

# **EDM and CPV in the $\tau$ system**

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# Outline

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## 1. Electric Dipole Moment

### 1.1 Definition

$$d_f^B$$

$B = \gamma, Z, g, \dots$

$f = e, \mu, \tau, \dots, b, t$

### 1.2 Experiments

## 2. Observables

### 2.1 High energies

### 2.2 Low energies

## 3. Conclusions

# 1. EDM

## 1.1 Definition

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P and T-odd interaction of a fermion with gauge fields:

Classical electromagnetism       $H_{EDM} = -\vec{d} \cdot \vec{E}$ ,       $\vec{d} = d \vec{\sigma}$   
Ordinary quantum mechanics

Relativistic quantum mechanics: Dirac equation

$$H = \bar{\Psi} (i(\partial + eA) - m) \Psi + \frac{i}{2} d \bar{\Psi} \gamma^5 \sigma^{\mu\nu} \Psi F_{\mu\nu}$$

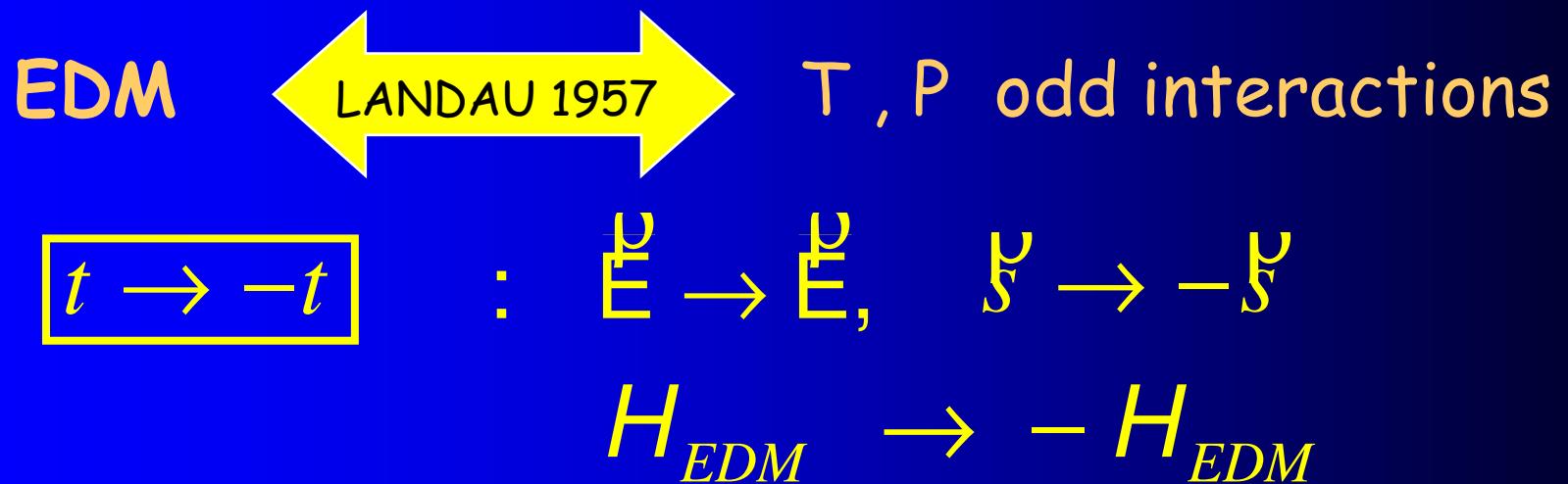
Non-relativistic limit:

