UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Escola Superior d'Enginyeries Industrial, Aeroespacial i Audiovisual de Terrassa

Nonlinear time series analysis
Master degree in Industrial Engineering
Master degree in Aeronautic Engineering
Course 2020-2021

Introduction

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SCHEDULE OF THE COURSE

Block 0: Matlab

Block 1:
Characterization of time series

- Introduction
- Univariate analysis
- Bivariate analysis
- Multivariate analysis

Block 2:
Data analysis tools

- Machine Learning techniques
- Classification methods
- Control, data assimilation and Kalman Filters

Learning objectives of the first module

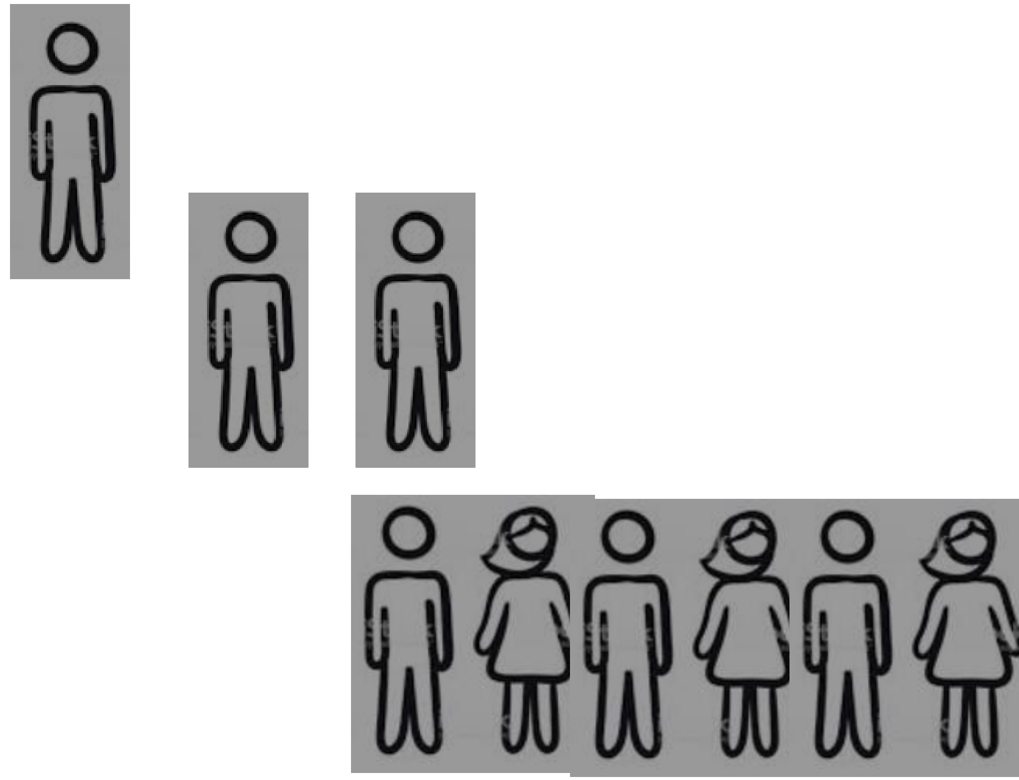
- Learn about the historic background and become familiar with current techniques (linear and nonlinear) for time series analysis.

- Became familiar with techniques for detecting statistical similarities and interdependencies in time series.

- Gain a broad knowledge of data-driven techniques for studying complex systems.

Outline of the first block

- Introduction
- Univariate analysis
- Bivariate analysis
- Multivariate analysis

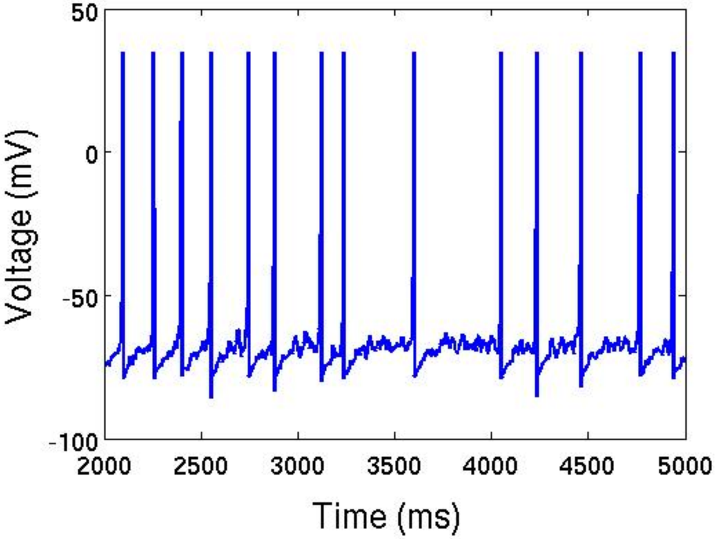


Time Series Analysis: what is it about? $X = \{x_1, x_2, \ldots x_N\}$

Optical spikes

Time (μs)

Neuronal spikes

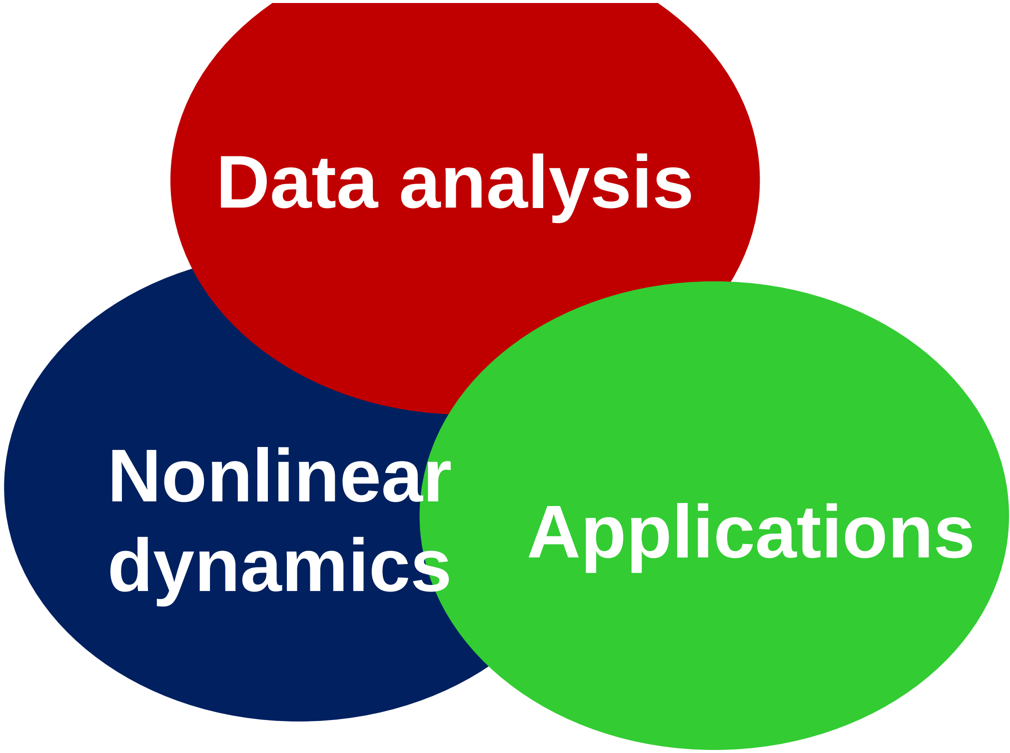


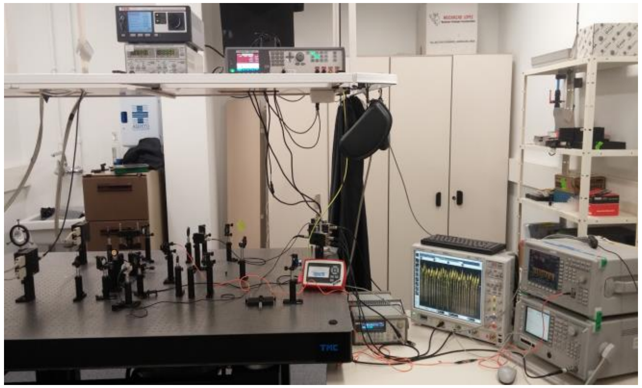
- Similar dynamical systems generate these signals?
- Ok, very different dynamical systems, but maybe similar statistical properties?
- Time series analysis can find "hidden similarities" in very different systems.

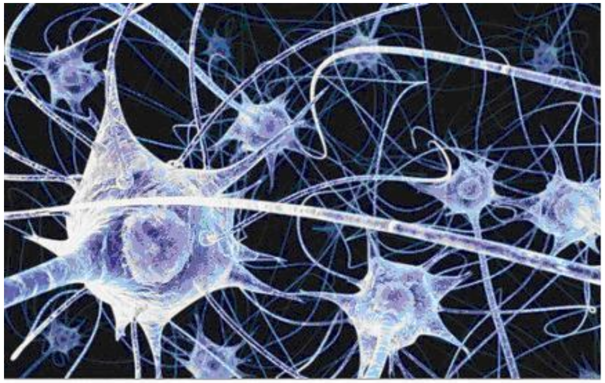
Time series analysis is a highly inter-disciplinary research field

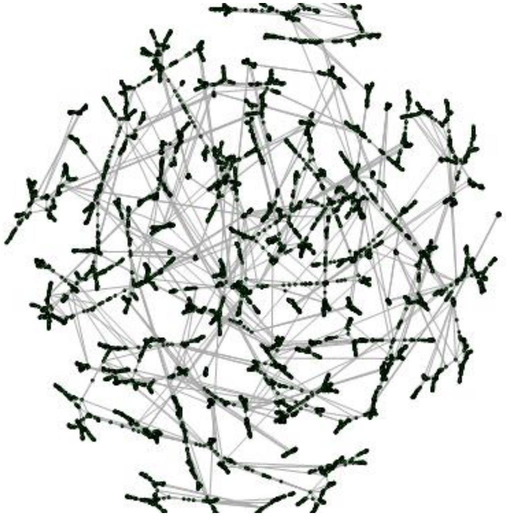
In our lab we work on

- lasers
- neurons
- complex networks
- climate data
- biomedical data



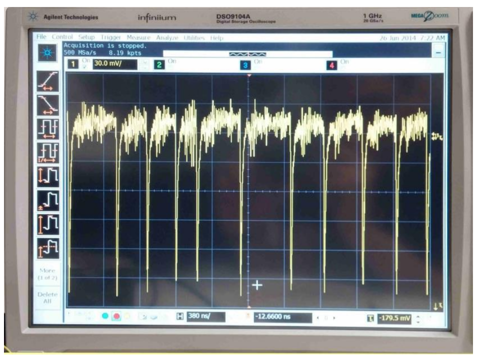




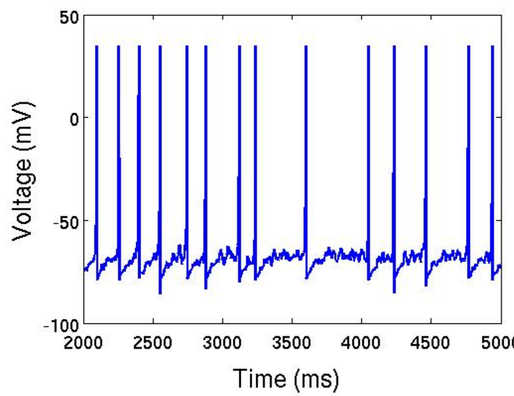


Lasers, neurons, climate, complex systems?

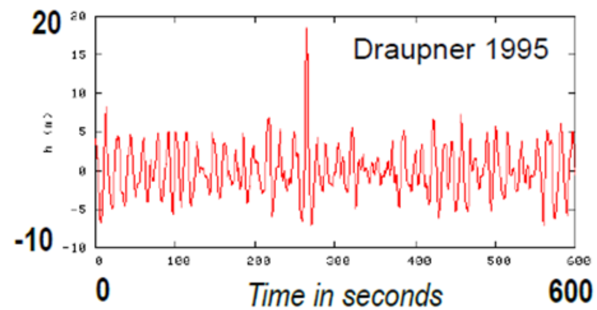
- Lasers allow us to study in a controlled way phenomena that occur in diverse complex systems.
- Laser experiments allow to generate sufficient data to test new methods of data analysis for prediction, classification, etc.



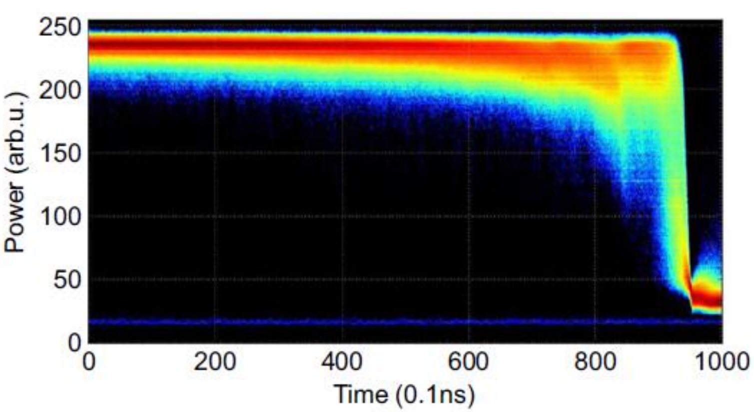
Laser & neuronal spikes



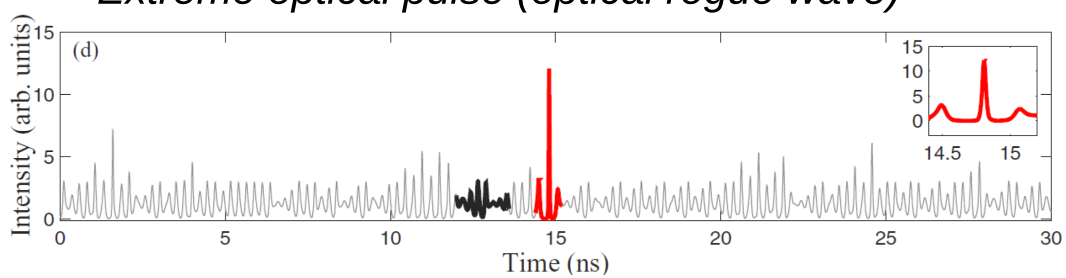
Ocean rogue wave (sea surface elevation in meters)



Abrupt transition

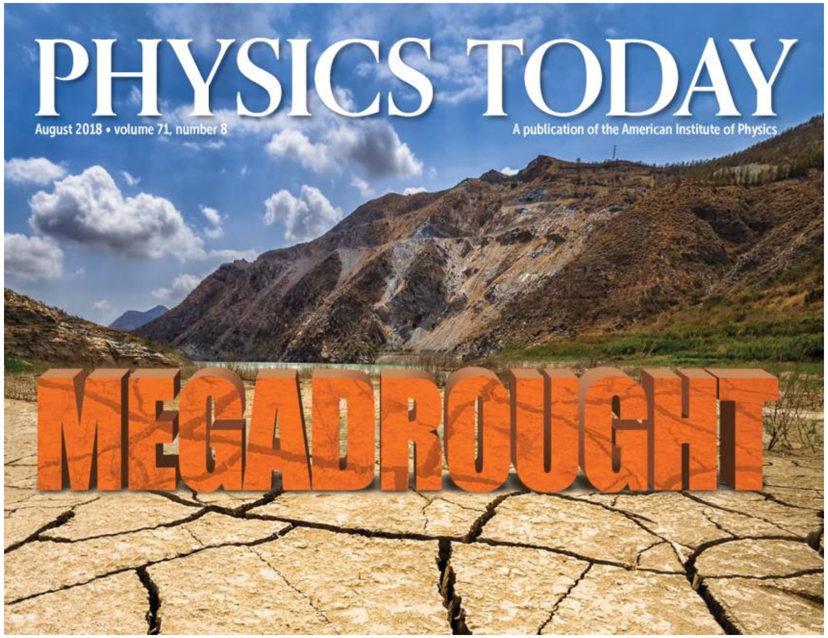


Extreme optical pulse (optical rogue wave)



Research question: how to identify (and predict) transitions between different dynamical regimes?

Are weather extremes becoming more extreme? more frequent?



At home (Blanes, Feb. 2020)

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Science & Environment

Rogue waves occurring less but 'becoming more extreme'

By Rebecca Morelle
Science Correspondent, BBC News

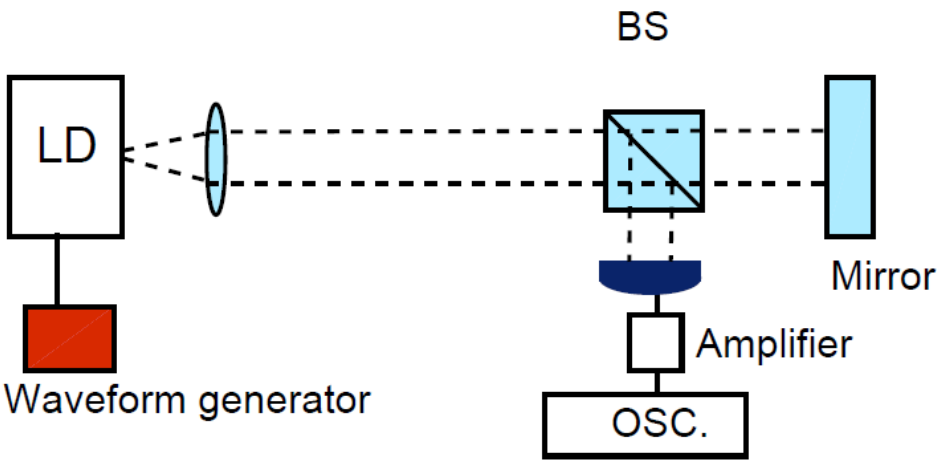
⇒ 21 March 2019   Share

Rogue waves are a growing threat for the global shipping industry

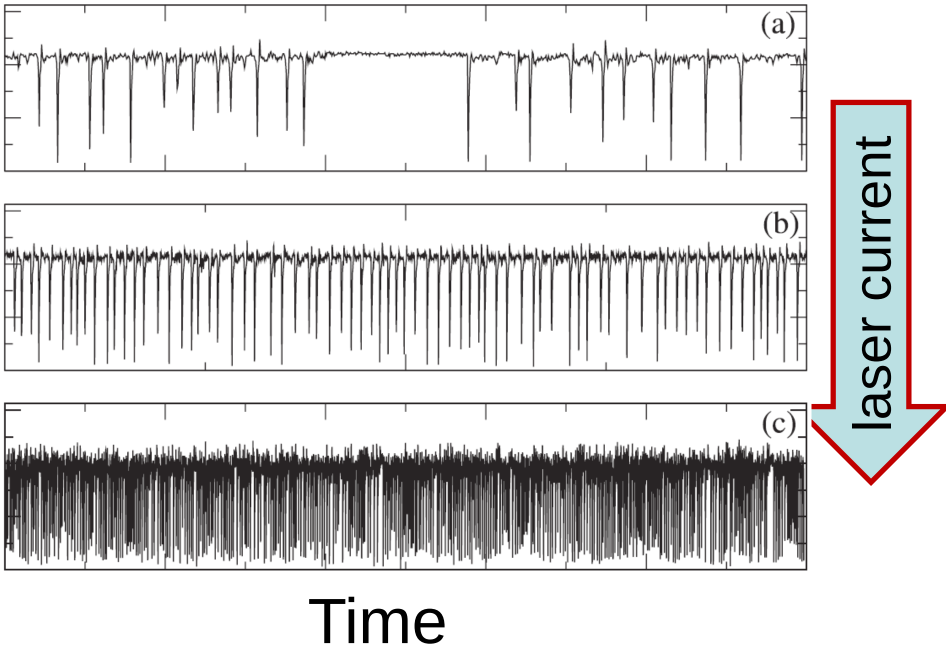
Big risk for traffic and off-shore platforms!

Example of an experimental system where we can study the gradual transition between two dynamical regimes

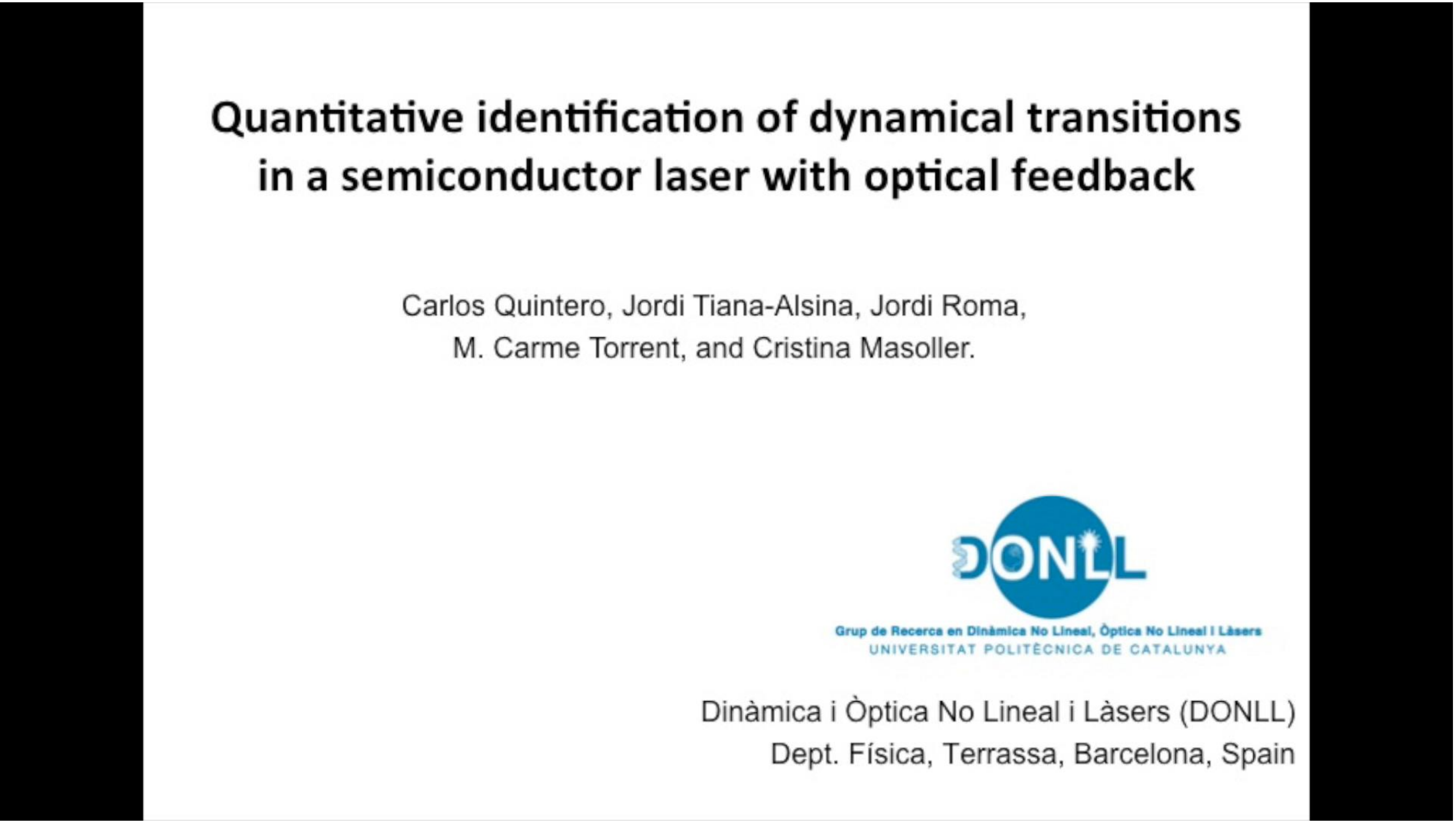
Laser diode (LD) with optical feedback from a mirror



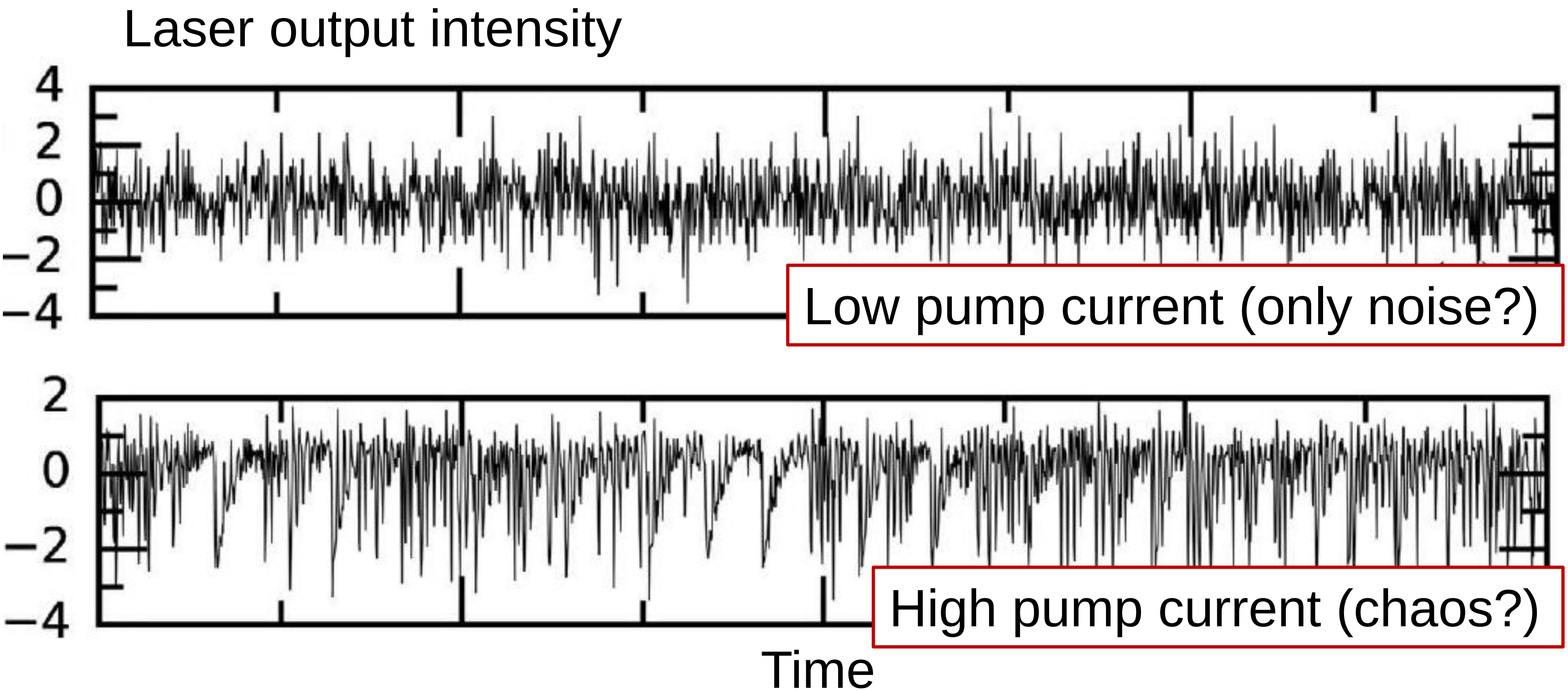
Output intensity recorded in the oscilloscope (osc)



How complex signals emerge from noise?

