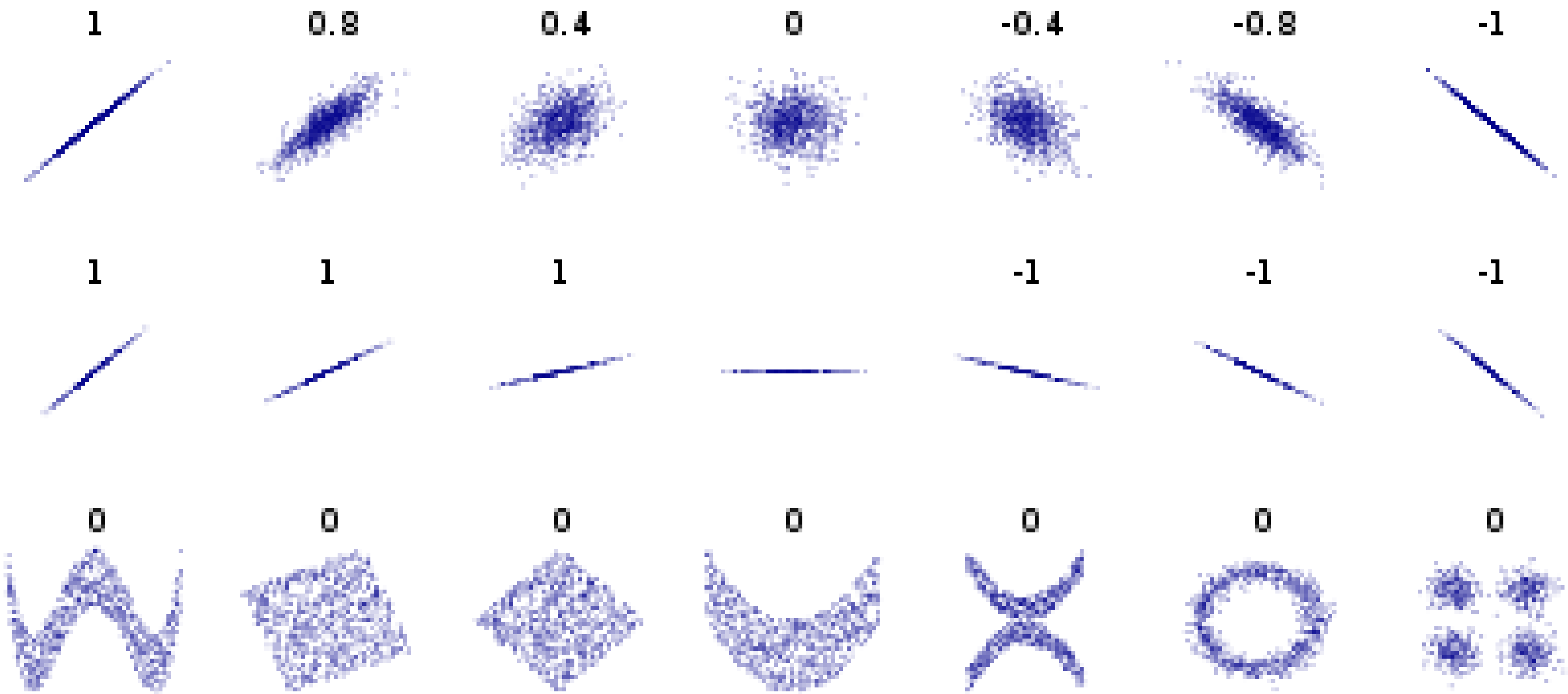
the two time series are normalized to zero-mean **μ=0** and unit variance, **σ=1**

- $-1 \leq C_{X,Y} \leq 1$
- $C_{X,Y} = C_{Y,X}$
- The maximum of $C_{X,Y}(\tau)$ indicates the **lag** that renders the time series *X* and *Y* best aligned.
- Pearson coefficient: $\rho = |C_{X,Y}(0)|$

Example: cross-correlation of cosine of Hilbert phase of SAT at a reference point (*), and all other regions



Cross-correlation analysis detects linear relationships only



Source: wikipedia

Nonlinear correlation measure based on information theory: the mutual Information

$$MI = \sum_{i \in x} \sum_{j \in y} p(x,y) \ \log \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

- *MI (x,y) = MI (y,x)*

- *p(x,y) = p(x) p(y)* $\Rightarrow$ *MI = 0*, else ***MI >0***

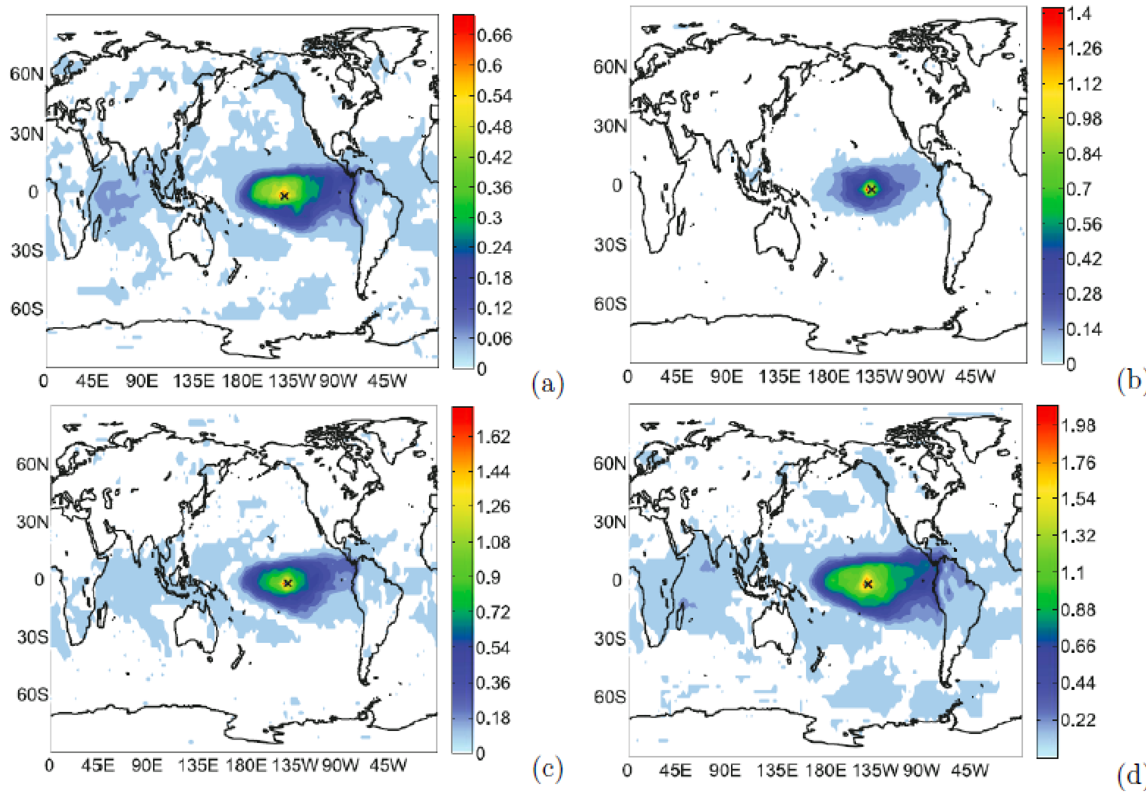
- *MI* can also be computed with a lag-time.