$$H \rightarrow H_{EDM} = -\vec{d} \cdot \vec{E}$$

# 1. EDM

## 1.1 Definition

**SYMMETRIES:** Time reversal T, Parity P  
Besides, chirality flip (some insight into the mass origin)



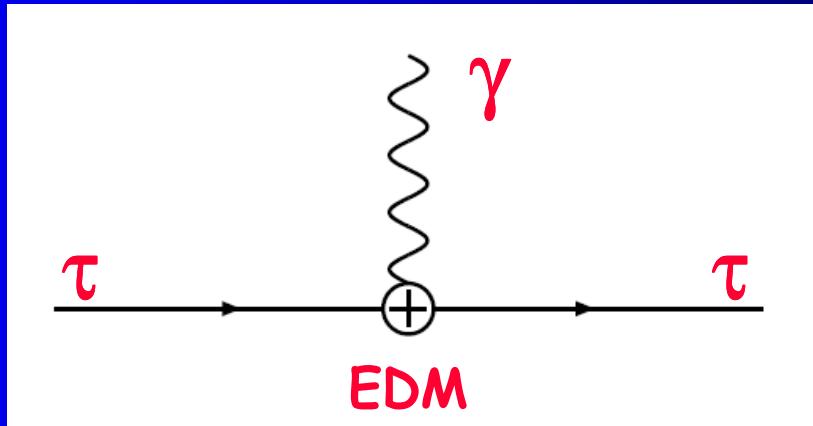
In QFT: CPT Invariance

$$\mathcal{CP} \approx \mathcal{T}$$

# 1. EDM

## 1.1 Definition

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SM :

- vertex corrections
- at least 4-loops

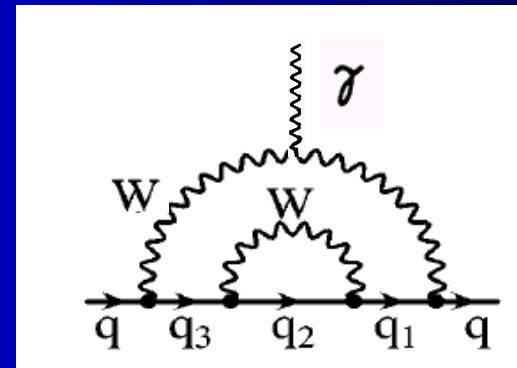
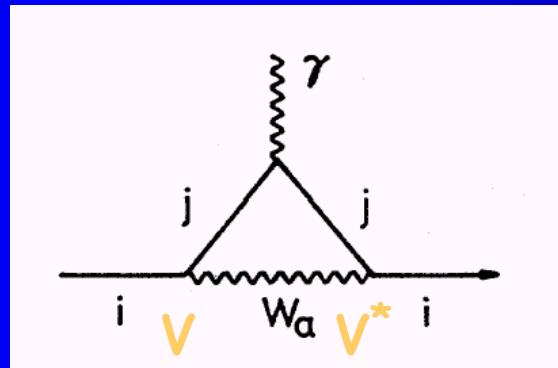
Beyond SM:

- one loop effect (SUSY, 2HDM, ...)
- dimension six effective operator

# 1. EDM

## 1.1 Definition

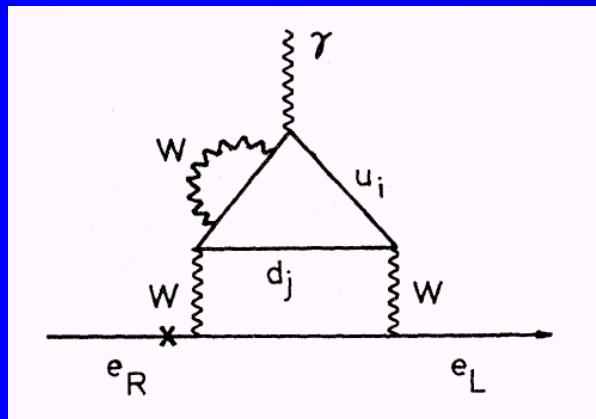
SM :



= 0 !!!

E.P. Shabalin '78

We need 3-loops for a quark-EDM, and 4 loops for a lepton...



$$d_e \simeq e G_F m_e \alpha^2 \alpha_s J / (4\pi)^5$$

J.F. Donoghue '78

I.B. Khriplovich, M.E. Pospelov '90

A.Czarnecki, B.Krause '97

# 1. EDM

## 1.1 Definition

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Fermion of mass  $m_f$  generated by physics at  $\Lambda$ - scale has

$$\text{EDM} \approx m_f/\Lambda^2$$

$T$ -odd  $T$ -EDM has particular interest and depends on the underlying physics of  $CP$  violation

# 1. EDM

## 1.1 Definition

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**Effective Lagrangian:**

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \frac{1}{\Lambda^2} \sum_i C_i O_i + \mathcal{O}\left(\frac{1}{\Lambda^4}\right),$$

**CPV:**

$$\mathcal{L}_{eff} = i\alpha_B \mathcal{O}_B + i\alpha_W \mathcal{O}_W + \text{h.c.}$$

**Operators:**

$$\mathcal{O}_W = \frac{g}{2\Lambda^2} \bar{L}_L \vec{\tau} \varphi \sigma_{\mu\nu} \tau_R \vec{W}^{\mu\nu}$$

$$\mathcal{O}_B = \frac{g'}{2\Lambda^2} \bar{L}_L \varphi \sigma_{\mu\nu} \tau_R B^{\mu\nu}$$