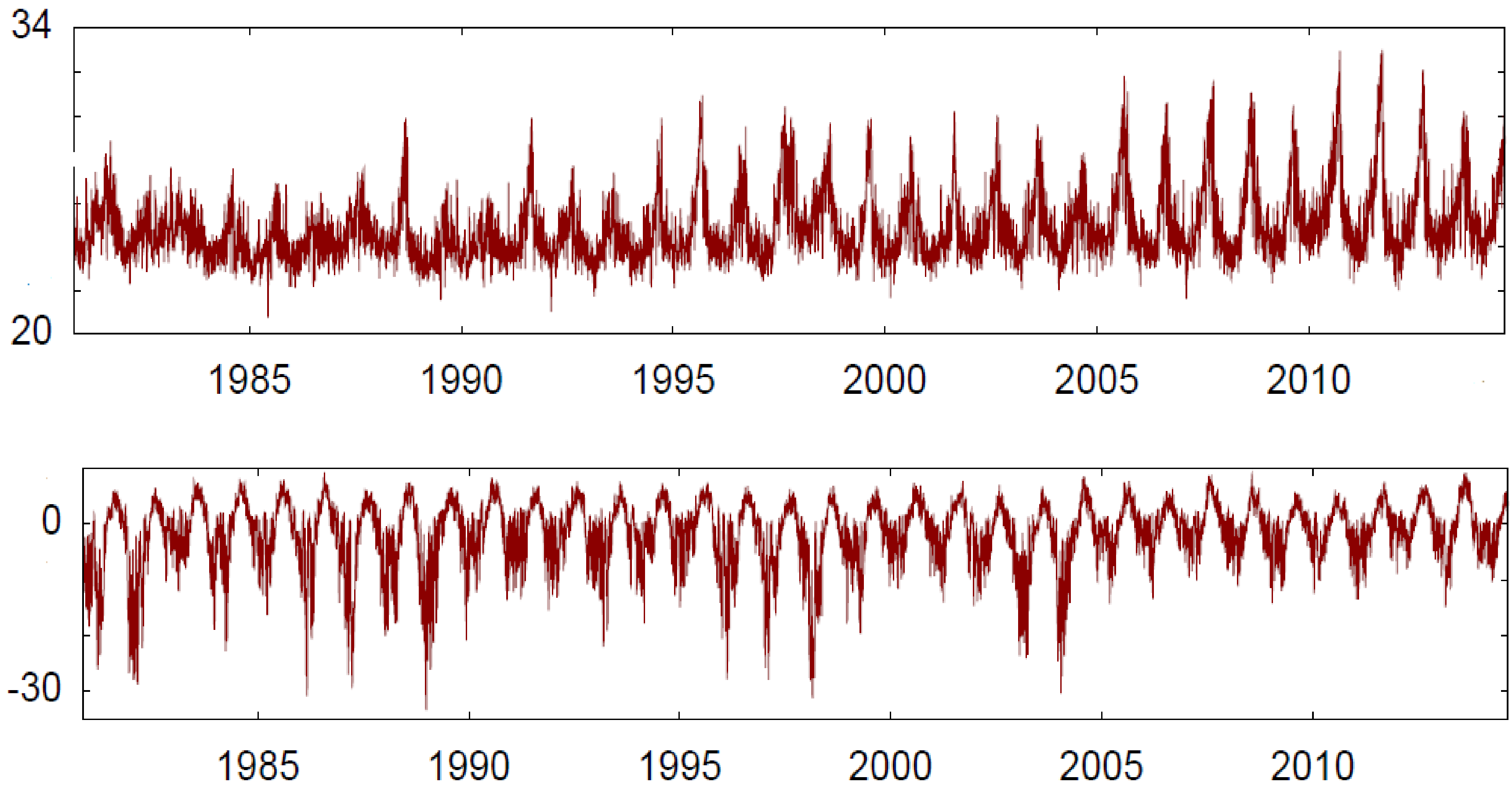


Video: how complex optical signals emerge from noisy fluctuations



Can differences be quantified? With what reliability?

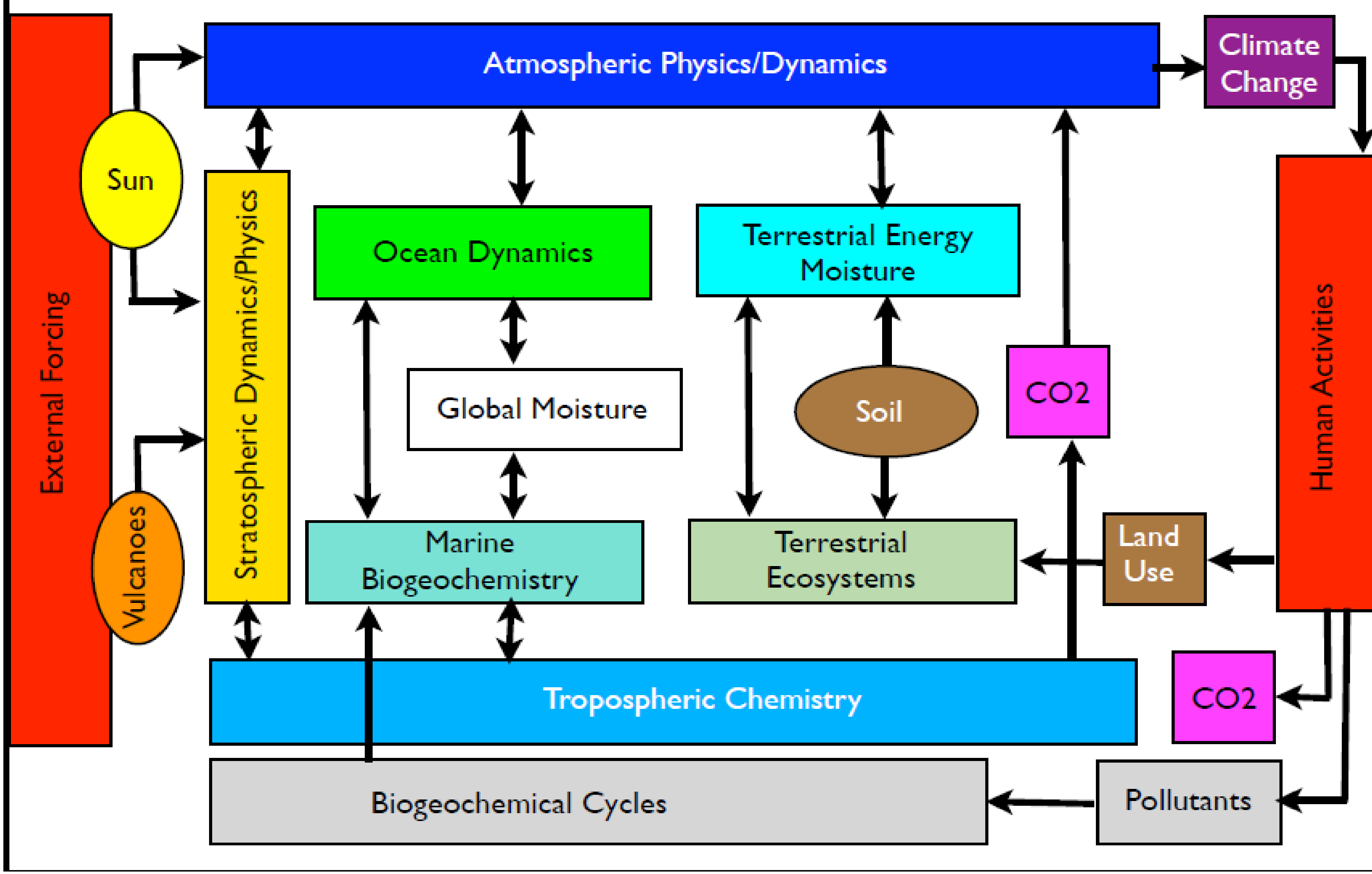
Another example of a gradual variation: Surface Air Temperature (SAT) in two geographical regions



Can changes be quantified? With what reliability?

D. A. Zappala et al., Earth Syst. Dynam. 9, 383 (2018)

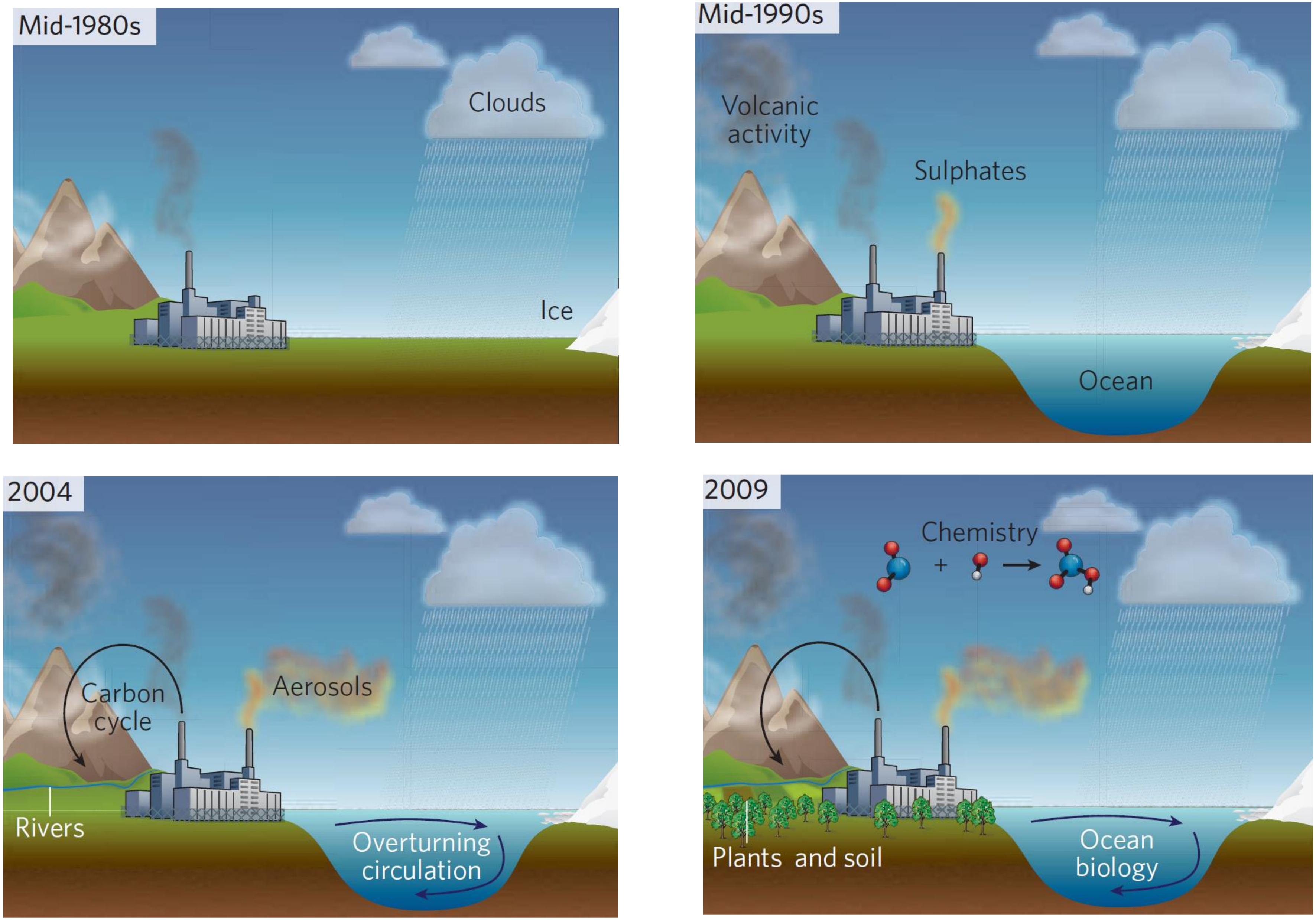
The Climate System is a “complex system”



Courtesy of Henk Dijkstra (Ultrech University)

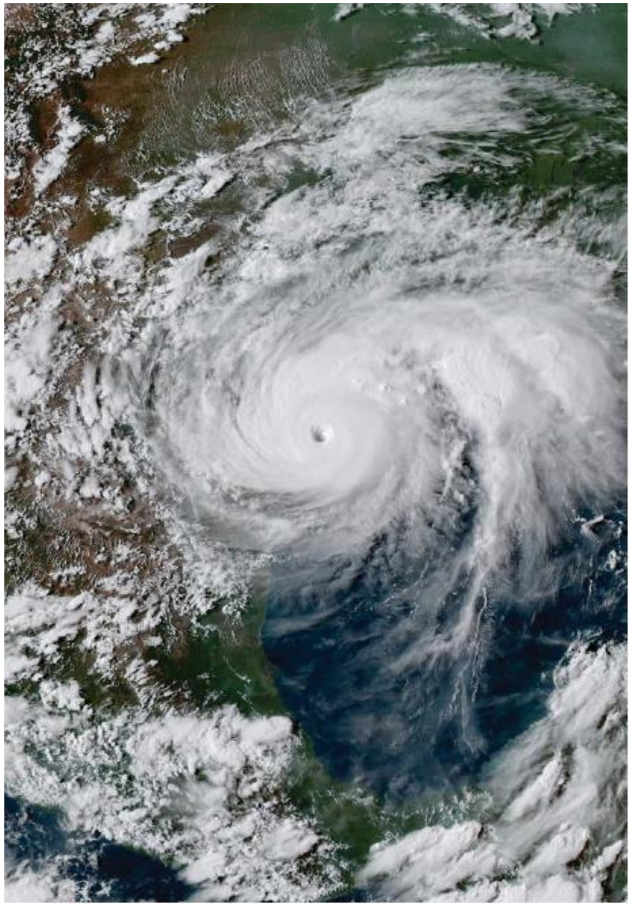
Thanks to advances in computer science, global climate models allow for “very good” weather forecasts

Nature, February 2010



Nature, February 2010

Nowadays: better weather forecasts with artificial intelligence



HURRICANE HARVEY killed 107 people and caused $125 billion in damage when it hit the southeastern US in August 2017.

At NOAA (US National Oceanic and Atmospheric Administration) scientists test AI and other emerging methods to help improve the prediction of severe weather events.
AI algorithms can be designed for forecasting a specific weather feature, such as hail or severe wind.

Physics Today May 2019

But AI and state-of-the-art climate models are not very useful for improving the understanding of our climate.

On the other hand, "over-simplified models" are not useful !

Did you hear about the "spherical cow"?

In early summer, 1996, milk production at a Wisconsin dairy farm was very low. The farmer wrote to the state university, asking help from academia. A multidisciplinary team of professors was assembled, headed by a theoretical physicist, and two weeks of intensive on-site investigation took place. A few weeks later, a physicist phoned the farmer, "I've got the answer," he said, "But it only works when you consider spherical cows in a vacuum. . . ."

Source:https://mirror.uncyc.org/wiki/Spherical_Cows

We need analysis tools that extract information directly from data (empirical or generated with high-dimensional models)

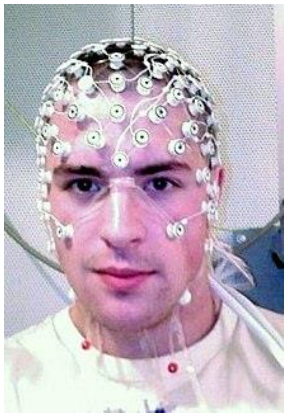
Main goal of Time Series Analysis: to extract information from data

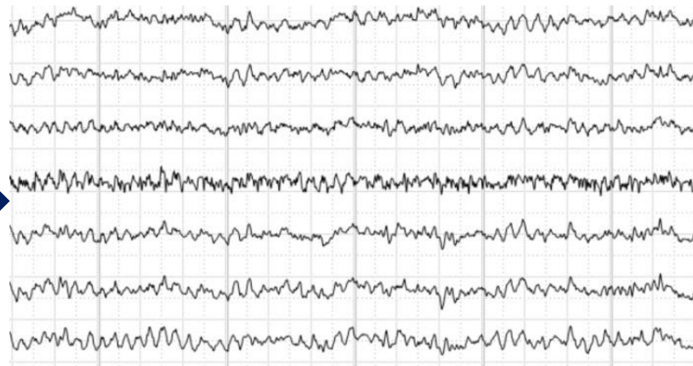
What for?

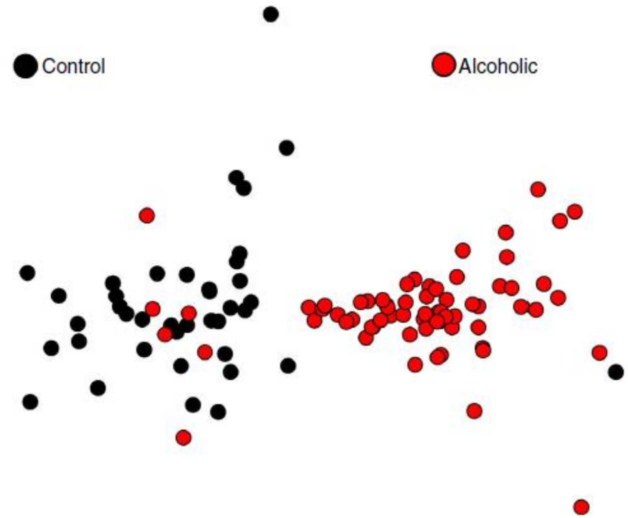
- Classification
- Prediction
- Model verification
- Parameter estimation

Example: analysis of EEG signals

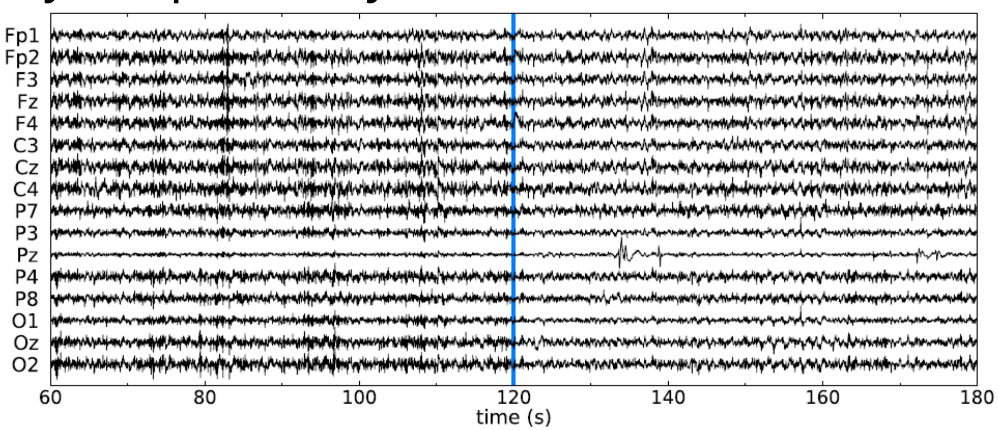
Distinguish alcoholic subjects from control subjects

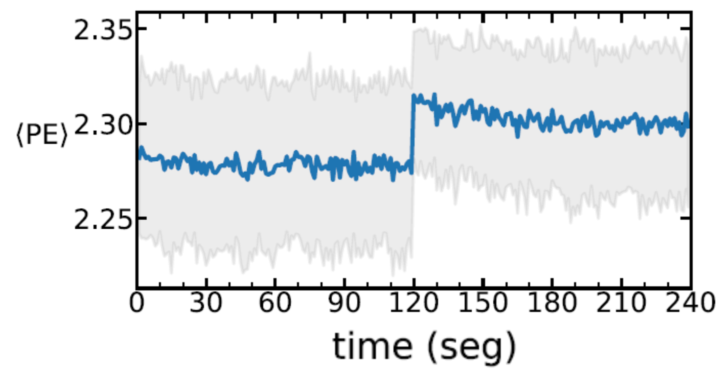






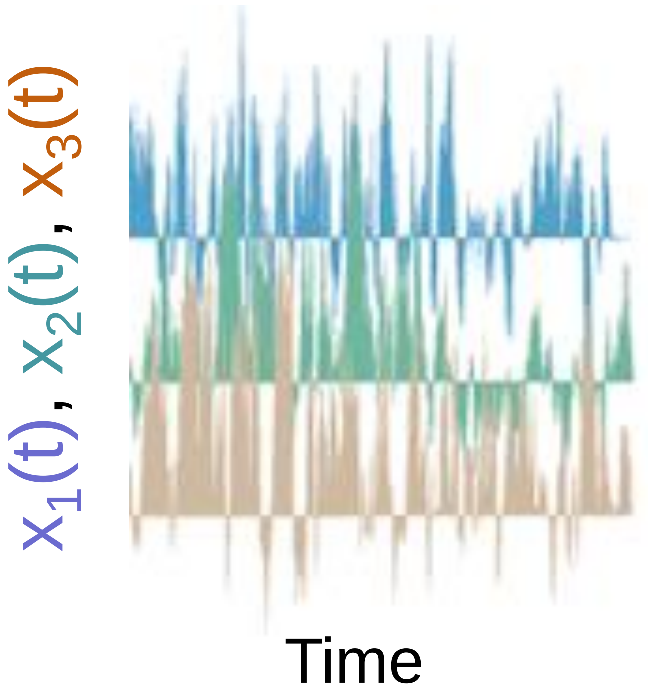
Eyes open or eyes closed?



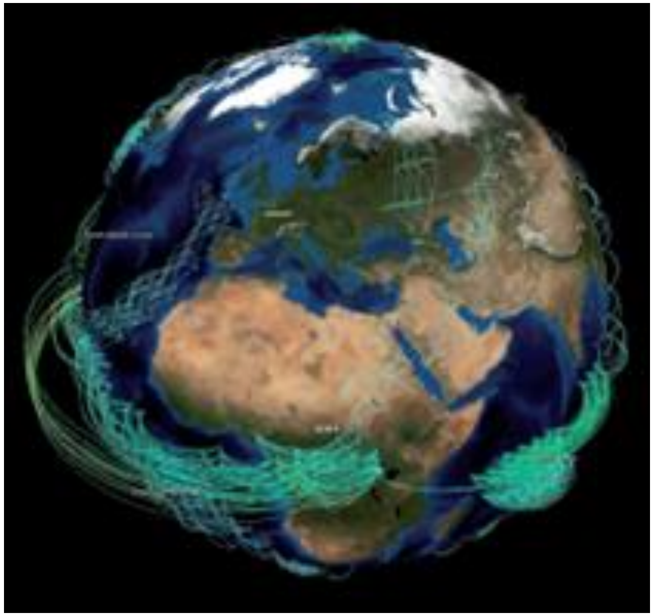


T. A. Schieber et al, Nature Communications 8,13928 (2017).
C. Quintero-Quiroz et al, Chaos 28, 106307 (2018).

Example: the analysis of climatic time series allows inferring statistical similarities and/or causal relations



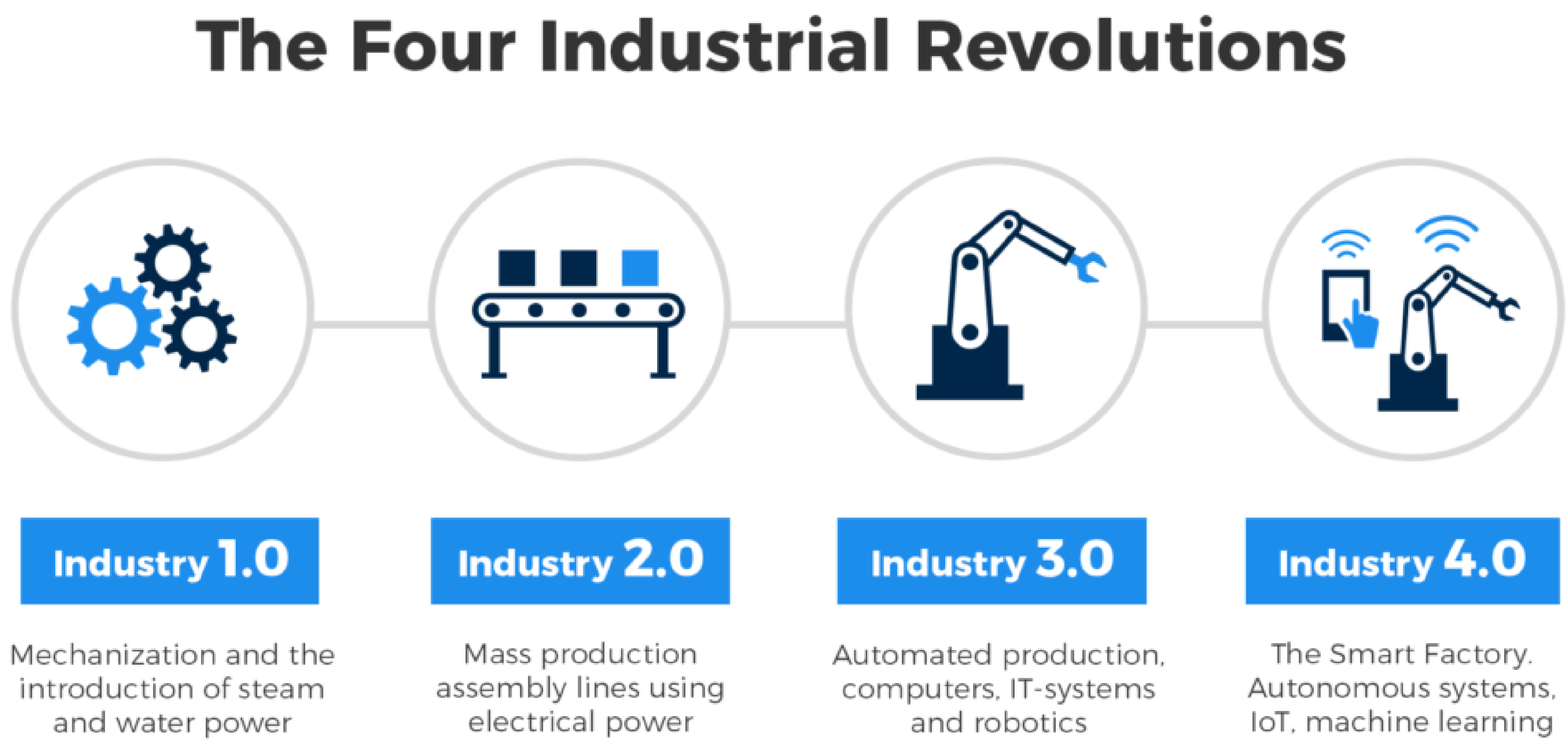
Bivariate statistical similarity analysis →



$x_i(t)$: <u>Anomaly</u> of a climatological variable in a geographical region "i"

Donges et al, Chaos 2015

Industry 4.0 (I4) = smart factories



Optical / photonic sensors + artificial intelligence + big data ⇒ I4 revolution

Role of big data analytics in Industry 4.0

- Help early detection of defects and production failures

⇒ enabling their prevention

⇒ increasing productivity, quality, and agility

⇒ increasing competitive value

- Big data analytics comprises (6C system):
  - Connection (sensor and networks)
  - Cloud (computing and data on demand)
  - Cyber (model & memory)
  - Content/context (meaning and correlation)
  - Community (sharing & collaboration)
  - Customization (personalization and value)

Role of signal processing and time series analysis in I4

- Connection (sensor and networks)
- Cloud (computing and data on demand)
- Cyber (model & memory)
- **Content/context (meaning and correlation)**
- Community (sharing & collaboration)
- **Customization (personalization and value)**

Strong need of analysis tools that extract reliable information directly from data

Example: advances in wireless sensor technologies enable the Internet of Things (IoT)

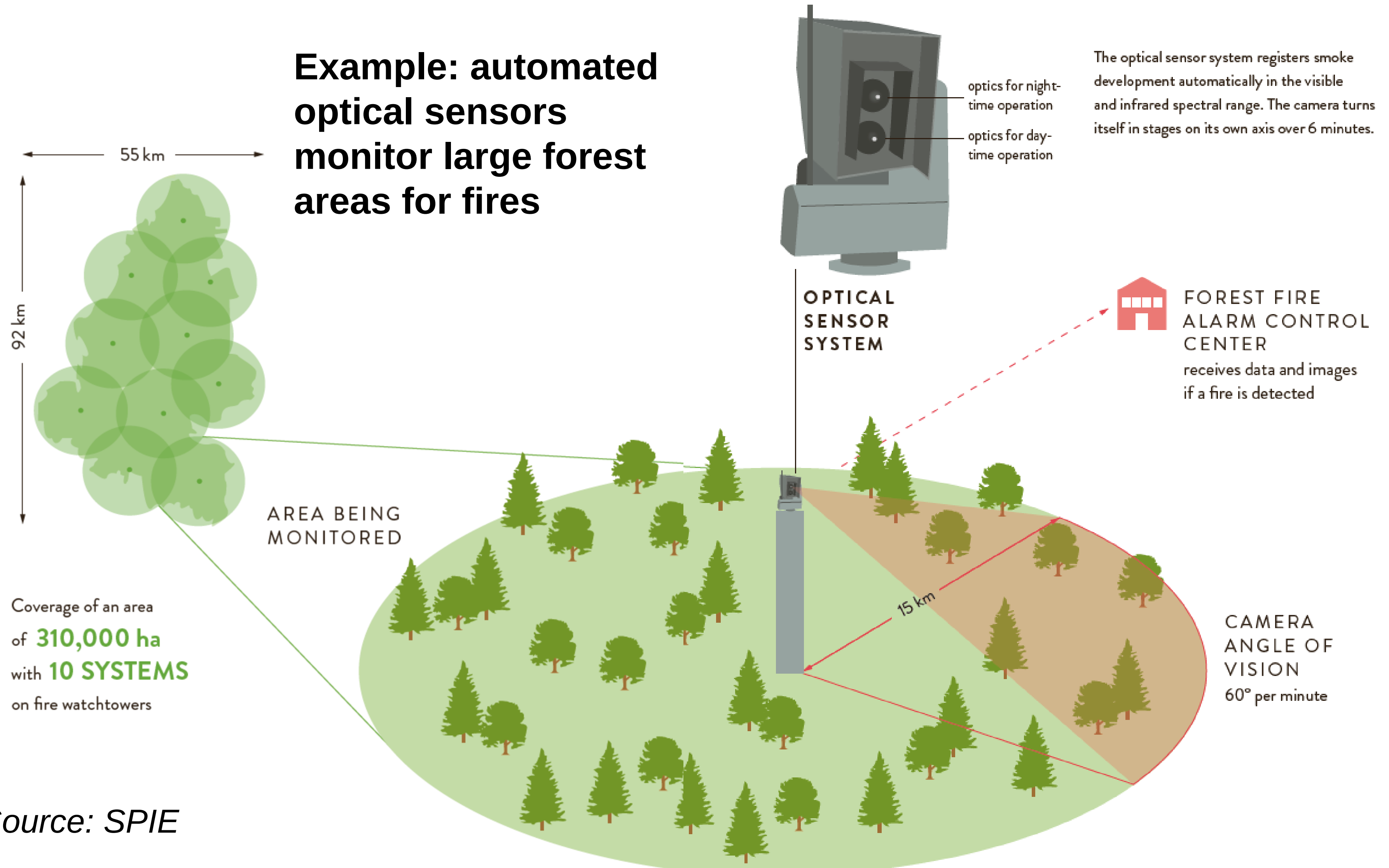
Smart electrical grids

Smart Farms: sensors connected to internet can allow control and forecasting (by taking into account weather information)

