Nonlinear time series analysis

Introduction

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About me

- Originally from Montevideo, Uruguay
- PhD in physics (lasers, Bryn Mawr College, USA)
- Since 2004 @ *Universitat Politecnica de Catalunya,* in the research group on *Dynamics, Nonlinear Optics and Lasers*.

Where are we?

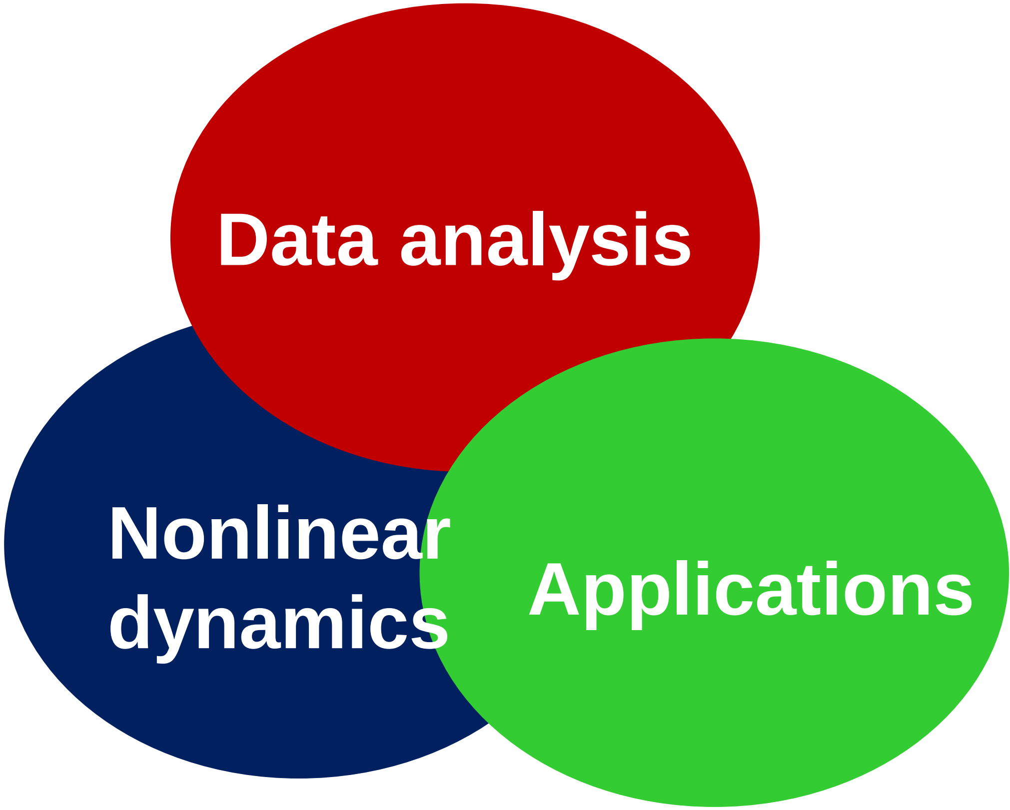
1. Barcelona
2. Castelldefels
3. Igualada
4. Manresa
5. Mataró
6. Sant Cugat del Vallès
7. Terrassa
8. Vilanova i la Geltrú

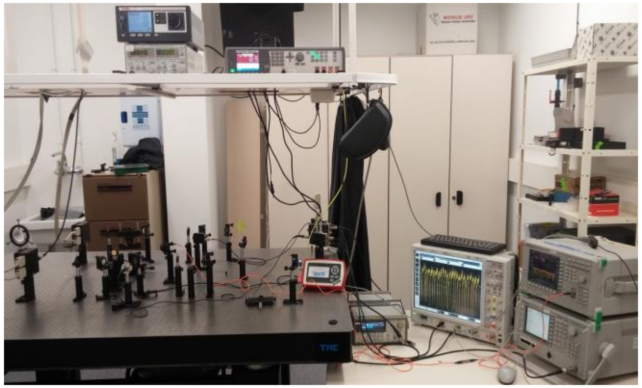
Viernes, 25 de septiembre de 2009 Diari de Terrassa

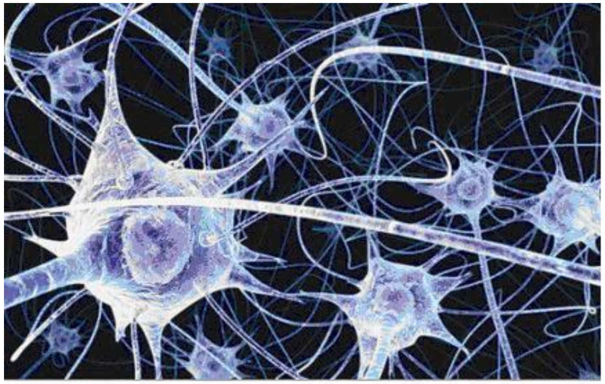
El edificio Gaia centraliza grupos científicos consolidados y emergentes.

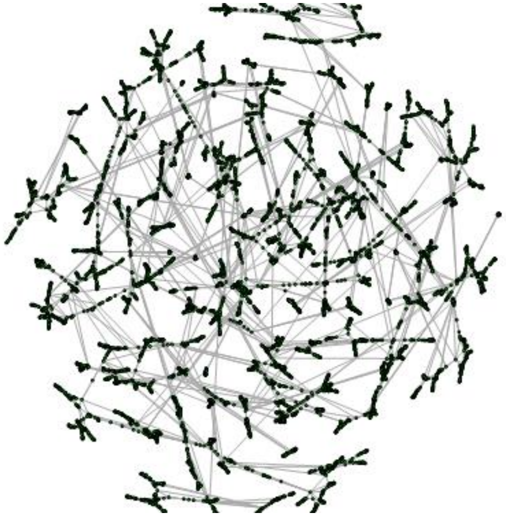
What do we study?

- Nonlinear and stochastic phenomena
  - laser dynamics
  - neuronal dynamics
  - complex networks
  - data analysis (climate, biomedical signals)





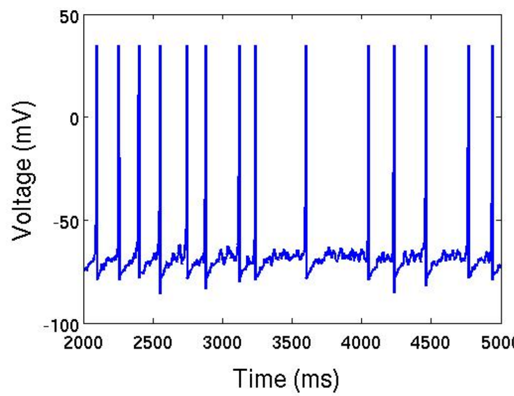




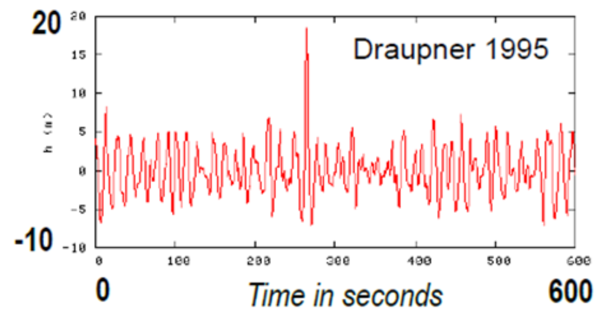
Lasers, neurons and complex systems?

- Lasers allow us to study in a controlled way phenomena that occur in diverse complex systems.
- Laser experiments allow to generate sufficient data to test new methods of data analysis for prediction, classification, etc.

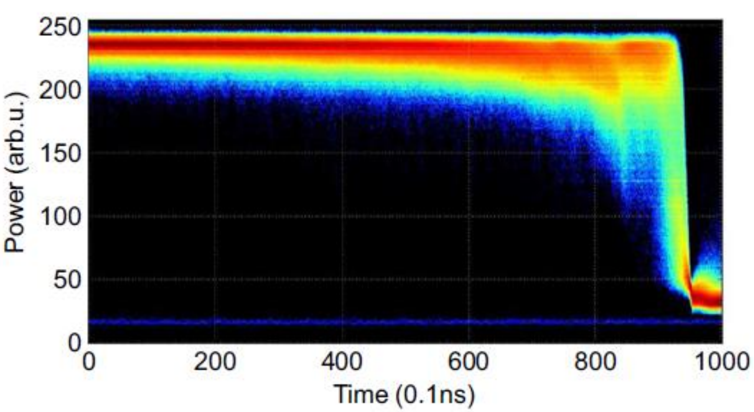
Laser & neuronal spikes



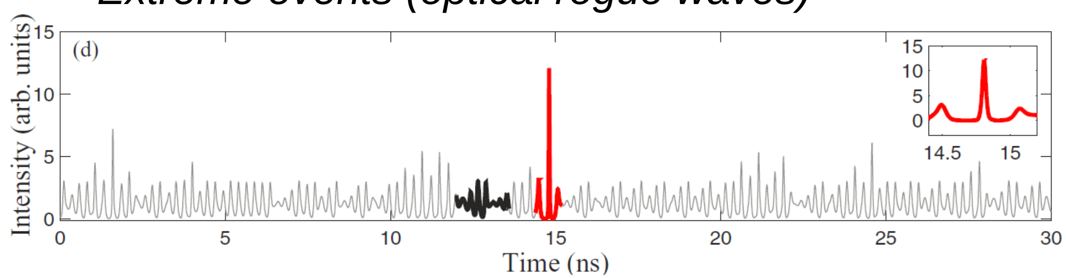
Ocean rogue wave (sea surface elevation in meters)



