Some topics on Solar System Dynamics

Tabaré Gallardo

www.fisica.edu.uy/∼gallardo/cospar/2015

Facultad de Ciencias
Universidad de la República
Uruguay

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Topics

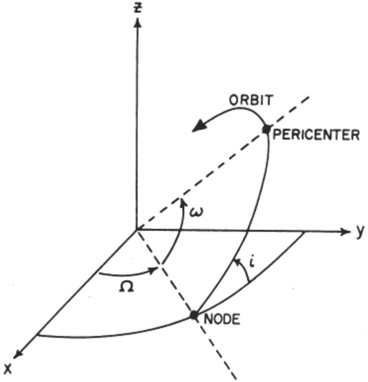
- Introduction
- Gauss equations
- Secular theory
- Chaos
- Kozai-Lidov
- Mean motion resonances
- Spin - Orbit resonances
- Lindblad resonances

Forces in the Solar System

Generated by

- **gravity** (Newtonian + relativistic) due to Sun, planets, satellites, asteroids.
  Model: N point masses + perturbations due to non-sphericity.
- **solar radiation**
  - radiation pressure (μm): in the direction of the radiation
  - Poynting-Robertson drag (cm): (Doppler) opposite to velocity generates migration to the Sun
  - Yarkovsky effect (from m to km): (thermal inertia) depending on rotation generates migration to or from the Sun
  - sublimation in comets NGF
- **medium**: solar wind, gas drag.
- **magnetic fields**: Lorentz forces.
- **collisions**

Orbital evolution



- given a Newtonian attraction

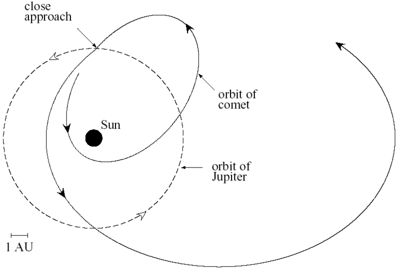
$$\ddot{\vec{r}} = -\frac{\mu}{r^2}\hat{r}$$

- and some initial conditions $(\vec{r}, \dot{\vec{r}})$
- an orbit is defined: $(a, e, i, \omega, \Omega, \tau)$
- constant energy $\varepsilon = -\mu/2a$
- constant angular moment $h = r^2\dot{f}$

The problem:

- given an extra acceleration $\vec{F}_p$ or **perturbation**
- we want to know $a(t), e(t), i(t), \ldots$

Orbital evolution WITH close encounters



- large perturbations
- drastic orbital changes
- only numerical methods (clones), statistical studies (Öpik)
- conserved quantity for RC3BP: Tisserand or Jacobi's constant
- no secular evolution

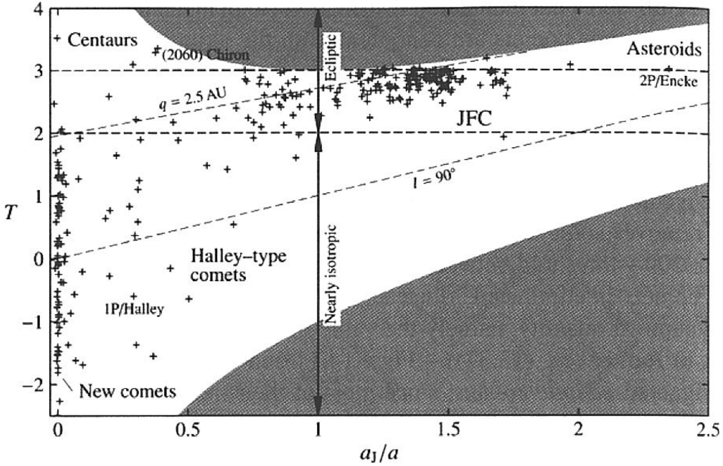
$$C \simeq \frac{a_p}{a} + 2\sqrt{\frac{a}{a_p}(1-e^2)}\cos i = T$$

Orbital evolution WITH close encounters

$$v_\infty \simeq \sqrt{3 - T} = U$$

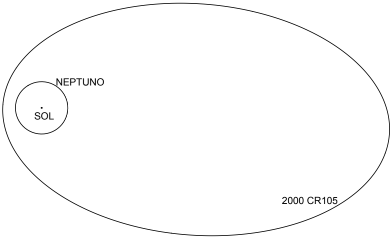
- U is the **encounter velocity** with the planet **before** the gravitational attraction is felt by the particle (that means "at infinity").
- The orbital elements (a, e, i) can evolve but T and U remain constant, only the orientation of $\vec{U}$ is modified.
- **It follows that when $T > 3$ encounters cannot exist.**

T is a good parameter for classification of small bodies.



(from Bertotti et al. 2003)

Orbital evolution WITHOUT close encounters

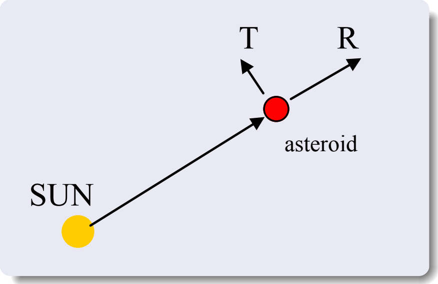


- small perturbations
- small orbital changes
- analytical methods $\Rightarrow$ theoretical predictions
- conserved quantities: "energy", z component of angular moment
- secular evolution

Gauss equations

Gauss equations: effects on $(a, e, i, \varpi, \Omega)$

$\vec{F}_p \Rightarrow$ effects on orbital elements



$$\ddot{\vec{r}} = -\frac{\mu}{r^2}\hat{r} + \vec{F}_p$$

$$\vec{F}_p = R\hat{r} + T\hat{t} + N\hat{n}$$

Energy: $\varepsilon = -\mu/2a$. Variation in energy:

$$\frac{d\varepsilon}{dt} = \vec{F}_p \cdot \frac{d\vec{r}}{dt} = \dot{r}R + r\dot{f}T = \frac{\mu}{2a^2}\frac{da}{dt}$$

Gauss equations

Instantaneous perturbation:

$$\Rightarrow \frac{da}{dt} = 2\frac{a^{3/2}}{\sqrt{\mu(1-e^2)}}[Re\sin f + T(1 + e\cos f)]$$

Mean over one orbital period P:

$$< \frac{da}{dt} >= \frac{1}{P}\int_0^P \frac{da}{dt} \cdot dt$$

If $< \frac{da}{dt} > \neq 0 \Rightarrow$ cumulative effect.

Gauss equations

Considering that angular momentum is:

$$h = r^2 \frac{df}{dt}$$

we change the variable:

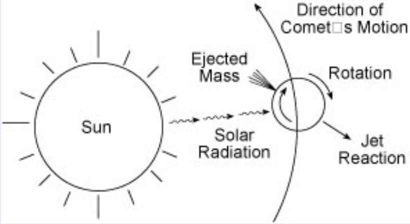
$$dt = \frac{r^2}{h} df$$

$$\Rightarrow < \frac{da}{dt} >= \frac{1}{P} \int_0^{2\pi} \frac{da}{dt} \frac{r^2}{h} df$$

Example 1: a comet with NGF

Consider a comet with

$$\vec{F}_p \sim \frac{1}{r^2}(R, T, N)$$



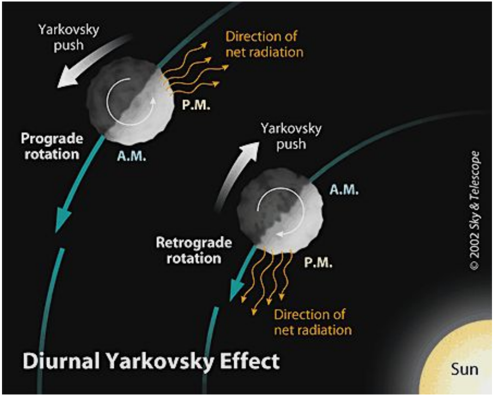
We obtain:

$$< \frac{da}{dt} >= \frac{2}{(1-e^2)\sqrt{a\mu}} T$$

$$< \frac{de}{dt} >= \frac{\left(1-\sqrt{1-e^2}\right)}{e\sqrt{a^3\mu}} T$$

The **radial component is irrelevant**, only T matters.

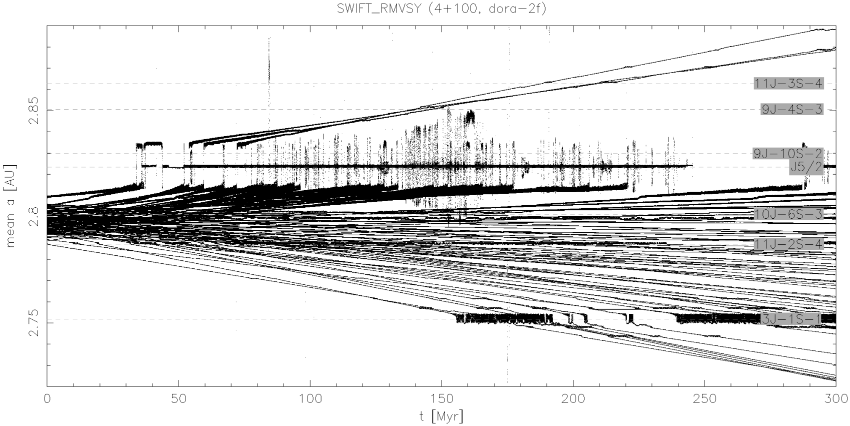
Example 2: Yarkovsky effect for an asteroid



$$< \frac{da}{dt} >= \frac{2}{(1-e^2)\sqrt{a\mu}} T$$

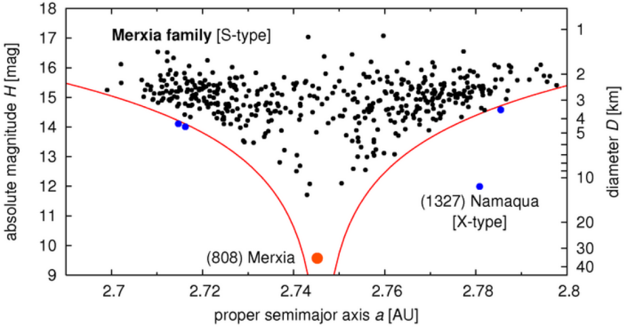
- prograde rotation: $T > 0$, then mean $da/dt > 0$, goes away
- retrograde: $T < 0$, then mean $da/dt < 0$, goes to the Sun

Yarkovsky effect: simulating a family



Broz, yarko-site

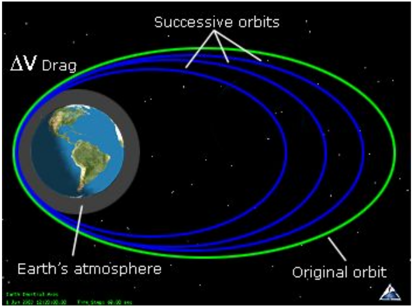
Yarkovsky effect



(Broz et al. 2005)

After a collision a family is generated: the smaller fragments (higher magnitude) are the most affected by Yarkovsky (so, the most dispersed). This effect can help us in the **determination of the age of the family**.