Smart homes, factories, cities, & medicine (patients, aging citizens)

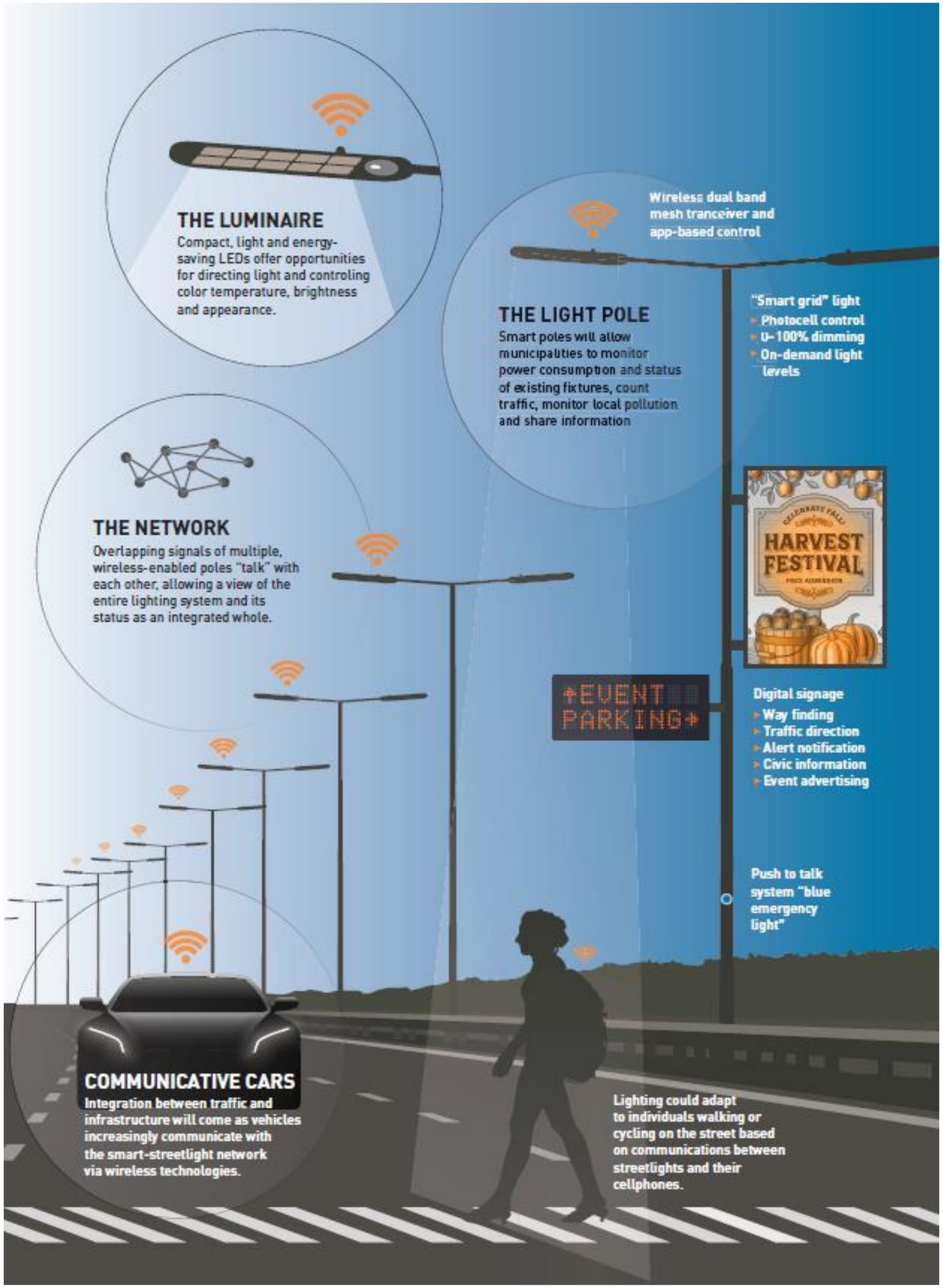
With the ubiquitous presence of sensors everywhere, time series analysis has found many applications

Example: automated optical sensors monitor large forest areas for fires



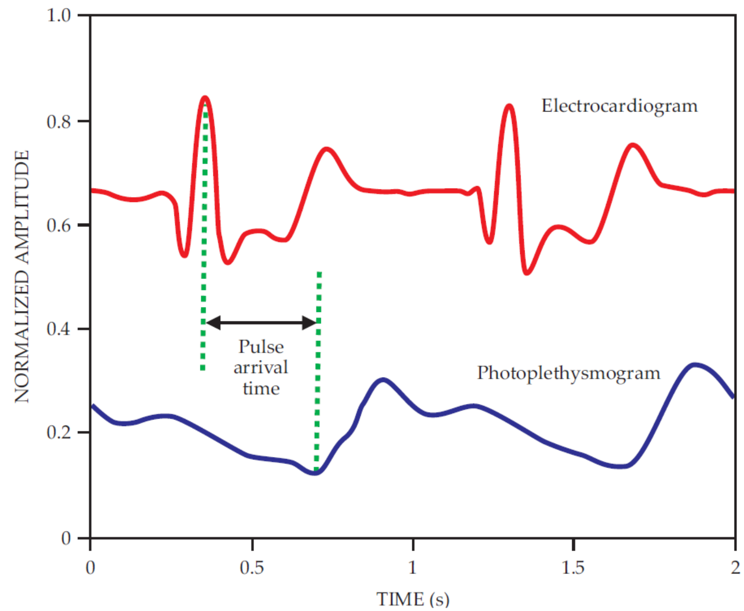
Source: SPIE

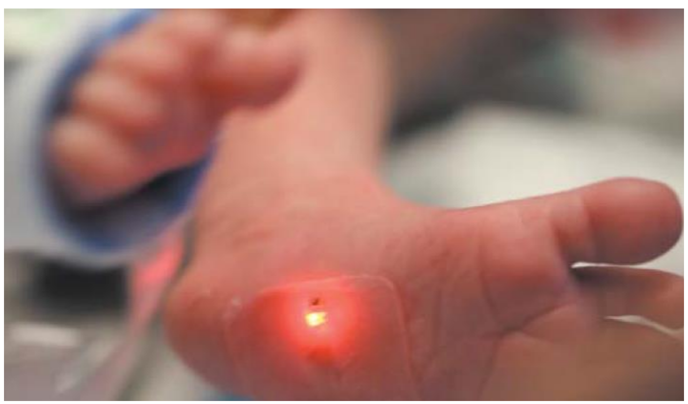
Example: LED-based sensors are enabling "Smart Cities"



Optics and Photonics News
November 2018

Example: wireless sensors for health monitoring



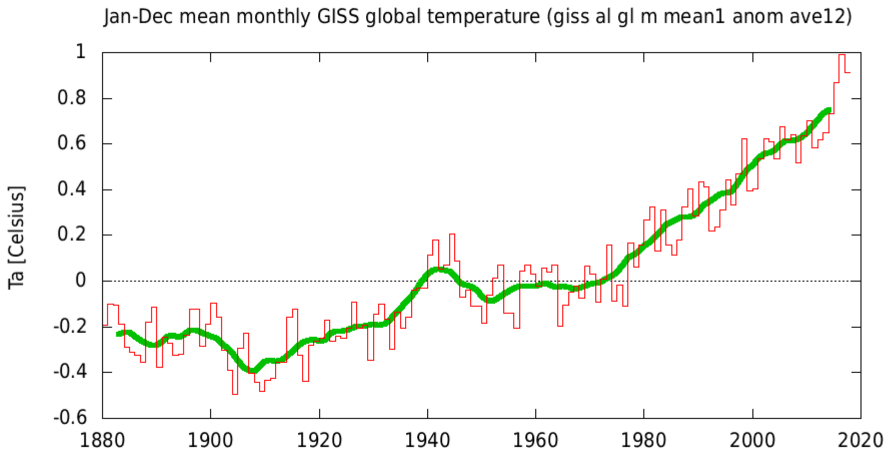


- Just two tiny wireless sensors, one on the foot and one on the chest, suffice to monitor all of an infant’s vital signs (but until the wireless devices are approved, patients need to wear the standard wired sensors too).
- From two measures of the heartbeat (an ECG recorded by the chest sensor and a PPG measured by a foot sensor) clinicians can extract the pulse arrival time from the heart to the foot, that is a measure of blood pressure.
- To reduce the amount of data that needs to be transmitted through the wireless link much of the ***signal processing***—for example, identifying the peaks in the ECG waveform—is done on the sensors themselves.

Physics Today may 2019

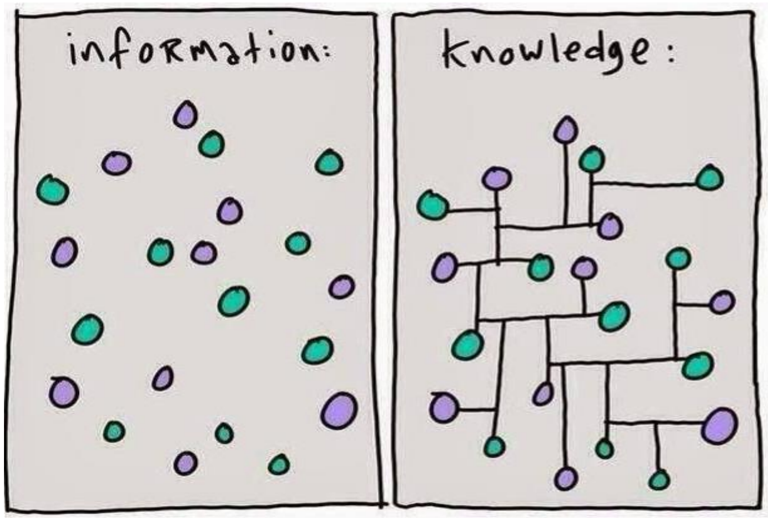
Methods of time series analysis $X = \{x_1, x_2, \ldots x_N\}$

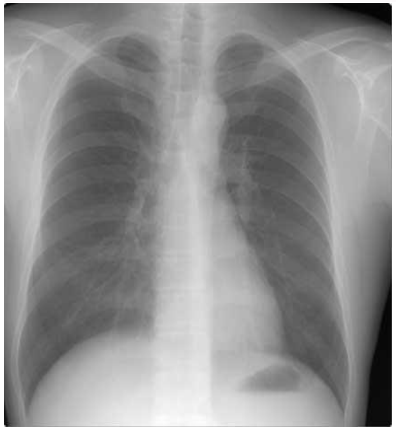
- Many methods have been developed to extract information from a time series.
- The methods to be used depend on the characteristics of the data
  - Length of the time series;
  - Stationarity;
  - Level of noise;
  - Temporal resolution;
  - etc.



- **Different methods provide complementary information.**

Good results depend on our *knowledge* of the system that generates the time series.





Where the data comes from?

Modeling assumptions about the system that generates the data:

- Stochastic or deterministic?
- Regular or chaotic or “complex”?
- Stationary or non-stationary? Time-varying parameters?
- Low or high dimensional?
- Spatial variable? Hidden variables?
- Time delays?

Historic background:

from dynamical systems to complex systems going through bifurcations and chaotic attractors

The beginning of the mathematical modelling of dynamical systems: Newtonian mechanics

- Mid-1600s: Ordinary differential equations (ODEs)

- **Isaac Newton**: studied planetary orbits and solved analytically the “two-body” problem (earth around the sun).

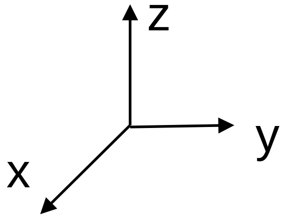
- Since then: a lot of effort for solving the “three-body” problem (earth-sun-moon) – Impossible.

Late 1800s

- **Henri Poincare** (French mathematician).

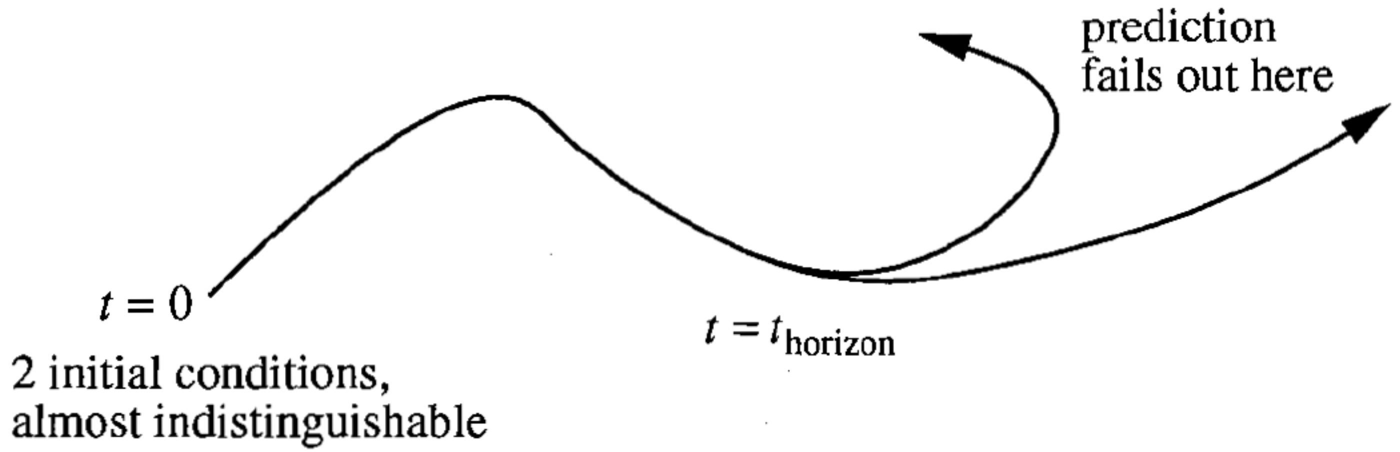
  Instead of asking "*which are the exact positions of planets (trajectories)*?"

  he asked: "*is the solar system* ***stable*** *for ever, or will planets eventually run away*?"
- He developed a **geometrical** approach to solve the problem.
- Introduced the concept of "phase space".



- *Poincaré recurrence theorem*: certain systems will, after a sufficiently long but finite time, return to a state very close to the initial state.
- He also had the intuition of the possibility of chaos.

*Poincare: "The evolution of a **deterministic** system can be aperiodic, unpredictable, and strongly depends on the initial conditions".*



Deterministic system: the initial conditions fully determine the future state.

Deterministic **chaotic** system: there is no randomness but the system can be, in the long term, unpredictable.

A problem in time series analysis: How to determine the prediction horizon? With what reliability?

1950s: First computer simulations

- Computes allowed to experiment with equations.
- Huge advance in the field of "*Dynamical Systems*".

- 1960s: **Eduard Lorenz** (American mathematician and meteorologist at MIT): simple model of convection rolls in the atmosphere.

$$\frac{dx}{dt} = -\sigma x + \sigma y,$$
$$\frac{dy}{dt} = -xz + rx - y,$$
$$\frac{dz}{dt} = xy - bz.$$

- Famous **chaotic** attractor.

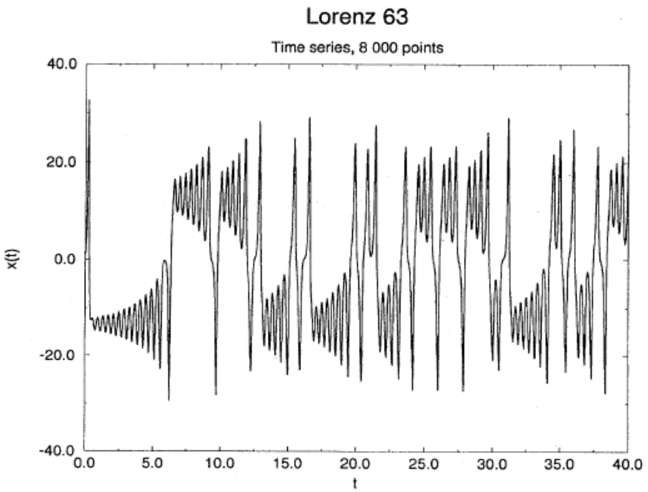
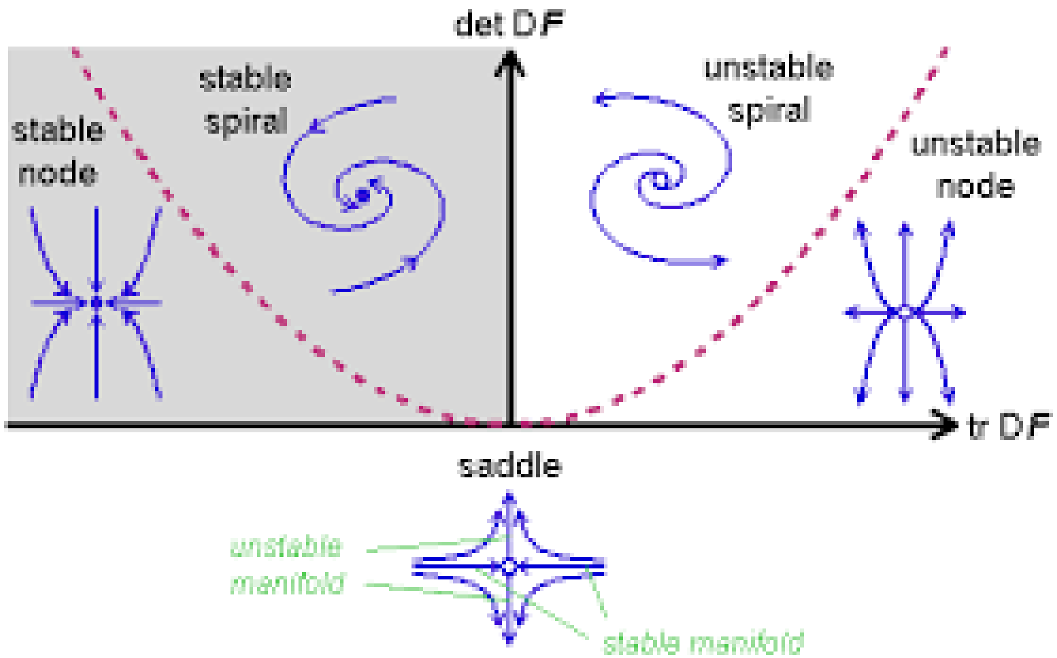
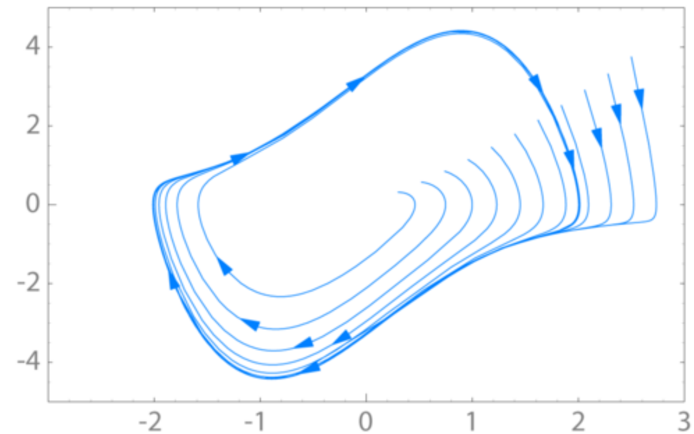
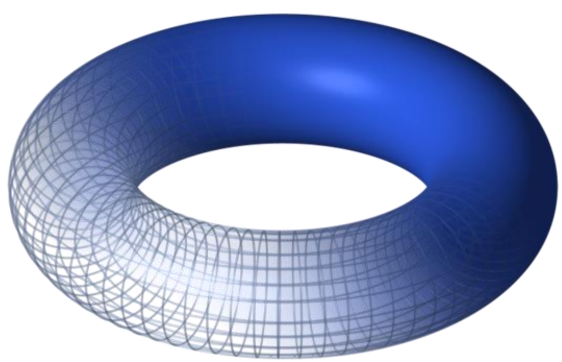


FIG. 1. Chaotic time series $x(t)$ produced by Lorenz (1963) equations (11) with parameter values $r=45.92$, $b=4.0$, $\sigma=16.0$.

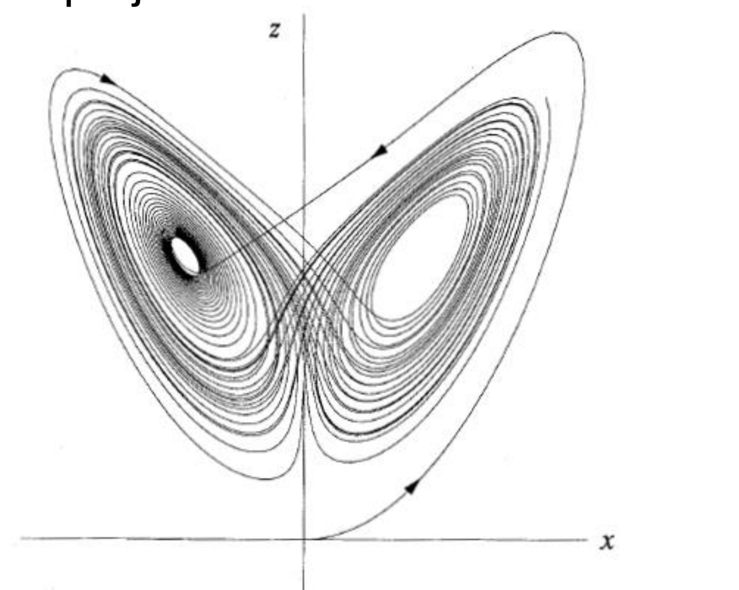
Attractors: fixed points, limit cycles, quasi-periodic torus, chaotic and “strange” (also known as fractal)







2D projection of 3D Lorenz attractor

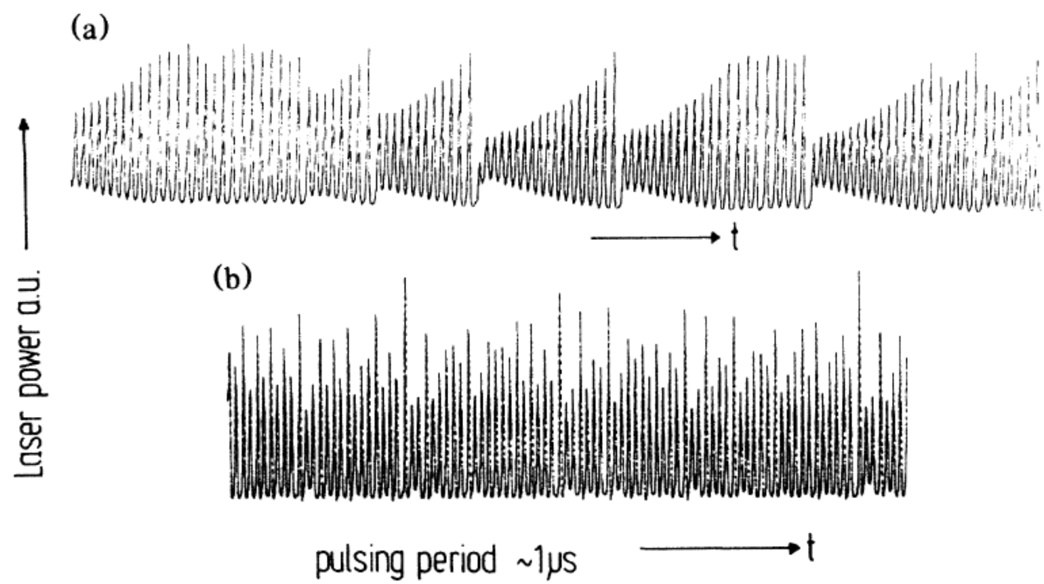


Can we observe chaos experimentally?

VOLUME 57, NUMBER 22 PHYSICAL REVIEW LETTERS 1 DECEMBER 1986

Evidence for Lorenz-Type Chaos in a Laser

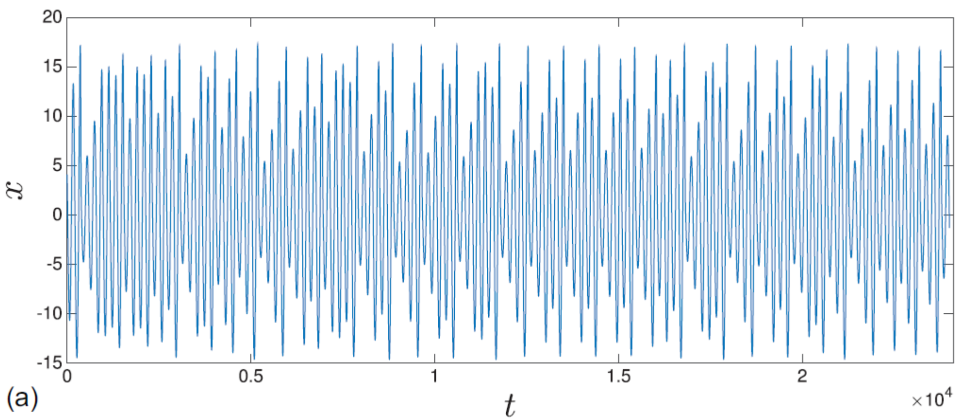
C. O. Weiss and J. Brock[a]
Physikalisch-Technische Bundesanstalt, D-3300 Braunschweig, Federal Republic of Germany
(Received 18 April 1986)

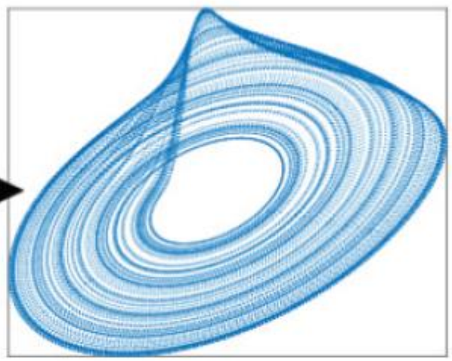


optically pumped NH_3 laser

A problem in time series analysis: how to "reconstruct" the phase space? How many dimensions?