**Other operators:**

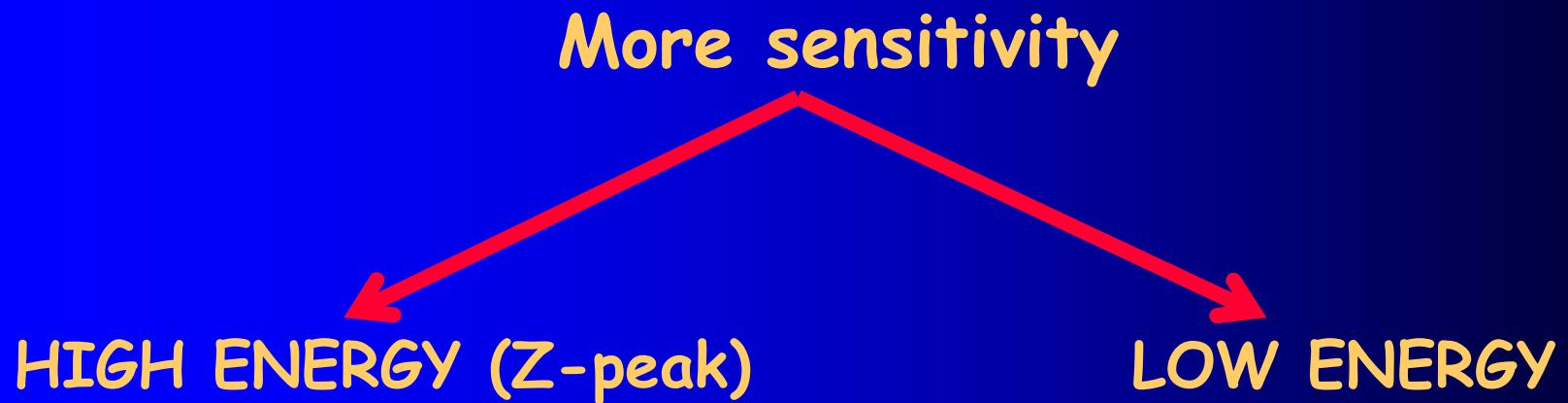
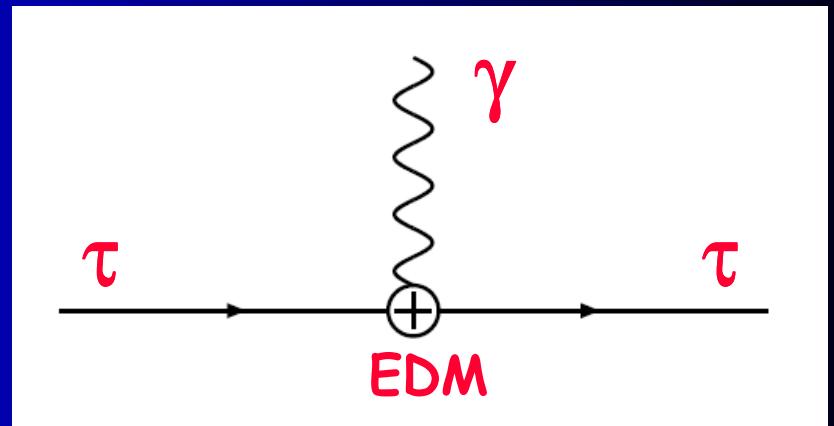
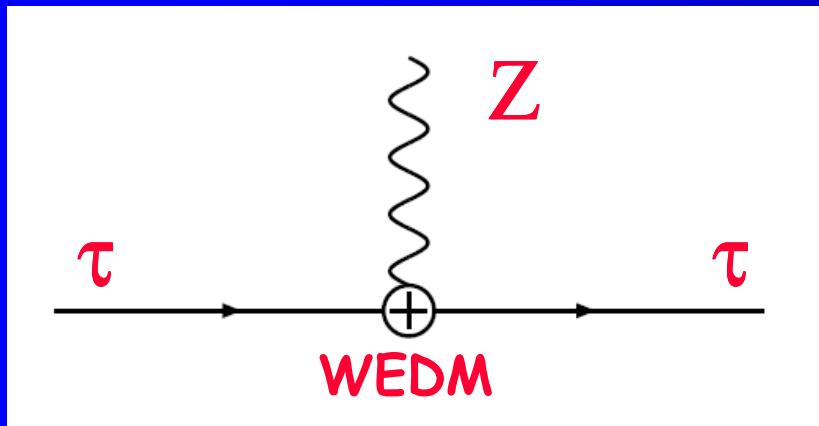
$$O_{LW} = i[\bar{L} \gamma^\mu \tau^I D^\nu L - \bar{D}^\nu L \gamma^\mu \tau^I L] W_{\mu\nu}^I,$$