Abrupt switching

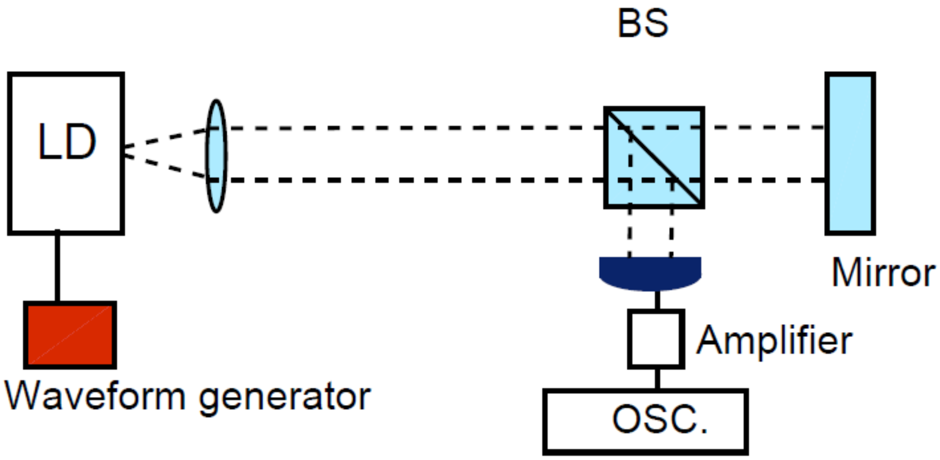


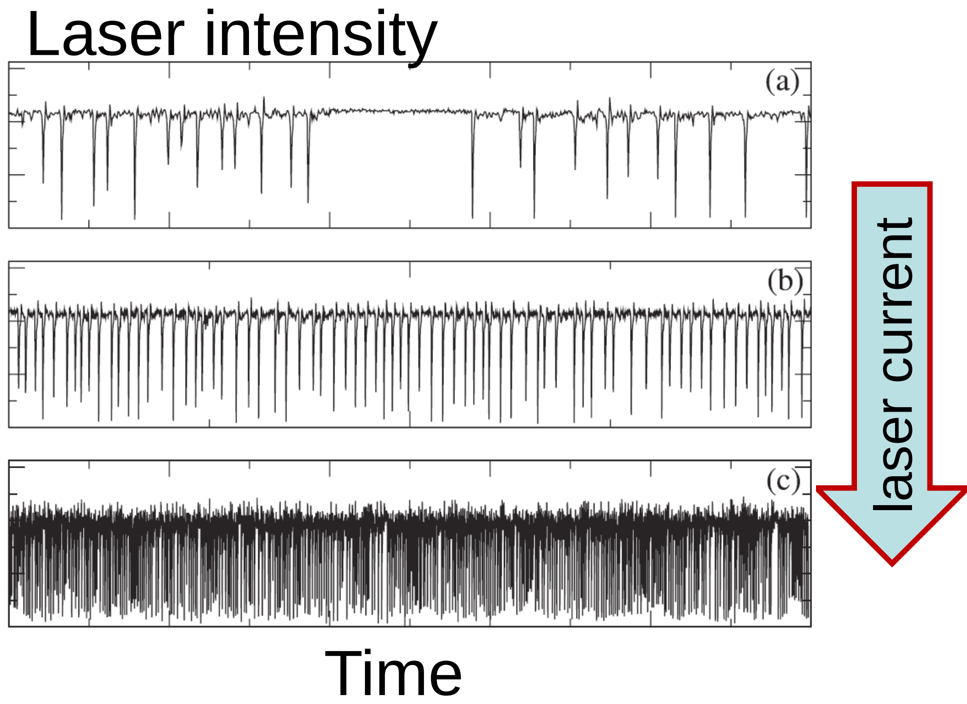
Extreme events (optical rogue waves)



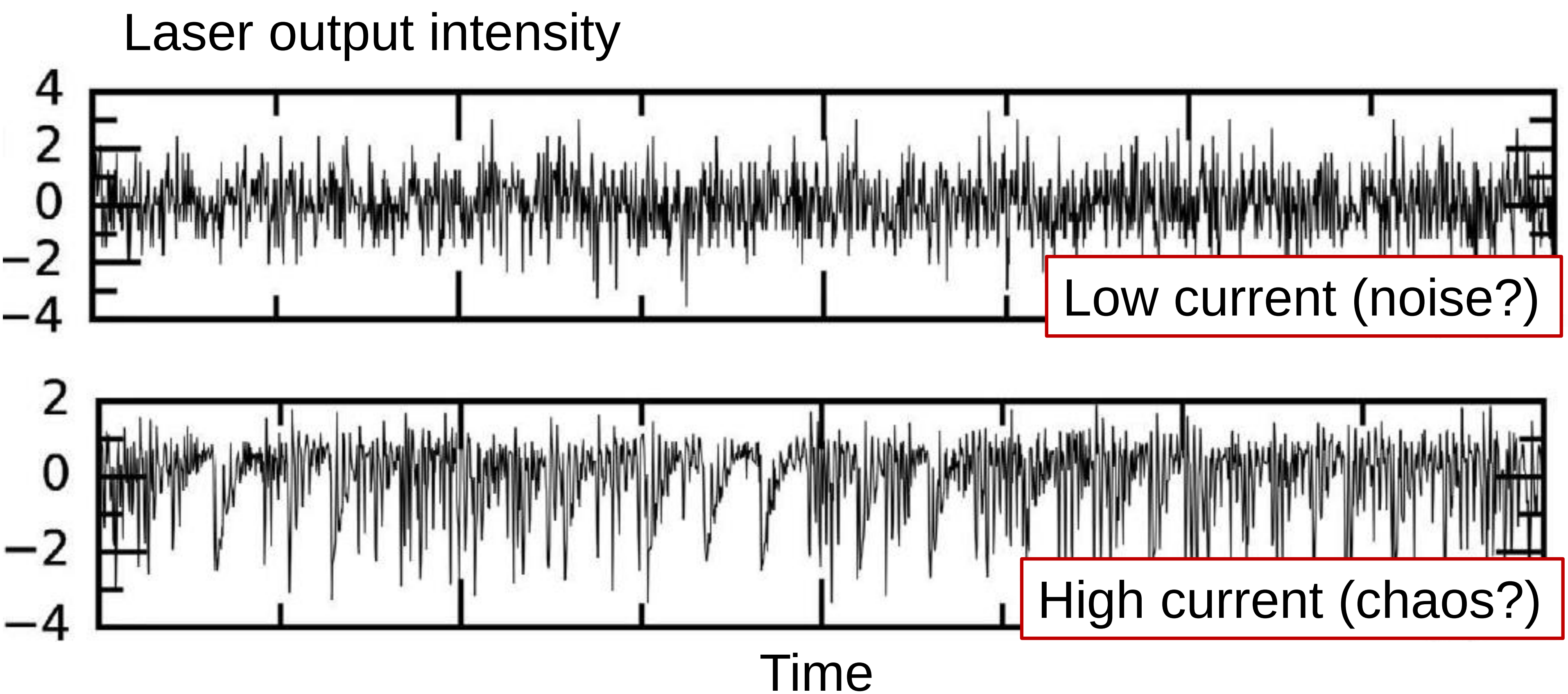
In complex systems dynamical transitions are difficult to identify and to characterize.

Example: laser with time delayed optical feedback





Video: [how complex optical signals emerge from noisy fluctuations]



Can differences be quantified? With what reliability?

Are weather extremes becoming more frequent?
more extreme?

Physics Today, Sep. 2017

ECMWF

Credit: Richard Williams, North Wales, UK

Strong need of reliable data analysis tools

Outline

- **Introduction**
  - Historical developments: from dynamical systems to complex systems

- **Univariate analysis**
  - Methods to extract information from a time series.
  - Applications to climate data.

- **Bivariate analysis**
  - Extracting information from two time series.
  - Correlation, directionality and causality.
  - Applications to climate data.

- **Multivariate analysis**
  - Many time series: complex networks.
  - Network characterization and analysis.
  - Climate networks.