Example 3: gas drag



- $da/dt < 0$ fall down
- $de/dt < 0$ circularization

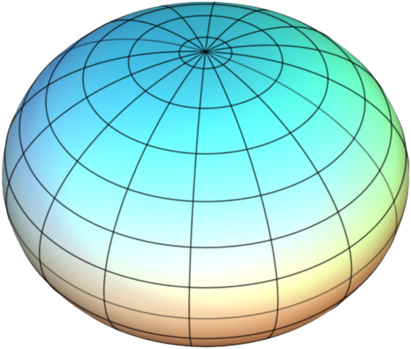
$$\vec{F}_p = (R, T, 0)$$

$$< \dot{a} > \propto T < 0$$

$$< \dot{e} > \propto T < 0$$

Example 4: axisymmetric oblate planet

$$V(r,\phi) = -\frac{GM}{r}\left[1 - J_2 P_2(\sin\phi)\left(\frac{R}{r}\right)^2 + \ldots\right]$$



acceleration:

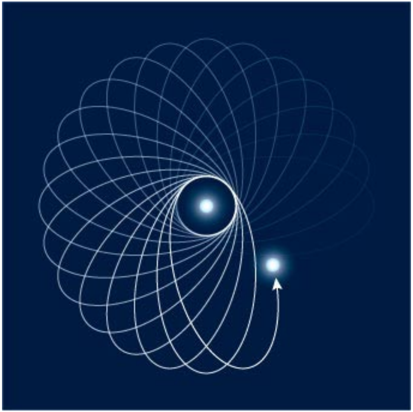
$$\vec{\alpha} = -\nabla V(r,\phi)$$

perturbation:

$$\vec{F_p} = \frac{R}{r^3}\hat{r} + \frac{N}{r^3}\hat{n}$$

$$T = 0 \qquad \text{(symmetry)}$$

Example 4: axisymmetric oblate planet



- $< da/dt >= 0$
- $< de/dt >= 0$
- $< di/dt >= 0$
- $d\varpi/dt > 0$, *advance of the perihelion*
- $d\Omega/dt < 0$, *precession of the nodes*

It is a very typical orbital behaviour.

Secular Theory

Euler

Laplace

Lagrange

Jacobi

LeVerrier

Hamilton

Birkoff

Poincaré

Perturbation Theory

Consider an asteroid at $\vec{r}$ perturbed by a planet at $\vec{r}_p$.
It is possible to write the equation of motion in the form

$$\ddot{\vec{r}} + \mu \frac{\vec{r}}{r^3} = \nabla R(\vec{r}, \vec{r}_p)$$

where R is the **Disturbing Function**. It is possible to transform this equation in another very different form due to Lagrange (+ Euler + Laplace):

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial R}{\partial \lambda}$$

$$\frac{de}{dt} = -\frac{\sqrt{1-e^2}}{na^2 e}\left(1-\sqrt{1-e^2}\right)\frac{\partial R}{\partial \lambda} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \varpi}$$

$$\frac{di}{dt} = -\frac{\tan\frac{i}{2}}{na^2\sqrt{1-e^2}}\left(\frac{\partial R}{\partial \lambda} + \frac{\partial R}{\partial \varpi}\right) - \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \Omega}$$

$$\frac{d\varpi}{dt} = \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} + \frac{\tan\frac{i}{2}}{na^2\sqrt{1-e^2}}\frac{\partial R}{\partial i}$$

$$\frac{d\Omega}{dt} = \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i}$$

R is a very unfriendly function

$$R = \sum_k C_k(a, e, i)\cos(\sigma_k)$$

where functions $\sigma_k(\lambda_p, \lambda, \varpi, \Omega)$ are linear combinations of $\lambda_p, \lambda, \varpi, \Omega$.

Perturbation Theory

The λs are quick varying angles, on the contrary ϖ, Ω are slow varying angles. Then:

$$R = R_{SP}(\varpi, \Omega, \lambda, \lambda_p) + R_{LP}(\varpi, \Omega)$$

Instead of full R we consider the **mean over the quick varying angles** λ, λ_p, then:

$$R \simeq R_{LP}(\varpi, \Omega)$$

this part of the disturbing function is the responsible for the long term **secular evolution** of the system.

Perturbation Theory: secular evolution

Taking $R \simeq R_{LP}$ the first of the Lagrange's planetary equations becomes:

$$\frac{da}{dt} \simeq \frac{2}{na} \frac{\partial R_{LP}}{\partial \lambda} = 0$$

$$\frac{de}{dt} \simeq -\frac{\sqrt{1-e^2}}{na^2 e} \frac{\partial R_{LP}}{\partial \varpi}$$

$\Rightarrow$ the semimajor axes of the planets do not change with time...

the planetary system do not shrinks nor expands

That was a very impacting result of the XVIII century due to Euler, Lagrange and Laplace.
In fact $a(t) = a_{sec} +$ small amplitude oscillations.
It is also possible to show that e and i do not grow systematically but oscillate.

Perturbation Theory: Delaunay canonical variables

Working in canonical variables $(M, \omega, \Omega, L, G, H)$ where:

- $L = \sqrt{\mu a}$, momentum conjugate of M
- $G = \sqrt{\mu a(1-e^2)}$, momentum conjugate of ω
- $H = \sqrt{\mu a(1-e^2)} \cos i$, momentum conjugate of Ω

$$\frac{dM}{dt} = \frac{\partial \mathcal{H}}{dL} \qquad \frac{dL}{dt} = -\frac{\partial \mathcal{H}}{dM}$$

$$\frac{d\omega}{dt} = \frac{\partial \mathcal{H}}{dG} \qquad \frac{dG}{dt} = -\frac{\partial \mathcal{H}}{d\omega}$$

$$\frac{d\Omega}{dt} = \frac{\partial \mathcal{H}}{dH} \qquad \frac{dH}{dt} = -\frac{\partial \mathcal{H}}{d\Omega}$$

$$\mathcal{H} = \frac{v^2}{2} - \frac{\mu}{r} + R = -\frac{\mu}{2a} + R = -\frac{\mu^2}{2L^2} + R$$

$$\mathcal{H}_{sec}(-, \omega, \Omega, L, G, H) = -\frac{\mu^2}{2L^2} + R_{sec}$$

$$\frac{dL}{dt} = -\frac{\partial \mathcal{H}_{sec}}{dM} = 0$$

$$\Rightarrow L = \sqrt{\mu a} = \text{constant}$$

that means

$$a = \text{constant}$$

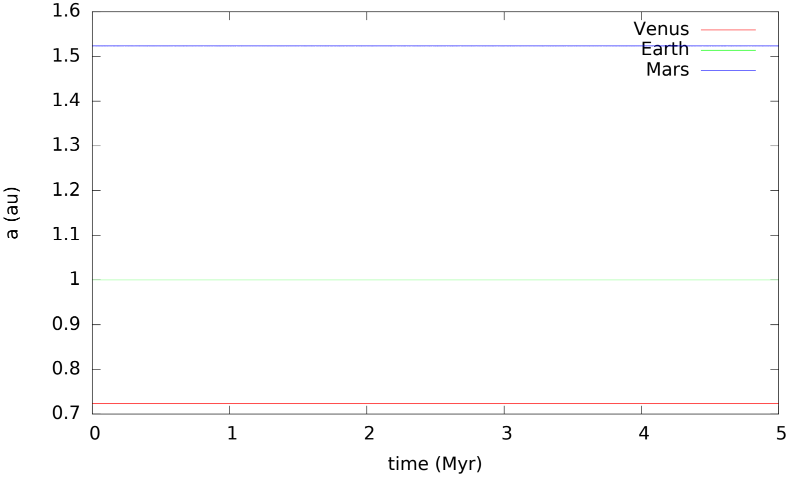
Then, secular evolution $\Rightarrow a = \text{constant}$.

Numerical integrations of the exact equations

Given a problem

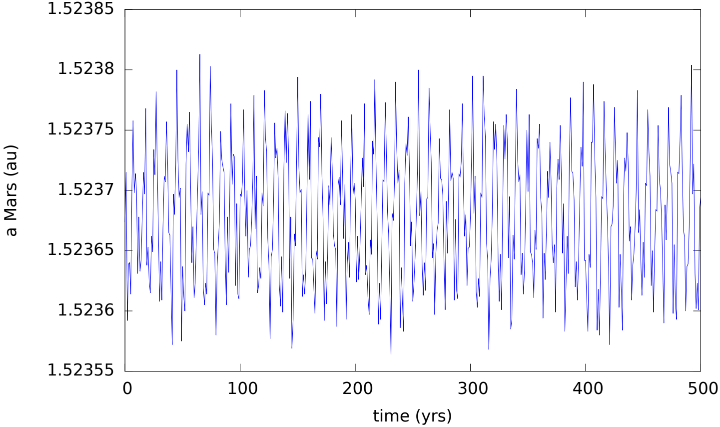
- write equations $\ddot{\vec{r}} = \vec{\alpha}$ for all bodies
- design an algorithm to calculate $\vec{r}(t + \Delta t)$ from $\vec{r}(t)$
- write in some computer language
- run in a computer
- we obtain $\vec{r}(t), \dot{\vec{r}}(t)$
- and $a(t), e(t), i(t), \ldots$

Numerical integrations of the exact equations

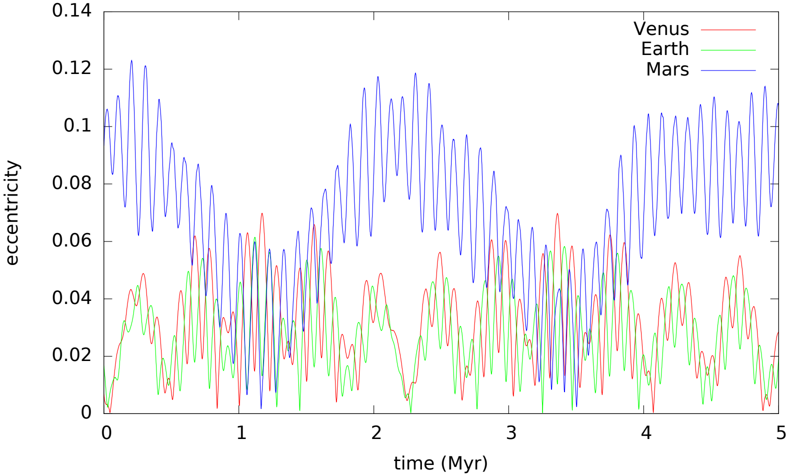


zoom

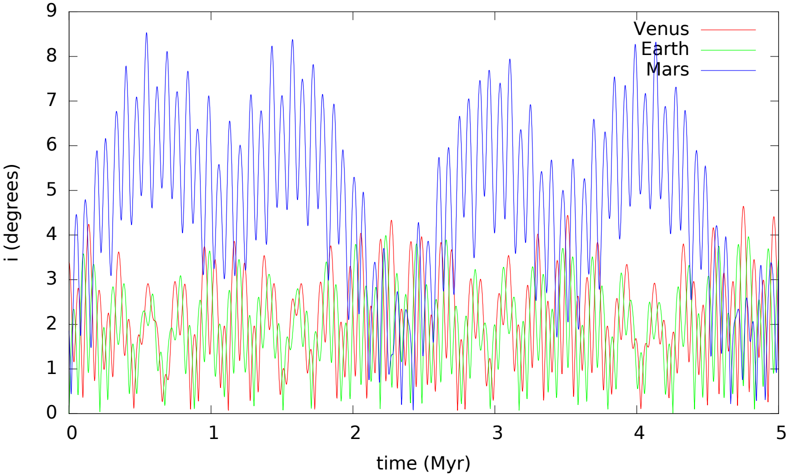
$a(t) = a_{sec} +$ small amplitude oscillations



Eccentricities



Inclinations



Secular evolution

There are 3 methods to obtain R_{sec} (or $\mathcal{H}_{sec}$) from R:

- canonical transformations
- scissors (just dropping SP terms)
- numerical averaging of the exact R:

$$R_{sec} = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} R \, d\lambda \, d\lambda_p$$