- *MI* can also be computed from symbolic probabilities (e.g., probabilities of ordinal patterns).

Example: MI maps computed from Surface Air Temperature anomalies at a reference point in El Niño, and other regions

Histograms

3 months ordinal patterns



Inter-annual ordinal patterns

3 years ordinal patterns

Ordinal analysis separates the times-scales of the interactions

Deza, Barreiro and Masoller, Eur. Phys. J. ST 222, 511 (2013)

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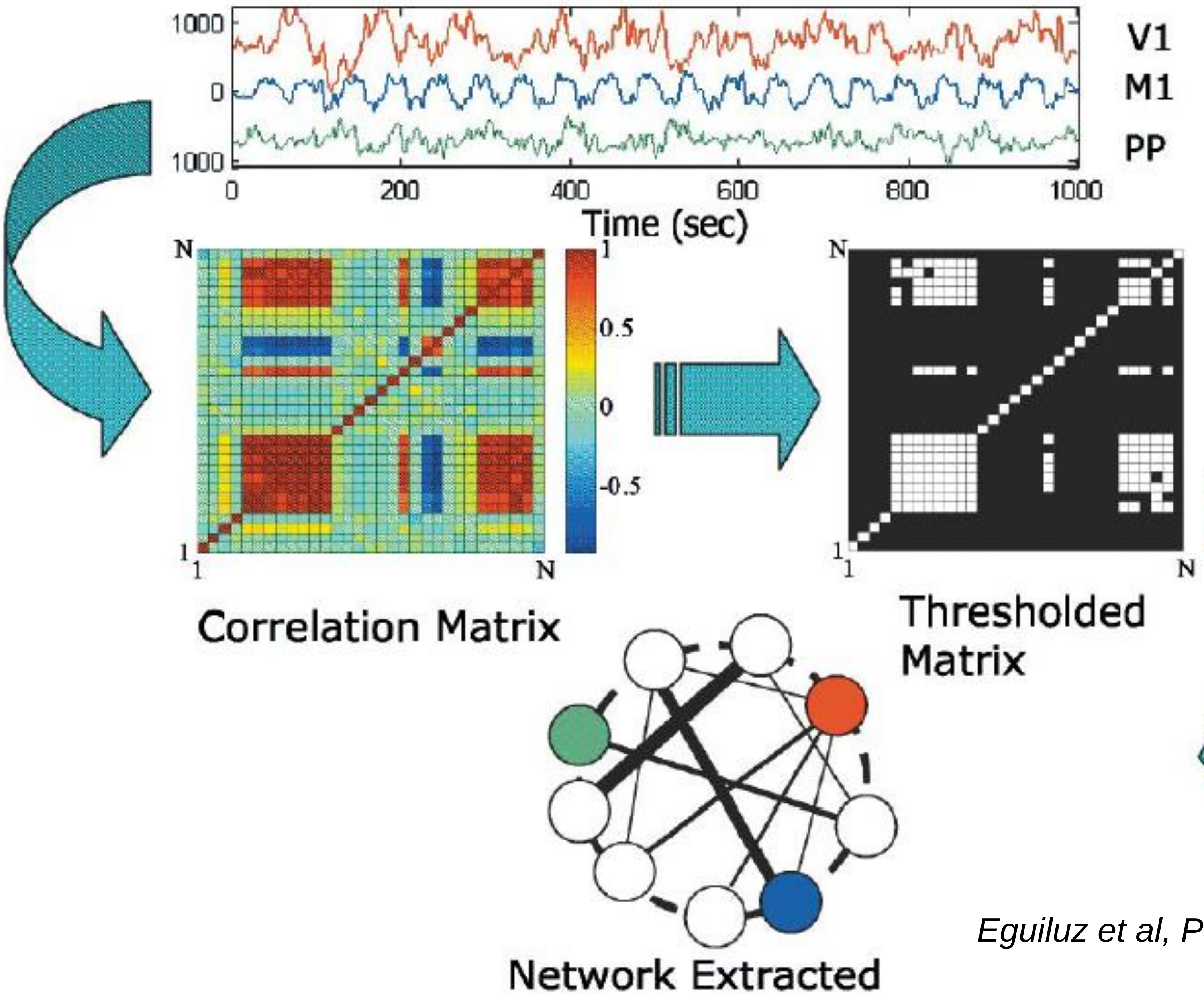
Brain functional network