Introduction:

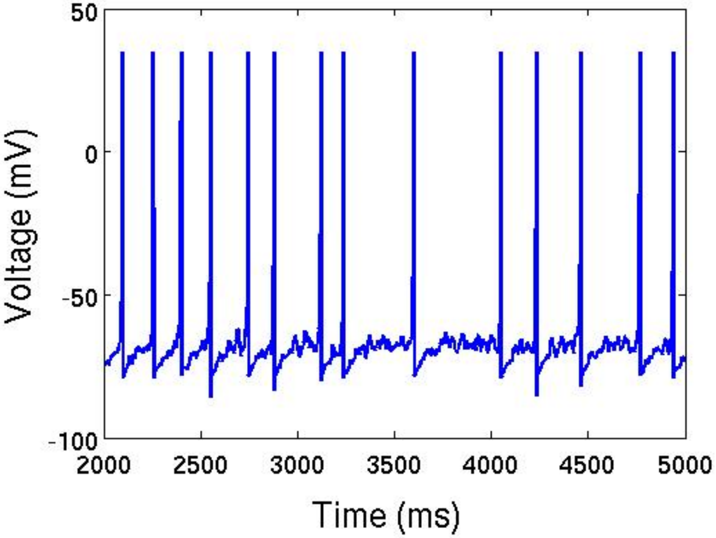
From dynamical systems to complex systems

Time Series Analysis: what is this about?

Optical spikes

Time (μs)

Neuronal spikes



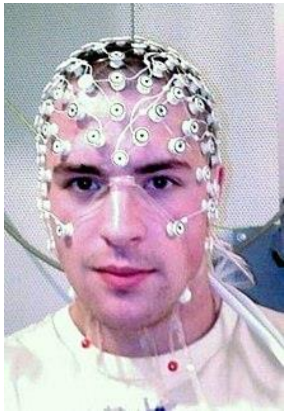
- Similar dynamical systems generate these signals?
- Ok, very different dynamical systems, but similar statistical properties?

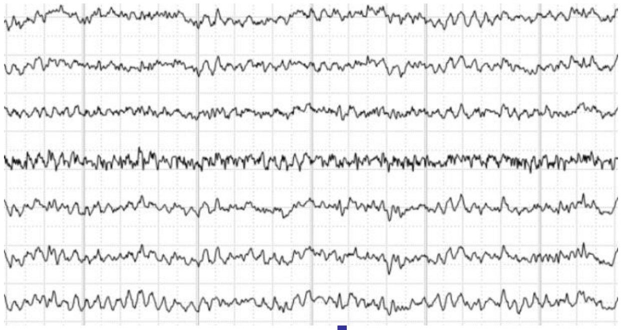
Time Series Analysis: main goal

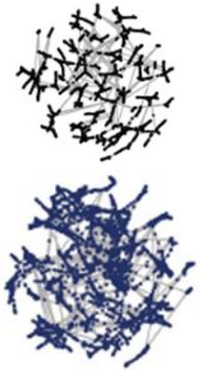
- Extract information from a time series $\{x_1, x_2, \dots x_N\}$.

- What for?
  - Classification
  - Prediction
  - Model verification & identification
  - Parameter estimation (assuming we have a good model).

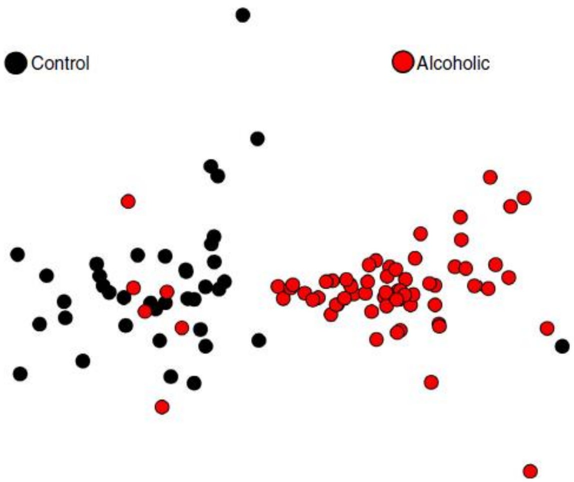
Classification: control vs alcoholic subjects