$$O_{LB} = i[\bar{L} \gamma^\mu D^\nu L - \bar{D}^\nu L \gamma^\mu L] B_{\mu\nu},$$

$$O_{\tau B} = i[\bar{\tau}_R \gamma^\mu D^\nu \tau_R - \bar{D}^\nu \tau_R \gamma^\mu \tau_R] B_{\mu\nu},$$

# 1. EDM

## 1.1 Definition



# 1. EDM

## 1.1 Definition

$$\Gamma_5^\mu(p_-, p_+) = ie \left[ \gamma^\mu \gamma^5 a_V(q^2) + \sigma^{\mu\nu} \gamma^5 q_\nu d_V(q^2) \right]$$

V = γ, Z

For  $q^2=0$  EDM

$$d_\gamma^\tau$$

GAUGE INVARIANT  
OBSERVABLE QUANTITIES

$q^2=M_Z^2$  WEDM

$$d_Z^\tau$$

Beyond SM EDM: Loop calculations

# 1. EDM 1.2 Experiments

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PDG '06 95% CL

EDM BELLE '02

$$\text{Re}(d_\gamma^\tau) : (-2.2 \text{ to } 0.45) \times 10^{-16} \text{ e cm}$$

$$\text{Im}(d_\gamma^\tau) : (-0.25 \text{ to } 0.008) \times 10^{-16} \text{ e cm}$$

WEDM ALEPH 1990-95 LEP runs

$$| \text{Re}(d_Z^\tau) | \leq 0.50 \times 10^{-17} \text{ e cm}$$

$$| \text{Im}(d_Z^\tau) | \leq 1.1 \times 10^{-17} \text{ e cm}$$

# 1. EDM 1.2 Experiments

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For other fermions....

$$d_\gamma^e = (0.069 \pm 0.074) \times 10^{-26} \text{ e cm}$$

$$d_\gamma^\mu = (3.7 \pm 3.4) \times 10^{-19} \text{ e cm}$$

$$d_\gamma^n < 0.63 \times 10^{-25} \text{ e cm, } 90\% \text{ CL}$$

## 2. Observables

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SM for EDM is well below within present experimental limits:

$$d_\gamma^q \approx 10^{-32} - 10^{-34} \text{ ecm}$$

CKM 3-loops

$$d_\gamma^e \approx 10^{-38} \text{ ecm}$$

SM prediction

CKM 4-loops

Hopefully

$$d_\gamma^\tau \approx \frac{m_\tau}{m_e} d_\gamma^e \approx 10^{-33} - 10^{-34} \text{ ecm}$$

in the SM

...15 orders of magnitude below experiments...

## 2. Observables

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Currents limits on electron EDM gives:

$$d_\gamma^\tau < 2.4 \times 10^{-24} \text{ ecm}$$

However, in many models the EDM do not necessarily scale as the first power of the masses.

Multihiggs models 1-loop contributions

Vectorlike leptons

EDM scale as the cube of the mass!

BSM  $\tau$ -EDM can go up to  $10^{-19}$  e cm

## 2. Observables

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Non-vanishing signal in a  $\tau$ -EDM observable



NEW PHYSICS

One expects:

$$d_\gamma^\tau \approx e \frac{m_\tau}{\Lambda^2}$$

special interest in  
heavy flavours

$\tau$ -EDM: In order to be able to measure a genuine non-vanishing signal one has to deal with a  $\cancel{CP}$ -observable

## 2. Observables    2.1 High energies

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Indirect arguments:

$$\left[ \frac{\Delta\sigma(e^+e^- \rightarrow \tau^+\tau^-)}{\Delta\Gamma(Z \rightarrow \tau^+\tau^-)} \right] \approx |d_Z^\tau|^2$$

any other physics may  
also contribute...