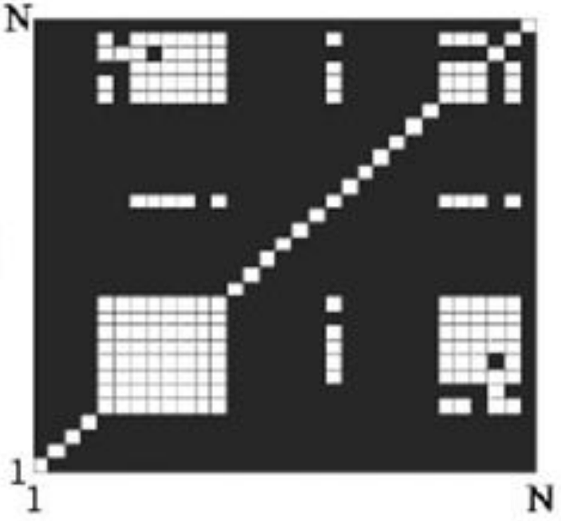
Adjacency matrix

S_{ij} > Th
$\Rightarrow A_{ij} = 1$,
else A_{ij}=0

Eguiluz et al, PRL 2005

Graphical representation

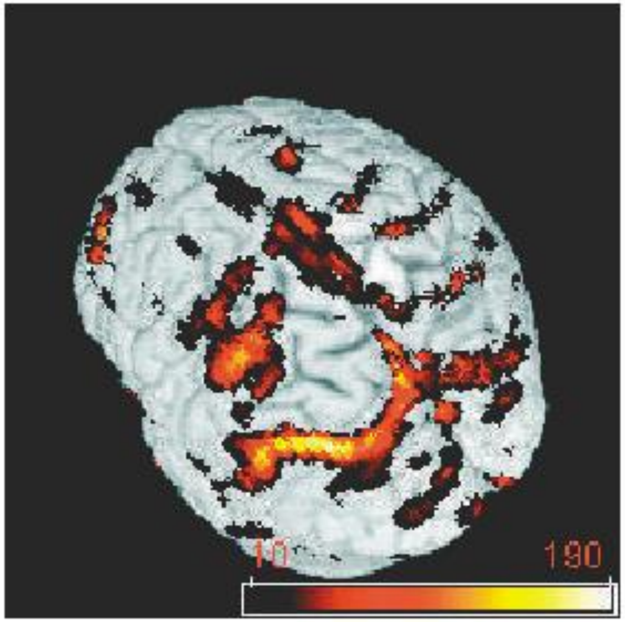
Adjacency matrix



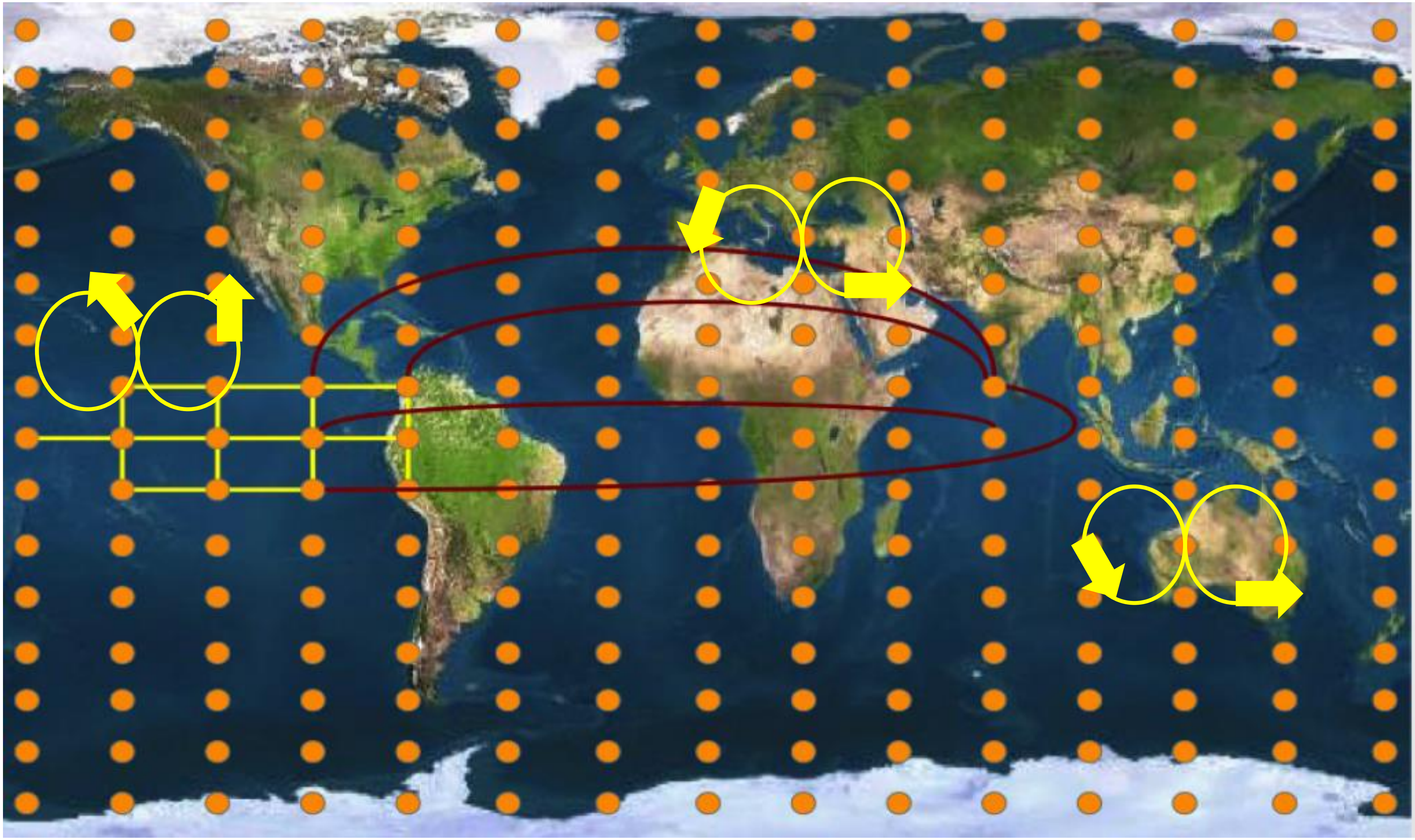
Thresholded matrix = inferred ("functional") network

Degree of a node: number of links

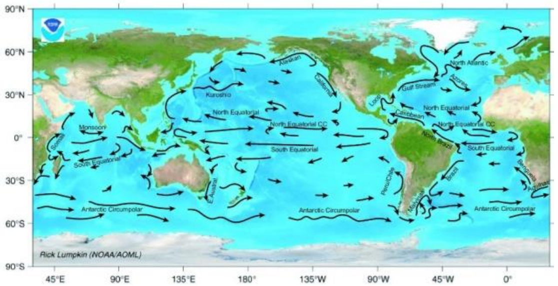
$$k_i = \Sigma_j A_{ij}$$



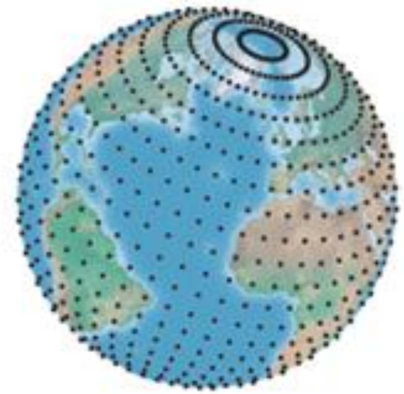
The climate system as a set of "interacting oscillators"



Complex network representation of the climate system

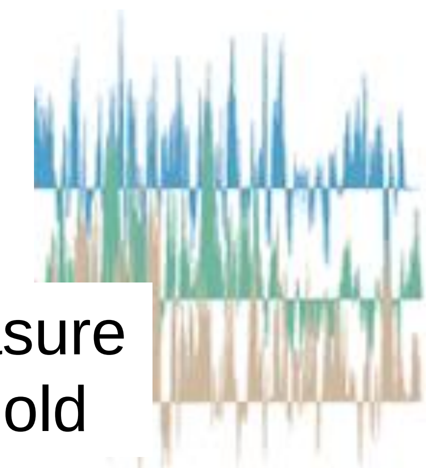


Back to the climate system: interpretation (currents, winds, etc.)