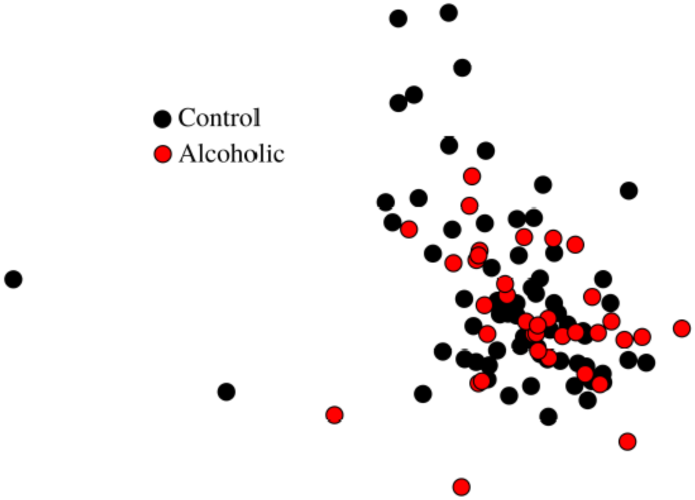




Dissimilarity measure

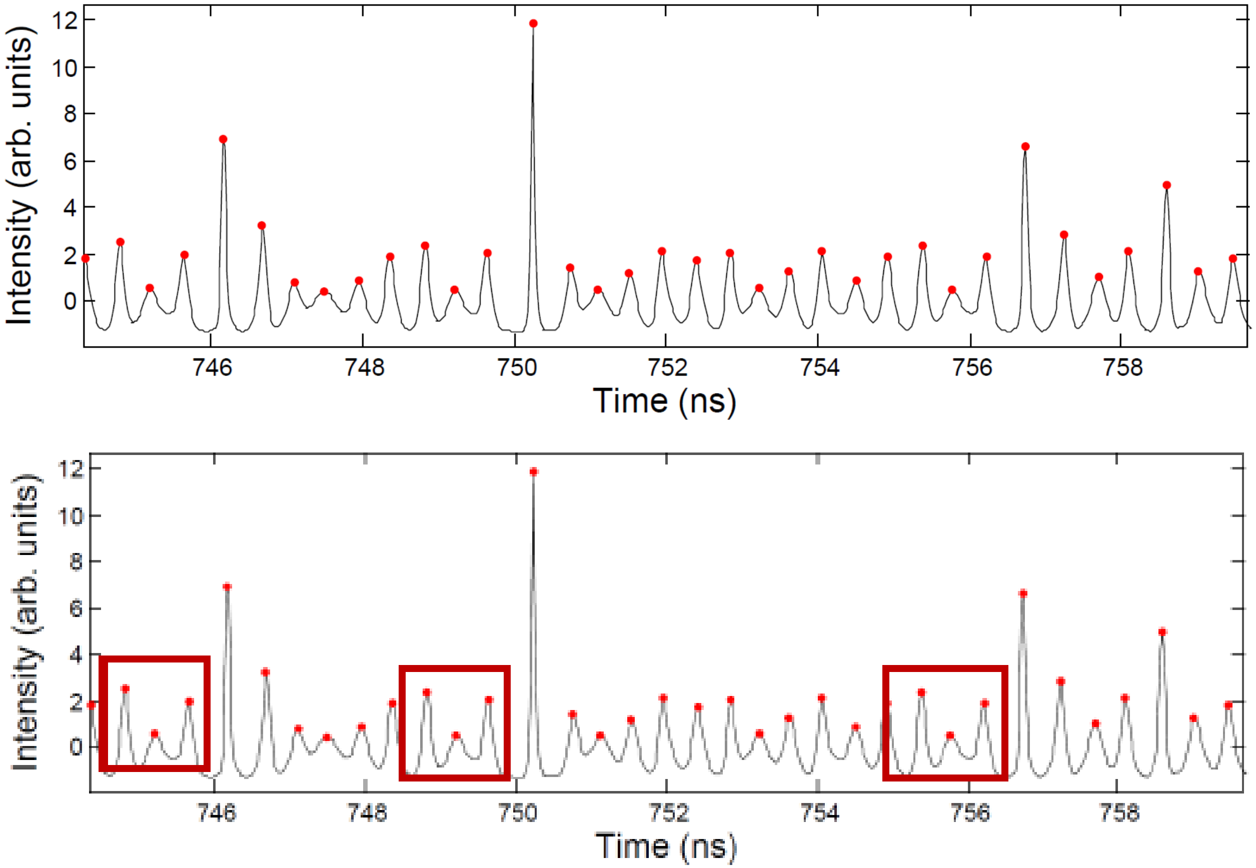


Hamming distance

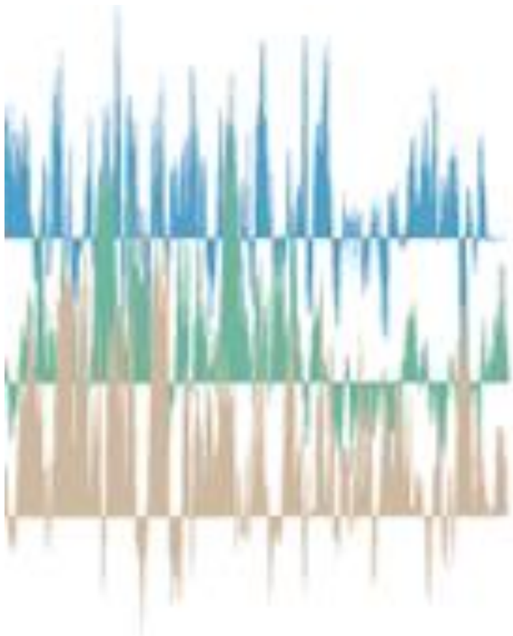


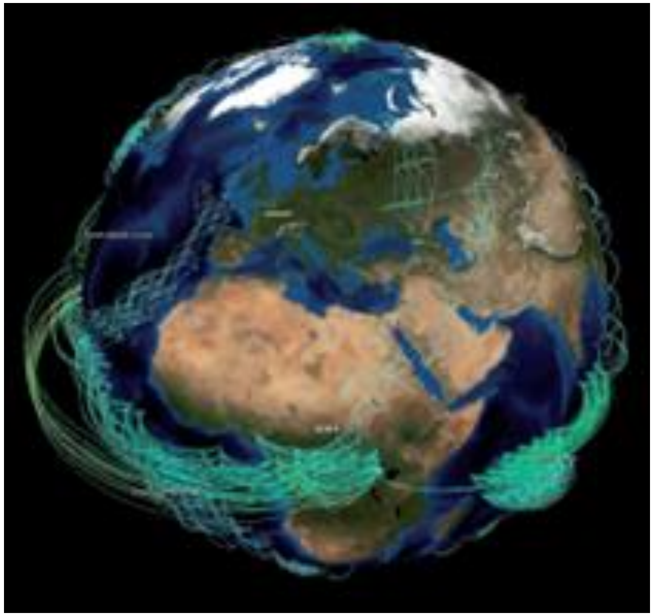
T. A. Schieber et al, Nat. Comm. 8:13928 (2017).

Prediction of extremes: Ultra-intense light pulses



Inferring underlying interactions



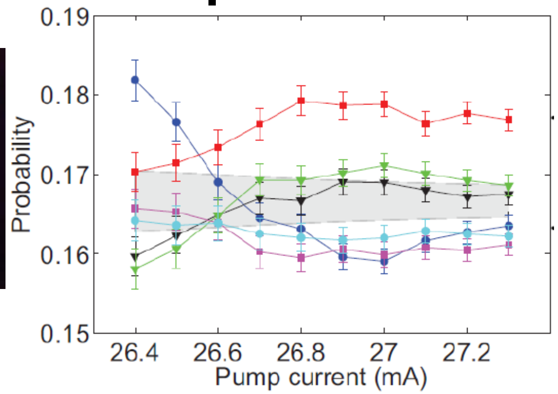


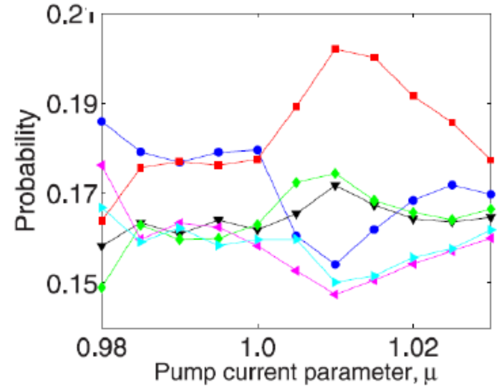
Surface Air Temperature <u>Anomalies</u> in different geographical regions

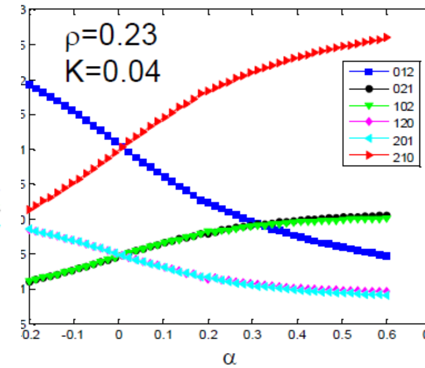
Donges et al, Chaos 2015

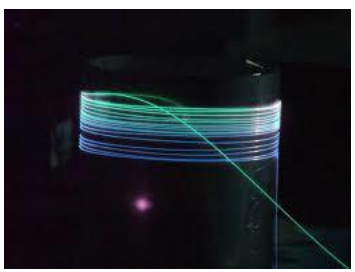
Model identification, parameter estimation

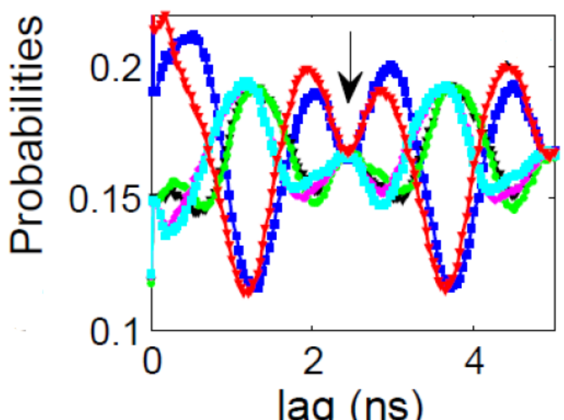
Empirical data | Known model | Minimal model











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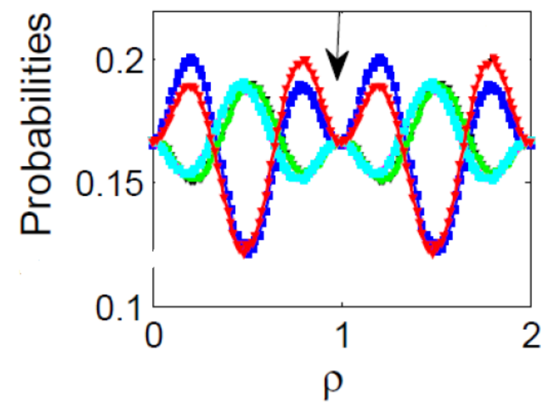
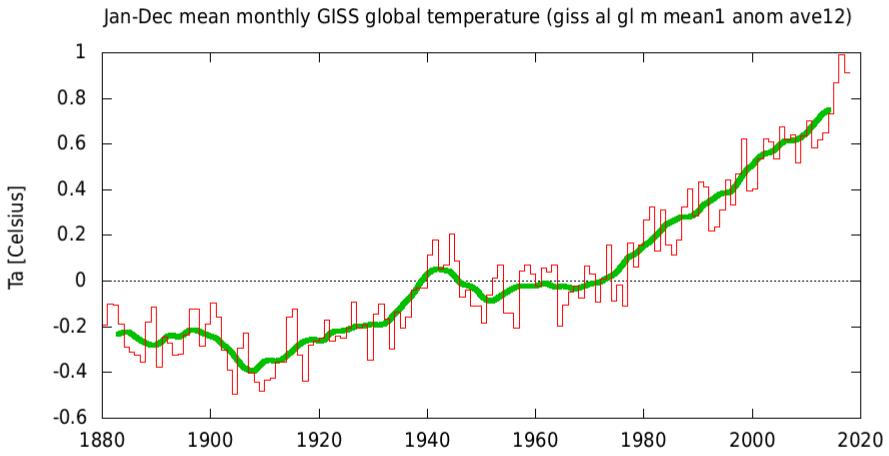


And much more, so let’s begin!

Aragoneses et al, Sci. Rep. 4, 4696 (2014)
Carpi and Masoller, Phys. Rev. A 97, 023842 (2018)

Methods

- Many methods have been developed to extract information from a time series.
- The method to be used depends on the characteristics of the data
  - Length of the time series;
  - Stationarity;
  - Level of noise;
  - Temporal resolution;
  - etc.



- **Different methods provide complementary information.**

Where the data comes from?

- Modeling assumptions about the **type of dynamical system** that generates the data:
  - Stochastic or deterministic?
  - Regular or chaotic or “complex”?
  - Stationary or non-stationary? Time-varying parameters?
  - Low or high dimensional?
  - Spatial variable? Hidden variables?
  - Time delays?
  - Etc.
- Brief historical tour: from **dynamical systems** to **complex systems.**

First studies of dynamical systems

- Mid-1600s: Ordinary differential equations (ODEs)

- **Isaac Newton**: studied planetary orbits and solved analytically the “two-body” problem (earth around the sun).

- Since then: a lot of effort for solving the “three-body” problem (earth-sun-moon) – Impossible.

Late 1800s

- **Henri Poincare** (French mathematician).

  Instead of asking "*which are the exact positions of planets (trajectories)*?"