Time series simulated with a model

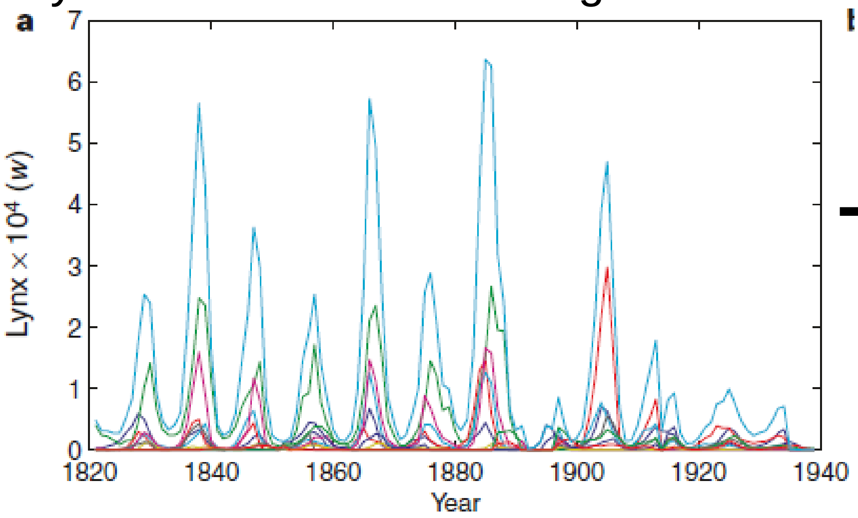


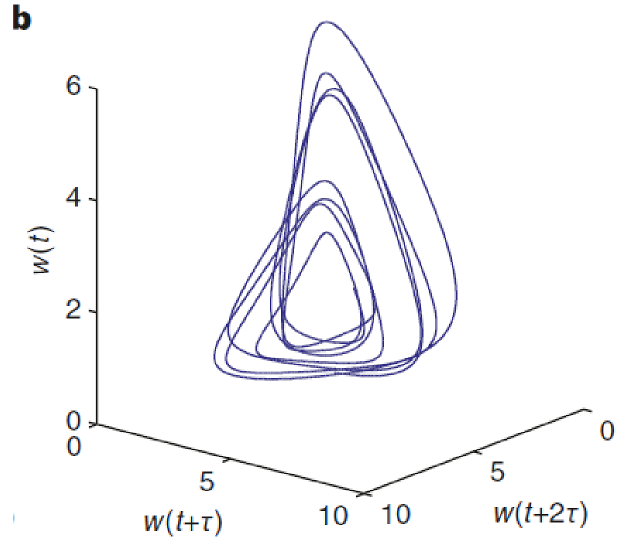


x(t)

y(t): an independent variable, obtained from x(t) [for example, x(t+**τ**)]

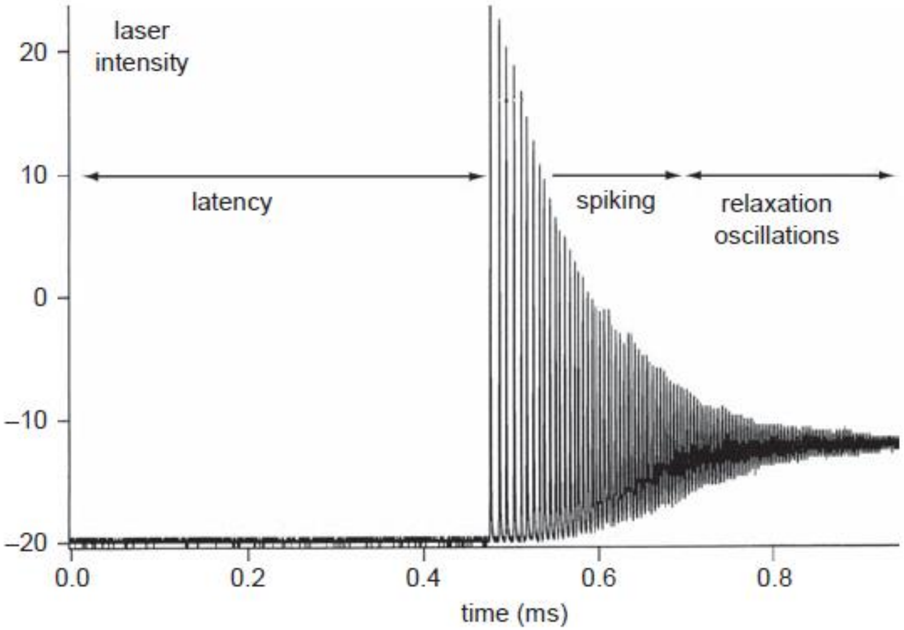
Example: Lynx abundances in six regions in Canada

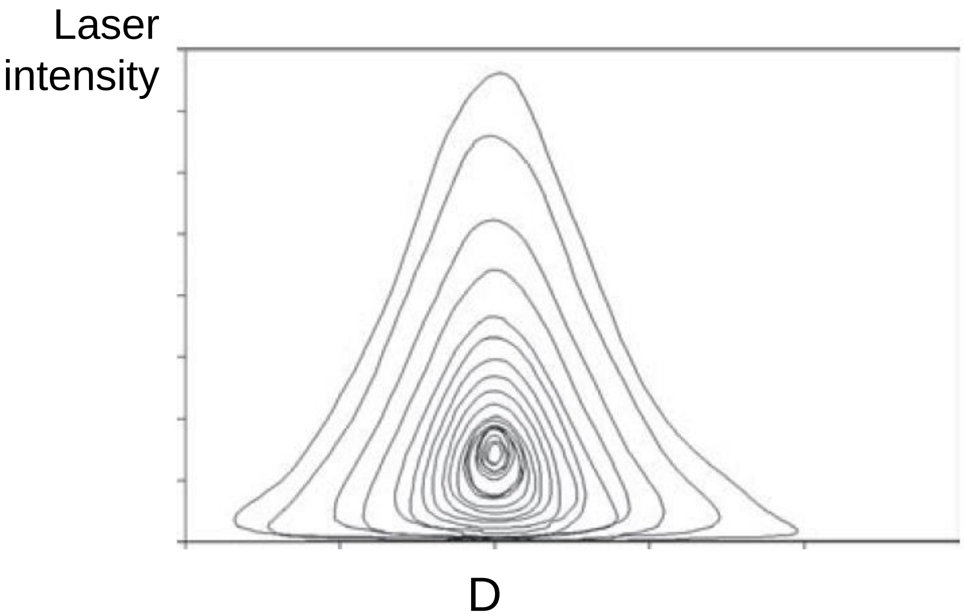




Bradley & Kantz, Chaos 25, 097610 (2015); B. Blasius et al, Nature 399, 354 (1999).

Example: Turn-on transient of a Nd3+:YAG laser

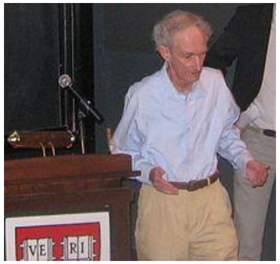




What is D?   D=(dI/dt)/I+1

T. Erneux and P. Glorieux, Laser Dynamics (Cambridge University Press 2010)

The 1970s



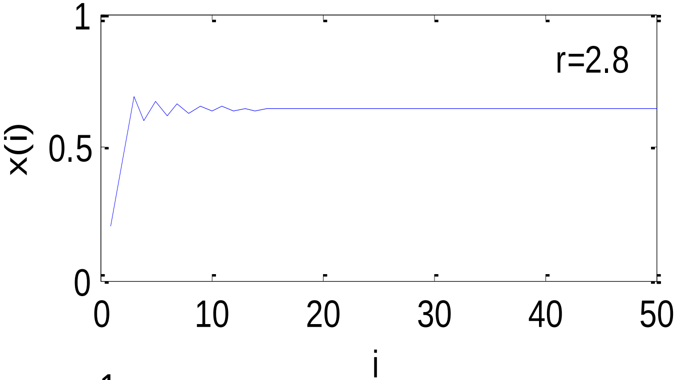
- **Robert May** (Australian, 1936): population biology
- "Simple mathematical models with very complicated dynamics“, *Nature* (1976).

$$x_{t+1} = f(x_t)$$

A classical example: The Logistic map $f(x) = r\,x(1-x)$
$x \in (0,1)$, $r \in (0,4)$

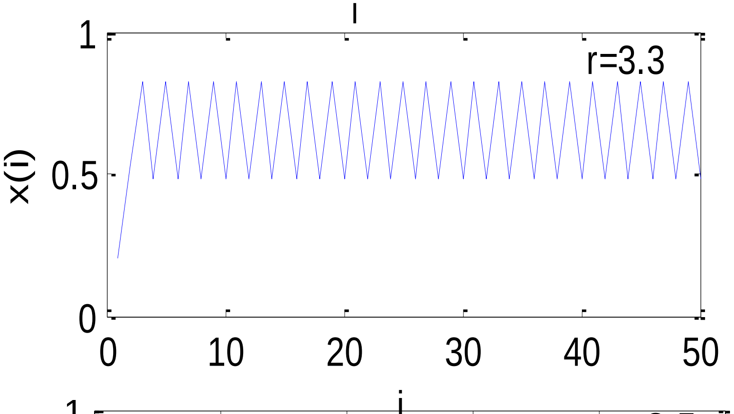
- Difference equations ("iterated maps"), in spite of being simple and deterministic, can exhibit: **stable points**, **stable cycles**, and **apparently random fluctuations**.

The logistic map: $x(i+1) = r\, x(i)[1 - x(i)]$ $x \in (0,1)$, $r \in (0,4)$

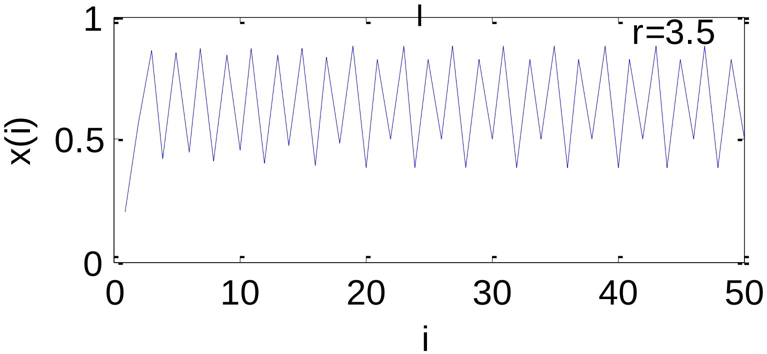


r=2.8, Initial condition: x(1) = 0.2

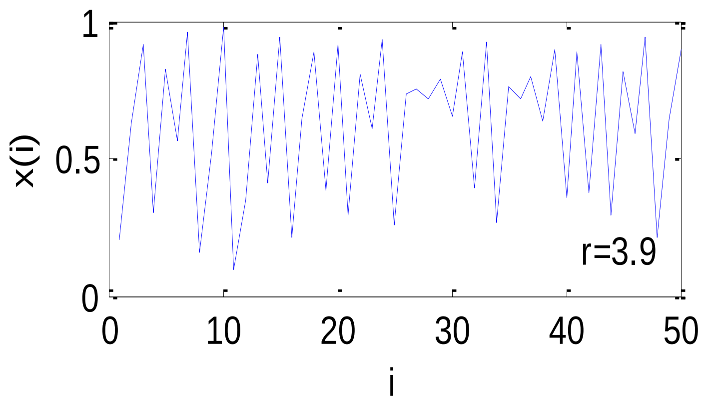
Transient relaxation → long-term stability



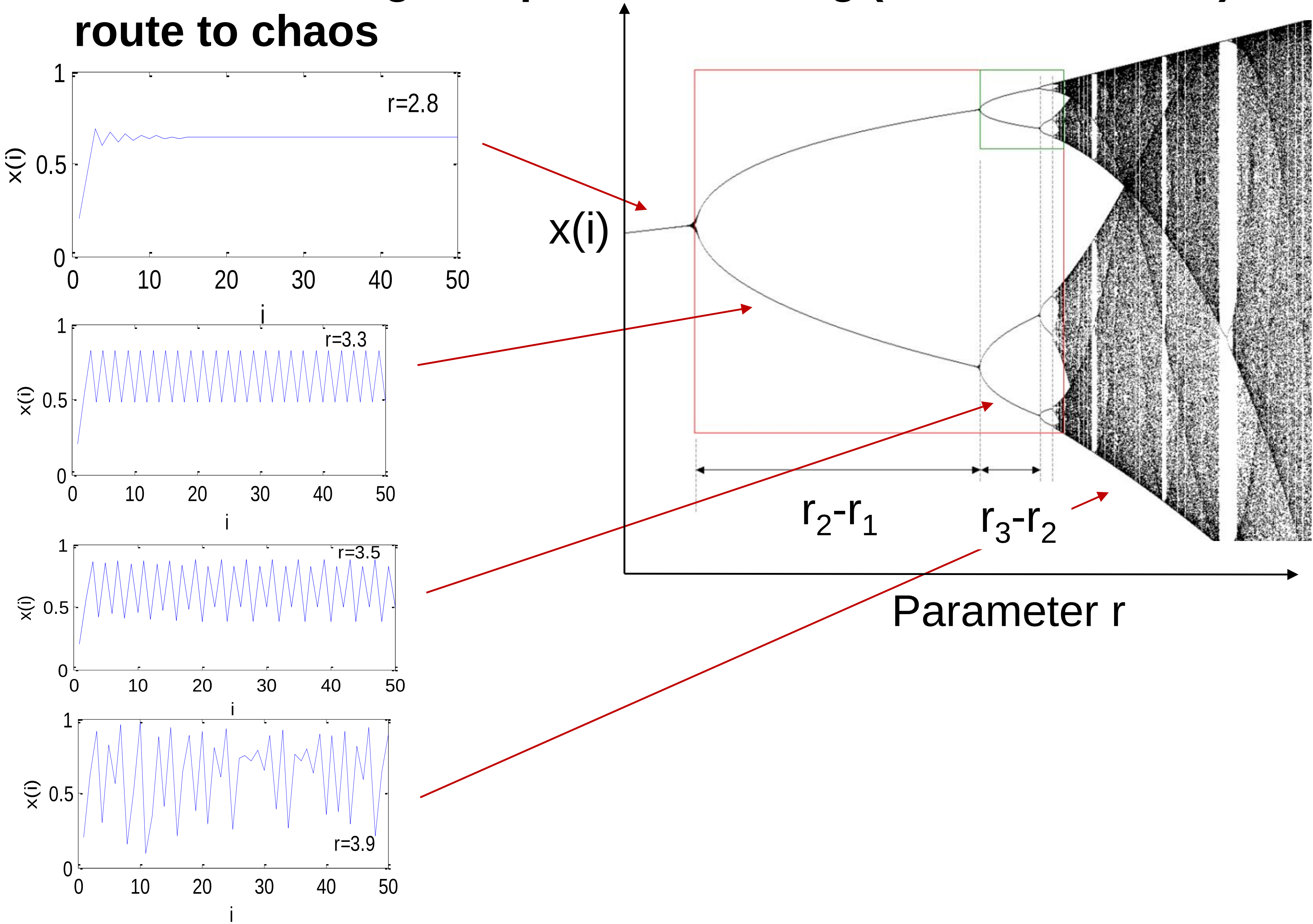
The fixed point is the solution of: $x = r\,x\,(1-x) \Rightarrow x = 1 - 1/r$



Transient dynamics → oscillations (regular or irregular)

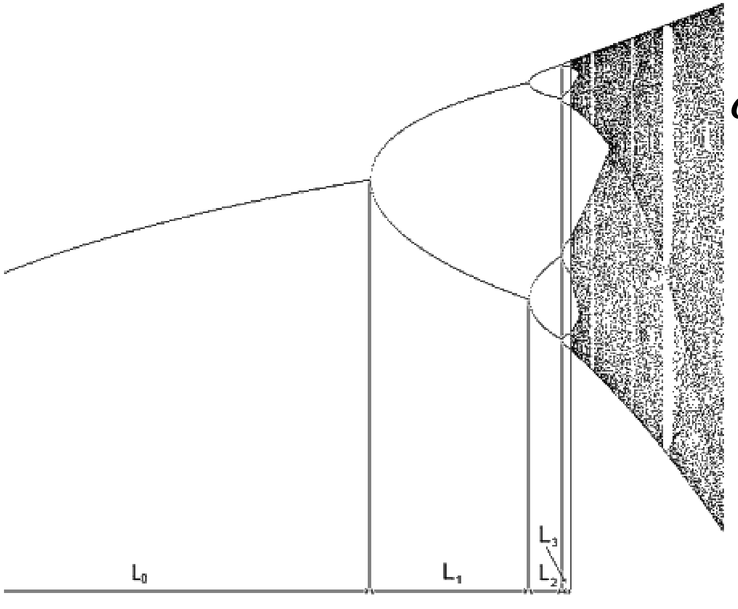


Bifurcation diagram: period-doubling (or subharmonic) route to chaos



Order within chaos and a "hidden" law in the subharmonic route to chaos

In 1975, **M. Feigenbaum** (American mathematician and physicist 1944-2019), using a small HP-65 calculator, discovered the <u>scaling law of the bifurcation points of the Logistic map.</u>



$$\delta = \lim \frac{L_i}{L_{i+1}} = 4.669201...$$

HP-65 calculator: the first magnetic card-programmable handheld calculator

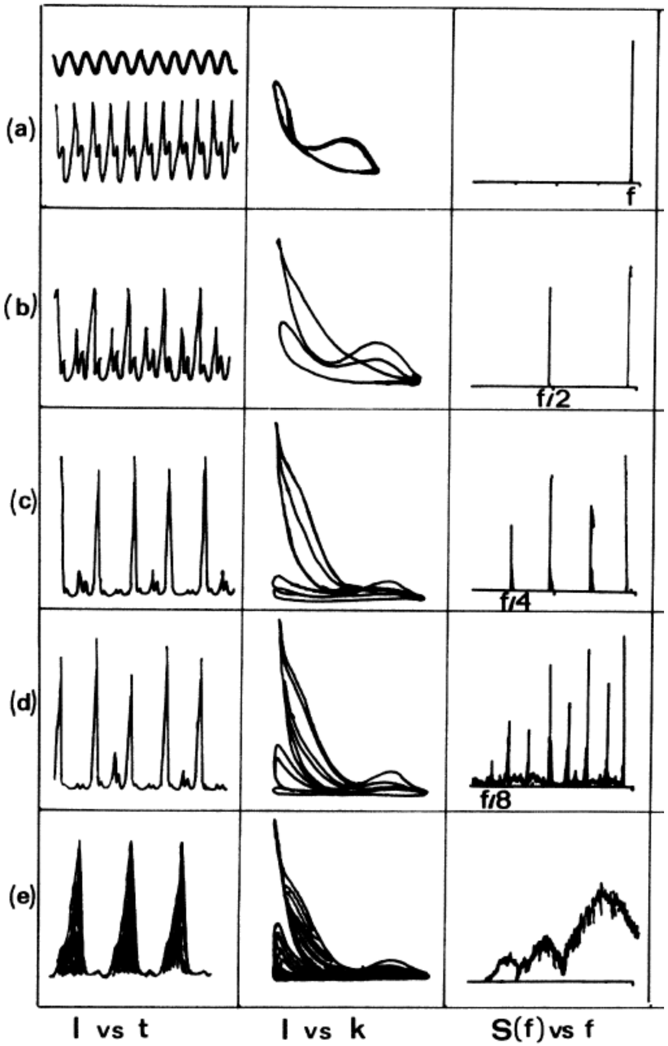
A universal law

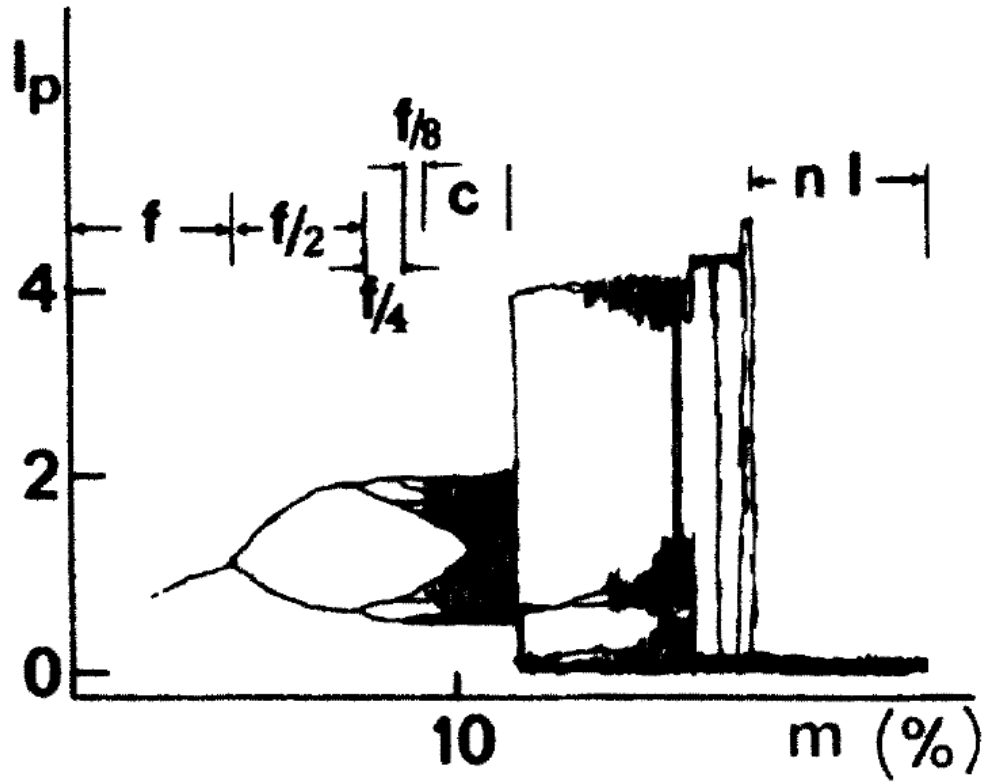
Feigenbaum showed that the same behavior, with the same mathematical constant, occurs for a wide class of functions (functions with a quadratic máximum).

Very different systems (in chemistry, biology, physics, etc.) go to chaos in the same way, quantitatively.

Early experiments: a periodically modulated CO_2 laser

Constant modulation frequency, increasing the modulation amplitude



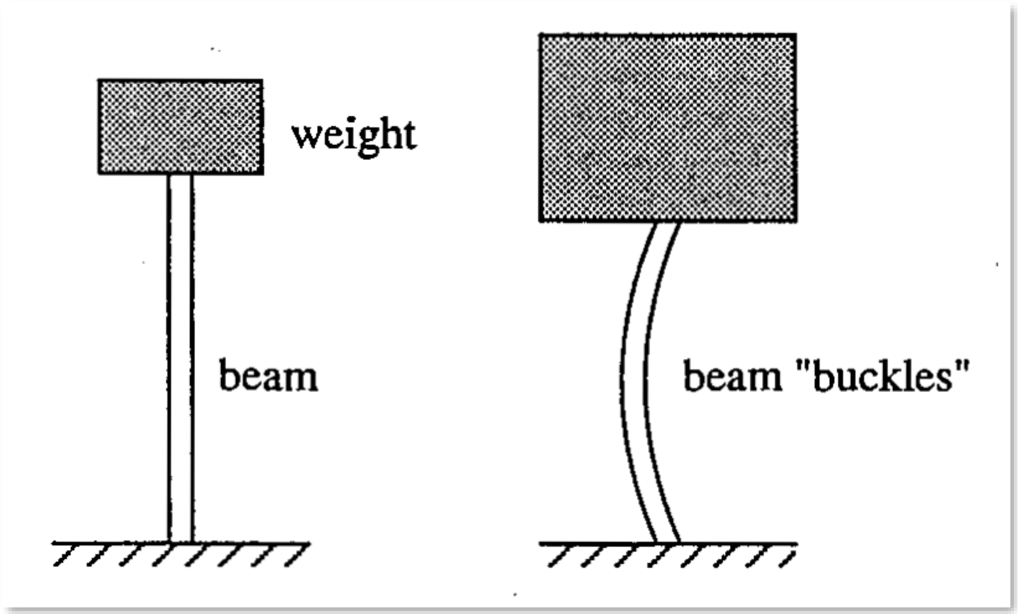


Tredicce et al, Phys. Rev. A 34, 2073 (1986).

We have seen how to make a bifurcation diagram, but, what *exactly* is a “bifurcation”?

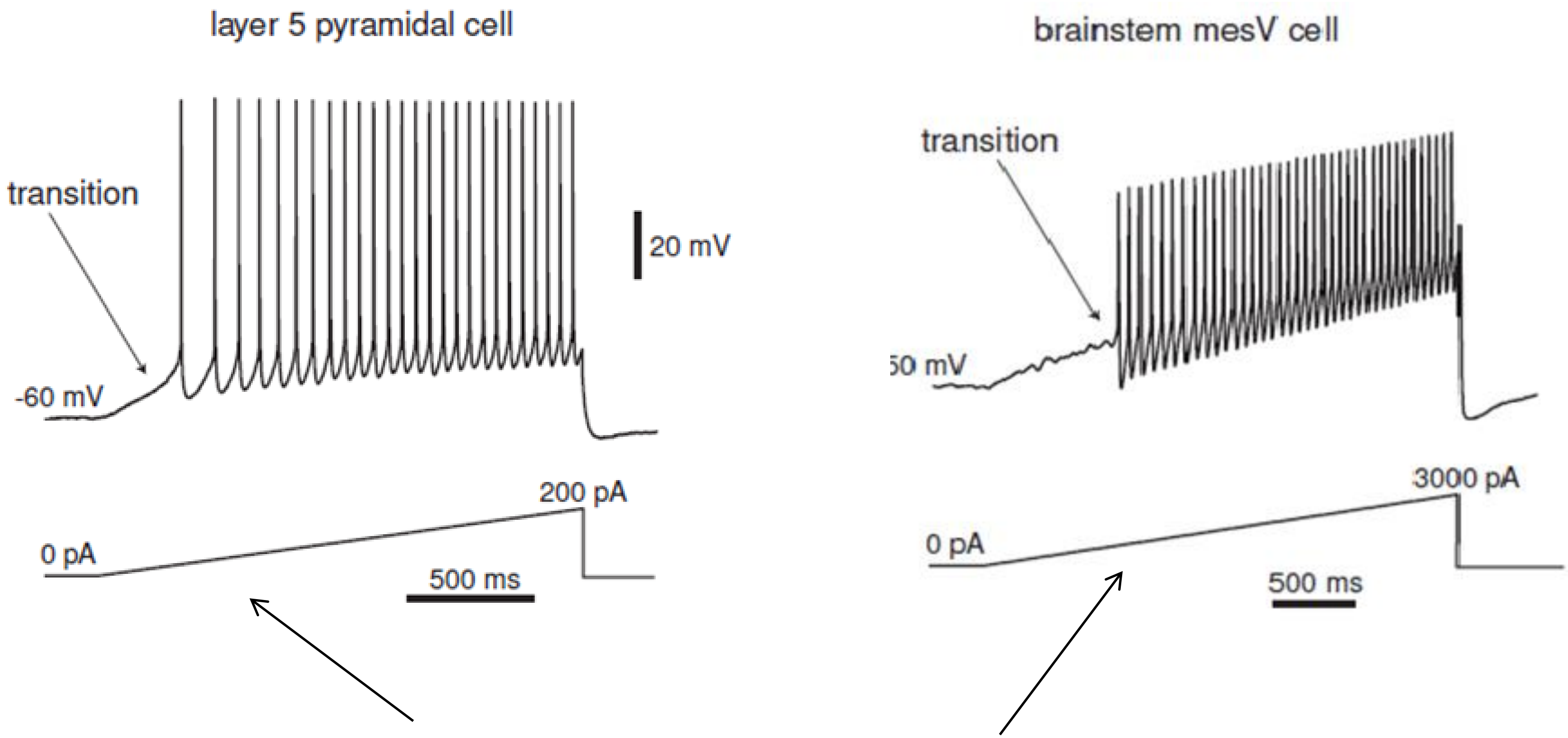
A **change** in the **structure of the phase space** when a control parameter is varied:

- Attractors can be created or destroyed
- The stability of an attractor can change



Strogatz, Nonlinear dynamics and chaos

Example: neuronal spikes

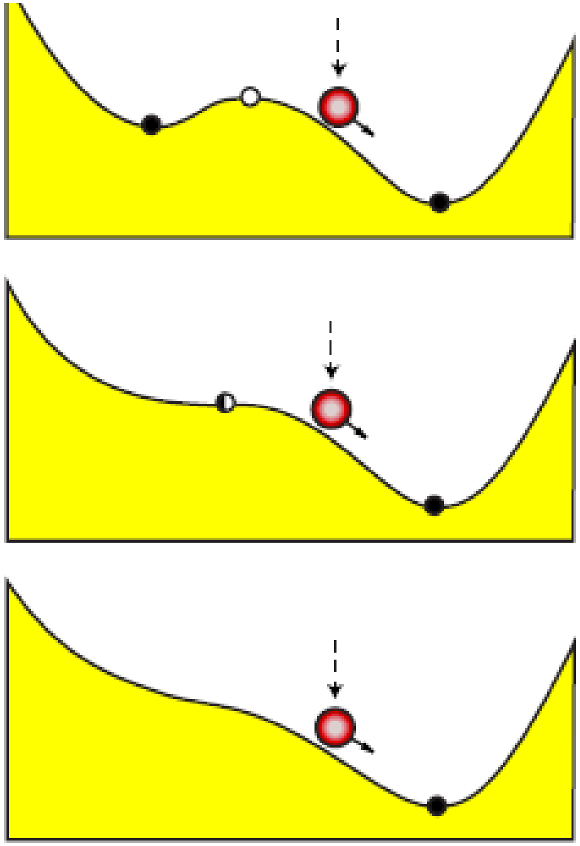


Control parameter increases in time

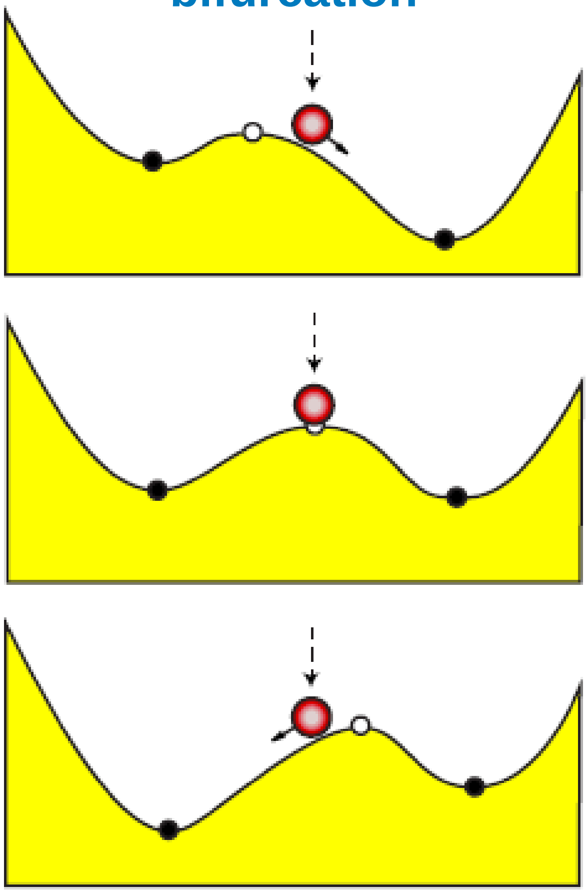
Eugene M. Izhikevich, Dynamical Systems in Neuroscience

A bifurcation is <u>not</u> equivalent to a change of behavior.