More than 10000 nodes (with different sizes).

Daily resolution: more than 13000 data points in each TS

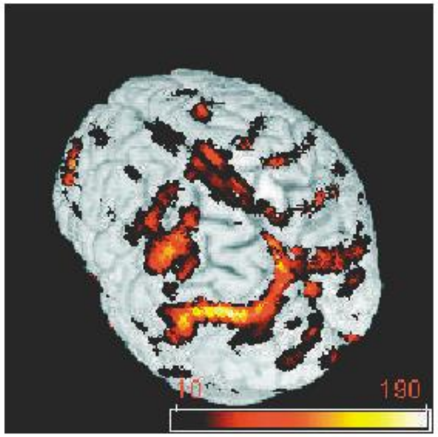


Surface Air Temperature <u>Anomalies</u> (solar cycle removed)

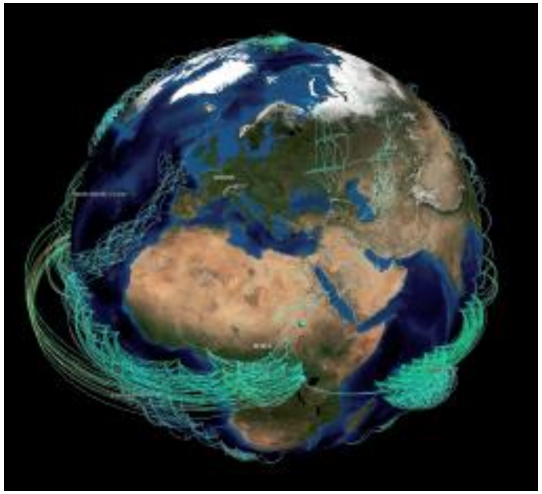
Sim. measure + threshold

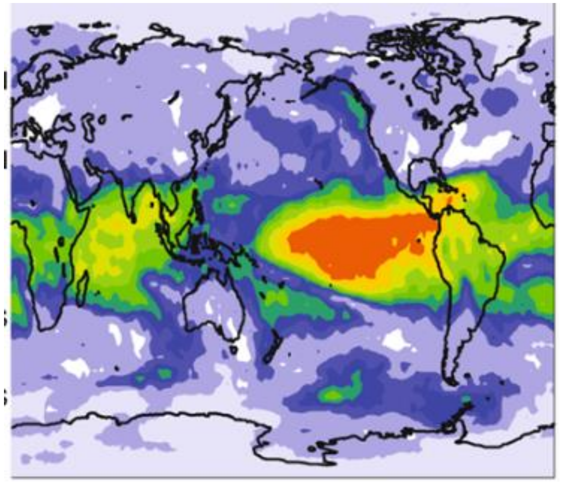
Donges et al, Chaos 2015

Brain network



Climate network





Area weighted connectivity (AWC):

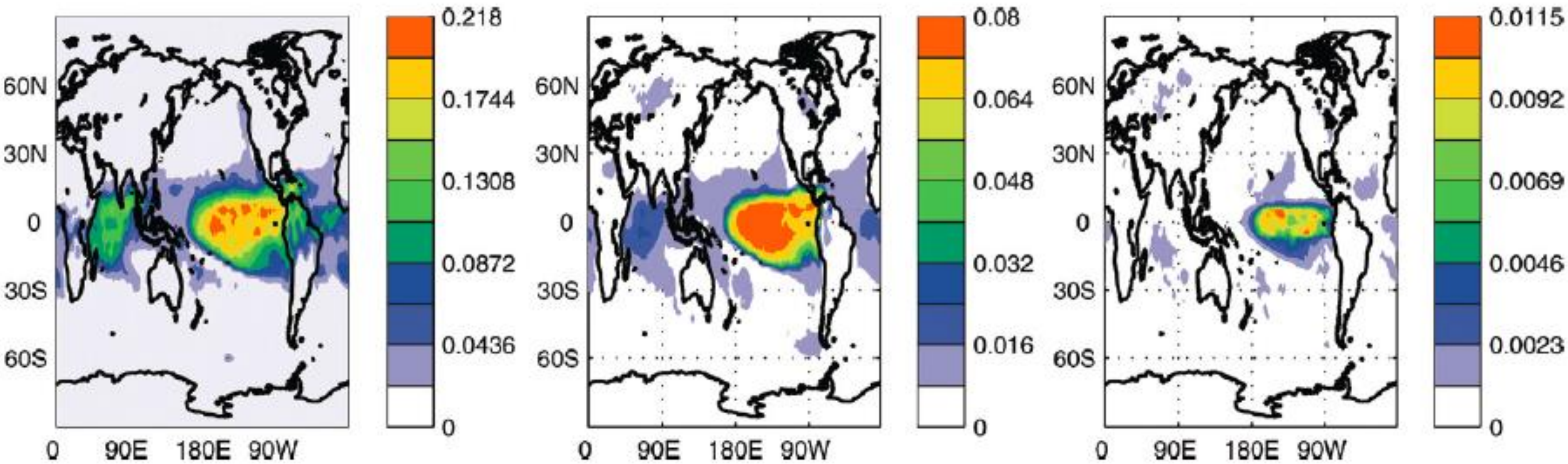
weighted degree (nodes represent areas with different sizes)

$$\mathrm{AWC}_i = \frac{\sum_j^N A_{ij} \cos(\lambda_j)}{\sum_j^N \cos(\lambda_j)}$$

How to select the threshold ?