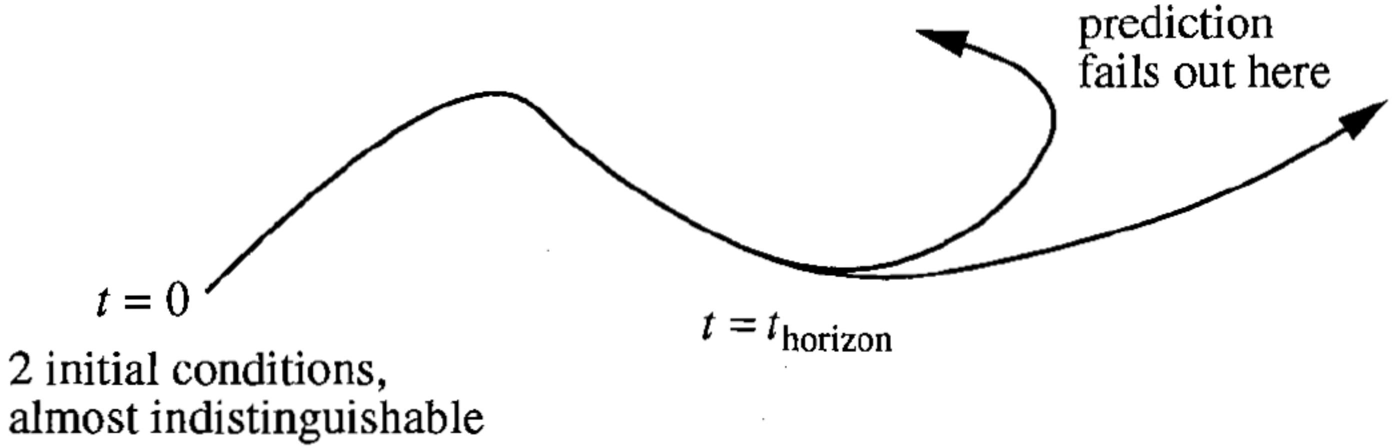
  he asked: "*is the solar system* ***stable*** *for ever, or will planets eventually run away*?"

- He developed a **geometrical** approach to solve the problem.

- Introduced the concept of "phase space".

- He also had an <u>intuition</u> of the possibility of **chaos**

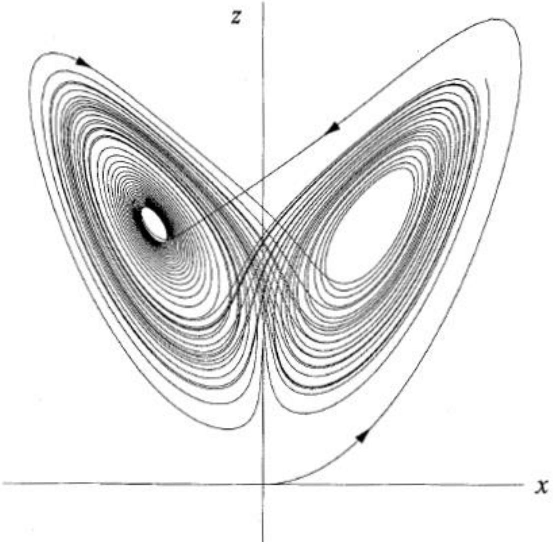
Poincare: "The evolution of a ***deterministic*** *system can be aperiodic, unpredictable, and strongly depends on the initial conditions"*



Deterministic system: the initial conditions fully determine the future state. **There is no randomness but the system can be unpredictable**.

1950s: First simulations

- Computes allowed to experiment with equations.
- Huge advance of the field of “*Dynamical Systems*”.
- 1960s: **Eduard Lorentz** (American mathematician and meteorologist at MIT): simple model of convection rolls in the atmosphere.
- **Chaotic** motion.



Lyapunov exponents

In the late 1800s **Aleksandr Lyapunov** (Russian mathematician) developed the (linear) stability theory of a dynamical system.

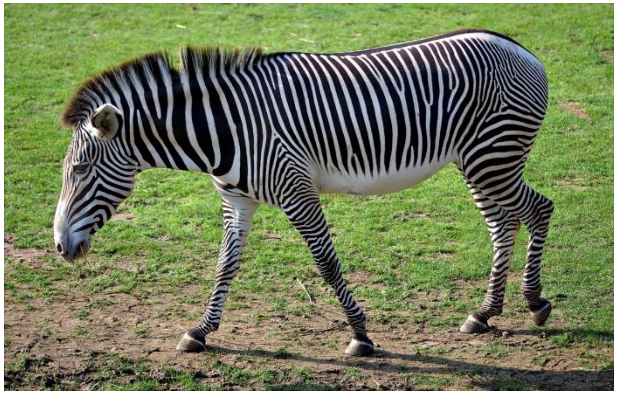
- The **Lyapunov exponent (LE)**: characterizes the rate of separation of infinitesimally close trajectories.

$$|\delta \mathbf{Z}(t)| \approx e^{\lambda t} |\delta \mathbf{Z}_0|$$

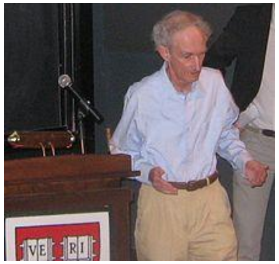
- The rate of separation can be different for different orientations of the initial separation vector → there is a spectrum of Lyapunov exponents; the number of LEs is equal to the dimension of the phase space.
- The largest LE quantifies the system’s predictability.
- More latter on how to compute LEs of real-world signals.

Order within chaos and self-organization

- **Ilya Prigogine** (Belgium, born in Moscow, Nobel Prize in Chemistry 1977)
- Thermodynamic systems far from equilibrium.
- Discovered that, in chemical systems, the interplay of (external) **input of energy** and **dissipation** can lead to “self-organized” patterns.



The 1970s



- **Robert May** (Australian, 1936): population biology
- "Simple mathematical models with very complicated dynamics“, *Nature* (1976).

$$x_{t+1} = f(x_t)$$ Example: $$f(x) = r\, x(1-x)$$

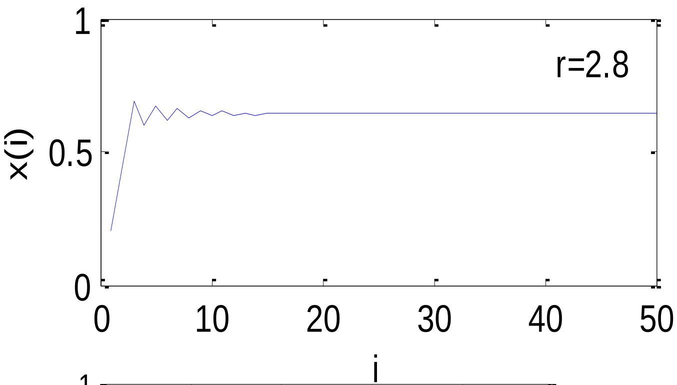
- Difference equations ("iterated maps”), even though simple and deterministic, can exhibit different types of dynamical behaviors, from **stable points**, to a bifurcating hierarchy of **stable cycles**, to **apparently random fluctuations**.

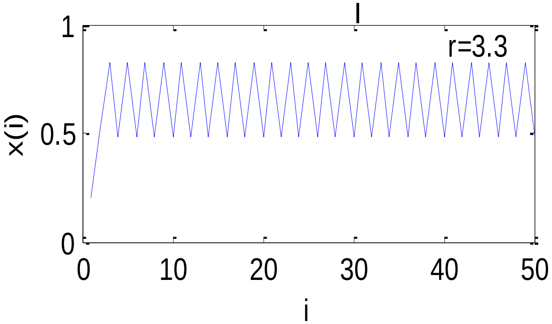
The logistic map

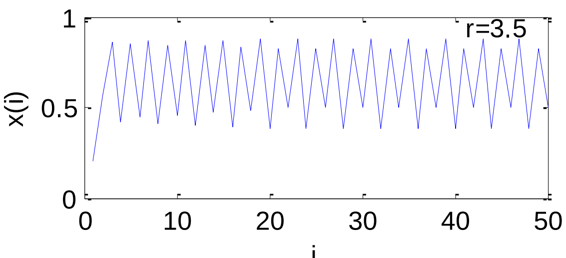
$$x(i+1) = r\,x(i)[1 - x(i)]$$

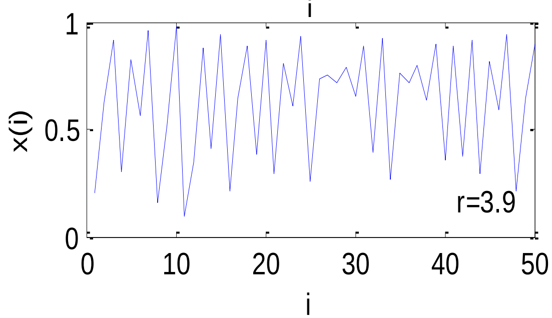
r=2.8, Initial condition: x(1) = 0.2

Transient relaxation → long-term stability



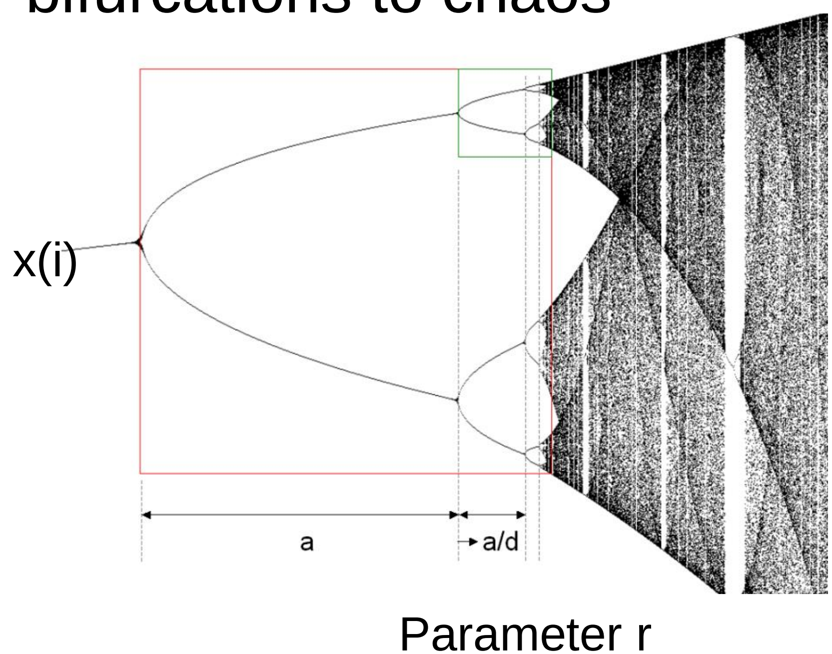






Transient dynamics → stationary oscillations (regular or irregular)