Bifurcation but no change of behavior

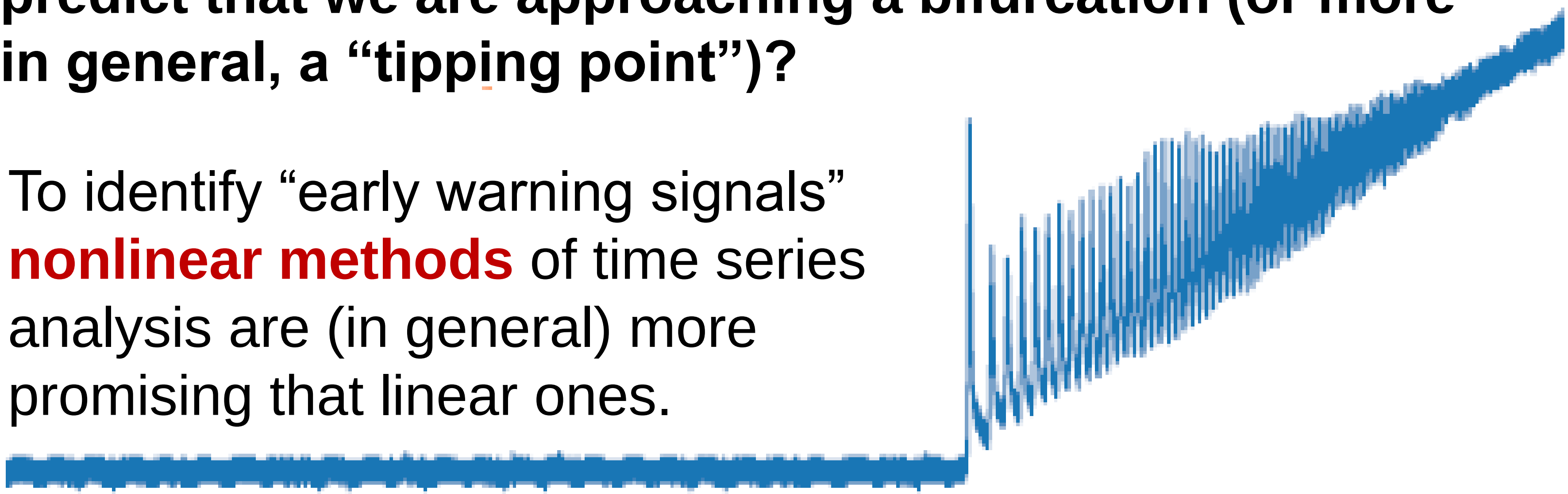


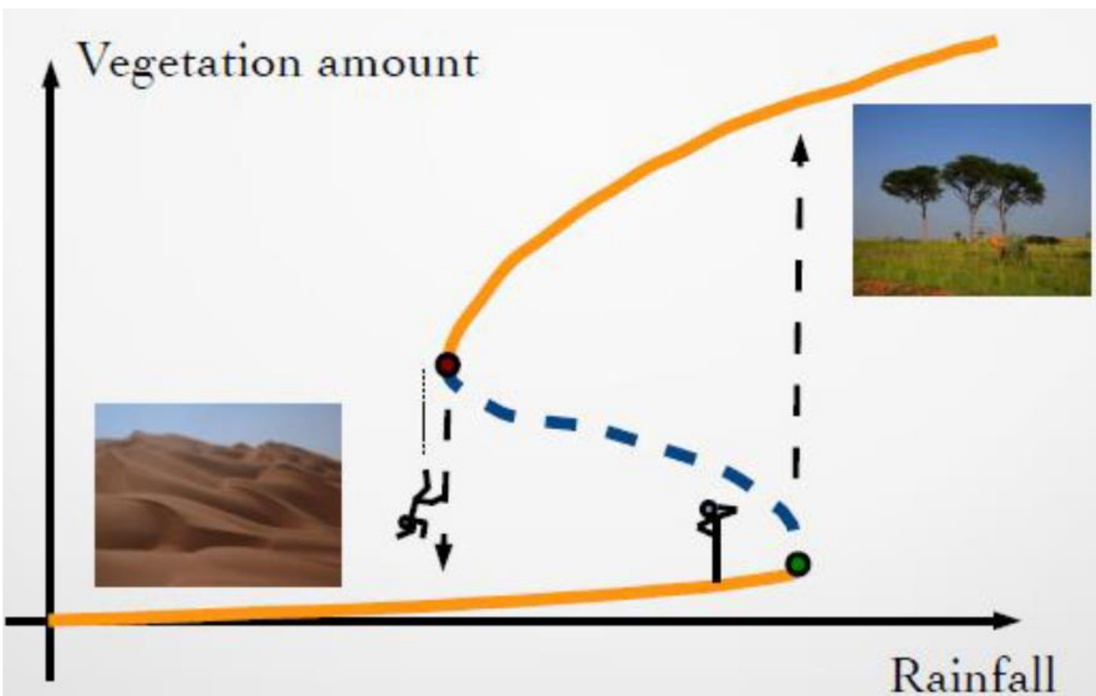
Change of behavior but no bifurcation



Another main problem of time series analysis: how to predict that we are approaching a bifurcation (or more in general, a “tipping point”)?

To identify “early warning signals” **nonlinear methods** of time series analysis are (in general) more promising that linear ones.





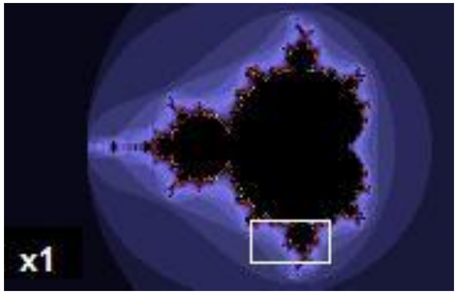
G. Tirabassi et al. Ecological Complexity 19, 148 (2014).
M. Marconi et al, Phys. Rev. Lett. 125, 134102 (2020).

The late 1970s

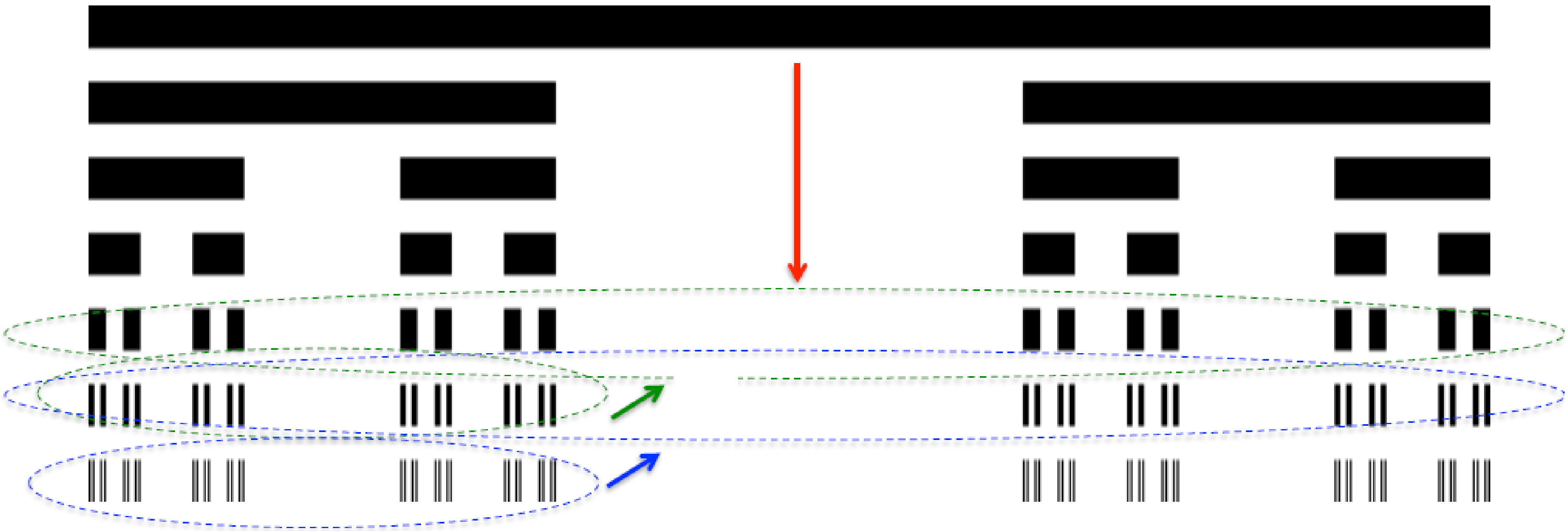
- **Benoit Mandelbrot** (Polish-born, French and American mathematician 1924-2010): “self-similarity” and **fractal objects:**

  each part of the object is like the whole object but smaller.

- Because of his access to IBM's computers, Mandelbrot was one of the first to use **computer graphics** to create and display fractal geometric images.



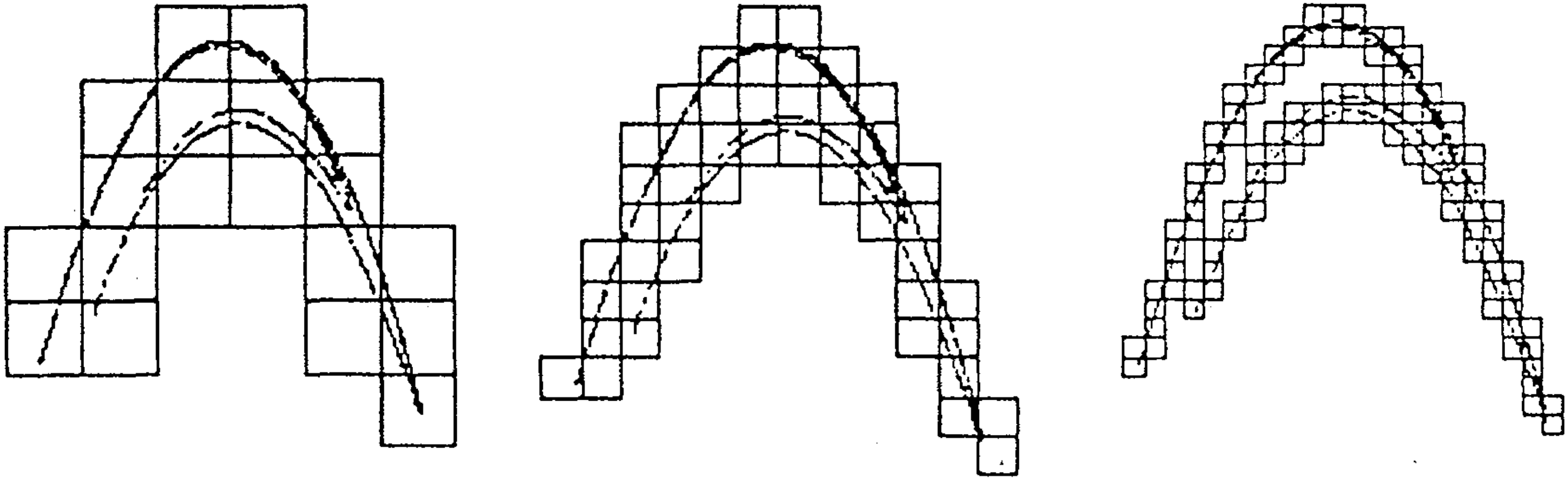
Cantor set (introduced by German mathematician Georg Cantor in 1883): remove the middle third of a line segment and then repeat the process with the remaining shorter segments



Fractal structure: a part of the object resembles the hole object.

D=0.63

How to estimate the dimension of a fractal?

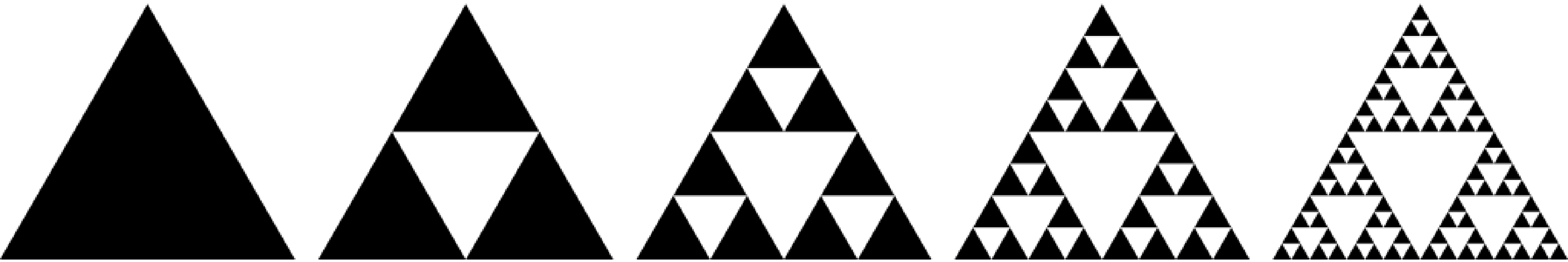


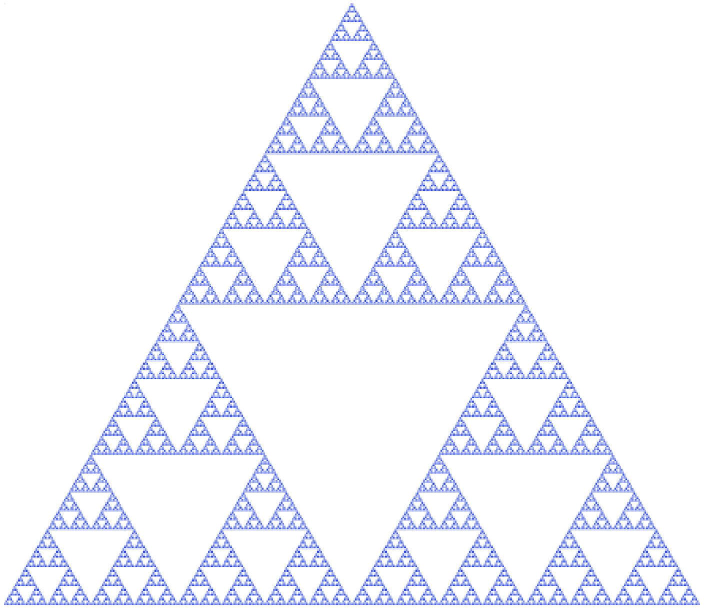
Box counting: $\textbf{bulk} \propto \textbf{size}^{\text{dimension}}$

(more latter)

Abarbanel et al, Reviews of Modern Physics 65, 1331 (1993).

Sierpiński triangle





D=1.585

Source: Wikipedia

Fractal objects: characterized by a “fractal” dimension that measures roughness.

Broccoli
D=2.66

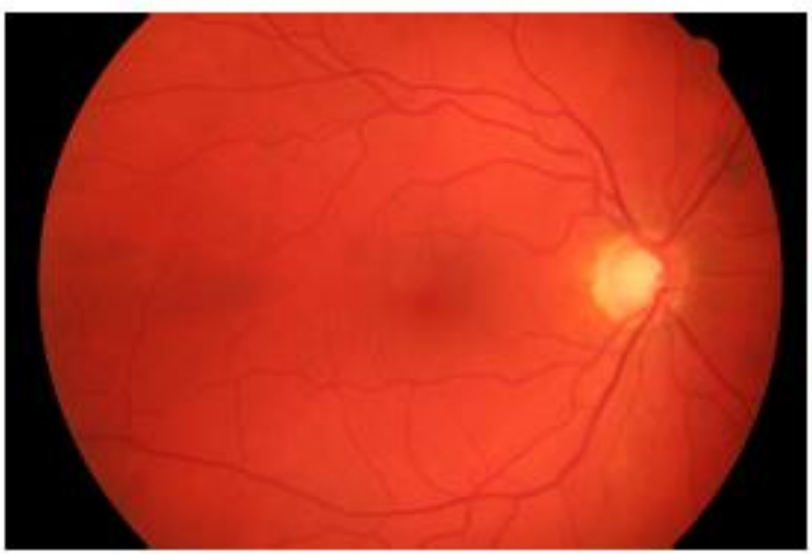
Human lung
D=2.97

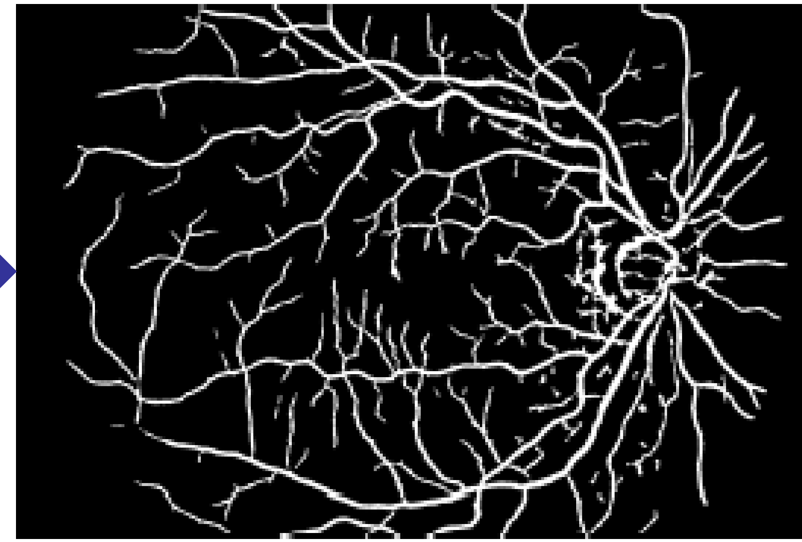
Coastline of Ireland
D=1.22

A lot of research is focused on detecting fractal behavior in observed data.

Video: http://www.ted.com/talks/benoit_mandelbrot_fractals_the_art_of_roughness#t-149180

Application of fractal analysis



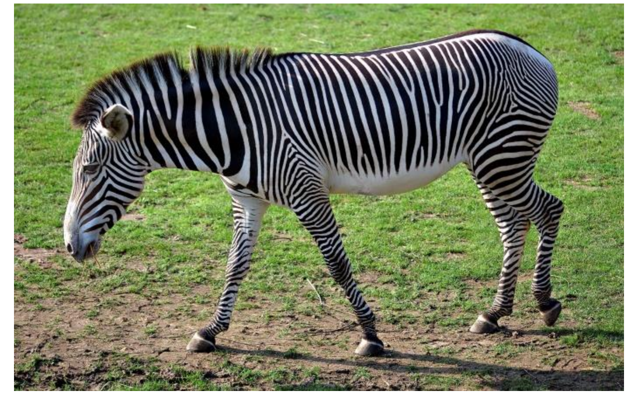


The fractal dimension of the blood vessels in the normal human retina is about 1.7 while it tends to increase with the level of diabetic retinopathy.

P. Amil et al., PLoS ONE 14, e0220132 (2019).

Spatio-temporal patterns: how "self-organization" emerges?





- **Ilya Prigogine** (Belgium, born in Moscow, Nobel Prize in Chemistry 1977).
- Studied thermodynamic systems far from equilibrium.
- Discovered that, in chemical systems, the interplay of (external) **input of energy** and **dissipation** can lead to "self-organized" patterns.


The study of spatio-temporal patterns has uncovered striking similarities in nature

Honey bees do a spire wave to scare away predators

https://www.youtube.com/watch?v=Sp8tLPDMUyg

Rotating waves occur in the heart during ventricular fibrillation

Hurricane Maria (Wikipedia)

https://media.nature.com/original/nature-assets/nature/journal/v555/n7698/extref/nature26001-sv6.mov

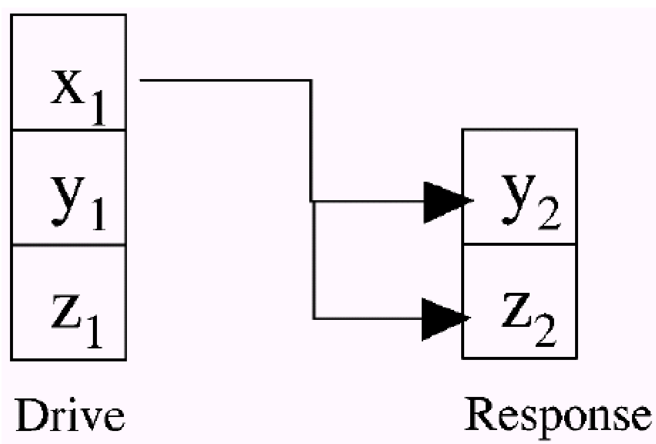
The 1990s: synchronization of two chaotic systems

VOLUME 64, NUMBER 8 PHYSICAL REVIEW LETTERS 19 FEBRUARY 1990

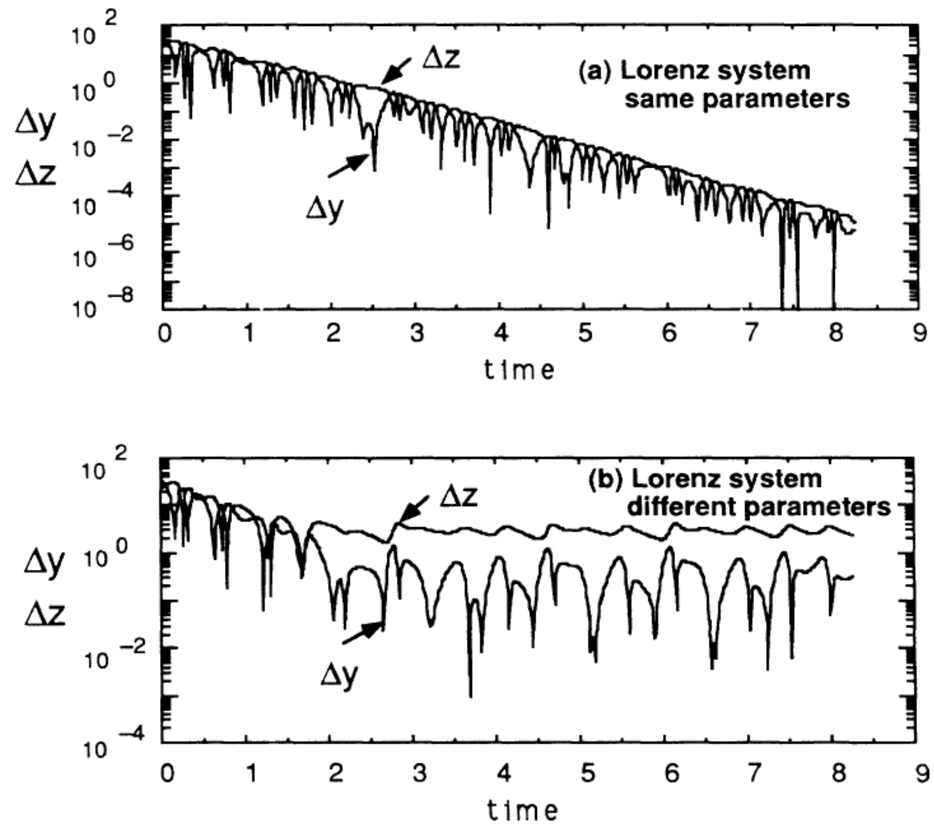
Synchronization in Chaotic Systems

Louis M. Pecora and Thomas L. Carroll
Code 6341, Naval Research Laboratory, Washington, D.C. 20375
(Received 20 December 1989)

Unidirectionaly coupled Lorenz systems



$$t \to \infty \quad |y_2 - y_1| \to 0, \; |z_2 - z_1| \to 0$$



Actually, the first observation of synchronization was much earlier (mutual *entrainment* of two pendulum clocks)

In mid-1600s **Christiaan Huygens** (Dutch mathematician) noticed that two pendulum clocks mounted on a common board synchronized and swayed in opposite directions (in-phase also possible).

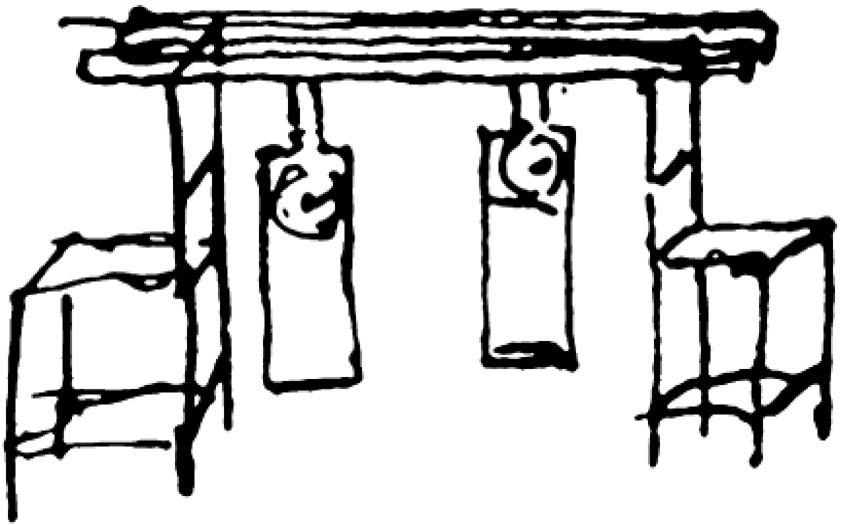
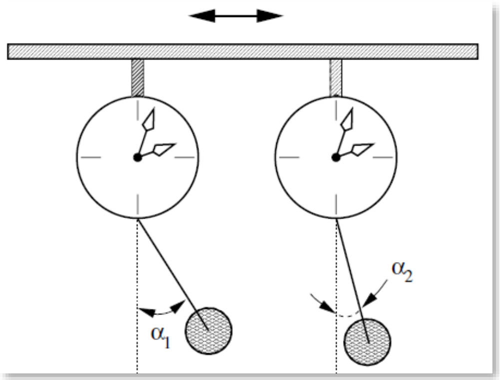


Figure 1.2. Original drawing of Christiaan Huygens illustrating his experiments with two pendulum clocks placed on a common support.