Asteroid: secular evolution for small e, i

- assuming e, i small
- define $I = \sin(i/2)$

the secular equations become

$$\frac{de}{dt} = -\frac{1}{na^2e}\frac{\partial R}{\partial \varpi}$$
$$\frac{dI}{dt} = -\frac{1}{na^2I}\frac{\partial R}{\partial \Omega}$$

$$\frac{d\varpi}{dt} = \frac{1}{na^2e}\frac{\partial R}{\partial e}$$
$$\frac{d\Omega}{dt} = \frac{1}{na^2I}\frac{\partial R}{\partial I}$$

with $a =$ constant

Changing variables $(e, \varpi, I, \Omega) \rightarrow (h, k, p, q)$

$$h = e \sin \varpi \qquad p = I \sin \Omega$$

$$k = e \cos \varpi \qquad q = I \cos \Omega$$

$$\Rightarrow R(h, k, p, q)$$

New equations:

$$\frac{dh}{dt} = C \frac{\partial R}{\partial k}$$
$$\frac{dk}{dt} = -C \frac{\partial R}{\partial h}$$

$$\frac{dp}{dt} = C \frac{\partial R}{\partial q}$$
$$\frac{dq}{dt} = -C \frac{\partial R}{\partial p}$$

With $C = \frac{1}{na^2}$ and discarding high order terms in R we obtain the solution $\Longrightarrow$

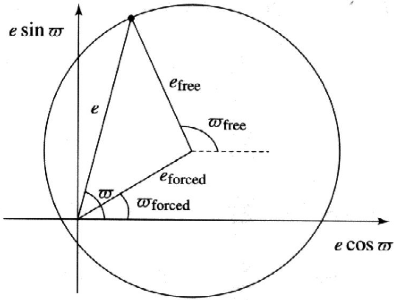
Asteroid: secular evolution

$h = e \sin \varpi, k = e \cos \varpi$

$$h(t) = e_{proper} \sin(At + \beta) + h_{forced}$$

$$k(t) = e_{proper} \cos(At + \beta) + k_{forced}$$

with $A(a)$, **proper frequency**



Murray and Dermott 1999

Osculating eccentricity:

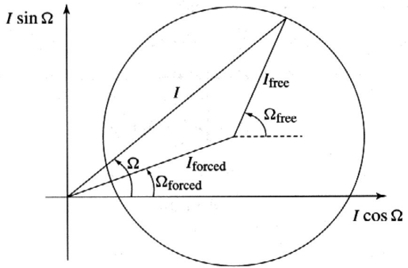
$$e_{osc}(t) = \sqrt{h^2 + k^2}$$

$p = I\sin\Omega, q = I\cos\Omega$

$$p(t) = I_{proper}\sin(Bt+\gamma) + p_{forced}$$

$$q(t) = I_{proper}\cos(Bt+\gamma) + q_{forced}$$

with $B(a)$, **proper frequency**

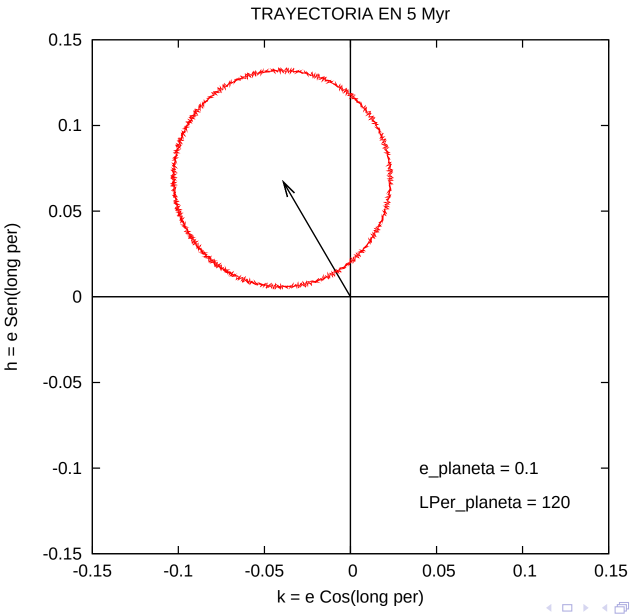


Murray and Dermott 1999

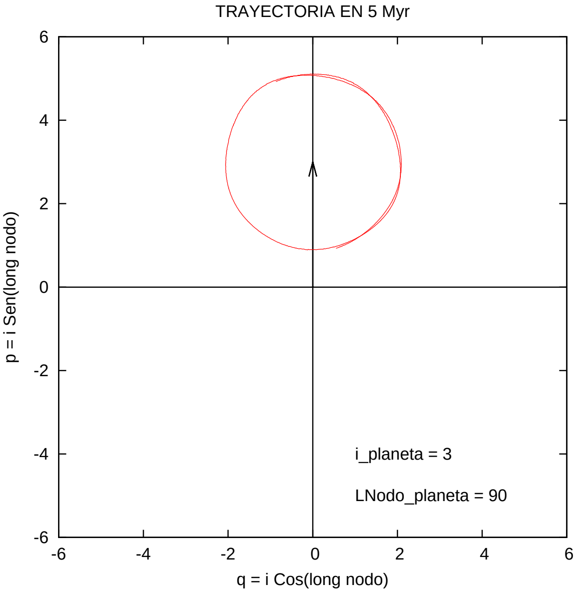
Osculating inclination:

$$I_{osc}(t) = \sqrt{p^2+q^2}$$

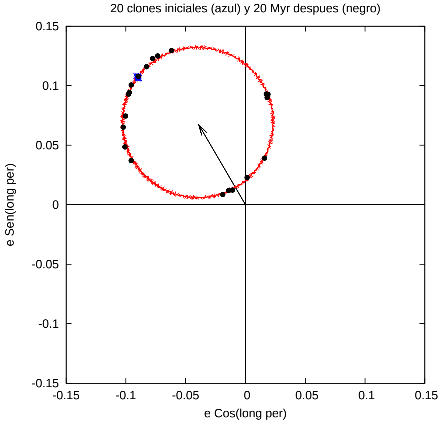
Example: TNO (h, k) and one planet



Example: TNO $(p = i \sin \Omega, q = i \cos \Omega)$



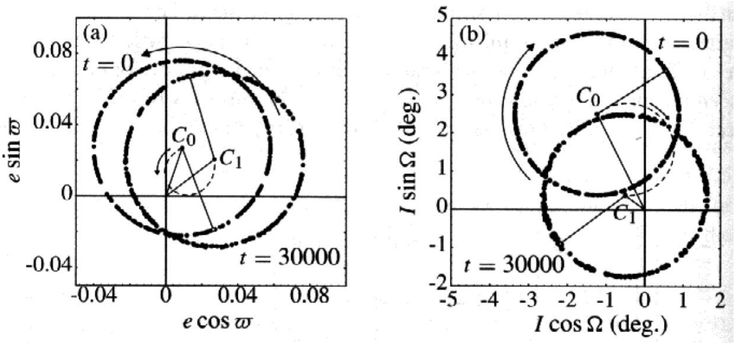
Example: collisional fragments



blue dot: initial conditions
black dots: final
red: intermediary positions

- collision generates several fragments
- small differences in a_i
- small differences in A_i, B_i
- after some time ϖ_p, Ω_p randomize
- osculating e, i changed
- proper e_p, I_p preserved

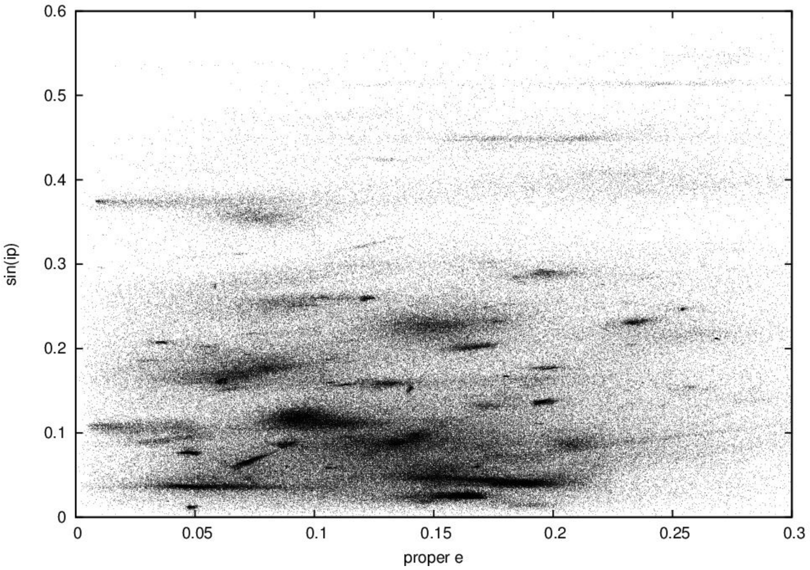
Family perturbed by several planets



Murray and Dermott 1999

The **forced modes vary with time** but proper values e_p, I_p (radius of the circle) are preserved.

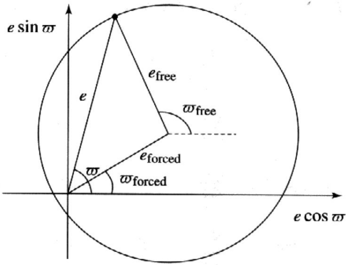
Asteroids' proper elements: memory of their origin



Planetary system: fundamental frequencies

$$h_{forced}(t) = -\sum_{i=1}^{N} \frac{\nu_i}{A - g_i} \sin(g_i t + \beta_i)$$

$$k_{forced}(t) = -\sum_{i=1}^{N} \frac{\nu_i}{A - g_i} \cos(g_i t + \beta_i)$$

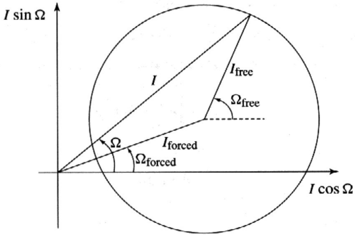


- g_i are **fundamental frequencies** of the system
- A is a **proper** frequency
- when $A = g_i$ we have a **secular resonance**

Planetary system: fundamental frequencies

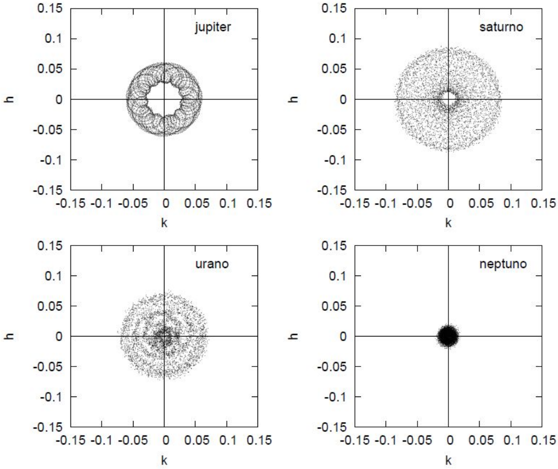
$$p_{forced}(t) = -\sum_{i=1}^{N} \frac{\mu_i}{B - f_i} \sin(f_i t + \gamma_i)$$

$$q_{forced}(t) = -\sum_{i=1}^{N} \frac{\mu_i}{B - f_i} \cos(f_i t + \gamma_i)$$



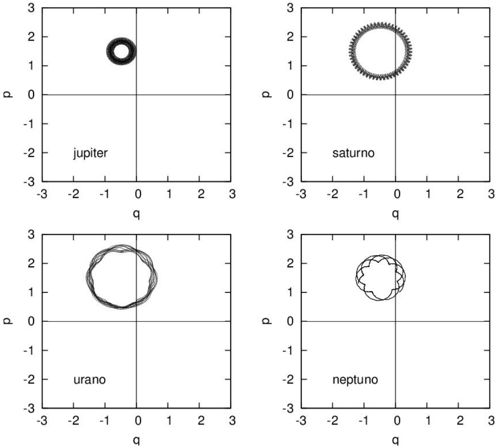
- f_i are **fundamental frequencies** of the system
- B is a **proper** frequency
- when $B = f_i$ we have a **secular resonance**

Planetary system: $h = e \sin \varpi, \qquad k = e \cos \varpi$



The Sun is a natural origin for computing positions. It is the focus.