→ Look for  $\not{P}$ ,  $\not{CP}$  linear effects

Genuine CPV observables in  $\tau$ -pair production:

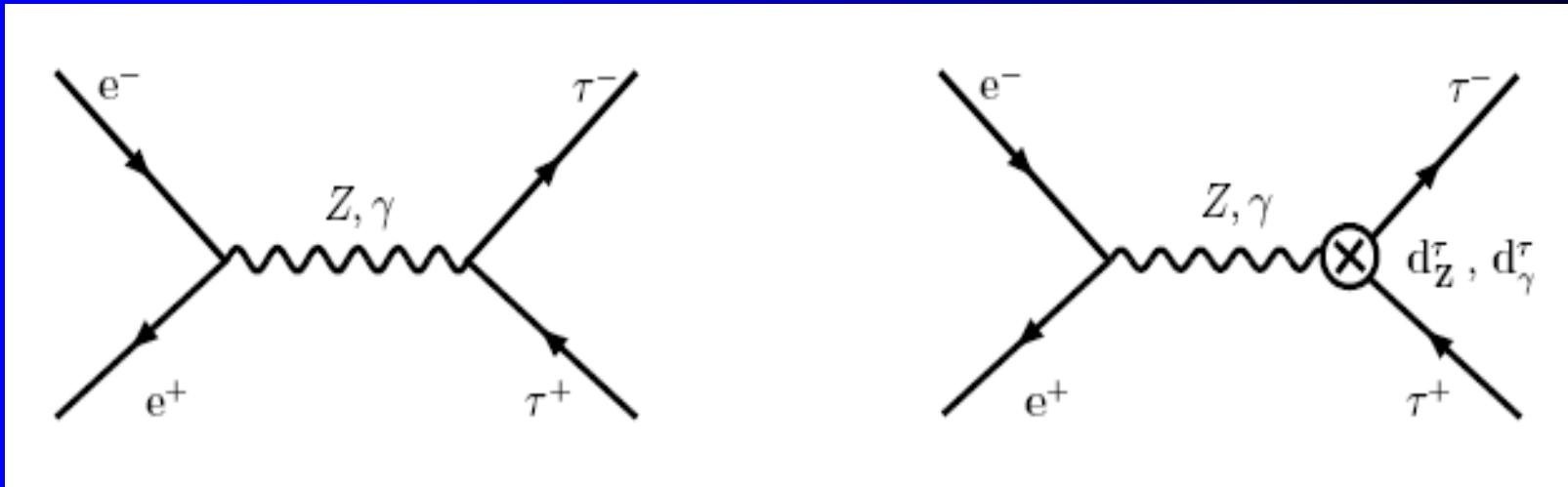
SPIN TERMS

and/or

SPIN CORRELATIONS

## 2. Observables

### 2.1 High energies



Spin terms



angular distribution of decay products

*CP*

- Asymmetries in the decay products
- Expectation values of tensor observables

# 2. Observables

## 2.1 High energies

Polarizations	P	CP	T
$(\mathbf{s}_1 + \mathbf{s}_2)_{x,z}$	-	+	+
$(\mathbf{s}_1 + \mathbf{s}_2)_y$	+	+	-
$(\mathbf{s}_1 - \mathbf{s}_2)_y$	+	-	-
$(\mathbf{s}_1 - \mathbf{s}_2)_{x,z}$	-	-	+

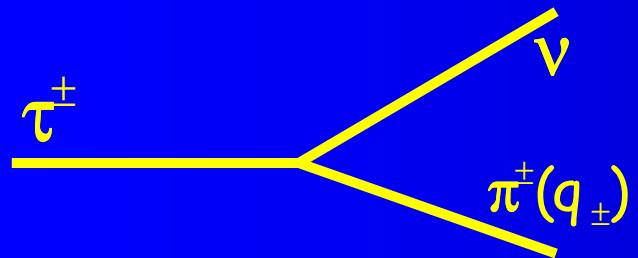
Correlations	P	CP	T
$s_{xx}, s_{yy}, s_{zz},$ $(s_{xz} + s_{zx})$	+	+	+
$(s_{xy} + s_{yx}),$ $(s_{yz} + s_{zy})$	-	+	-
$(\mathbf{s}_1 \times \mathbf{s}_2)_{x,z}$	-	-	-
$(\mathbf{s}_1 \times \mathbf{s}_2)_y$	+	-	+

x: Transverse y: Normal z: Longitudinal

## 2. Observables 2.1 High energies

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Tensor observables:  $T_{ij} = (q_+ - q_-)_i (q_+ \times q_-)_j + (i \leftrightarrow j)$