Different types of synchronization

Unidirectional coupling:

$$\frac{dx}{dt} = f(x)$$

$$\frac{dy}{dt} = f(y) + \eta\, g(x-y)$$

Bidirectional (mutual) coupling:

$$\frac{dx}{dt} = f(x) + \rho\, h(y-x)$$

$$\frac{dy}{dt} = f(y) + \eta\, g(x-y)$$

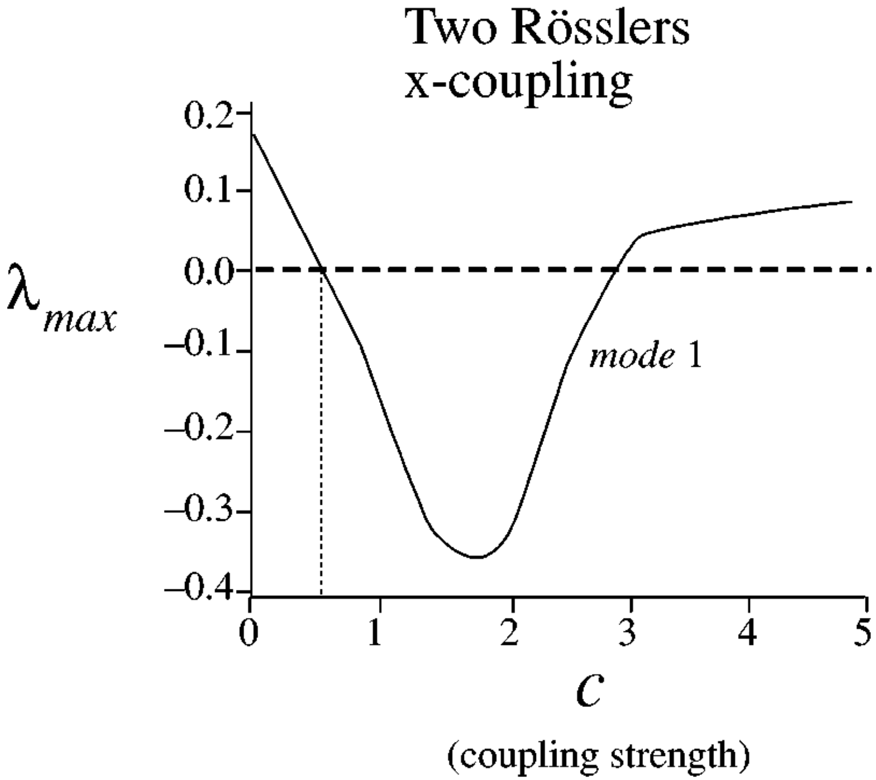
- Complete: $y(t) = x(t)$ (identical systems)
- Phase: the phases of the oscillations are synchronized, but the amplitudes are not.
- Lag: $y(t+\tau) = x(t)$
- Generalized: $y(t) = F(x(t-\tau))$ (*F and* τ can depend on the coupling strengths, η and ρ)

Another problem of time series analysis:

How to detect synchronization? How to quantify it?

Synchronization can occur for a range of coupling strengths

Max. transverse Lyap. Exponent, determines the stability of the synchronized (x=y) solution



Pecora et al, Chaos 7, 520 (1997).

Experimental observation of synchronization of chaotic systems

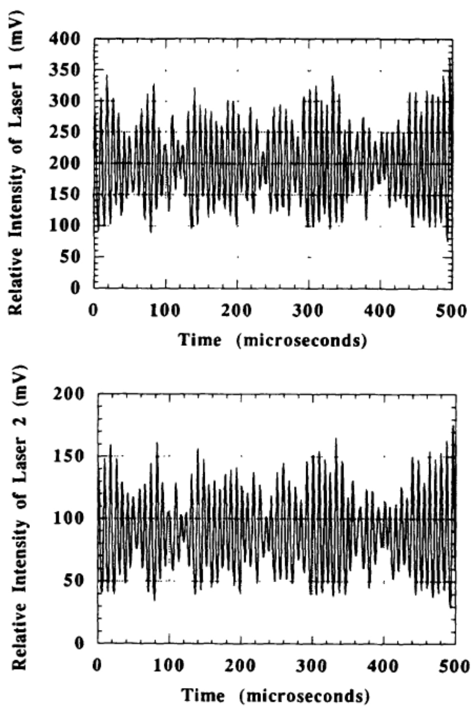
VOLUME 72, NUMBER 13 PHYSICAL REVIEW LETTERS 28 MARCH 1994

Experimental Synchronization of Chaotic Lasers

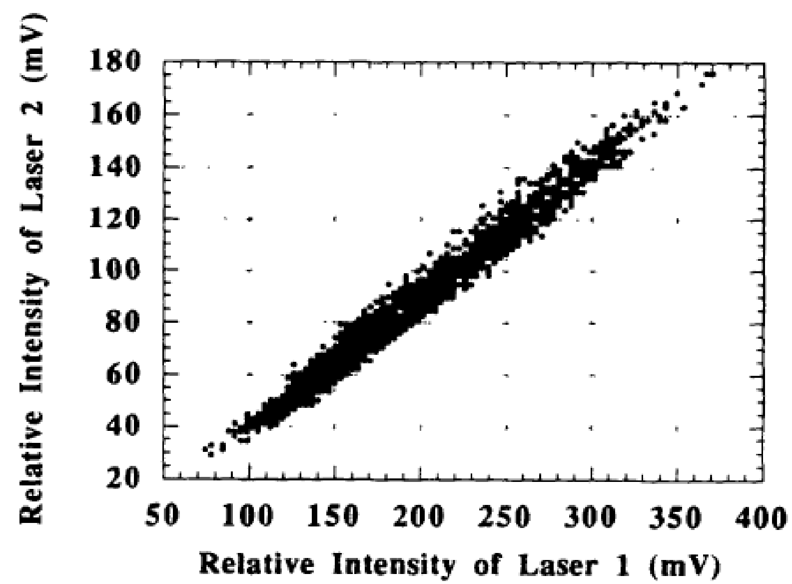
Rajarshi Roy and K. Scott Thornburg, Jr.
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332
(Received 30 August 1993)

We report the observation of synchronization of the chaotic intensity fluctuations of two Nd:YAG lasers when one or both the lasers are driven chaotic by periodic modulation of their pump beams.

(a)

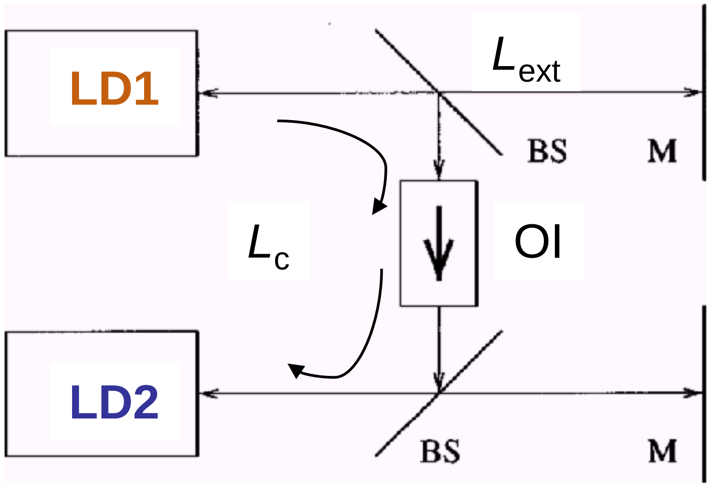


(b)

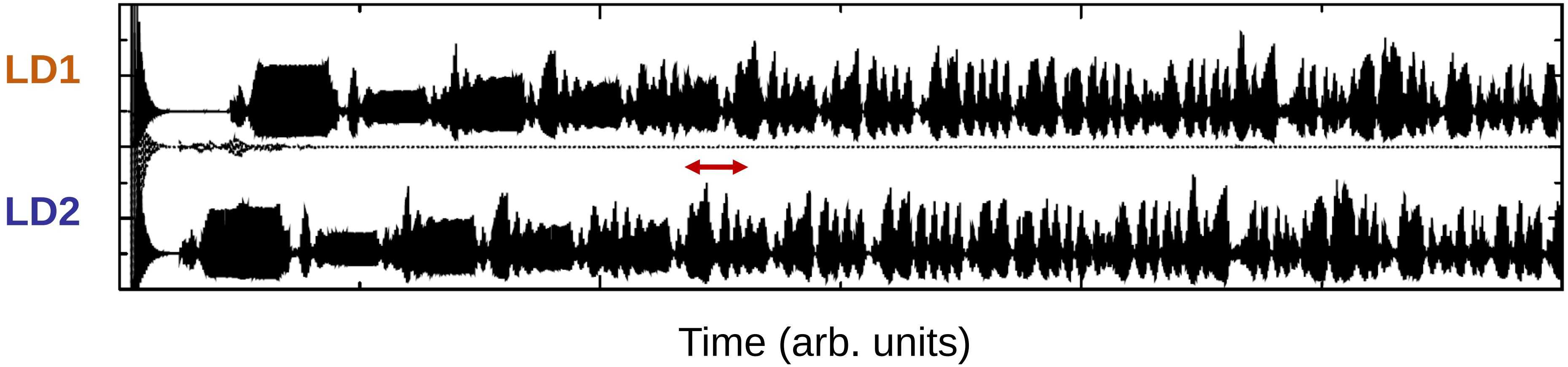


An example of lag synchronization

Two laser diodes (LD), with "feedback" from a mirror (M) with time delay τ, and coupled with time delay τ_c (uni-directionally or bi-directionally with or without an optical isolator, OI). The lag depends on $\tau - \tau_c$.



$$\tau = \frac{2L_{ext}}{c}; \quad \tau_c = \frac{L_c}{c}$$

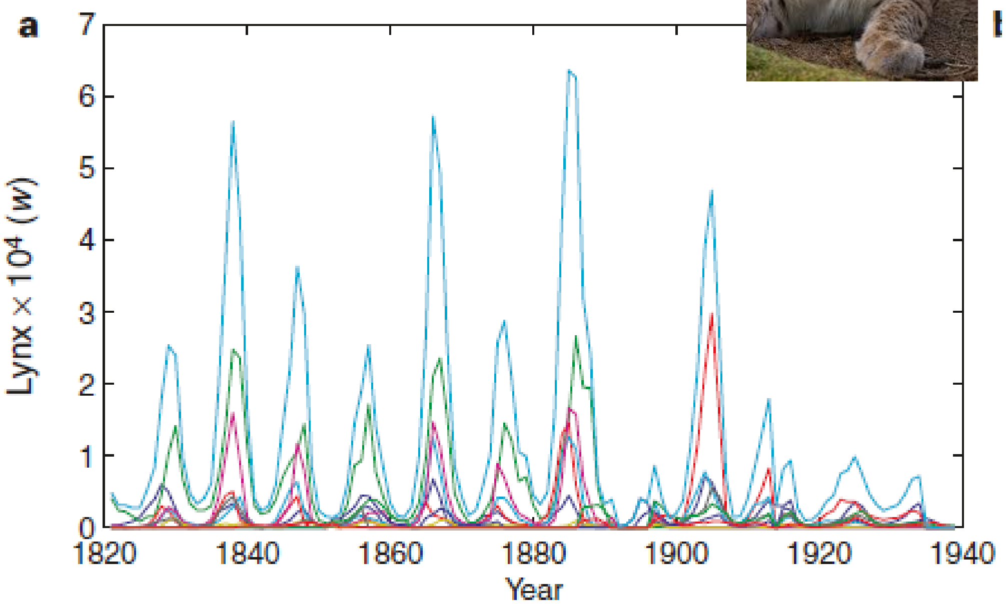


Other problems in time series analysis: How to detect the presence of feedback loops? How to detect delays in the interactions?

C. Masoller, Phys. Rev. Lett 86, 2782 (2001).

Back to the abundances of the Lynx populations in six regions in Canada

This is an example of *phase synchronization*: populations oscillate regularly and periodically in phase, but with irregular and uncorrelated chaotic peaks.



Foodwebs (that represent the interactions of vegetation and populations of herbivores and predators) can display very complex oscillatory behaviors.

B. Blasius et al, Nature 399, 354 (1999).

Lotka–Volterra predator–prey model (early 1900s)

Simplest version:

$$\frac{dx}{dt} = \alpha x - \beta x y,$$
$$\frac{dy}{dt} = \delta x y - \gamma y,$$

x is the number of prey (for example, rabbits);

y is the number of predators (for example, foxes).

Two equations $\Rightarrow$ only stable or periodic oscillations. Aperiodic (chaotic) behavior occurs when other variables or spatial effects are included in the model.

Role of noise in nonlinear systems? (80’ and early 90’)

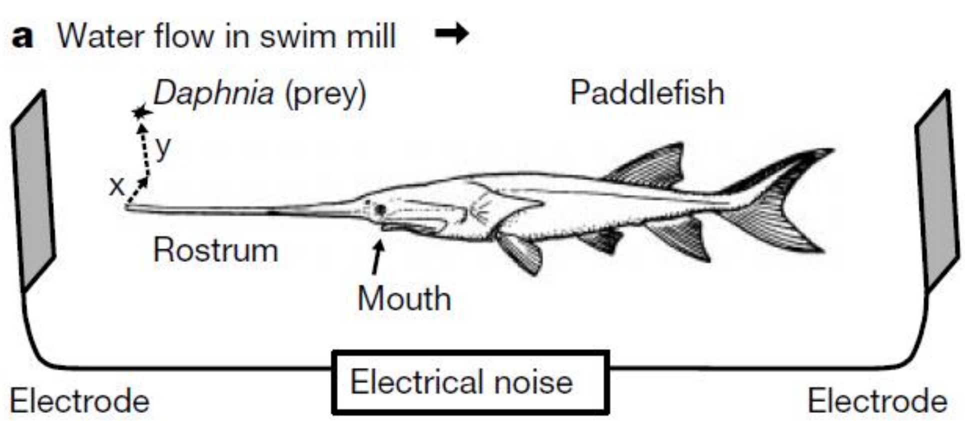
Stochastic resonance: the addition of an optimal level of noise to a weak input signal can, in some **nonlinear** systems, enhance the detection of the signal, improving the “output” performance of the system.

NATURE | VOL 402 | 18 NOVEMBER 1999 |

Use of behavioural stochastic resonance by paddle fish for feeding

David F. Russell, Lon A. Wilkens & Frank Moss

Center for Neurodynamics, University of Missouri at St. Louis, St Louis, Missouri 63121, USA



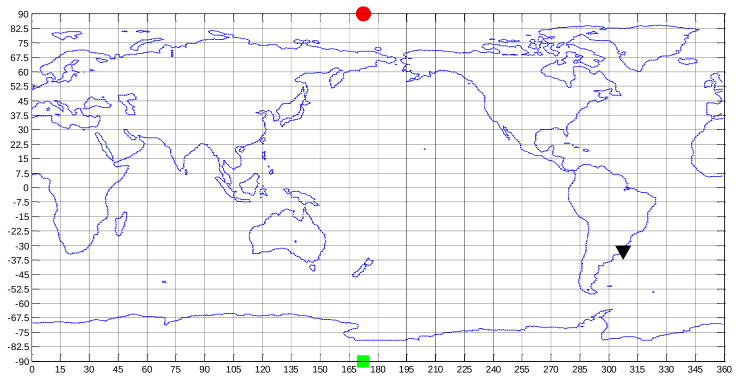
“We demonstrate significant broadening of the spatial range for the detection of plankton when a noisy electric field of optimal amplitude is applied in the water. We also show that swarms of Daphnia plankton are a natural source of electrical noise.”

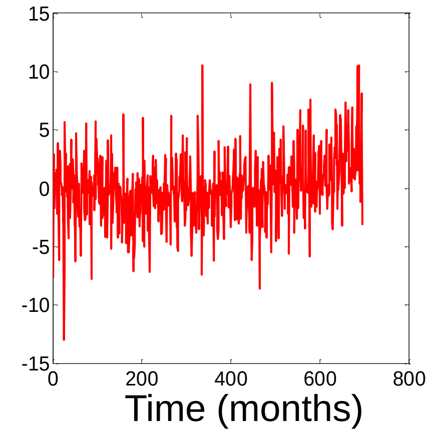
But what is “noise”?

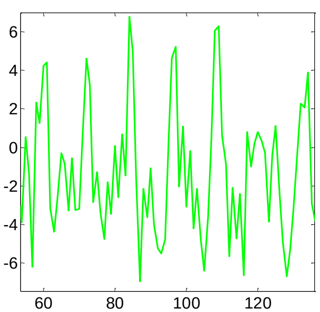
Let’s consider an example: monthly sampled surface air temperature (SAT).

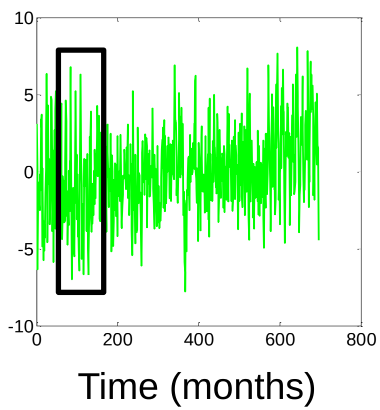
Anomaly = annual solar cycle removed

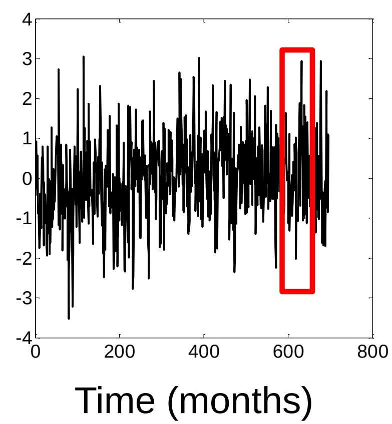
In each grid point we have a time series with 768 data points (1949-2013: 64 x 12). How does the data look like?

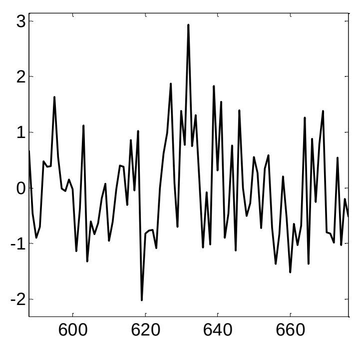












Where does the data come from?

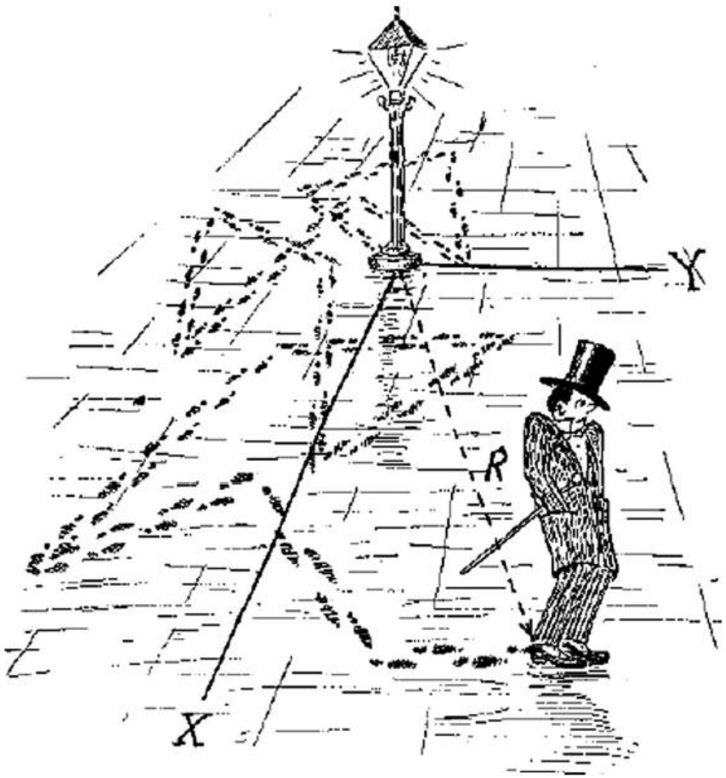
- Reanalysis of National Center for Environmental Prediction, National Center for Atmospheric Research (NCEP-NCAR).

- Reanalysis = run a sophisticated model of general atmospheric circulation and feed it with the available experimental data, in the different points of the Earth, at their corresponding times (*data assimilation: second block*).

- This process restricts the solution of the model to one as close to reality as possible in regions where there are data available, and to a solution physically “plausible” in regions where no data is available.

So, what is "noise"?

Someone's noise is another one's signal.
For a climatologist "weather" is noise.



Gaussian noise (uncorrelated, memory less) is well known, but many other types of noises have been discovered.

A main problem of time series analysis: find the signal (i.e., filter out noise, preprocess the signal).

Another problem: to quantify the degree of determinism (i.e., to distinguish "noise" from "chaos").

*Cartoon of a two-dimensional **random walk** or drunkard's walk.*
From Gamow (The Viking Press, New York,1955)

In the late 90s early 2000s: synchronization of a large number of coupled oscillators

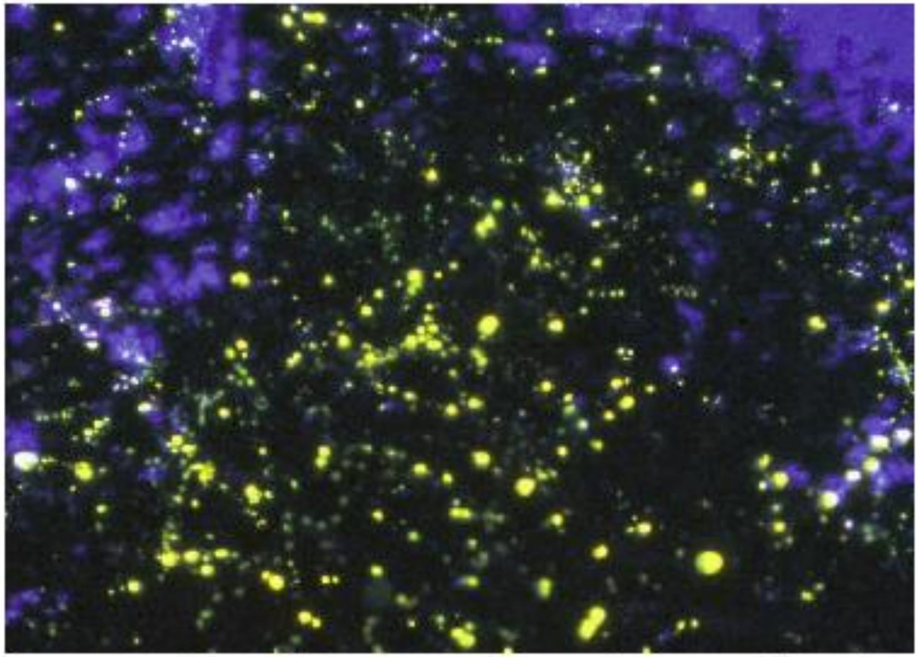


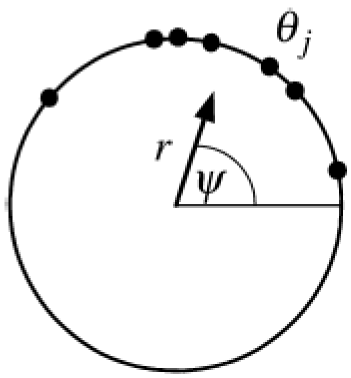
Figure 1 | Fireflies, fireflies burning bright. In the forests of the night, certain species of firefly flash in perfect synchrony — here *Pteroptyx malaccae* in a mangrove apple tree in Malaysia. Kaka *et al.*[2] and Mancoff *et al.*[3] show that the same principle can be applied to oscillators at the nanoscale.

London Millennium Bridge Opening

A model proposed some time ago is now a "classic": the Kuramoto model (Japanese physicist, 1975)

Model of **all-to-all** coupled **phase oscillators**.

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) + \xi_i, \qquad i = 1 \ldots N$$



K = coupling strength, ξ_i = stochastic term (noise)

Describes the emergence of collective behavior

How to quantify?

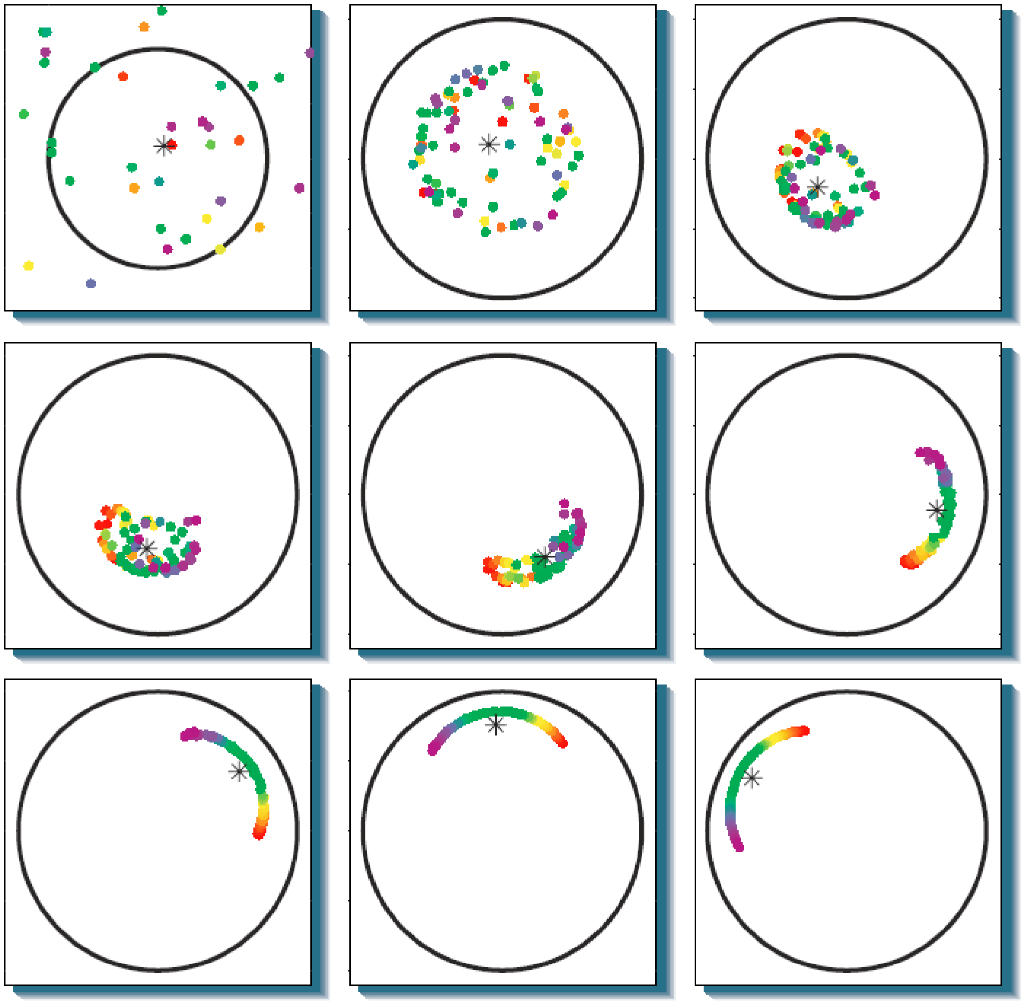
With the **order parameter**:

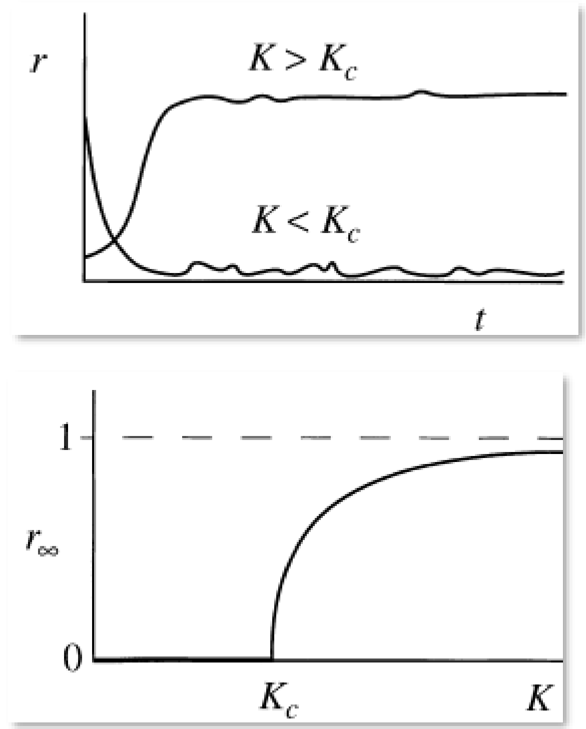
$$re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

r =0 incoherent state (oscillators scattered in the unit circle)

r =1 all oscillators are in phase ($\theta_i = \theta_j \ \forall\ i,j$)