“period-doubling” bifurcations to chaos



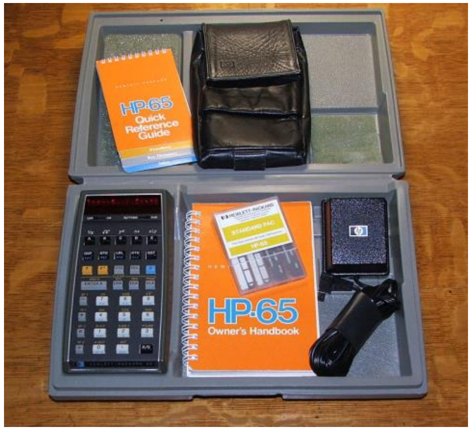
Universal route to chaos

- In 1975, **Mitchell Feigenbaum** (American mathematical physicist), using a small HP-65 calculator, discovered the scaling law of the bifurcation points

$$\lim_{n\to\infty} \frac{r_{n-1} - r_{n-2}}{r_n - r_{n-1}} = 4.6692...$$

- Then, he showed that the same behavior, with the same mathematical constant, occurs within a wide class of functions, prior to the onset of chaos (**universality**).

Very different systems (in chemistry, biology, physics, etc.) go to chaos in the same way, quantitatively.



HP-65 calculator: the first magnetic card-programmable handheld calculator

The late 1970s

- **Benoit Mandelbrot** (Polish-born, French and American mathematician 1924-2010): "self-similarity" and **fractal objects:**

  each part of the object is like the whole object but smaller.

- Because of his access to IBM's computers, Mandelbrot was one of the first to use **computer graphics** to create and display fractal geometric images.

Fractal objects

- Are characterized by a “fractal” dimension that measures roughness.

Broccoli
D=2.66

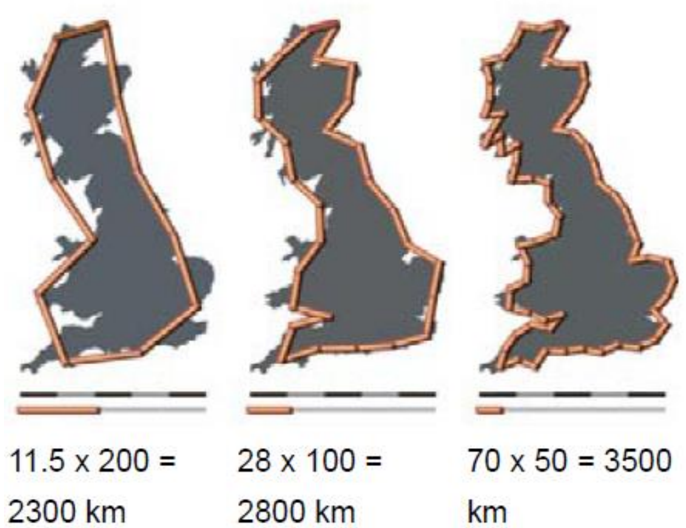
Human lung
D=2.97

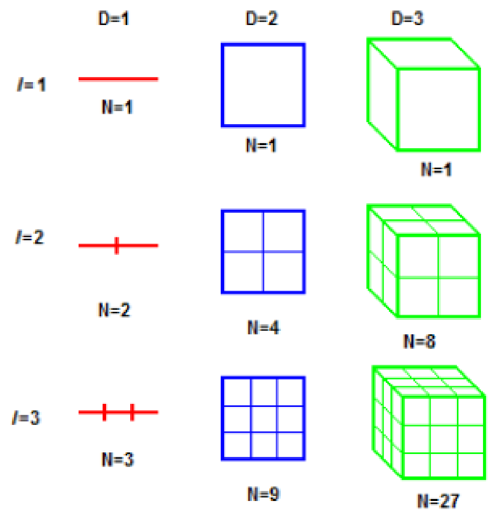
Coastline of Ireland
D=1.22

Video: http://www.ted.com/talks/benoit_mandelbrot_fractals_the_art_of_roughness#t-149180

Fractal dimension

- Example: the fractal dimension of a coastline quantifies how the number of scaled measuring sticks required to measure the coastline changes with the scale applied to the stick.

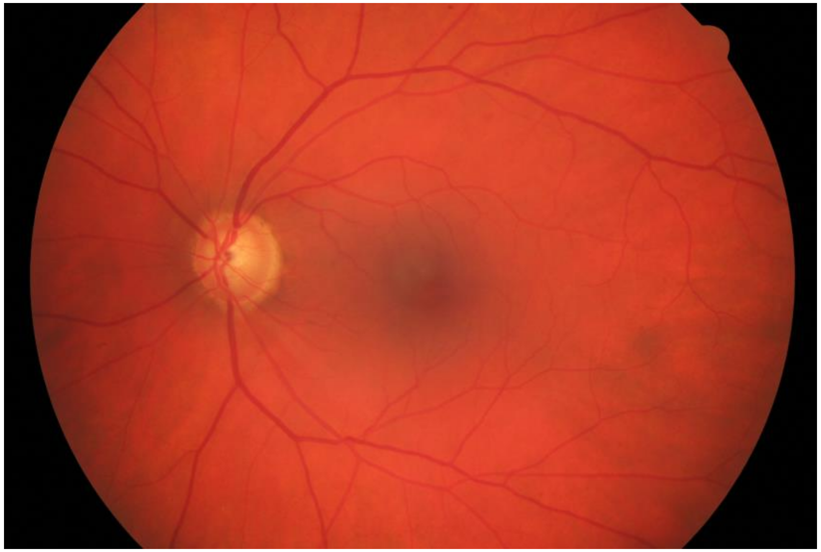




Source: wikipedia

- Fractal dimension: $N \propto \epsilon^{-D} \longrightarrow D_0 = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log \frac{1}{\epsilon}}$

Example of application of fractal analysis: distinguishing between diabetic retinopathy and normal patients



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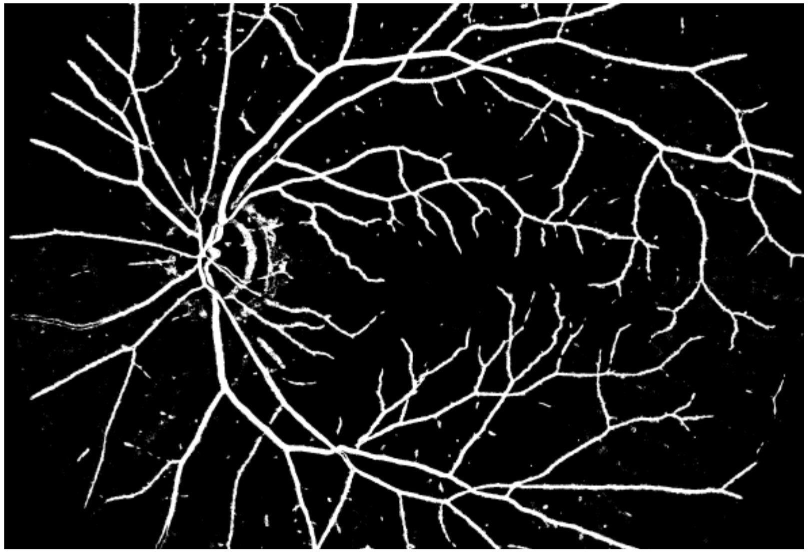


Source: Pablo Amil, UPC

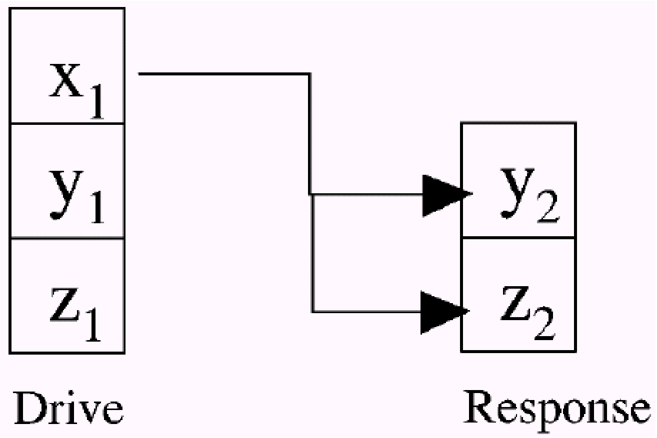
The fractal dimension of the blood vessels

- in the normal human retina is ≈1.7
- tends to increase with the level of diabetic retinopathy
- varies considerably depending on the image quality and the technique used for measuring the fractal dimension

The 1990s: synchronization of chaotic systems

Pecora and Carroll, PRL 1990

Unidirectional coupling of two Lorenz systems: the '*x*' variable of the response system is **replaced** by the '*x*' variable of the drive system.



$$t \to \infty \quad |y_2 - y_1| \to 0, \ |z_2 - z_1| \to 0$$

First observation of synchronization: *entrainment* of pendulum clocks

In mid-1600s **Christiaan Huygens** (Dutch mathematician) noticed that two pendulum clocks mounted on a common board synchronized with their pendulums swinging in opposite directions (in-phase also possible).