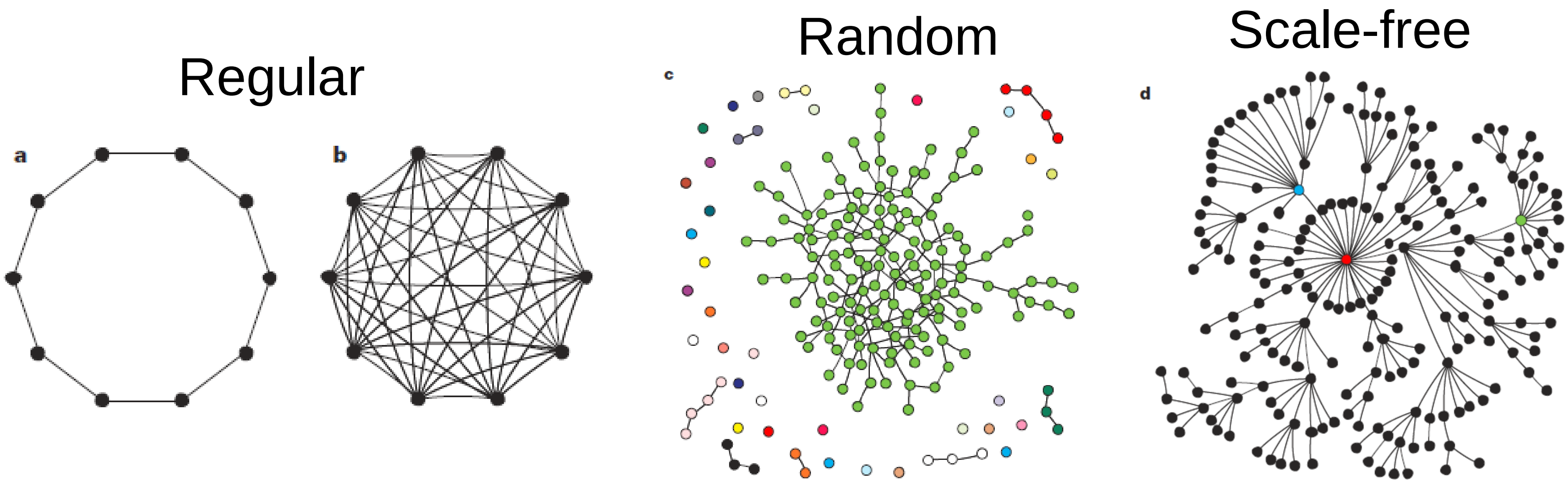
If S_{ij} > Th $\Rightarrow$ A_{ij} = 1,
else A_{ij}=0

Th $\Rightarrow$



M. Barreiro, et. al, Chaos 21, 013101 (2011)

An important challenge in time-series analysis: how to infer the structure of the network from observed data



Strogatz, Nature 2001

A link classification problem

$$S_{ij} > Th \Rightarrow A_{ij} = 1, \text{ else } A_{ij}=0$$

- There are many **statistical similarity measures** to infer interactions from observations, i.e., to classify:
  - the interaction exists (is significant)
  - the interaction does not exists (or is not significant)
- How to select the threshold?
- In "spatially embedded networks", nearby nodes have the strongest links.
- How to keep **weak-but-significant** links?

Goal: use a system with known connectivity to test the performance of statistical similarity measures

Observed time series in nodes i and j: $a_i(t)$, $a_j(t)$, $t=1, \ldots, T$ *(normalized* $\mu=0$, $\sigma=1$*)*

Lagged |cross correlation|: $$CC_{ij}(\tau) = \frac{1}{T-\tau_{\max}} \left| \sum_{t=0}^{T-\tau_{\max}} a_i(t) a_j(t+\tau) \right|$$

Statistical Similarity Measure:

$$S_{ij} = \max | CC_{ij}(\tau) | = | CC_{ij}(\tau_{ij}) | \qquad \tau_{ij} \text{ in } [0, \tau_{\max}]$$

We compare with the Mutual Information, computed from probabilities of "raw" values and from ordinal probabilities

G. Tirabassi et al., "*Inferring the connectivity of coupled oscillators from time-series statistical similarity analysis*", Sci. Rep. **5** 10829 (2015).

Kuramoto oscillators in a random network

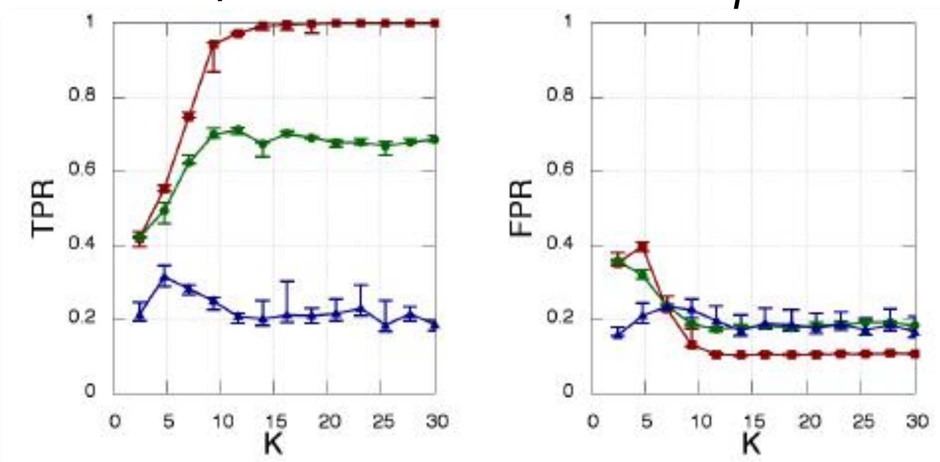
$$d\theta_i = \omega_i dt + \frac{K}{N}\sum_{j=1}^{N} A_{ij}\sin(\theta_j - \theta_i)dt + D\, dW_t^i$$

A_{ij} is a symmetric random matrix; N=12 time-series, each with 10^4 data points.