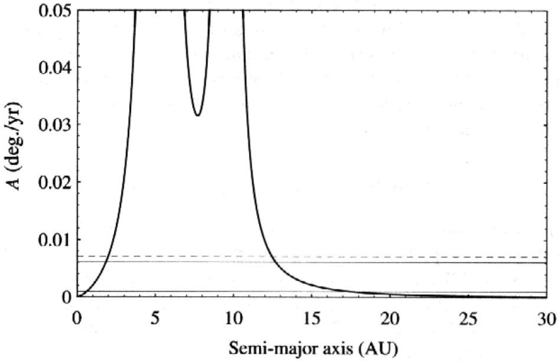
Planetary system: $p = i \sin \Omega, \qquad q = i \cos \Omega$



The ecliptic it is NOT a natural plane for defining inclinations.

Secular resonances: small (e, I) case

For **small** e, I it results that A, B only depend on a, then there are specific values of a for which some terms of the forced mode diverge: when $A(a) = g_i$ or $B(a) = f_i$.

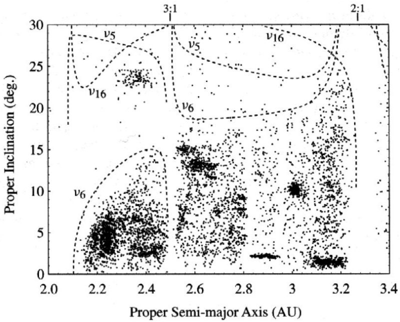


Murray and Dermott 1999

Secular resonances: general case

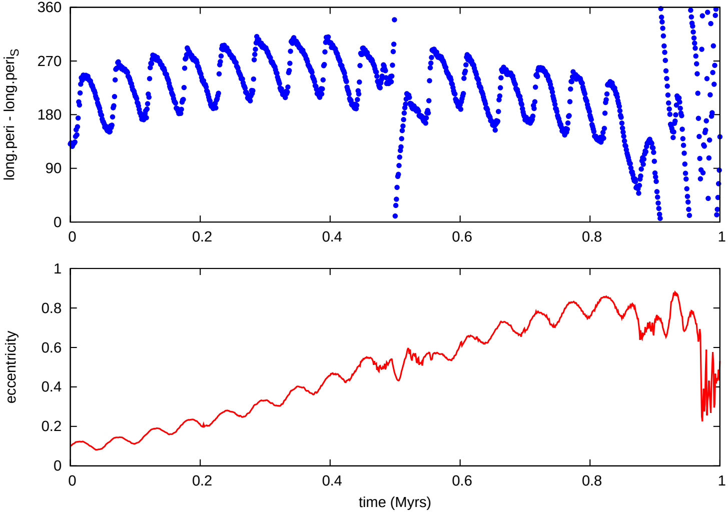
For **large** e, I it results that $A(a,e,i)$ and $B(a,e,i)$ so the divergence occurs in the surfaces $A(a,e,i) = g_i$ and $B(a,e,i) = f_i$.

- ν_i corresponds to $A(a,e,i) = g_i$
- ν_{1i} corresponds to $B(a,e,i) = f_i$

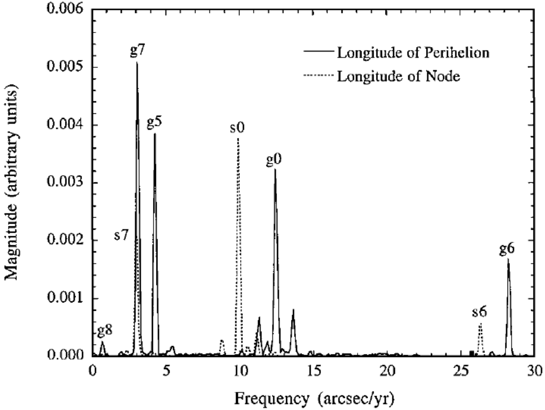


Murray and Dermott 1999

Example: particle in secular resonance ν_6



Computing the spectra of $h(t), k(t), p(t), q(t)$ all frequencies appear: **fundamental** (g_i, f_i) and **proper** (A, B).



Fundamental and proper frequencies

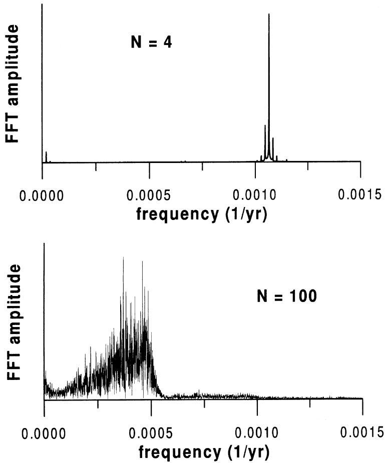
In the secular evolution of an asteroid perturbed by N planets we have:

- N (low) **fundamental** frequencies g_i related to oscillations in e_i, ϖ_i
- N (low) **fundamental** frequencies f_i related to oscillations in i_i, Ω_i
- 2 (low) **proper** frequencies A, B related to oscillations in e, ϖ and i, Ω

In the spectra obtained from a numerical integration we will observe also $N + 1$ **high** frequencies related to the orbital motion of the bodies and associated with small amplitude oscillations in the semimajor axes. They do not appear in a secular theory.

Chaos

Regular versus chaotic motion



Michtchenko and Ferraz-Mello 2001

- well defined frequencies: **regular** motion
- poorly defined frequencies, varying with time: **chaotic** motion

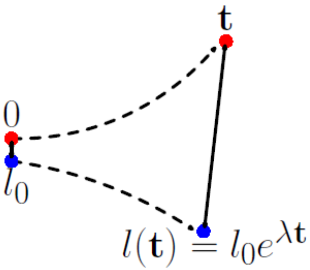
Chaos versus regular

The planetary system is stable and chaotic ...

- fundamental frequencies of the planetary system have small variations in time scales of 10^9 years
- the future of the system is DETERMINED (only one solution exists) but CHAOTIC (hard to predict)
- the planetary system is under STABLE CHAOS: we can predict reasonably well the orbital evolution but not the exact position of the planets in their orbits

Dynamical regimes

- with close encounters: highly chaotic, unpredictable
- without close encounters
  - regular motion: fixed frequencies, very predictable
  - chaotic motion: varying frequencies, predictable in some timescale

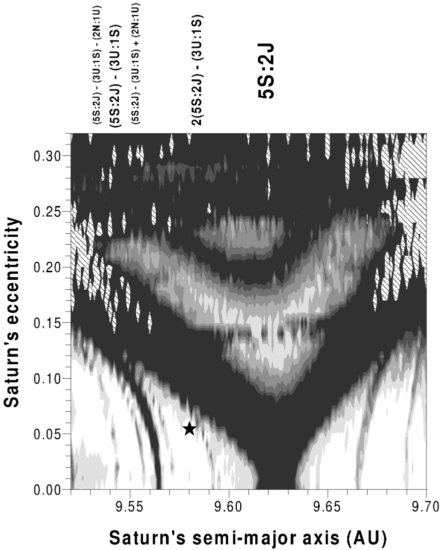


Consider the time evolution of the difference between 2 trajectories in the phase space:

$$\mathbf{X}_a - \mathbf{X}_b = l(t) \approx l_0 \exp(\lambda t)$$

we calculate the **Lyapunov exponent**, λ, which is a measure of the chaos. The timescale of the **dynamical memory** is $1/\lambda$.

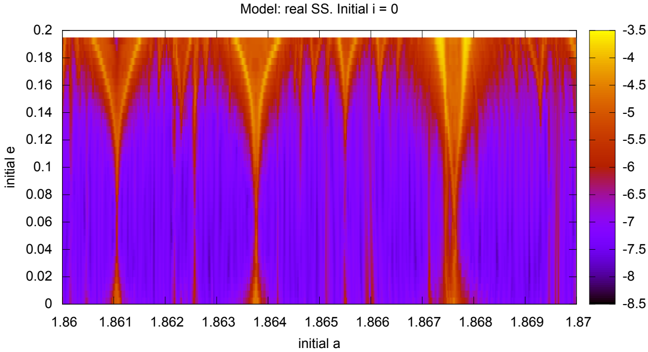
Dynamical maps: chaotic regions



Michtchenko and Ferraz-Mello 2001

Dynamical maps: non secular regimes

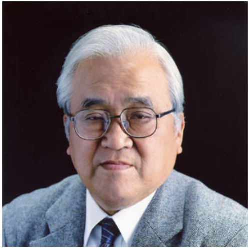
- take set of initial values (a, e)
- calculate the mean $\bar{a}$ in some interval
- calculate the variation $\Delta\bar{a}$ (running window)
- surface plot of $\Delta\bar{a}(a, e)$



Theory and numerical studies

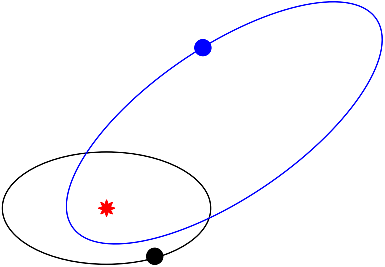
- Nowadays theoretical analysis is used not just to obtain analytical solutions but to **provide theoretical explanations** to the very precise solutions obtained with the numerical integrators.
- Everybody can obtain a precise numerical solution of a dynamical problem but only with the understanding of the theory we can **explain the numerical results**.

Kozai-Lidov Mechanism



Kozai-Lidov mechanism

Consider a comet at $\vec{r}$ perturbed by a planet at $\vec{r_p}$ in circular orbit:



$$\mathcal{H}(t) = \frac{v^2}{2} - \frac{\mathcal{G}m_\odot}{r} + R(\vec{r}, \vec{r_p})$$

$$\mathcal{H}_{sec} = -\frac{\mathcal{G}m_\odot}{2a} + R_{sec}$$

Kozai-Lidov mechanism

$$R_{sec} = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} R(\lambda, \lambda_p) \, d\lambda \, d\lambda_p$$

- The numerical method by Bailey et al. (1992) and Thomas & Morbidelli (1996) allows to consider several perturbing planets.
- $R_{sec}(a, e, i, \omega)$ is independent of the variable Ω, then $\mathcal{H}$ is also independent and the momentum canonically conjugated $H = \sqrt{a(1-e^2)} \cos i$ is constant.

Kozai-Lidov mechanism

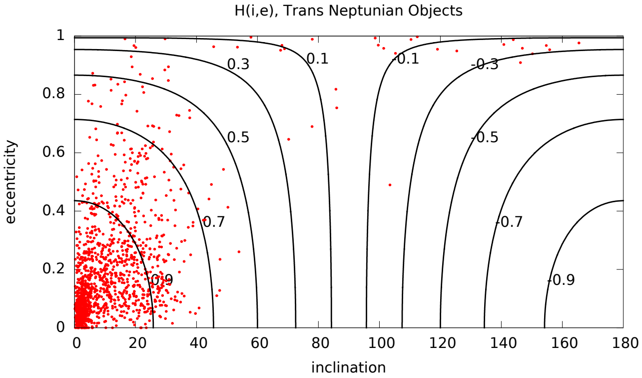
Then

- $\mathcal{H}(e, i, \omega) =$ constant, energy constant
- $a =$ constant, because it is a secular motion
- $\sqrt{(1-e^2)}\cos i =$ constant, because $H =$ constant

$\Rightarrow$ coupled oscillations e, i and also ω.