Z-peak:  $\langle T_{ij} \rangle_{\pi\pi} \approx \frac{m_Z}{e} c_{\pi\pi} s_{ij} d_Z^\tau$

W.Bernreuther,O.Nachtmann '89

Normal polarization:  $P_N^\tau \leftrightarrow T\text{-odd}, P\text{-even}$

(and needs helicity-flip)

Genuine CPV if

$$P_N^{\tau^+} \leftrightarrow P_N^{\tau^-}$$

J.Bernabeu,GGS,J.Vidal '93

## 2. Observables

### 2.1 High energies

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$$P_N^\tau \propto a\gamma\beta\sin\theta_\tau [2v^2 + (v^2 + a^2)\beta\cos\theta_\tau] m_\tau \frac{d_\tau}{e}$$

EDM (and  $P_N^\tau$ ) is proportional to angular asymmetries,  
to extract  $\sin\theta_\tau \cos\theta_\tau \sin\phi_h$

$$A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \quad (\text{more details latter..})$$

One can measure  $A$  for  $\tau^+$  and/or  $\tau^-$

$\mathcal{CP}$ :

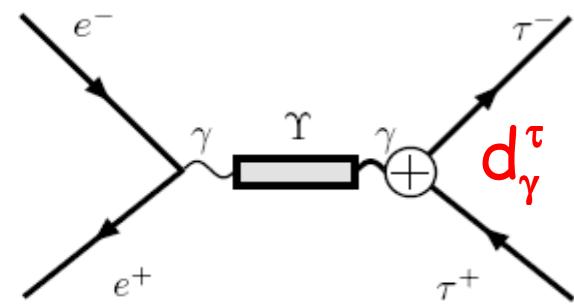
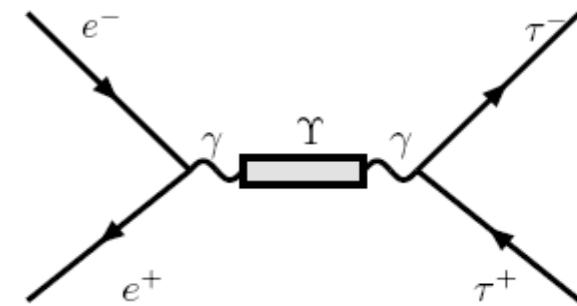
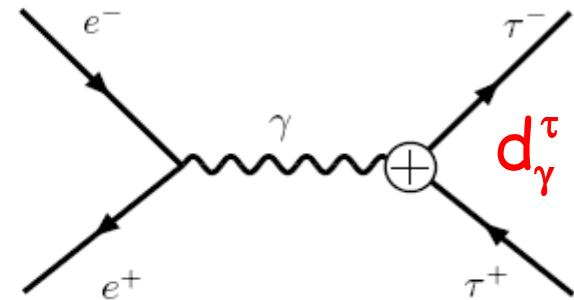
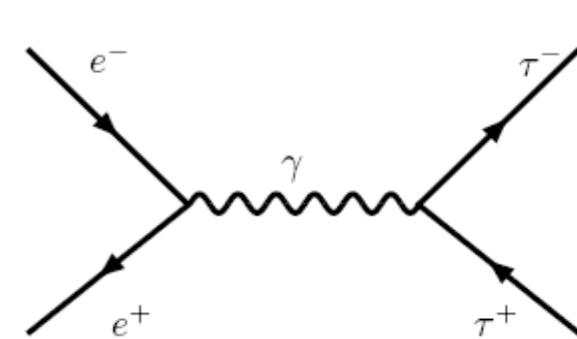
$$A^{CPV} \equiv \frac{1}{2}(A^+ + A^-)$$

## 2. Observables

2.2 Low energies

$$e^+ e^- \rightarrow \gamma, \Upsilon \rightarrow \tau^+(s_+) \tau^-(s_-)$$

Diagrams:



## 2. Observables    2.2 Low energies

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EDM  $\longleftrightarrow$  Spin correlation terms only

NORMAL-TRANSVERSE T-odd  $(s_+ \times s_-)_{N,T}$

NORMAL-LONGITUDINAL  $(s_+ \times s_-)_{N,L}$

Discrete symmetries: P, CP, T and helicity flip.

J.Bernabeu, GGS, J.Vidal '04

What about the linear terms?

Normal polarization: P-even, T-odd  $(s_+ - s_-)$

vs.

EDM lagrangian P and T-odd.