CC MI MIOP

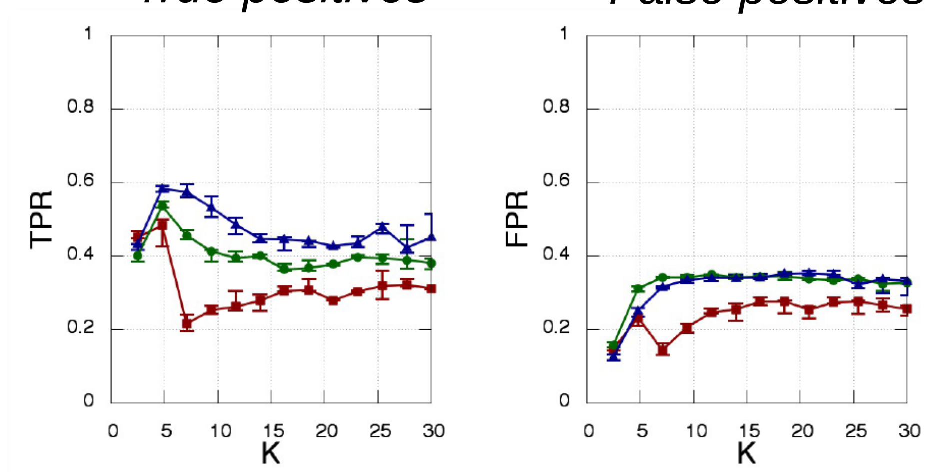
Phases (θ)

True positives *False positives*



“Observable” Y=sin(θ)

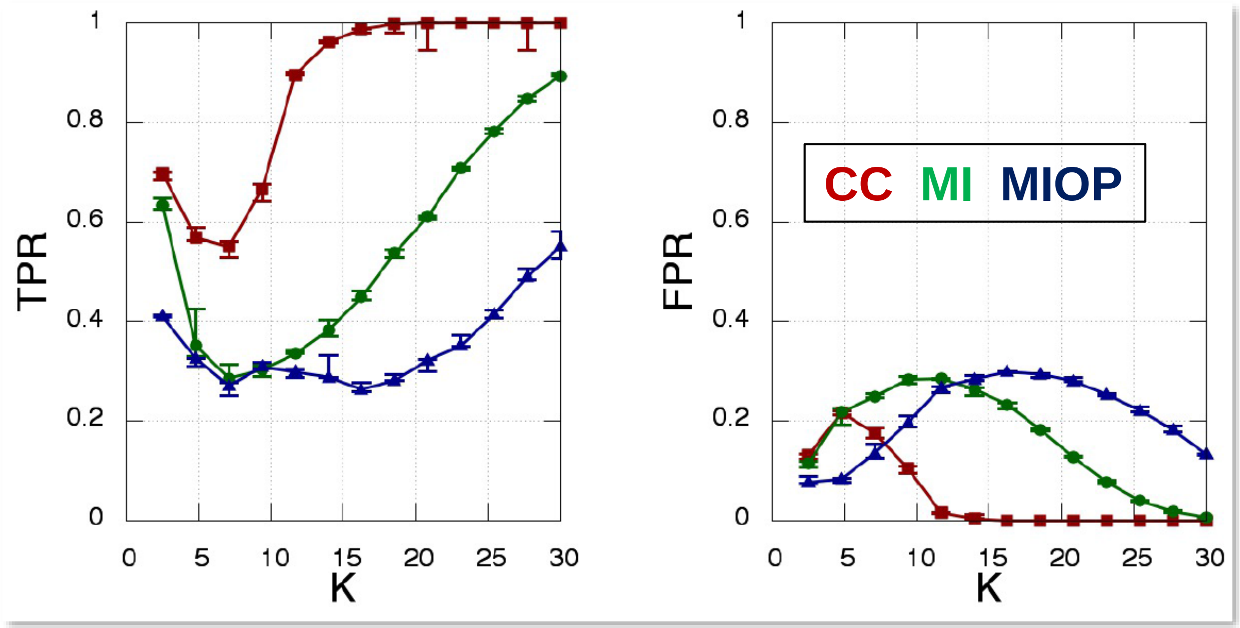
True positives *False positives*



Results of a 100 simulations with different oscillators’ frequencies, random matrices, noise realizations and initial conditions.

For each K, the threshold was varied to obtain optimal reconstruction.

Instantaneous frequencies (dθ/dt)



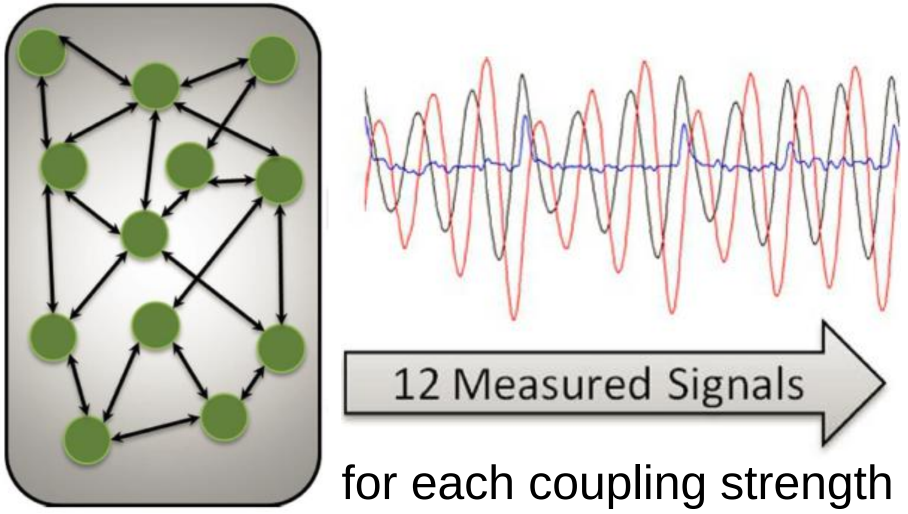
Perfect network inference is possible!

BUT

- the number of oscillators is small (12),
- the coupling is symmetric ($\Rightarrow$ only 66 possible links) and
- the data sets are long (10^4 points)