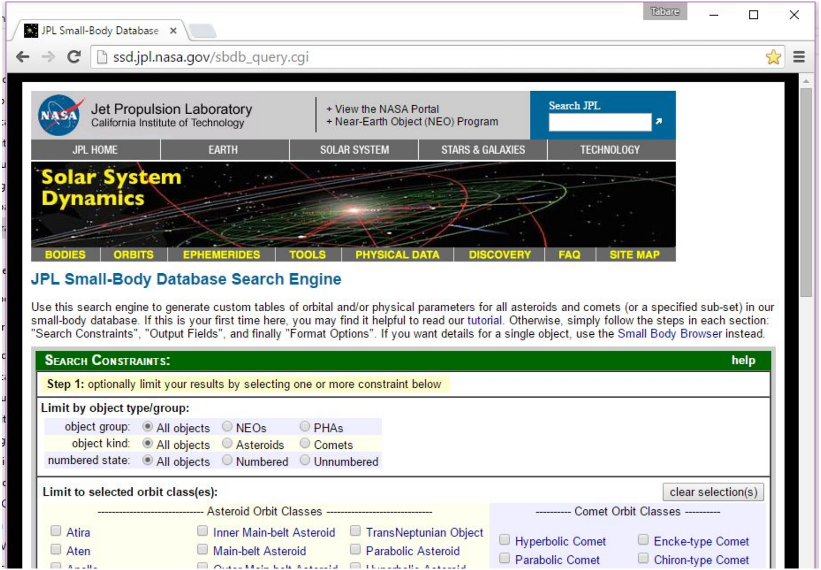
from now on we will call $H = \sqrt{(1-e^2)}\cos i$

$H = \sqrt{(1 - e^2)} \cos i = \text{constant}$

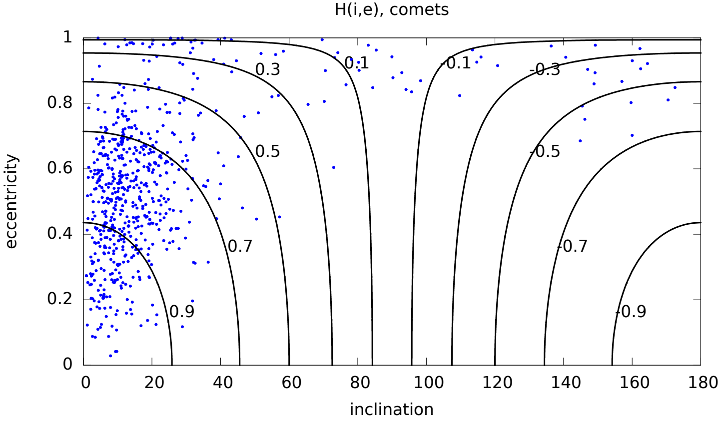


- $\mid H \mid \sim 1 \Rightarrow$ small variations in (e, i)
- $H \sim 0 \Rightarrow$ large variations in (e, i) allowed

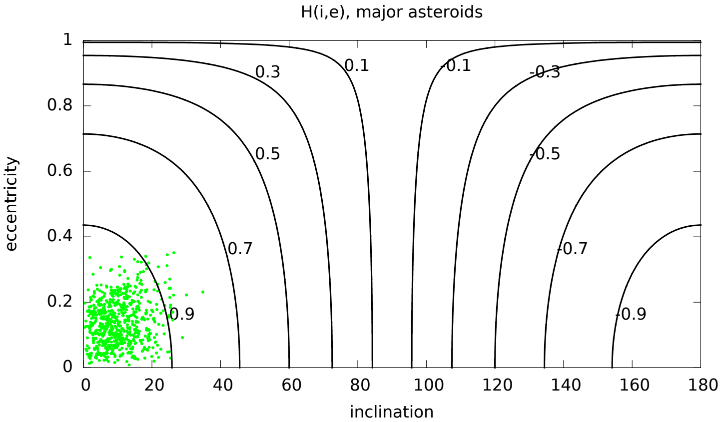
Orbital elements data base



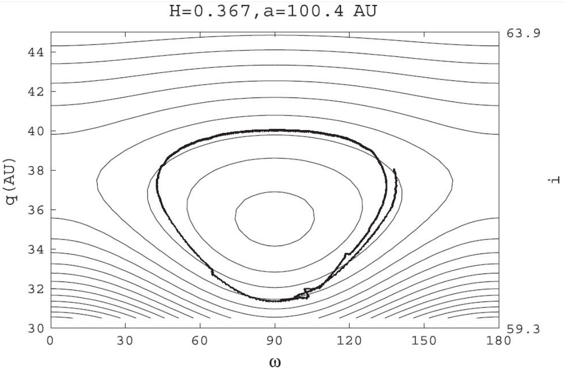
Kozai-Lidov mechanism: comets



Kozai-Lidov mechanism: asteroids



Actual variations in (e, i) are limited by the energy level curves $\mathcal{H}(e, i, \omega) = \text{constant}$



(Gallardo et al. 2012).

Kozai-Lidov mechanism: analytical

Particle evolving **always** inside or outside the perturbers $\Rightarrow$ acceptable analytical approximation to R.

$$\Rightarrow \frac{di}{dt} \propto \sin(2\omega)$$

$$\Rightarrow \text{equilibrium points at } \omega = 0^\circ, 90^\circ, 180^\circ, 270^\circ$$

Asteroid perturbed by Jupiter or satellite perturbed by an exterior satellite:

$$\frac{d\omega}{dt} \propto (3 - 3e^2 - 5\cos^2(i))$$

critical inclination $i \sim 39^\circ$

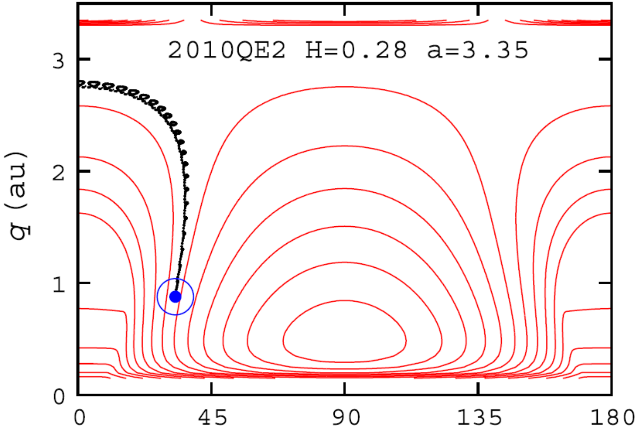
TNO perturbed by Neptune or satellite perturbed by J_2 or interior satellite:

$$\frac{d\omega}{dt} \propto (3 + 5\cos(2i))$$

critical inclination $i \sim 63^\circ$

Kozai-Lidov mechanism: sungrazers

It is an efficient mechanism to generate **sungrazers**:



Fernandez et al. 2014

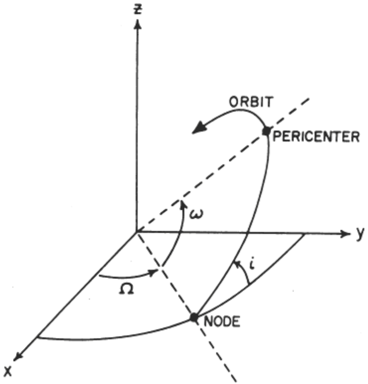
Kozai-Lidov mechanism

- KL mechanism implies large coupled changes in e, i
- is typical for orbits with large e or i ($H \sim 0$)
- for low (e, i) orbits ($\mid H \mid \sim 1$) there are very small orbital variations
- KL mechanism and secular resonances also appear inside MMRs

Mean Motion Resonances

Orbital Resonances

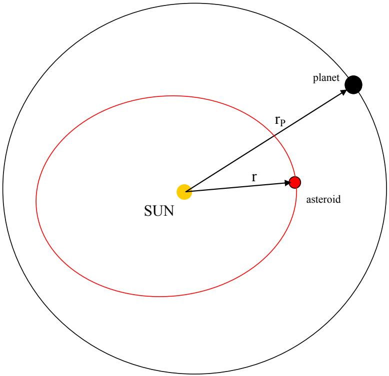
Commensurability between **frequencies** associated with orbital motion: mean motion, nodes and pericenters



- mean motion resonances
  - two-body
  - three-body
- secular resonances
- Lindblad resonances
- spin-orbit resonances

Some examples

- Io-Europa-Ganymede
- Saturn satellites
- Saturn rings
- Uranus satellites
- asteroids with Jupiter, Mars, Earth, Venus...
- Trans Neptunian Objects with Neptune
- Pluto - Neptune
- comets - Jupiter
- Pluto satellites: Styx, Nix, and Hydra



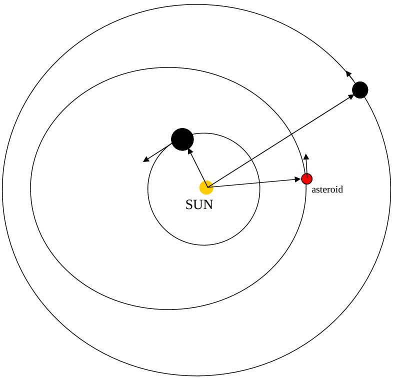
p, q integers
q is the **order**

$$pn_{ast} \simeq (p+q)n_{pla}$$

$$a_{ast} \simeq \left(\frac{p}{p+q}\right)^{2/3} a_{pla}$$

- particle (asteroid, comet, TNO, ring) with **a massive body** (planet, satellite)
- between two massive bodies (planets, satellites)
- strength $\propto$ $\mathbf{m_{perturber}}$

Three body resonances

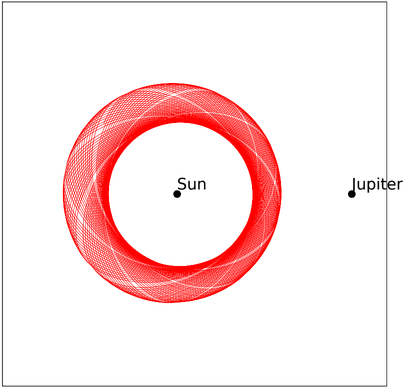


$$k_0 n_0 + k_1 n_1 + k_2 n_2 \simeq 0$$

- particle (asteroid, comet, TNO) with **two massive bodies** (planets, satellites)
- between three massive bodies (planets, satellites)
- strength $\propto \mathbf{m_1 m_2}$

Non resonant asteroid in rotating frame

Mean perturbation is radial: Sun-Jupiter



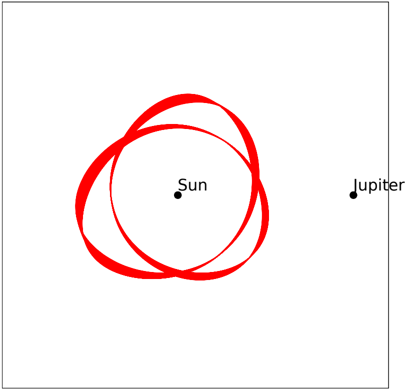
$$< F_p > \propto R\hat{r}$$

$$T = 0 \Rightarrow a = \text{constant}$$

$\Rightarrow$ typical secular evolution

Resonant asteroid

Mean perturbation has a **transverse** component.