Synchronization transition as the coupling strength increases



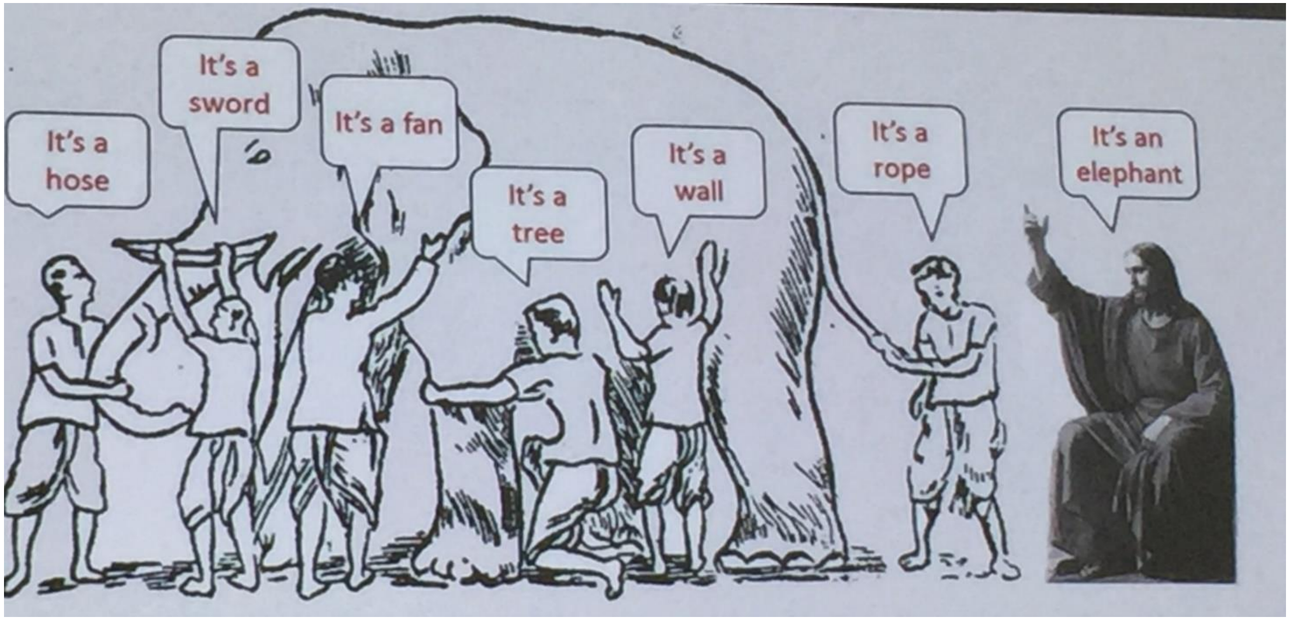


Strogatz, Nature 2001

Video: https://www.ted.com/talks/steven_strogatz_on_sync

2000s to present: from chaotic systems to complex systems

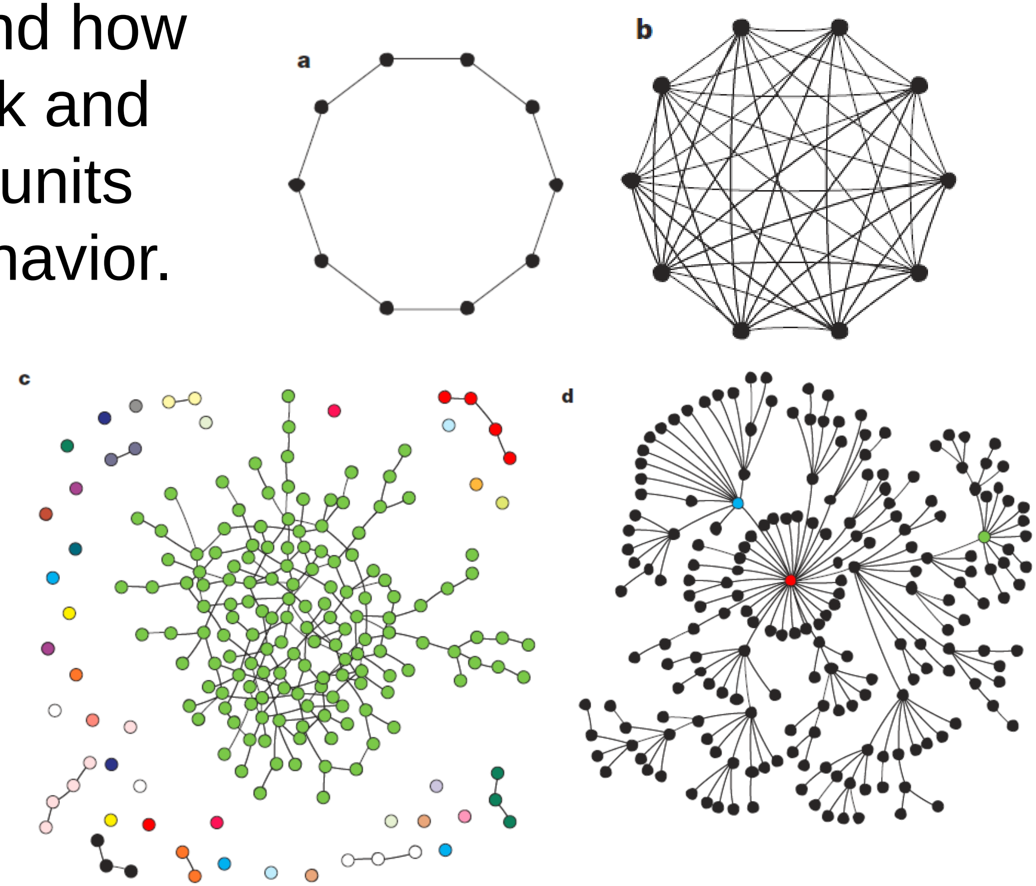
- Complicated systems (large sets of linear elements) are not complex.
- Complex systems: large number of elements, where the elements and/or their interactions are **nonlinear**.
- Main difference: *the whole is not equal to the sum of the parts*.



(a <u>good meal</u> is another example: it is much more than the sum of its ingredients)

Network science

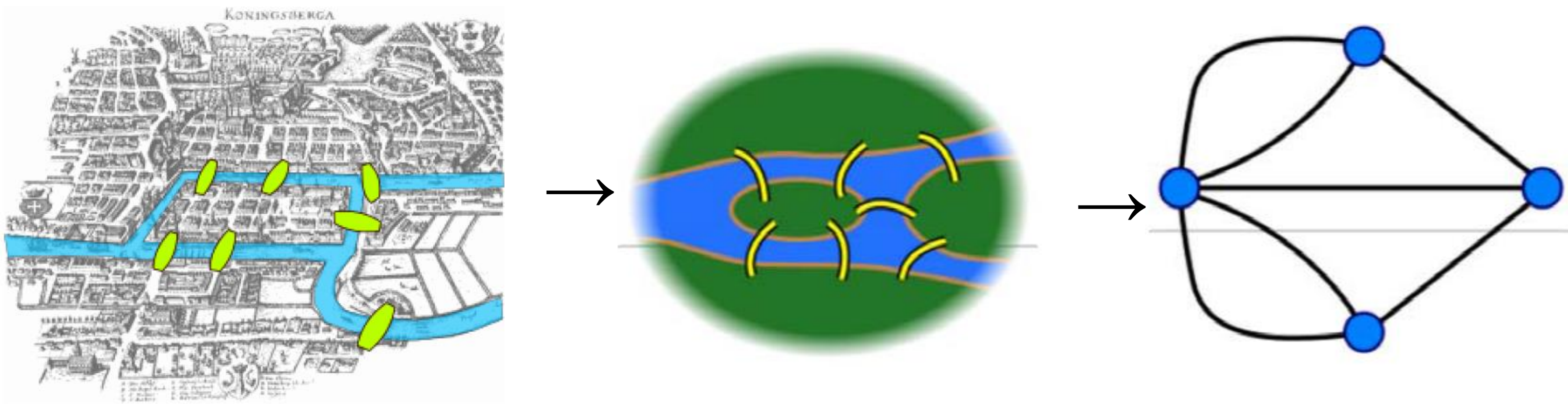
- **Networks** (or **graphs**) are used for mathematical modelling of complex systems.
- **Complexity science**: study of the emergent properties, not present in the individual elements.
- The challenge: to understand how the **structure** of the network and the **dynamics** of individual units determine the collective behavior.
- Applications
  - Epidemics
  - Rumor spreading
  - Transport networks
  - Financial crises
  - Brain, physiology, etc.



S. Strogatz, Nature 2001

The start of Graph Theory: The Seven Bridges of Königsberg
(Prussia, now Russia)

- The problem was to devise a walk through the city that would cross each of those bridges once and only once.



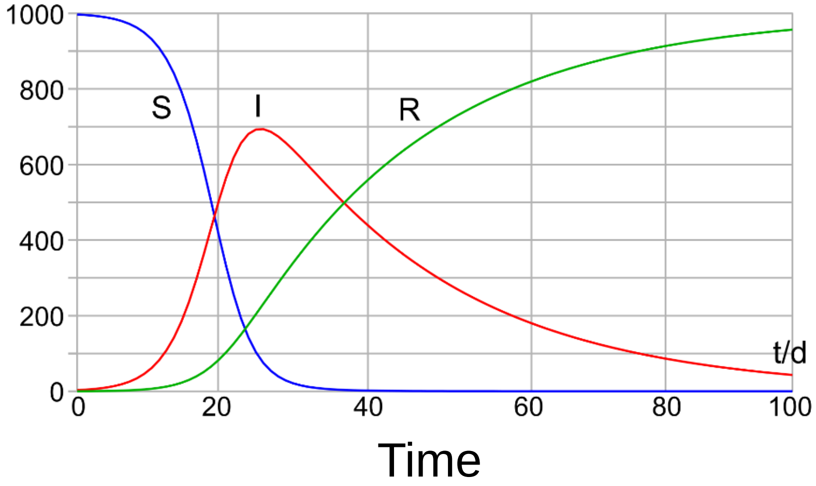
Source: Wikipedia

- By considering the number of odd/even links of each “node”, **Leonhard Euler** (Swiss mathematician) demonstrated in 1736 that is impossible.

The SIR epidemic model (early 1900s)

In its simplest version the SIR model consists of three rate equations for

- S(t): individuals not yet infected (susceptible).
- I(t): infected individuals that are capable of spreading the disease to those susceptible.



- R(t): individuals that have been infected and can’t be re-infected nor transmit the infection to others (either due to immunization or due to death).
- N = S(t) + I(t) + R(t) constant.
- The model predicts the existence of a threshold that separates grow from extinction.

Source: Wikipedia

Many extensions of the SIR model

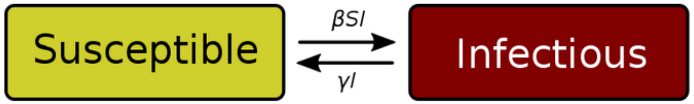
- Immunity that lasts only a certain time interval (after which individuals are back in the susceptible group).
- Additional populations
  - E: exposed people that could have been infected;
  - C: susceptible people that are protected in a confinement compartment;
  - Q: infected people in quarantine;
  - B, D: births and deaths
  - Etc.
- Many extensions of the model to take into account **diffusion in “networks”.**

https://www.investigacionyciencia.es/revistas/investigacion-y-ciencia/una-crisis-csmica-798/cmo-modelizar-una-pandemia-18561?utm_source=Facebook&utm_medium=Social&utm_campaign=fb+web

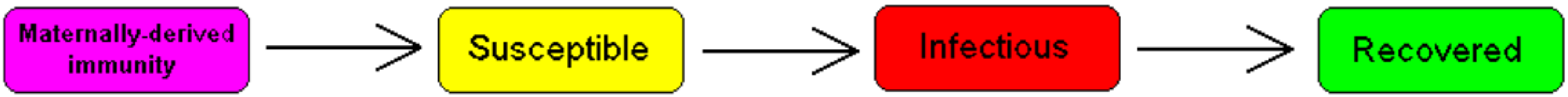
Pastor-Satorras et al, Rev. Mod. Phys. 87, 925 (2015)

A few examples of epidemic models

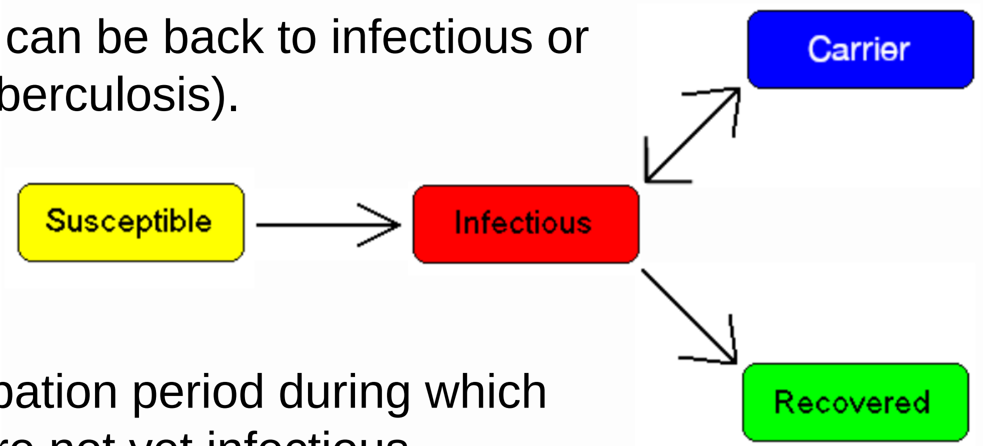
SIS: No long lasting immunity (example: cold).



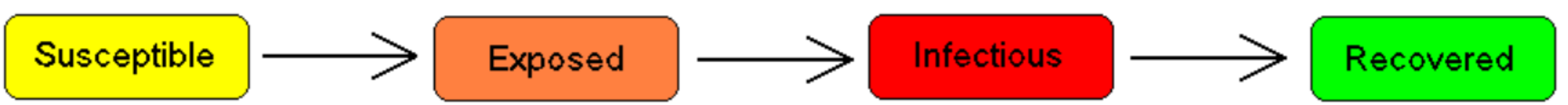
MSIR: Babies have some initial immunity.



Some people might not recover and can be back to infectious or carry disease with symptoms (ex: tuberculosis).

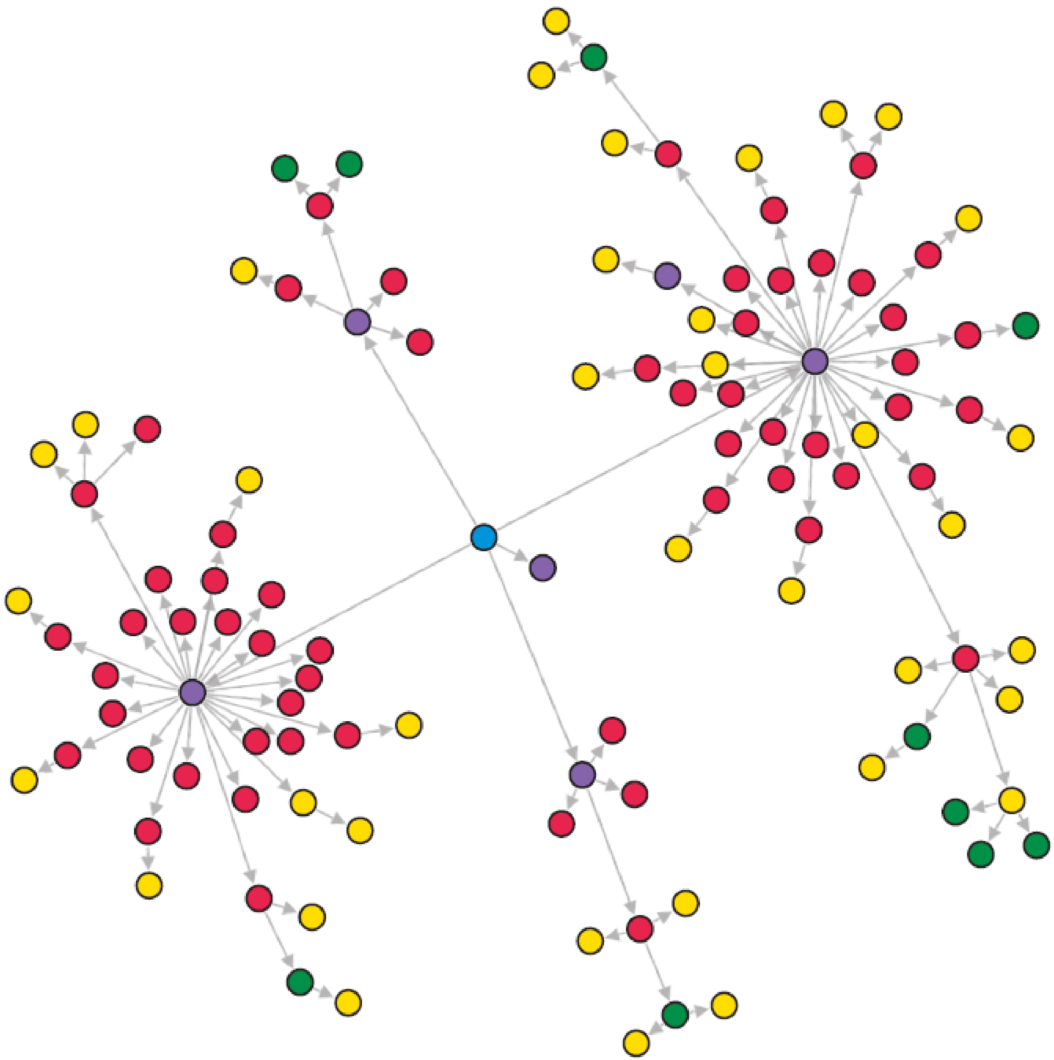


For some infections there is an incubation period during which individuals have been infected but are not yet infectious.



Source: Wikipedia

Example of transmission network of Covid-19



Transmission network seeded by an unknown infected individual (**blue**) who attended a training course with other fitness instructors (**purple**).

The fitness instructors spread the infection to students in their classes (**red**), to family (yellow), and to coworkers (**green**).

Source: Alison Hill, The math behind epidemics, https://physicstoday.scitation.org/doi/10.1063/PT.3.4614

Revisiting the Kuramoto model

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_j \sin(\theta_j - \theta_i) + \xi_i \quad \Rightarrow \quad \boxed{\frac{d\theta_i}{dt} = \omega_i + \sum_j A_{ij} G(\theta_i, \theta_j) + \xi_i}$$

Different synchronization regimes can occur, depending on:

- The coupling function (attractive / repulsive).
- The network topology (homogeneous / heterogeneous).
- The number of units ("crowd synchrony")
- The properties of the individual units, in relation to the network:
  - relation between the # of links an element has and the # of links the neighbors have.
  - relation between the # of links that an element has and its properties.
- The synchronization transition can be gradual or explosive.
- Synchronized and unsynchronized oscillations can co-exist ("chimera states").
- Bi-stability: the network can synchronize, depending on the initial conditions.

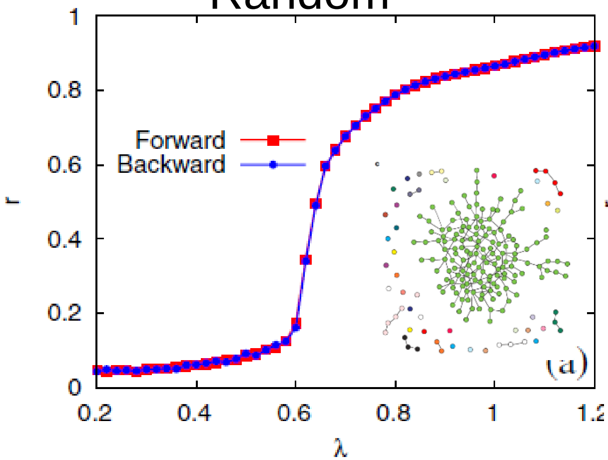
Example: “Explosive” synchronization

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j=1}^{N} A_{ij} \sin(\theta_j - \theta_i)$$

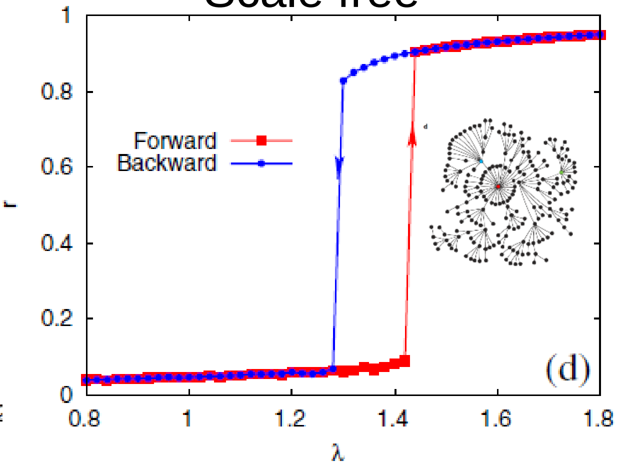
$$\omega_i = k_i = \sum_{j=1}^{N} A_{ij}$$

Fast oscillators have many links; slow oscillators only few links

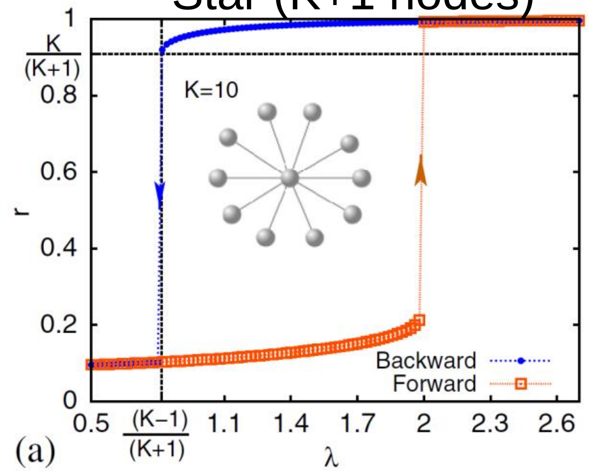
Random



Scale free



Star (K+1 nodes)



Explosive sync. has been found in coupled lasers and in electronic circuits.

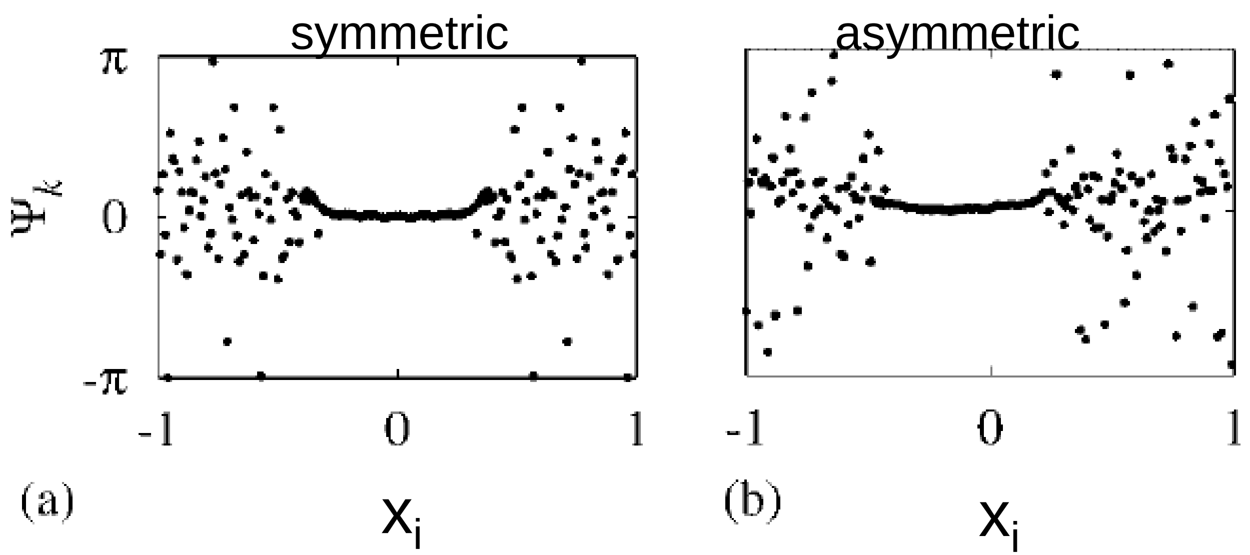
ω_1=K
ω_i=1 i≠1

J. Zamora et al., Phys. Rev. Lett. 105, 264101 (2010).
J. Gomez-Gardeñes et al., Phys. Rev. Lett. 106, 128701 (2011).
I. Leyva et al, Phys. Rev. Lett. 108, 168702 (2012).

“Chimera states”: spatiotemporal patterns in which coherence coexists with incoherence

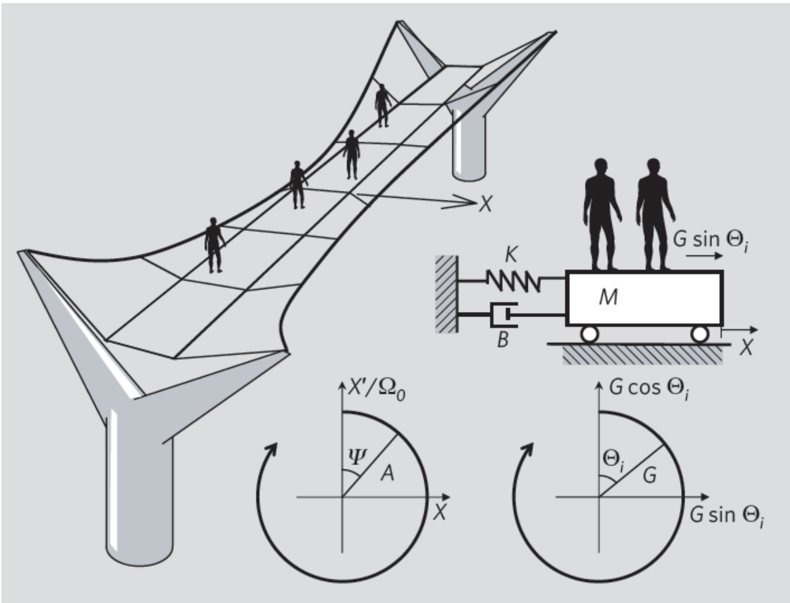
$$\dot{\theta}_i = \frac{1}{N}\sum G_{ij}\sin(\theta_j - \theta_i - \alpha)$$

- N identical oscillators (ω_i=0) with spatial coordinates x_i that are uniformly distributed in the interval (-1, 1).
- G nonlocal positive coupling: $G_{ij} = 1 + r\cos\pi\left(x_i - x_j\right)$
- α (“frustration parameter”) <≈ π/4



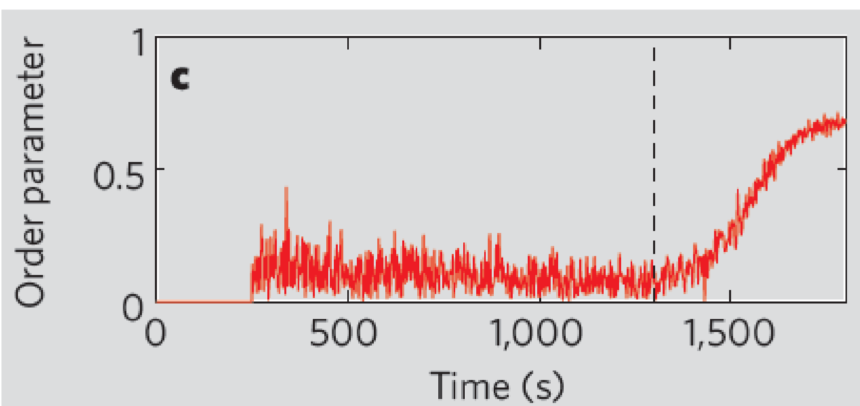
Omel’chenko et al, Phys. Rev. E 81, 065201R (2010).