G. Tirabassi et al, Sci. Rep. **5** 10829 (2015)

We also analyzed **experimental data** recorded from 12 chaotic Rössler electronic oscillators (symmetric and random coupling)



for each coupling strength

The Hilbert Transform was used to obtain phases from experimental data

- Kuramoto Oscillators' Network

$$\theta_i$$

$$f_i = \dot{\theta}_i$$

$$Y_i = \sin(\theta_i)$$

- Rössler Oscillators' Network

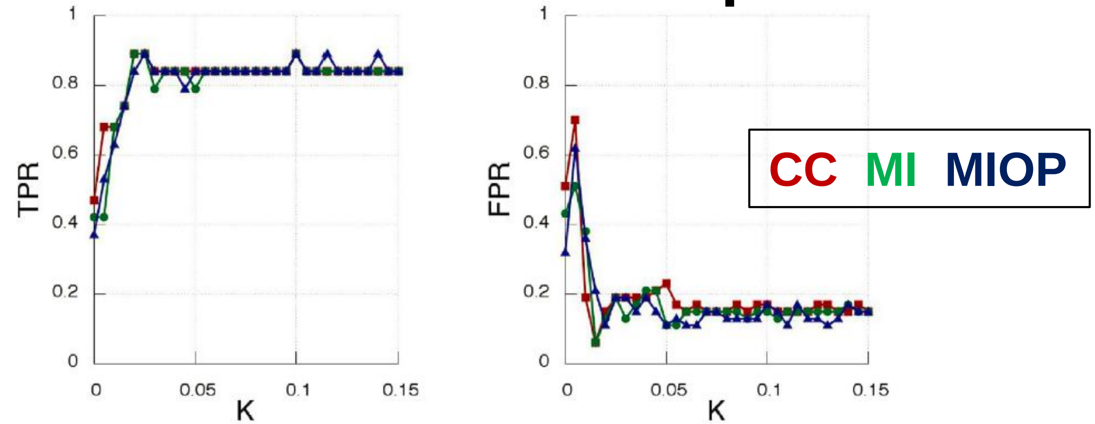
$$\varphi_i = HT(x_i)$$

$$f_i = \dot{\varphi}_i$$

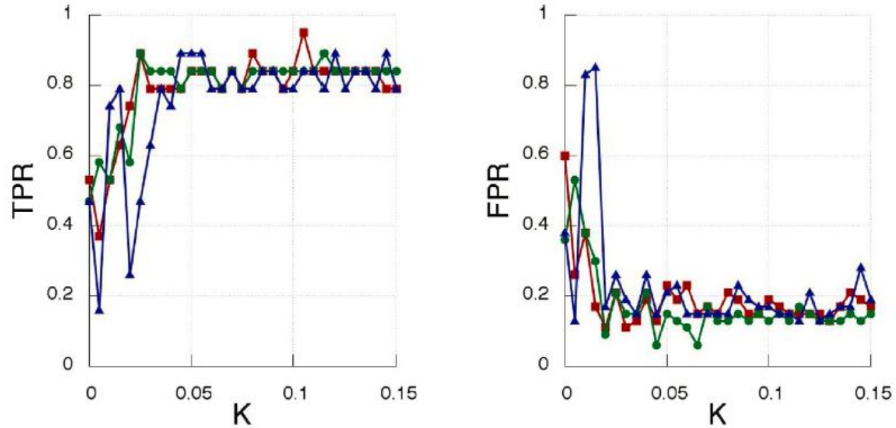
$$x_i$$

Results obtained with experimental data

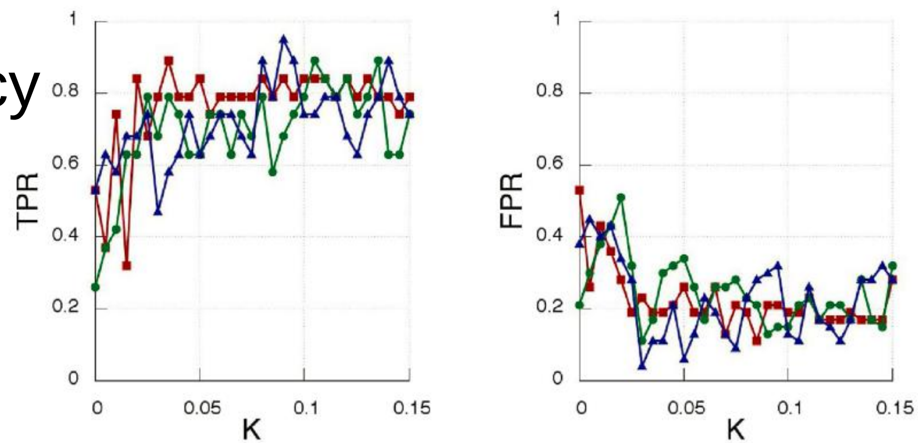
Observed variable (x)