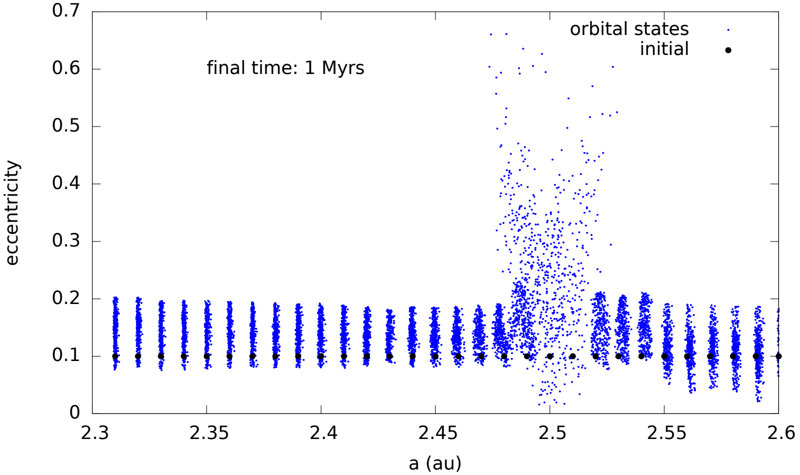


$$< F_p > \propto R\hat{r} + T\hat{t}$$

$$T \neq 0 \Rightarrow a = \text{oscillating}$$

$\Rightarrow$ different from secular evolution

Dynamical effects: a numerical exercise

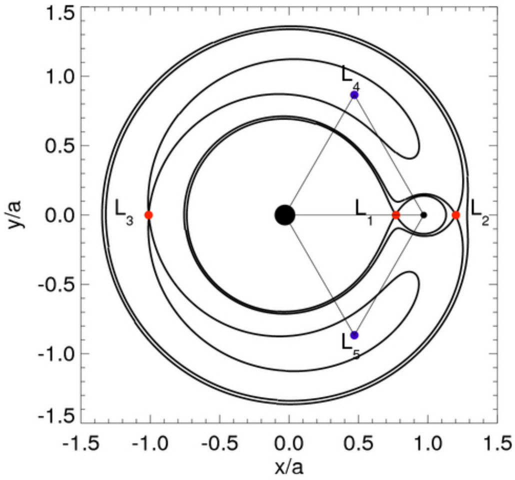


Mean motion resonances

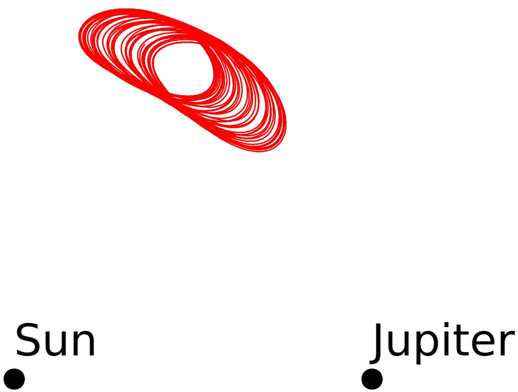
It has sense to find asteroids inside resonances because:

- there are several resonances
- resonances have some **strength** and **stickiness**, they can "attract" trajectories to them
- there are mechanisms (like Yarkovsky, tides, gas drag) that drive the objects to the resonances and there is a chance to be captured by them

1772: Lagrange equilibrium points



1906: (588) Achilles by 500 yrs



1784: Laplacian resonance

$$3\lambda_{Europa} - \lambda_{Io} - 2\lambda_{Ganymede} \simeq 180^\circ$$

$$3n_{Europa} - n_{Io} - 2n_{Ganymede} \simeq 0$$

They are also in commensurability by pairs:

$$2n_{Europa} - n_{Io} \simeq 0$$

$$2n_{Ganymede} - n_{Europa} \simeq 0$$

$$\Downarrow$$

It must be the consequence of some physical mechanism.

Quasi resonances

quasi resonance Uranus - Neptune:

$$n_{Uranus} \sim 2n_{Neptune}$$

quasi resonance Saturn - Uranus:

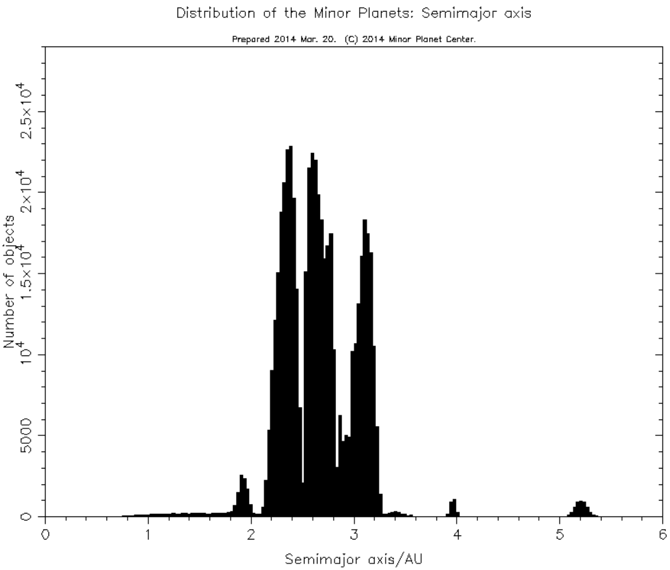
$$n_{Saturn} \sim 3n_{Uranus}$$

quasi resonance: Jupiter - Saturn

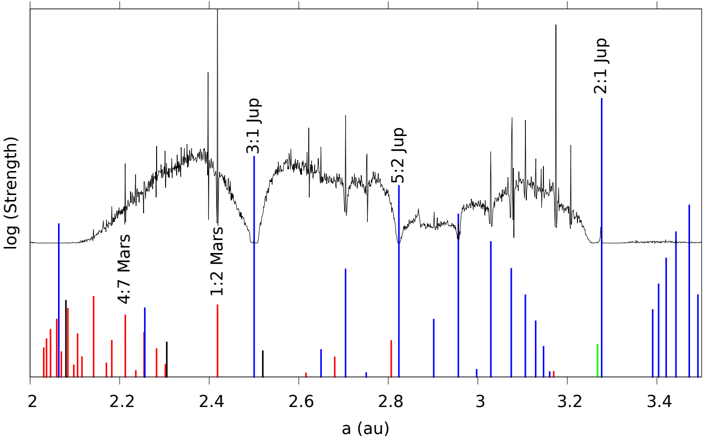
$$2n_{Jupiter} \sim 5n_{Saturn}$$

Why the planets are close to resonance?
Hint: **planetary migration**

1866: Kirkwood gaps



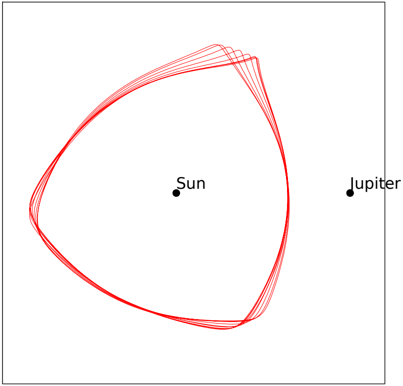
Distribution of asteroids semimajor axes



Main belt of asteroids is *sculpted* by resonances.

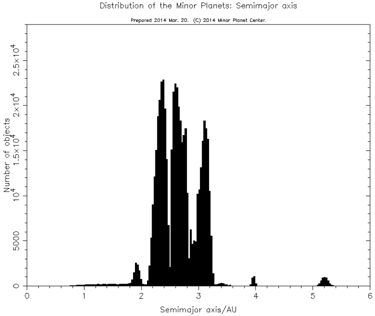
1875: resonant asteroids (153) Hilda 3:2

rotating frame



$$2n_{Hilda} \simeq 3n_{Jup}$$

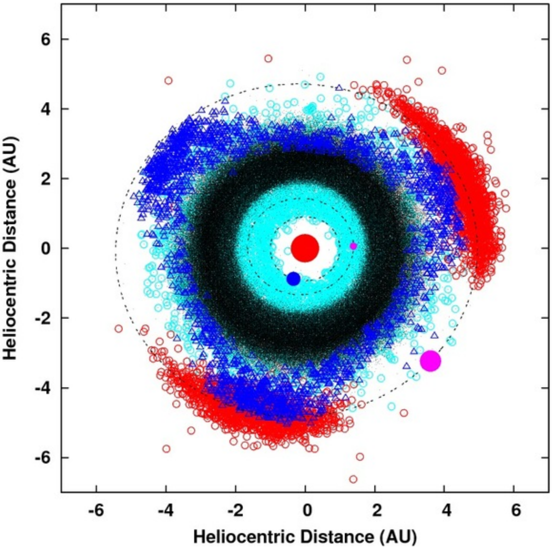
$$a_{Hilda} \simeq \left(\frac{2}{3}\right)^{2/3} a_{Jup} \simeq 3.97\text{au}$$



Gaps and concentrations

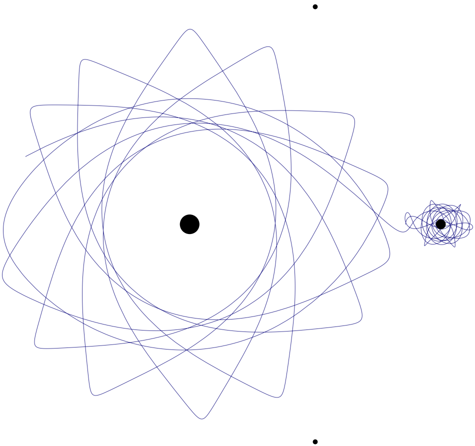
- resonances excite e
- perihelion diminishes
- close encounter with Mars, Earth, Venus
- ejection from the resonance $\Rightarrow$ gap
- asteroids with large a cannot reduce their q enough, no encounters $\Rightarrow$ remain trapped in resonance

Hildas and Trojans



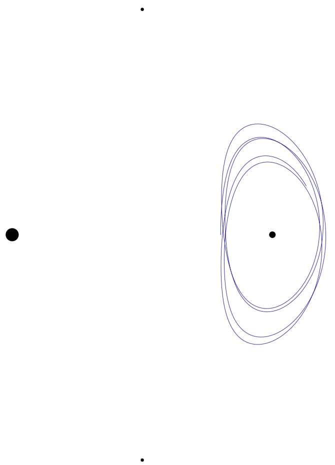
www.astro.ncu.edu.tw

Temporary satellite capture

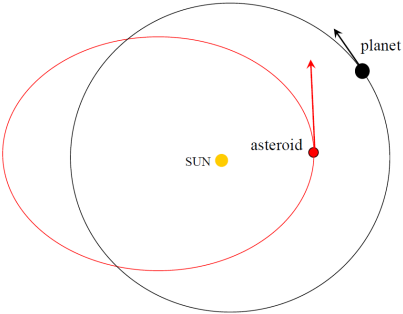


the most probable origin of the irregular satellites

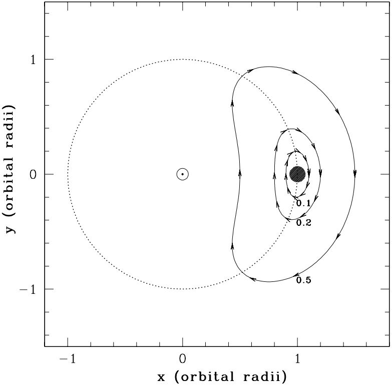
Quasi satellite, resonance 1:1



Quasi satellite, resonance 1:1



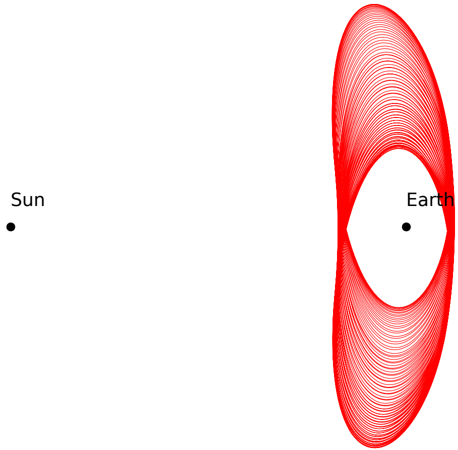
inertial frame



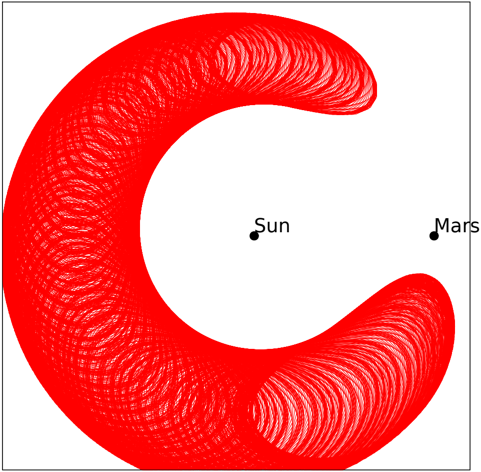
Wiegert et al. 2000

rotating frame

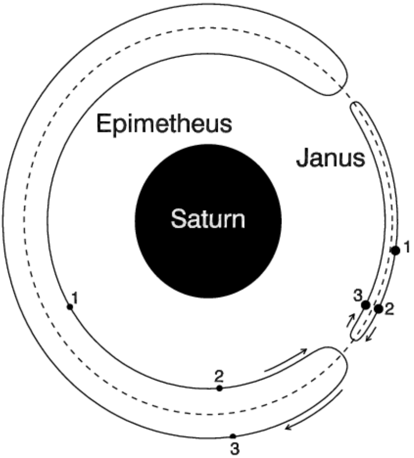
2004 GU9: Earth quasi satellite, resonance 1:1