"Crowd synchrony": the millennium footbridge starts to sway when packed with pedestrians that synchronize their steps.



Model the bridge as a weakly damped and driven harmonic oscillator:

$$M\frac{d^2X}{dt^2} + B\frac{dX}{dt} + KX = G\sum_{i=1}^{N}\sin\Theta_i$$

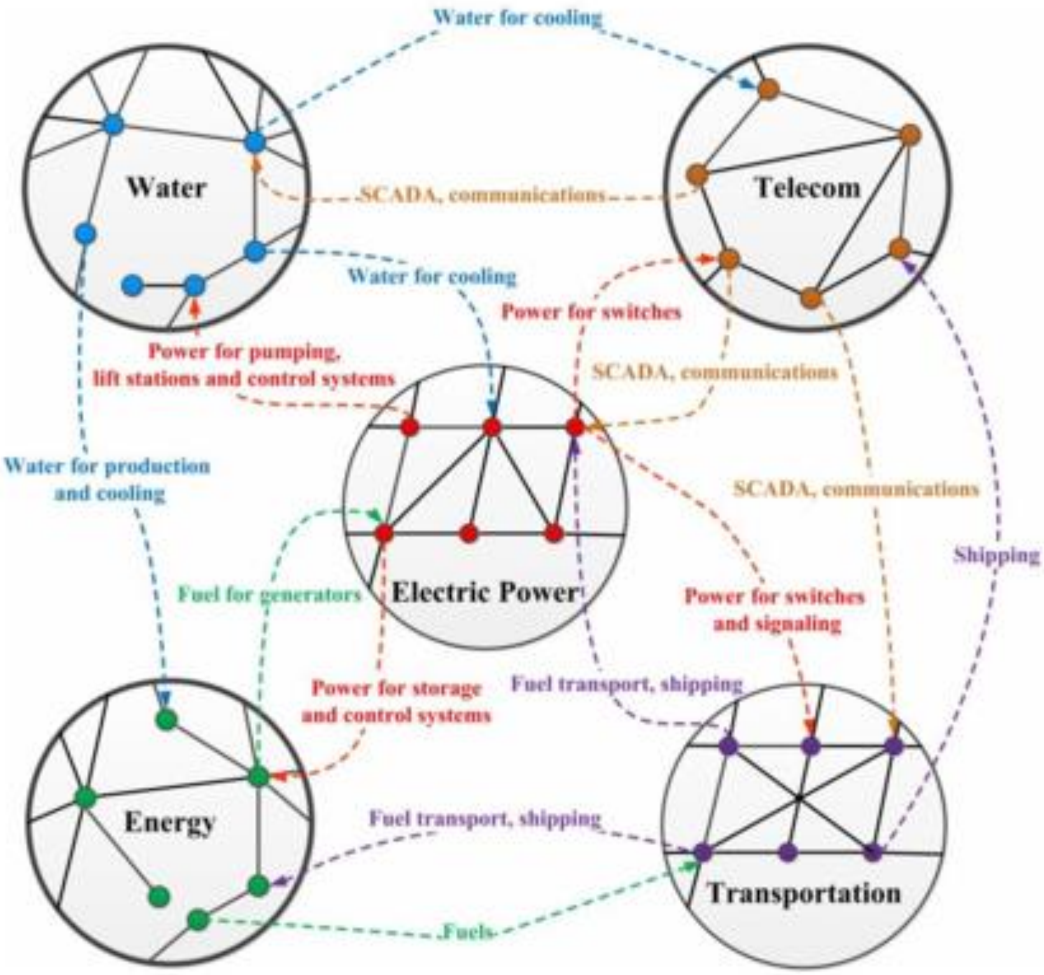


The bridge's movement alters each pedestrian's gait:

$$\frac{d\Theta_i}{dt} = \Omega_i + CA\sin(\Psi - \Theta_i + \alpha)$$

S. Strogatz, Nature 438, 43 (2005).

Interactions between networks: interdependent networks



Time series analysis problems: how to predict a critical (or extreme) event in one network? (a failure of a link or a node) How will it affect other networks?

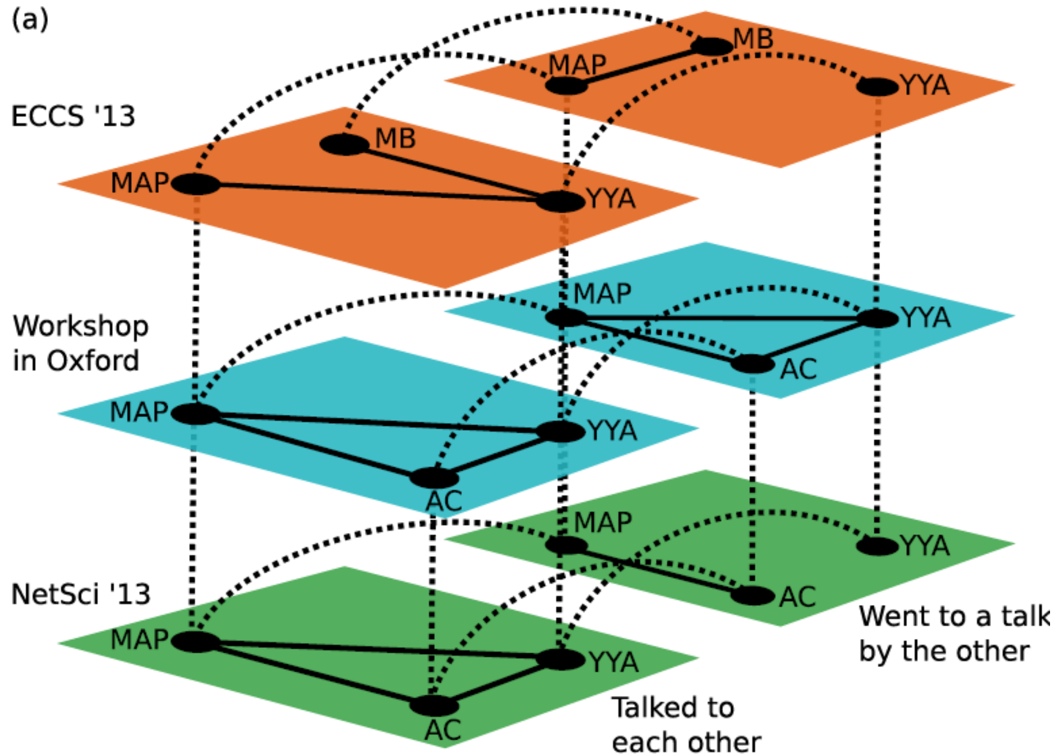
Source: Wikipedia

Multilayer networks

Facebook

Twitter

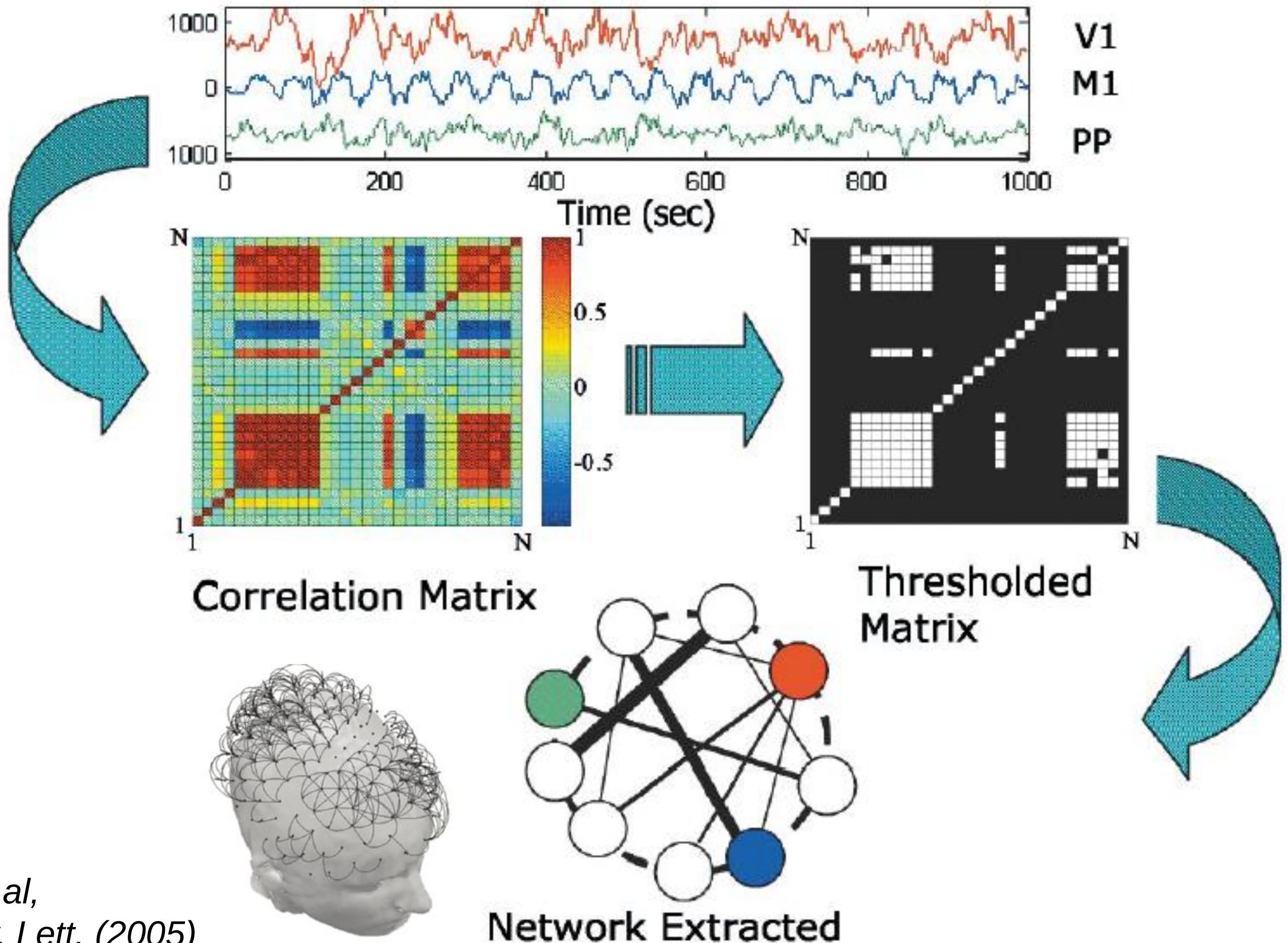
Linkedin



Time series analysis problem: how to predict the existence of a link?

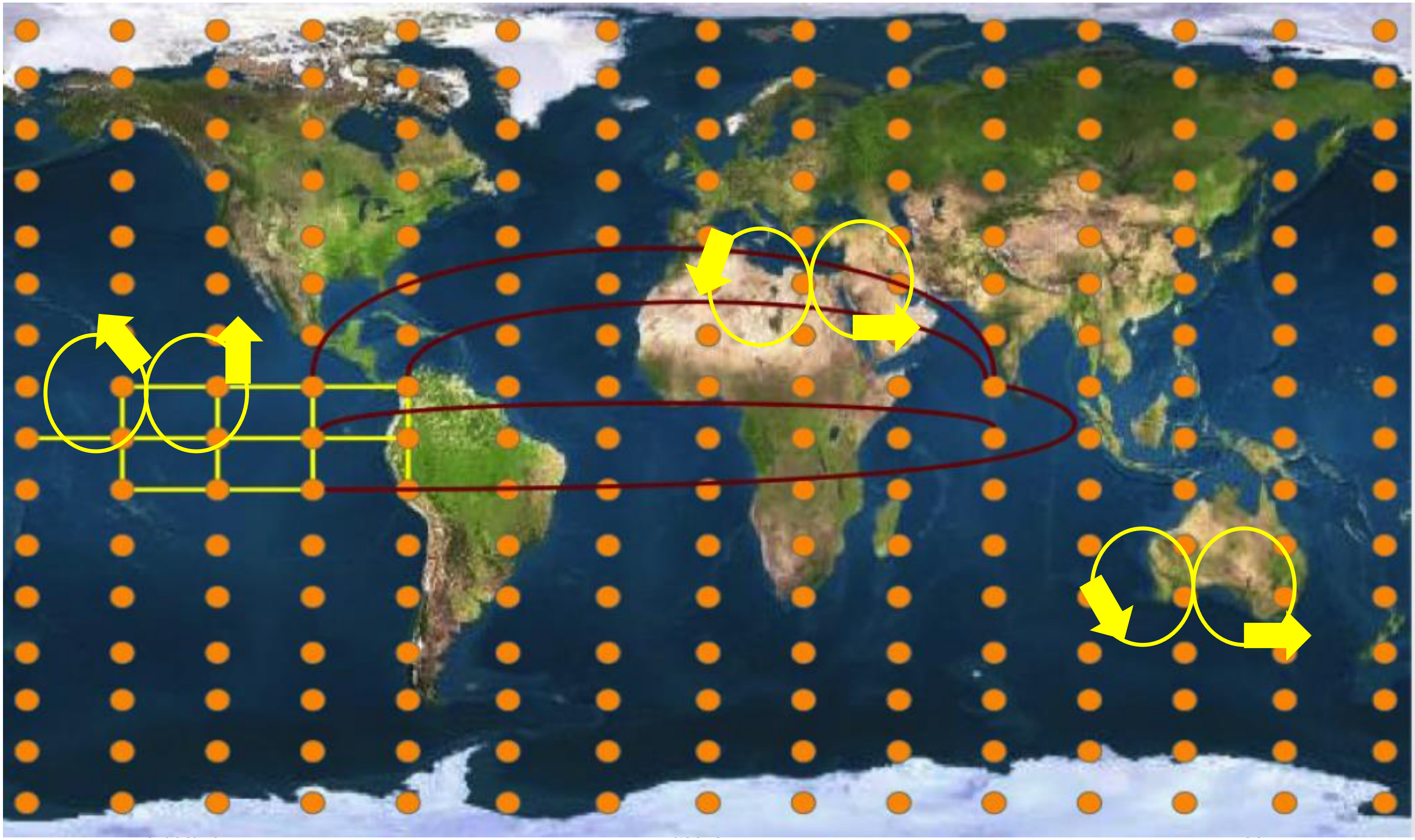
Kivela et al, J. Complex Netw. 2, 203 (2014).

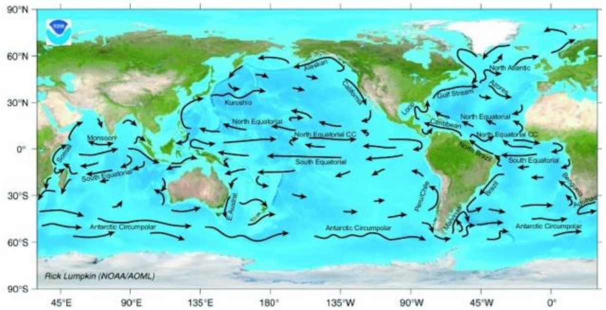
“Functional networks”: inferred from “bivariate” analysis of time series recorded in the nodes

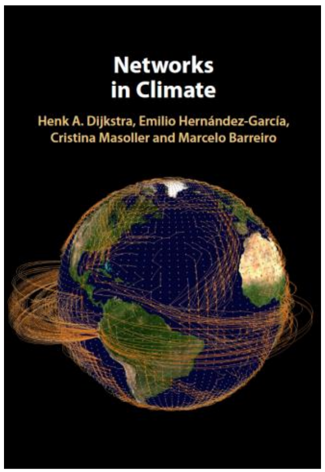


Eguiluz et al,
Phys. Rev. Lett. (2005)

Same procedure used to analyze climate data ⇒ climate network

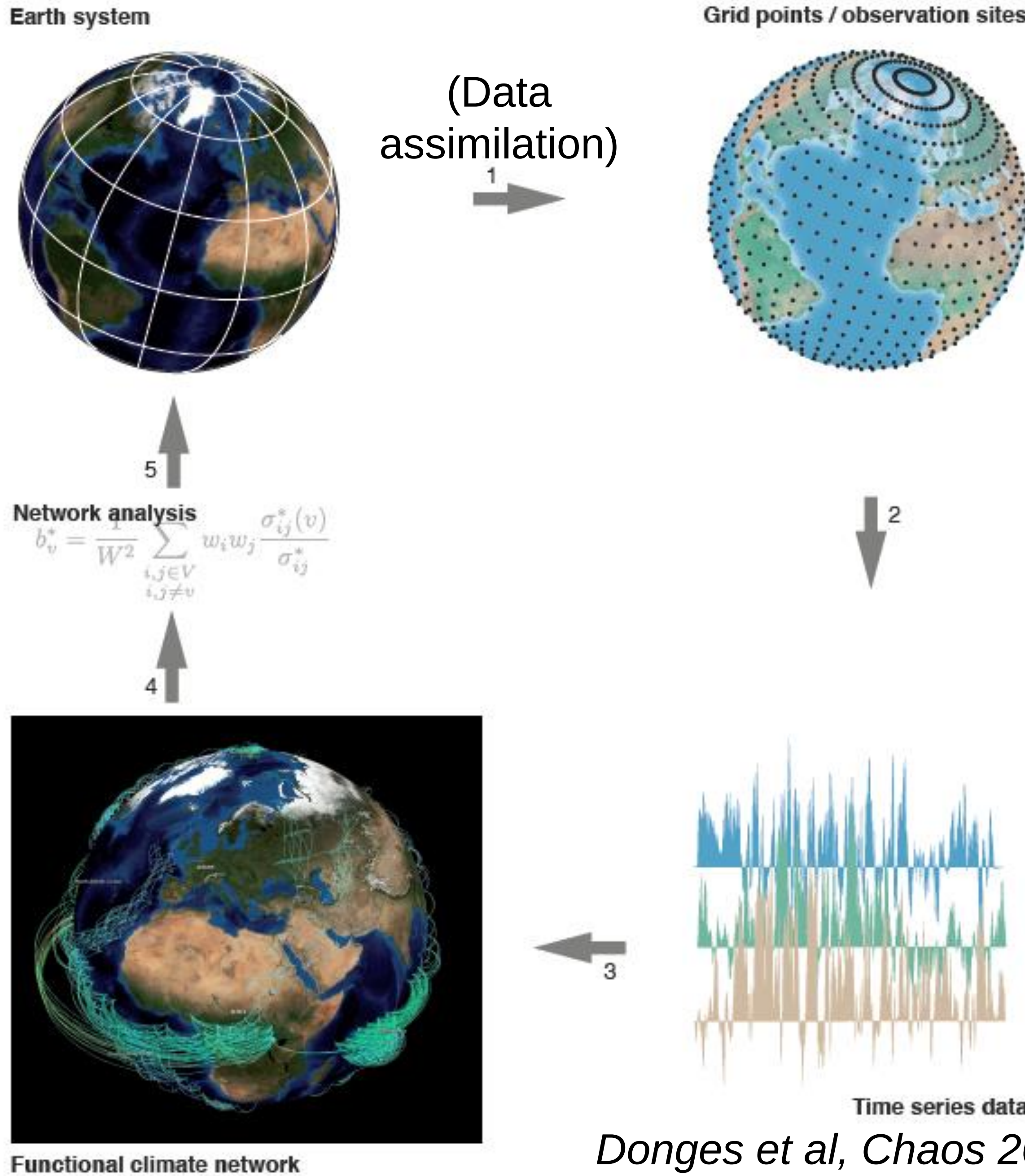






Cambridge University Press 2019

(Data assimilation)



Donges et al, Chaos 2015

Time series analysis + complex systems ⇒ Big Data

- For a given time series, by using different methods of analysis we can a large number of “features”, *M*.

- Examples of “features”:
  - statistical properties (mean value, standard deviation, etc.),
  - Fourier properties (main frequencies),
  - fractal dimension, Lyapunov exponent, etc. etc.

- If we have a large set of time series to analyze (*N*), we end up with a huge number of features (*N x M*).

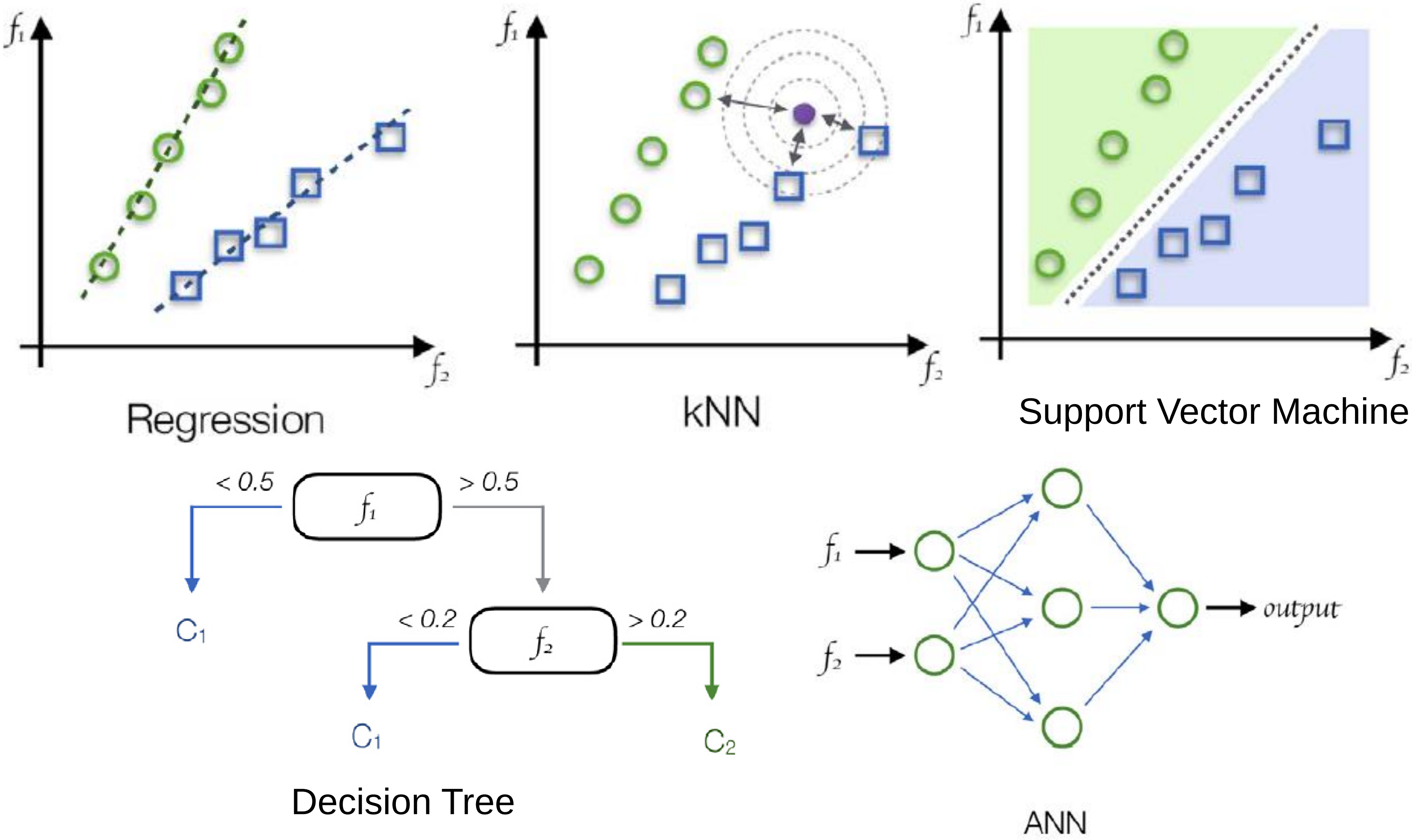
What is “Big data”?

- Is a field that treats ways to analyze, systematically extract information from, or otherwise deal with data sets that are too large or complex to be dealt with by traditional data-processing application software (Wikipedia)

- It seeks to identify complex and evolving relationships among data.

- How? “Data mining”: the process of finding anomalies, patterns and correlations within large data sets.

Five "data mining" classification algorithms *(second block)*



M. Zanin et al, Physics Reports 635, 1 (2016).

Summary

- Dynamical systems allow to
  - understand low-dimensional systems,
  - uncover patterns and “order within chaos”,
  - characterize attractors, uncover universal features
- Synchronization emerges in interacting dynamical systems.
- Complexity and network science: phenomena in large sets of nonlinear interacting units.
- Time series analysis develops methods to characterize signals and to obtain “features”.
- Data science: feature selection and analysis.
- Time series analysis is an interdisciplinary field with many applications.

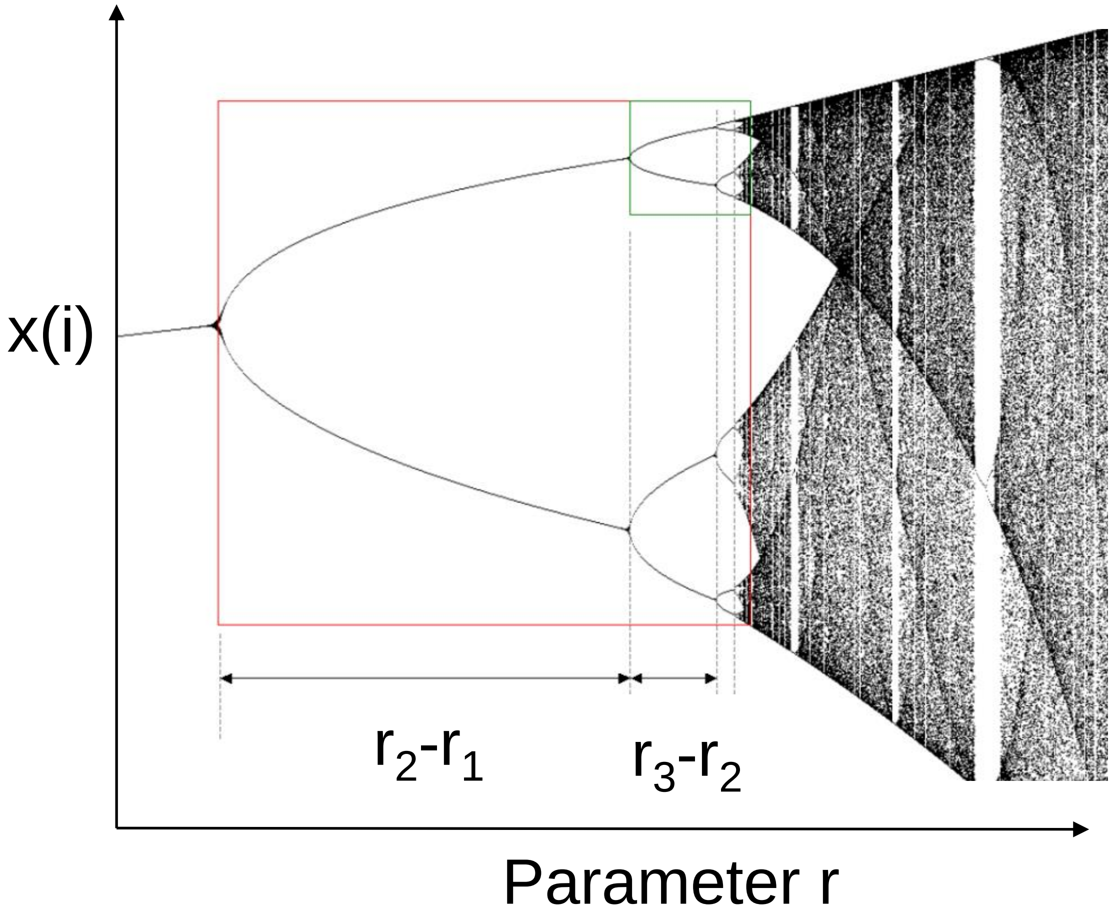
Summary of some relevant time series analysis problems

- “Reconstruct” the phase space of a low-dimensional dynamical system from (incomplete) observed data.
- Is the signal just noise? Has a degree of determinism?
- Can the signal be predicted? Which is the prediction horizon?
- Is the system approaching a dynamical transition (“tipping point”)?
- Are two (or more) oscillatory systems synchronized?
- Are there time delays in the interactions? Feedback loops?
- Are systems interdependent? How extreme events in one system propagate to other systems?

Holger Kantz: “*Every data set bears its own difficulties: data analysis is never routine*”

Hands-on exercise 1: analyze the logistic map

$$x(i+1) = r\,x(i)[1 - x(i)]$$



- Plot the bifurcation diagram.
- Estimate $\delta = (r_2 - r_1)/(r_3 - r_2)$

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