Hilbert phase



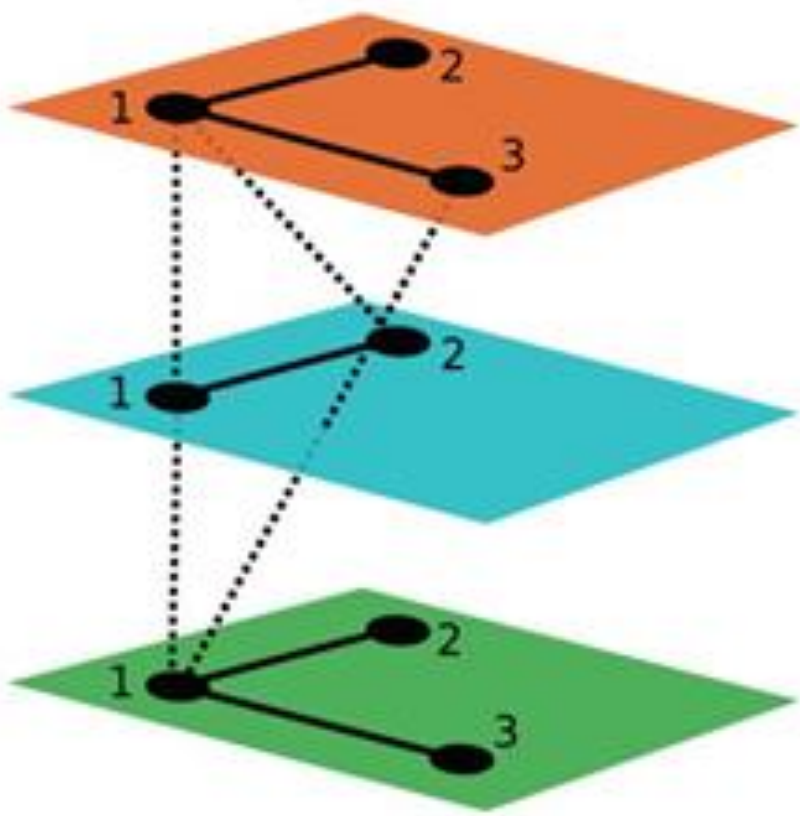
Hilbert frequency

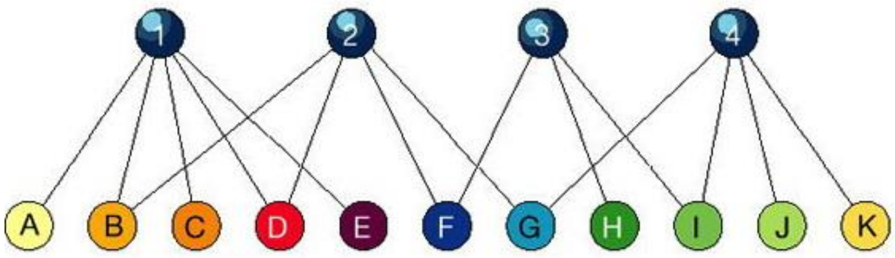


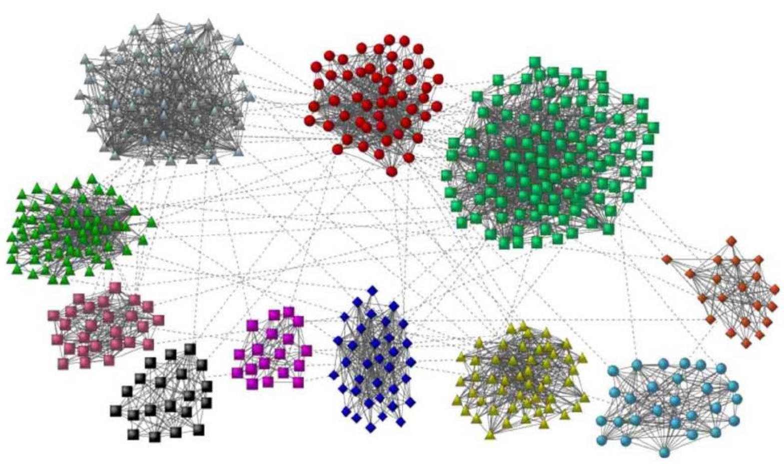
- No perfect reconstruction
- No important difference among the 3 methods & 3 variables

Generalizations of complex network analysis

Network structures:
Multilayer, multiplex, bipartite, networks of networks and many others



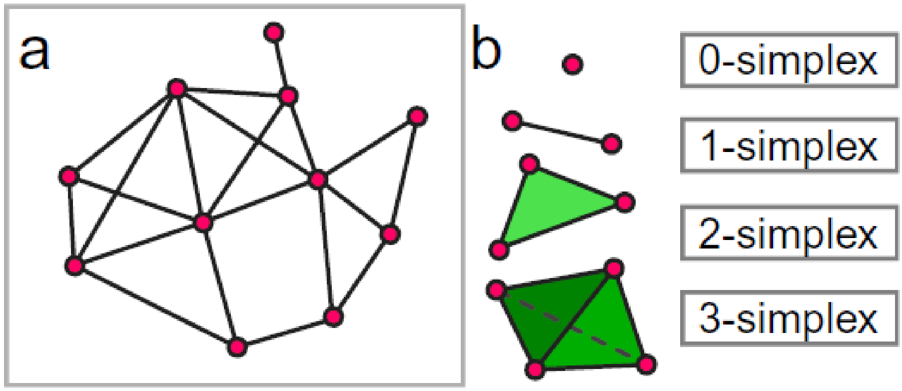




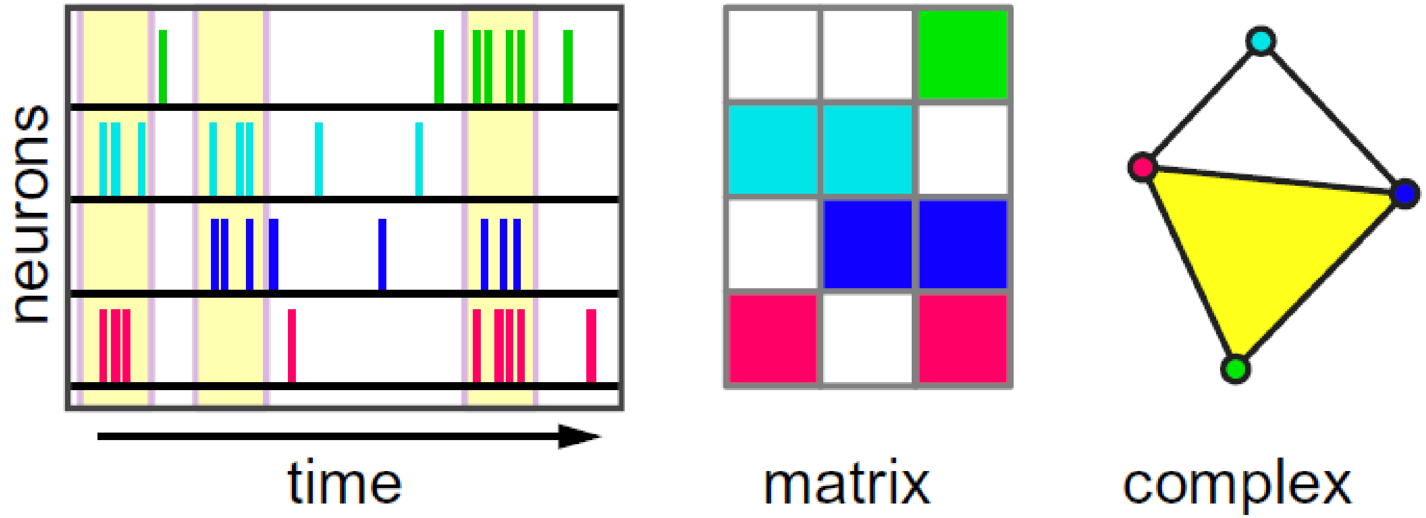
Limitations of complex network analysis

Interactions are not limited to pairs of elements

- Links represent interactions between pairs of nodes.
- ***Simplicial complexes*** represent interactions among several nodes.



Example



Giusti et al., J Comput Neurosci (2016) 41:1–14

And many more time series analysis methods

- Wavelets
- Detrended fluctuation analysis
- Sample entropy, approximate entropy
- Multifractality
- Topological data analysis
- Etc. etc.

Take home messages

- Many methods are available for investigating complex signals.
- Different methods provide *complementary* information.
- Time series analysis allows us to obtain sets of “features”.
- Data science, machine learning: feature selection and analysis.
- Time series analysis is an interdisciplinary field with many applications.

<cristina.masoller@upc.edu>

Universitat Politecnica de Catalunya

http://www.fisica.edu.uy/~cris/

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