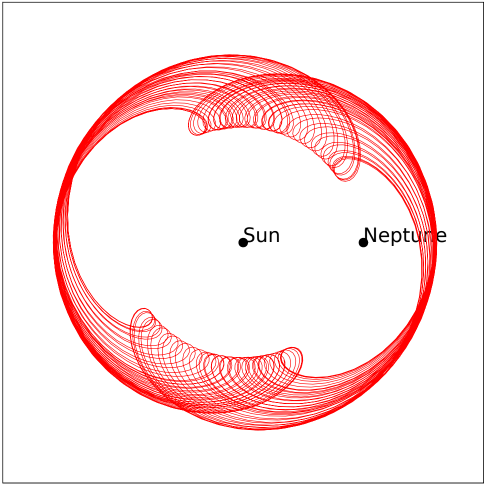
1999 ND43: Mars horseshoe, resonance 1:1



Janus - Epimetheus 1:1



(134340) Pluto in exterior resonance 2:3



Theory: a simple model for resonance $(p+q):p$

$$R = R_{ShortP} + R_{LongP} + R_{Res}(\sigma)$$

Assuming

- Jupiter in circular orbit
- coplanar orbits ($i = 0$)

Resonant disturbing function

$R_{Res}(\sigma)$ depends on the **critical angle**:

$$\sigma = (p+q)\lambda_J - p\lambda_{ast} - q\varpi_{ast}$$

planetary equations

$$\frac{da}{dt} \propto \frac{\partial R}{\partial \sigma}$$

where

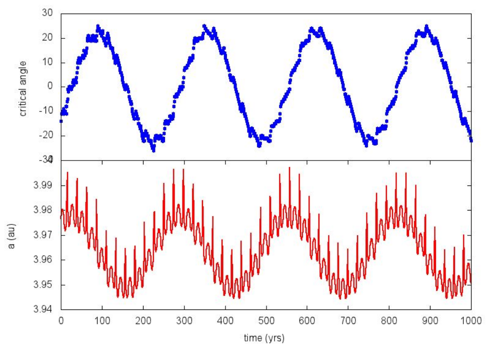
$$R \propto m_J e^q \cos\sigma$$

$\Longrightarrow$

$$\Rightarrow \frac{da}{dt} \propto m_J e^q \sin\sigma$$

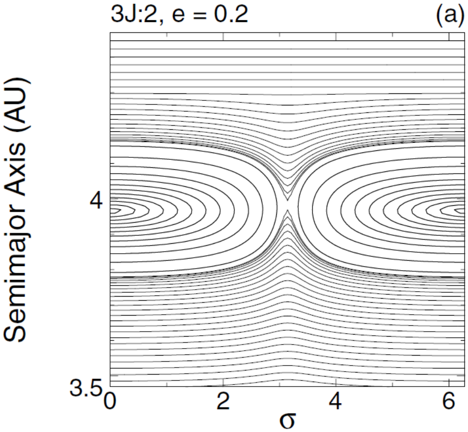
- equilibrium points: $\sigma = 0°, 180°$
- amplitude $\propto e^q$

153 Hilda: num. integration



libraties

Semimajor axis: width



Nesvorny et al. in Asteroids III

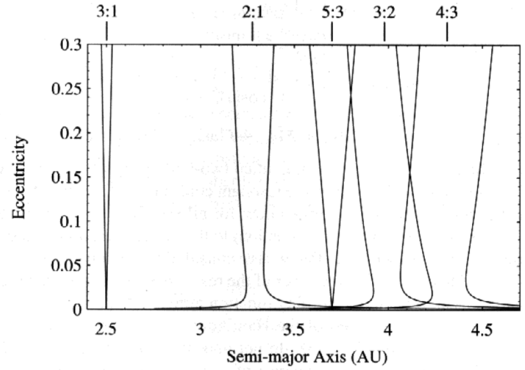
- circulation:

$$\frac{d\sigma}{dt} \neq 0$$

- librations:

$$\frac{d\sigma}{dt} \sim 0$$

Widths $\propto e^q$

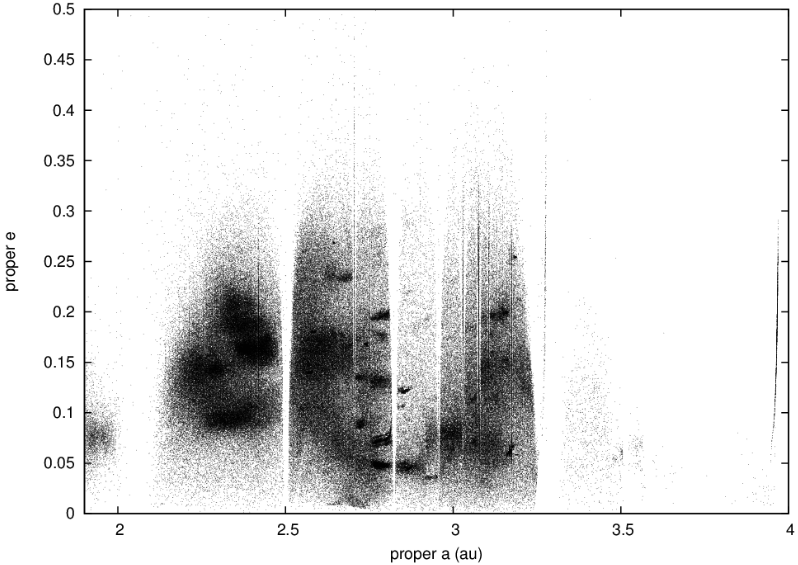


Murray and Dermott in Solar System Dynamics

Chaos:

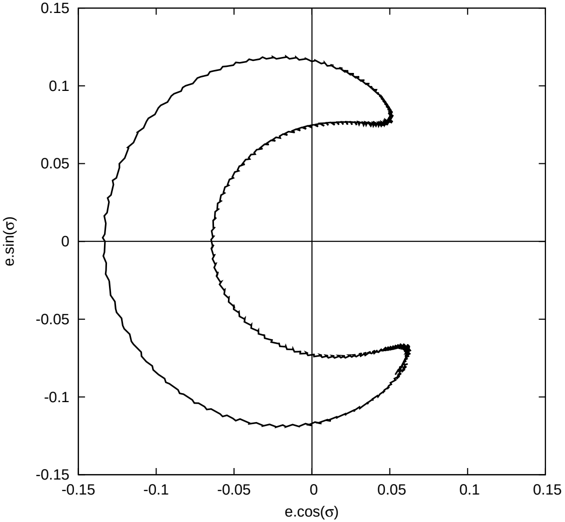
- at resonance borders
- superposition of resonances

350.000 asteroids

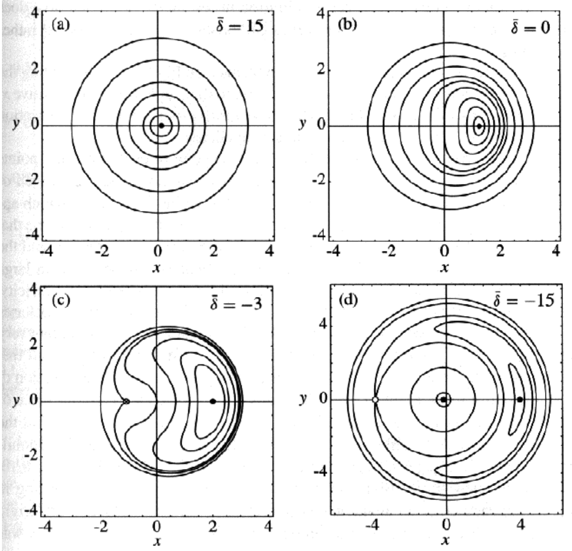


AstDyS database

Librations in eccentricity: bananas

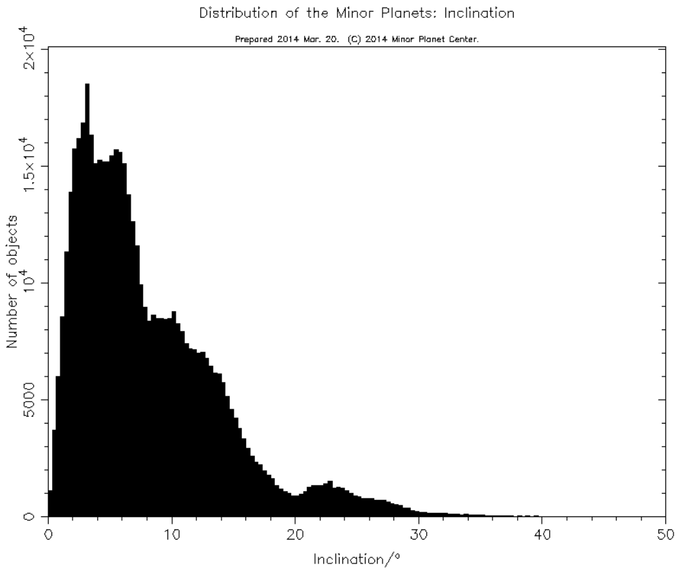


Topology $(e\cos\sigma, e\sin\sigma)$



Murray and Dermott in Solar System Dynamics

Orbital inclinations of asteroids



SPATIAL Resonant Disturbing Function

Asteroid perturbed by Jupiter in circular orbit:

$$R = \sum_j C_j(a, e, i) \cos(\sigma_j)$$

$$\sigma_j = k_1\lambda + k_2\lambda_J + k_3\varpi + k_4\Omega$$

the arbitrary set of k_i must verify:

$$k_1 + k_2 + k_3 + k_4 = 0$$

principal term:

$$C(a, e, i) \propto e^{|k_3|}(\sin i)^{|k_4|}$$

- eccentricity type: $C \propto e^{|k_3|}$
- inclination type: $C \propto (\sin i)^{|k_4|}$
- mixed: $C \propto e^{|k_3|}(\sin i)^{|k_4|}$

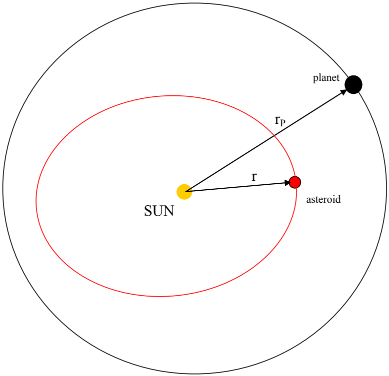
$\Longrightarrow$ we look for small k_3, k_4

Semi analytical methods

Reasons for numerical methods:

- analytical methods are very complex (R is complicated)
- interest to have a general view of all resonances in the SS
- quick estimation of locations and strengths
- identification of the strongest resonance in an interval of (a, e, i)

Numerical calculation of $R(\sigma)$



$$R = k^2 m_P \left(\frac{1}{\mid \mathbf{r}_P - \mathbf{r} \mid} - \frac{\mathbf{r} \cdot \mathbf{r}_P}{r_P^3} \right)$$

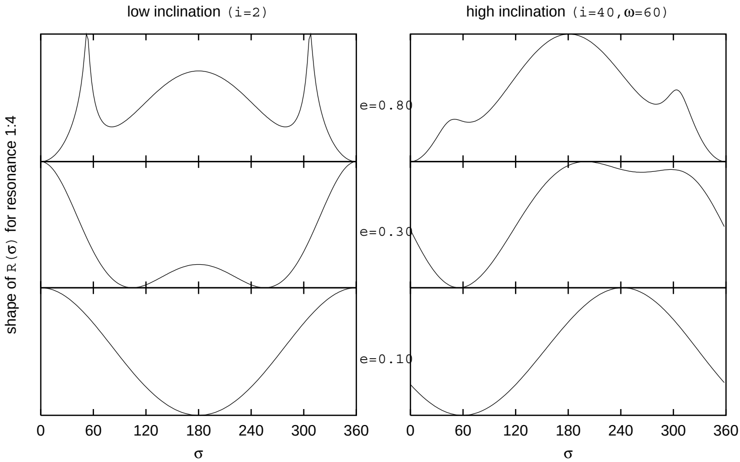
$$R(\sigma) \simeq \int R(\lambda_P, \lambda) d\lambda_P$$

where $\lambda = \lambda(\lambda_P, \sigma)$ assuming

$$\sigma = (p+q)\lambda_P - p\lambda - q\varpi$$

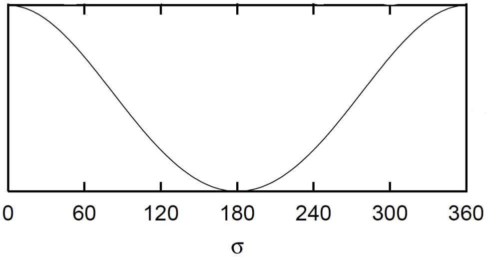
$R(\sigma)$ is mean R imposing the resonant link: $\sigma =$ constant.