Interference with the axial part of Z-exchange,  
suppressed by  $q^2/M_Z^2$

# 2. Observables

## 2.2 Low energies

SPIN CORRELATIONS



ANGULAR DISTRIBUTION OF TAU  
DECAYS

$\tau \rightarrow h\nu$  kinematic variables :

- $e^-\tau^-$  CM angle  $\theta$
- Azimuthal  $\phi_{h+}$ ,  $\phi_{h'-}$
- Polar  $\theta_{h+}$ ,  $\theta_{h'-}$  angles of the produced hadrons  $h^+$  and  $h'^-$

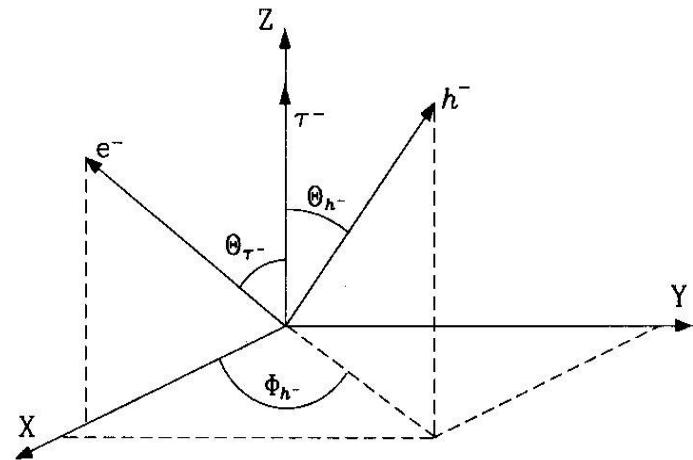


Fig. 2. Reference system for the process  $e^+ e^- \rightarrow \tau^+ \tau^- \rightarrow h_1^+ h_2^- + X$

# 2. Observables

## 2.2 Low energies

$\tau^-$  momenta  $\leftrightarrow$  LONGITUDINAL  $\leftrightarrow z$  axe

$\mathbf{p}_{\tau^-} \times \mathbf{p}_{e^-}$   $\leftrightarrow$  NORMAL  $\leftrightarrow y$  axe

CORRELATIONS:

$$\frac{d\sigma^{corr}}{d\Omega_{\tau^-}} = \frac{\alpha^2}{16s} \beta \left( s_+^x s_-^x C_{xx} + s_+^y s_-^y C_{yy} + s_+^z s_-^z C_{zz} + (s_+^x s_-^y + s_+^y s_-^x) C_{xy}^+ + (s_+^x s_-^z + s_+^z s_-^x) C_{xz}^+ + (s_+^y s_-^z + s_+^z s_-^y) C_{yz}^+ + (s_+ \times s_-)_x C_{yz}^- + (s_+ \times s_-)_y C_{xz}^- + (s_+ \times s_-)_z C_{xy}^- \right)$$

$$C_{xx} = (2 - \beta^2) \sin^2 \theta$$

$$C_{yy} = -\sin^2 \theta$$

$$C_{zz} = \beta^2 + (2 - \beta^2) \cos^2 \theta$$

$$C_{xz}^+ = \frac{1}{\gamma} \sin 2\theta$$

$$C_{xy}^- = 2\beta \sin^2 \theta d\gamma$$

$$C_{yz}^- = \gamma \beta \sin^2 \theta d\gamma$$

## 2. Observables 2.2 Low energies

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$\mathcal{T}, \mathcal{P}$

NORMAL-TRANSVERSE CORRELATION

CPV

$C_{xy}^-$  term in  $d\sigma(e^+e^- \rightarrow \gamma \rightarrow \tau^+\tau^- \rightarrow h^+\bar{\nu}h'\bar{\nu})$

$$\frac{d\sigma^8}{d\Omega_\tau d^3q_-^* d^3q_+^*} \Big|_{C_{xy}^-} = \frac{\alpha^2 \beta^2}{128\pi^3 s^2} Br_+ Br_- d\gamma_\tau \\ \sin^2 \theta (n_{+x}^* n_{-y}^* - n_{+y}^* n_{-x}^*) \\ \delta(q_-^* - P_-) \delta(q_+^* - P_+)$$