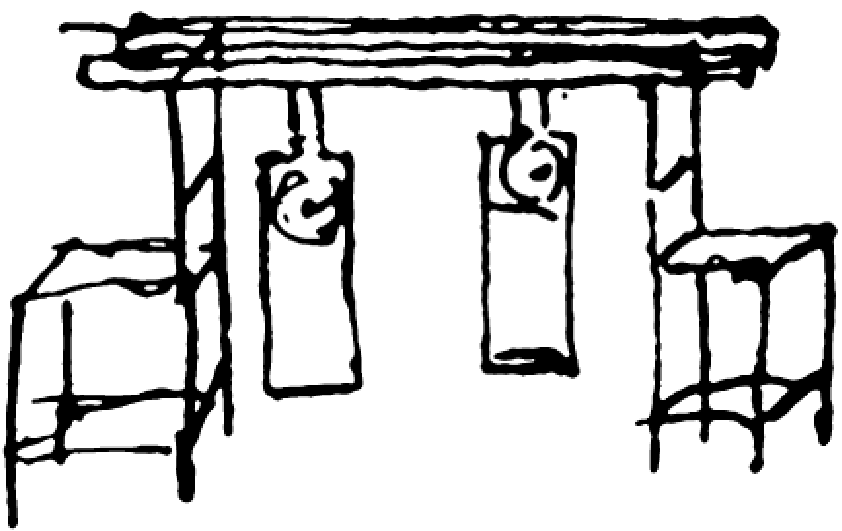
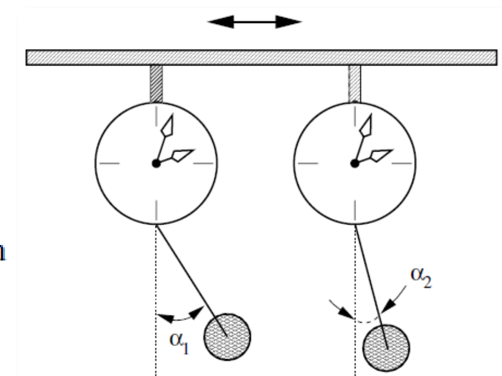


Figure 1.2. Original drawing of Christiaan Huygens illustrating his experiments with two pendulum clocks placed on a common support.



http://www.youtube.com/watch?v=izy4a5erom8

Different types of synchronization

$$dx_1 / dt = F(x_1)$$

$$dx_2 / dt = F(x_2) + \alpha E(x_1 - x_2)$$

- <u>Complete</u>: $x_1(t) = x_2(t)$ (identical systems)
- <u>Phase</u>: the phases of the oscillations synchronize, but the amplitudes are not.
- <u>Lag</u>: $x_1(t+\tau) = x_2(t)$
- <u>Generalized</u>: $x_2(t) = f(x_1(t))$ (*f* can depend on the strength of the coupling)

A lot of work is being devoted to develop methods able to detect synchronization in real-world signals.

Synchronization of a large number of coupled oscillators

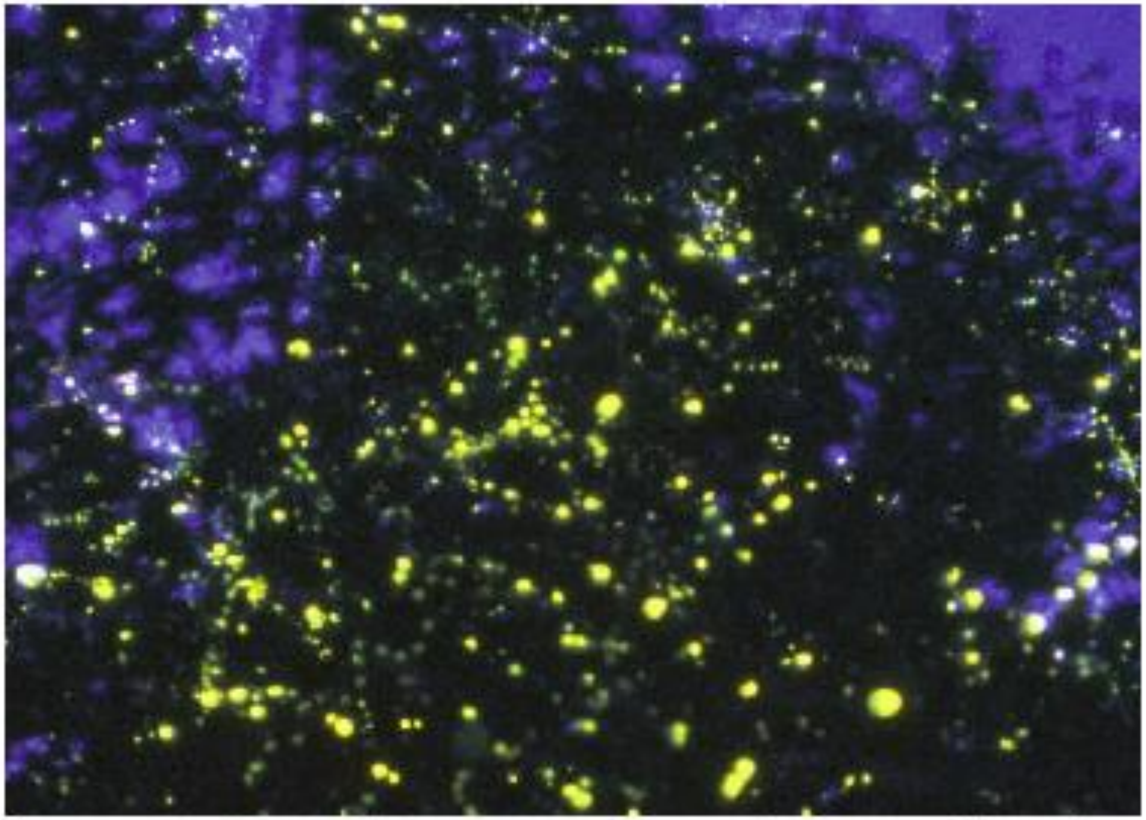
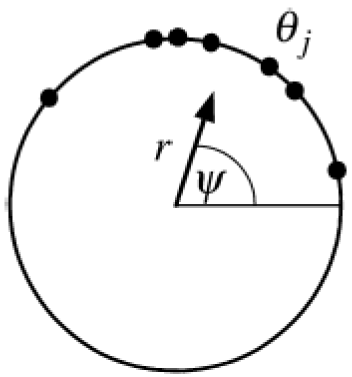


Figure 1 | Fireflies, fireflies burning bright. In the forests of the night, certain species of firefly flash in perfect synchrony — here *Pteroptyx malaccae* in a mangrove apple tree in Malaysia. Kaka *et al.*[2] and Mancoff *et al.*[3] show that the same principle can be applied to oscillators at the nanoscale.

Kuramoto model

(Japanese physicist, 1975)

Model of **all-to-all** coupled **phase oscillators**.



$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) + \xi_i, \qquad i = 1\ldots N$$

K = coupling strength, ξ_i = stochastic term (noise)

Describes the emergence of collective behavior

How to quantify?

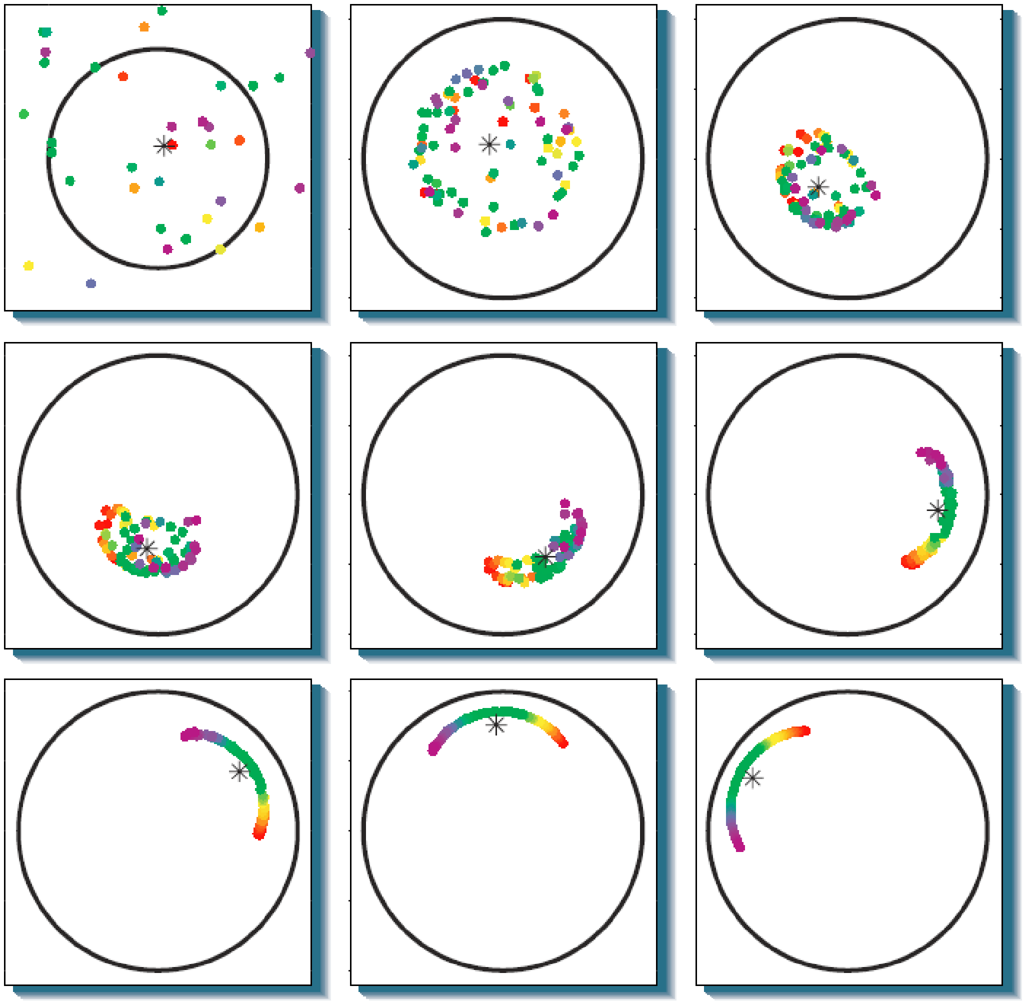
With the **order parameter**:

$$re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

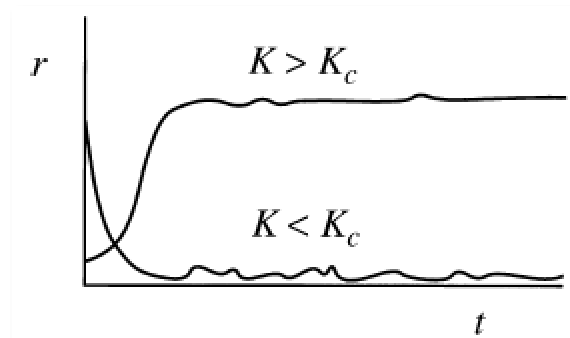
r =0 incoherent state (oscillators scattered in the unit circle)

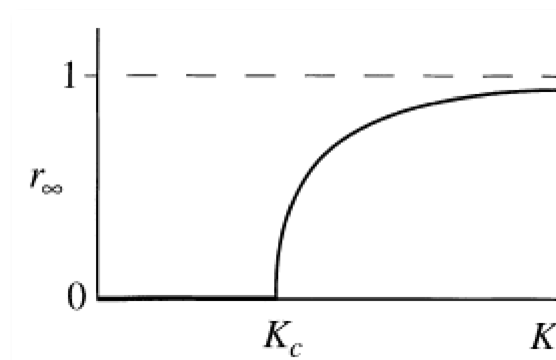
r =1 all oscillators are in phase ($\theta_i = \theta_j \ \forall$ i,j)

Synchronization transition as the coupling strength increases



Strogatz and others, late 90’





Strogatz, Nature 2001

Video: https://www.ted.com/talks/steven_strogatz_on_sync

End of 90's - present

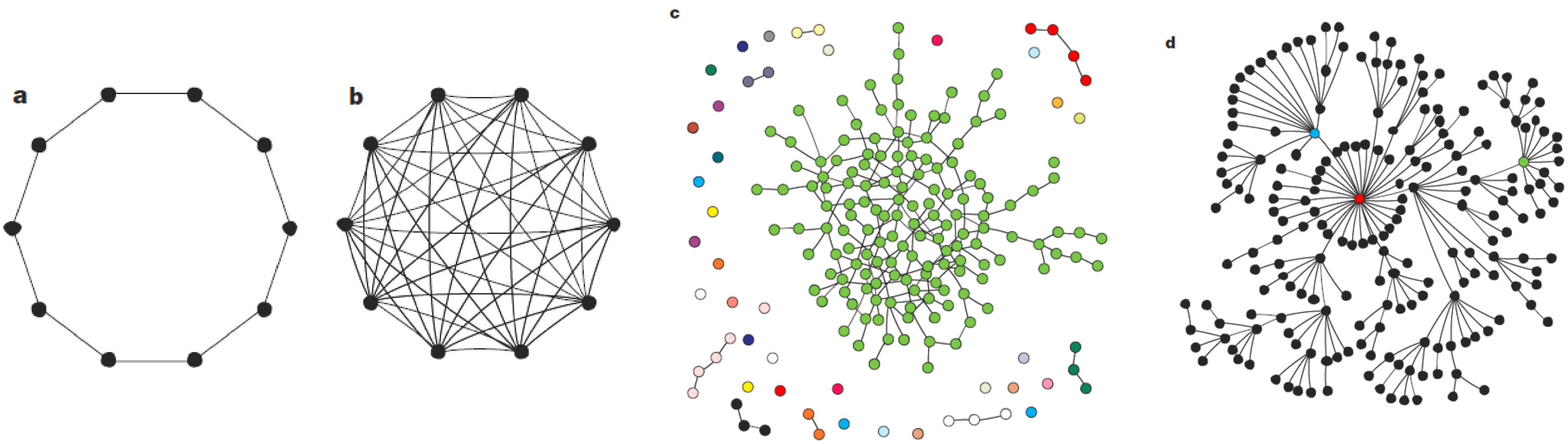
- Interest moves from <u>chaotic systems</u> to <u>complex systems</u> (small vs. very large number of variables).

- Networks (or graphs) of interconnected systems

- **Complexity science**: dynamics of emergent properties
  - Epidemics
  - Rumor spreading
  - Transport networks
  - Financial crises
  - Brain diseases
  - Etc.

Network science

The challenge: to understand how the network **structure** and the **dynamics** (of individual units) determine the collective behavior.

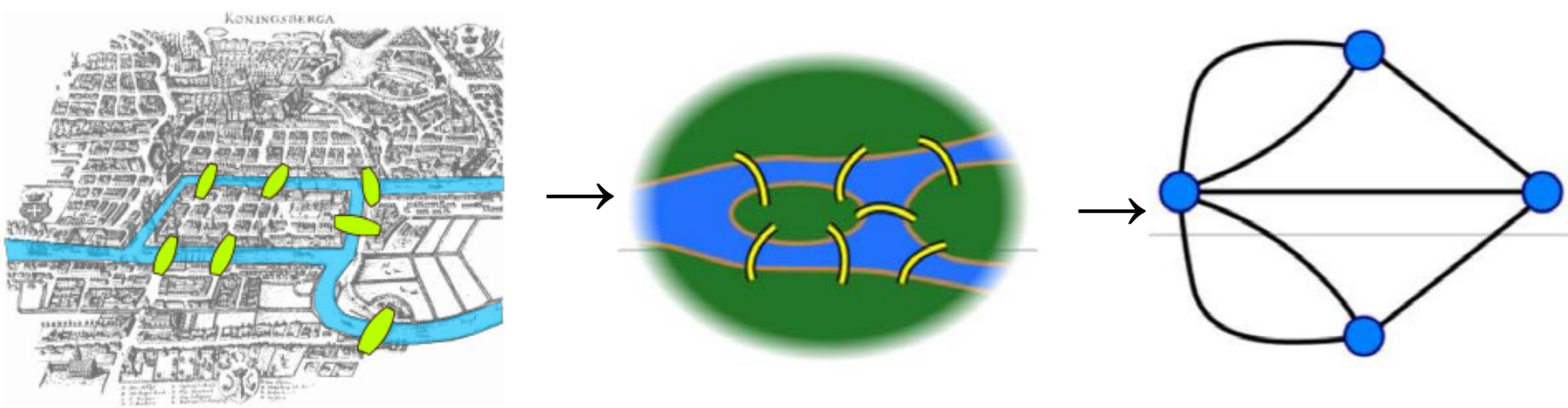


Source: Strogatz
Nature 2001

The start of Graph Theory: The Seven Bridges of Königsberg

(Prussia, now Russia)

- The problem was to devise a walk through the city that would cross each of those bridges once and only once.



Source: Wikipedia

- By considering the number of odd/even links of each “node”, **Leonhard Euler** (Swiss mathematician) demonstrated in 1736 that is impossible.

Summary

- Dynamical systems allow to
  - understand low-dimensional systems,
  - uncover patterns and “order within chaos”,
  - characterize attractors, uncover universal features
- Synchronization: emergent behavior of interacting dynamical systems.
- Complexity and network science: emerging phenomena in large sets of interacting units.
- Complexity science is an interdisciplinary research field with many applications.

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