$$n_\pm^* = \pm \alpha_\pm \bar{q}_\pm^*$$

$q_\pm$  are the momentum of the hadrons

$$P_\pm = \frac{m_\tau^2 - m_\pm}{2m_\tau}$$

## 2. Observables 2.2 Low energies

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$$\frac{d^2\sigma}{d\phi_-^* d\phi_+^*} = \frac{\alpha^2 \beta^2}{192 s^2} Br_- Br_+ \alpha_- \alpha_+ \sin(\phi_-^* - \phi_+^*) d\gamma_\tau$$

$$A_{NT} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

$$\sigma^\pm = \int_{\substack{w \geq 0 \\ \gtrless}} \frac{d^2\sigma}{d\phi_-^* d\phi_+^*} d\phi_-^* d\phi_+^*$$
$$w = \sin(\phi_-^* - \phi_+^*)$$

$$A_{NT} = \frac{4\beta}{\pi} \frac{\alpha_- \alpha_+}{3 - \beta^2} d\gamma_\tau$$

# 2. Observables

## 2.2 Low energies

### NORMAL-LONGITUDINAL CORRELATION

$$\frac{d\sigma^8}{d\Omega_\tau d^3q_-^* d^3q_+^*} \Big|_{C_{yz}^-} = \frac{\alpha^2 \beta^2}{128\pi^3 s^2} Br_+ Br_- \gamma \textcolor{red}{d_\tau^\gamma}$$
$$\sin 2\theta (n_{+z}^* n_{-y}^* - n_{+y}^* n_{-z}^*)$$
$$\delta(q_-^* - P_-) \delta(q_+^* - P_+)$$

$$A_{NL}^- = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

$$\sigma^\pm = \int_{w \gtrless 0} \frac{d^3\sigma}{d\phi_-^* d\theta_-^* d\theta_+^*} d\phi_-^* d\theta_-^* d\theta_+^*$$

$$A_{NL} = \frac{1}{2} (A_{NL}^+ - A_{NL}^-) = \frac{\beta\gamma}{4(3-\beta^2)} \alpha_h^2 \frac{2m_\tau}{e} d_\tau^\gamma$$

## 2. Observables

### 2.2 Low energies

$e^+e^-$  at  $\Upsilon$  energies

$\tau$  pair production:  $e^+e^- \rightarrow \Upsilon \rightarrow \tau^+\tau^-$

- Multiplicative factor appears in the cross section
- Interference of diagrams (a) and (d) plus the interference of diagrams (b) and (c) is exactly zero.
- Only the interference of diagrams (b) and (d) contributes

$$\left(\frac{e^2 Q_b^2 |F_\Upsilon|^2}{s \Gamma_\Upsilon M_\Upsilon}\right)^2 = \left(\frac{3}{\alpha} Br(\Upsilon \rightarrow e^+e^-)\right)^2$$

The asymmetries do not change at the  $\Upsilon$  peak

# 2. Observables

## 2.2 Low energies

EDM imaginary part:

LONGITUDINAL AND TRANSVERSE  
POLARIZATION TERMS

$$A_T^\pm = \frac{\sigma_+^\pm - \sigma_-^\pm}{\sigma_+^\pm + \sigma_-^\pm}$$

$$\sigma_\pm^\pm = \int_{w>0} \frac{d^2\sigma}{d\cos\theta_{\tau^-} d\phi_\pm} d\cos\theta_{\tau^-} d\phi_\pm$$

$$w = \sin 2\theta_{\tau^-} \cos \phi_\pm$$

$$A_T^\pm = -\frac{\beta\gamma}{2(3-\beta^2)} \alpha_\pm \frac{2m_\tau}{e} \text{Im}[d_\tau^\gamma]$$

$$A^{\text{CPV}} = \frac{1}{2} (A_T^+ + A_T^-)$$

## 2. Observables    2.2 Low energies

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Bounds:

For  $10^{6/7} \tau$  and at low/Y energies

upper bound

$$|Re(d_\gamma^\tau)| < 10^{-16/-17} \text{ ecm}$$

# 3. Conclusions

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- $\tau$  EDM can be well bounded at low energy experiments
- Different CP-odd asymmetries allow to study the correlation and linear spin terms
- Bounds from these observables are competitive with present limits

→ Polarized beams open the possibility for new observables

→ Improving the number of  $\tau$ -pairs ( ... $10^{11}$ ? ) allows to lower the EDM bound by many (...2?) orders of magnitude

→ These new bounds may be important for beyond the SM  $\tau$ -physics

# 3. Summary

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Discussions with J.Bernabeu, J.Vidal and A.Santamaría  
are gratefully acknowledged

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