Numerical calculation of $R(\sigma)$ for resonance 1:4



Gallardo 2006

Strength of the resonance

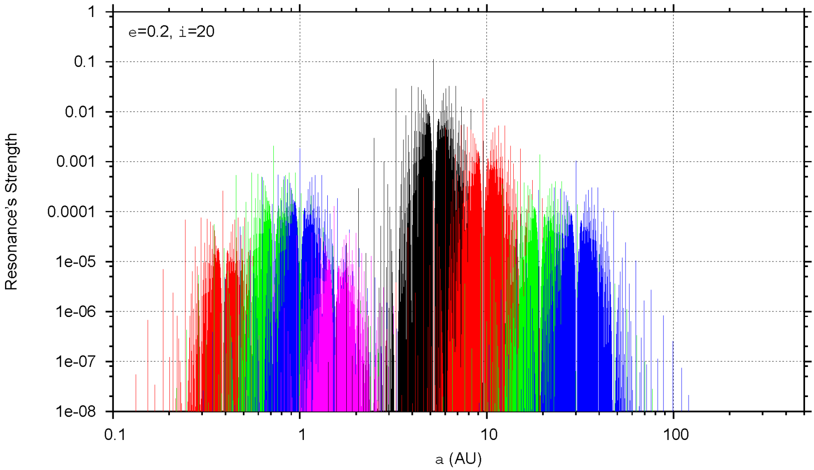


Strength:

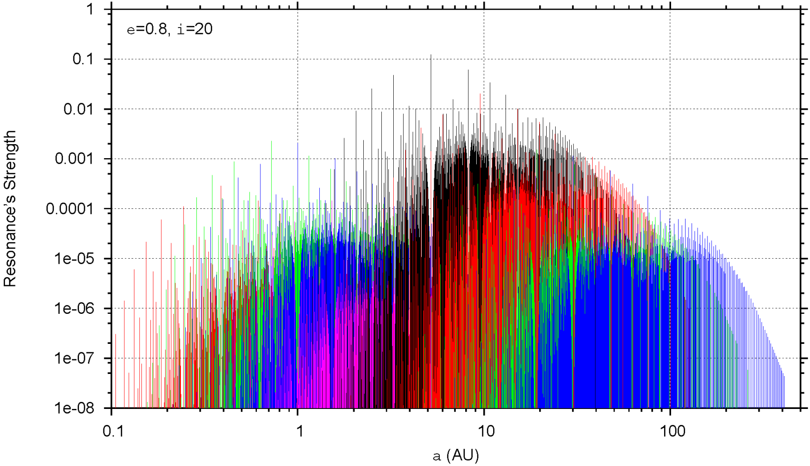
$$S = \Delta R(\sigma)$$

The perturbation necessary to eject an asteroid from the resonance is proportional to the amplitude of $R(\sigma)$.

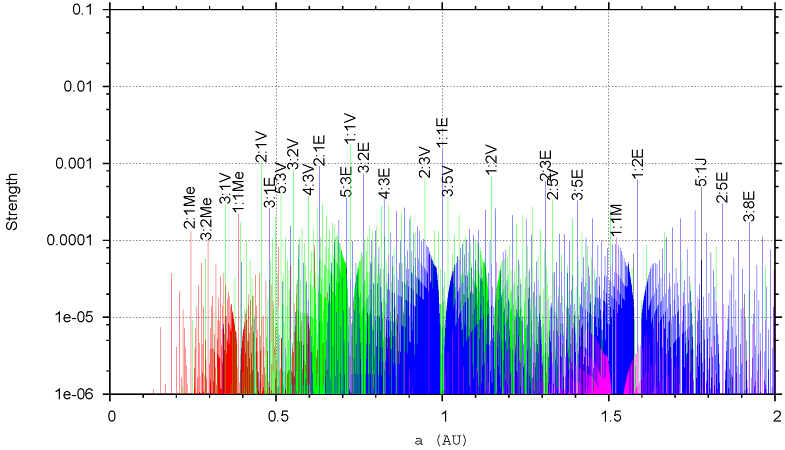
Atlas of resonances in the Solar System, low e



Atlas of resonances in the Solar System, high *e*

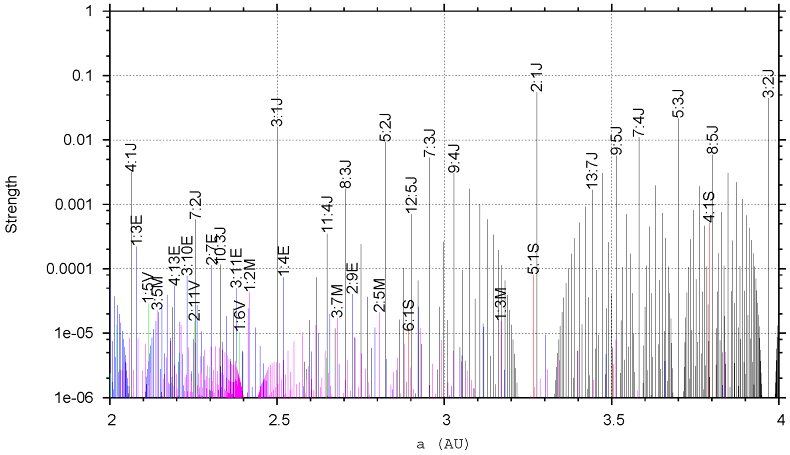


Atlas from 0 to 2 au



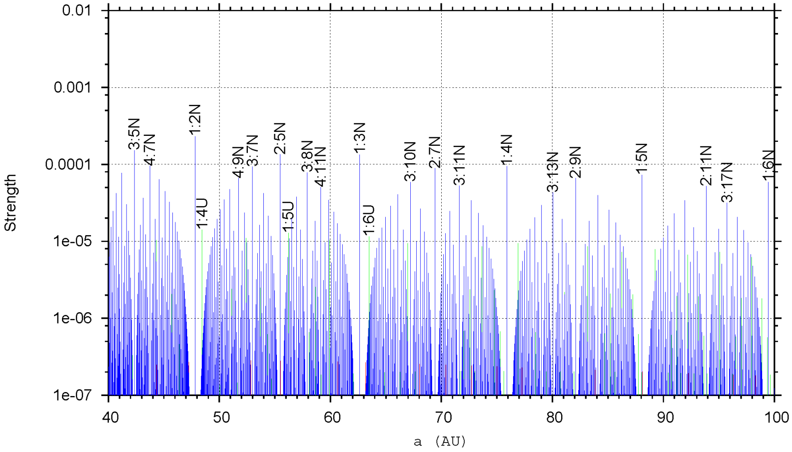
Gallardo 2006

Atlas in the asteroids region



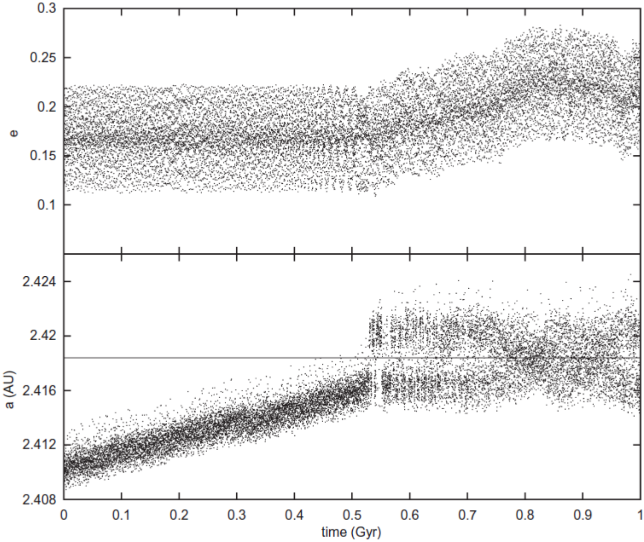
Gallardo 2006

Atlas in the Trans Neptunian Region



Gallardo 2006

Stickiness: ability to capture particles



Gallardo et al. 2011

Retrograde resonances

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

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Asteroids in retrograde resonance with Jupiter and Saturn

M. H. M. Morais[1]★ and F. Namouni[2]★

[1]*Department of Physics & I3N, University of Aveiro, Campus Universitário de Santiago, P-3810-193 Aveiro, Portugal*
[2]*Université de Nice, CNRS, Observatoire de la Côte d'Azur, BP 4229, F-06304 Nice, France*

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ABSTRACT
We identify a set of asteroids among Centaurs and Damocloids, which orbit contrary to the common direction of motion in the Solar system and which enter into resonance with Jupiter and Saturn. Their orbits have inclinations $I \gtrsim 140^\circ$ and semimajor axes $a < 15$ au. Two objects

Retrograde resonance in the planar three-body problem

M. H. M. Morais · F. Namouni

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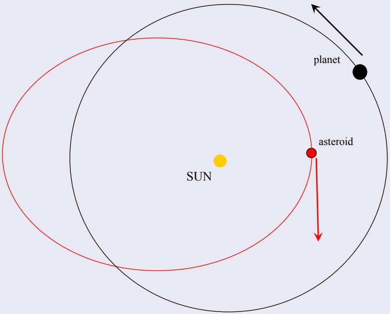
Published online: 12 October 2013

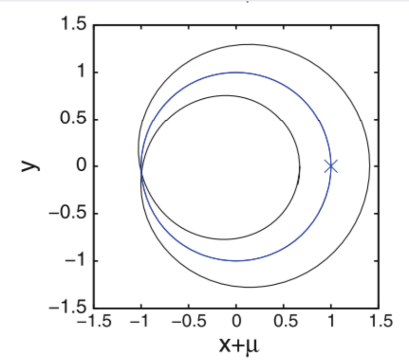
Abstract We continue the investigation of the dynamics of retrograde resonances initiated in Morais and Giuppone (Mon Notices R Astron Soc 424:52–64, doi:10.1111/j.1365-2966.

Coorbital retrograde

heliocentric motion



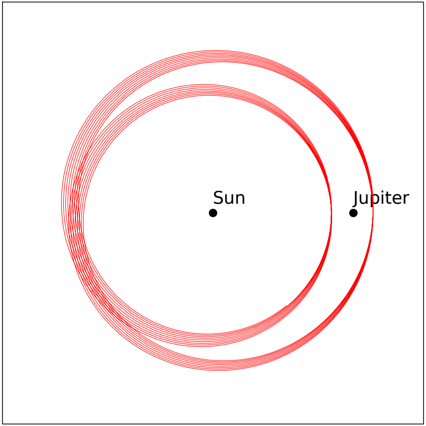
relative motion



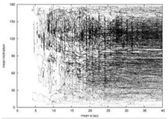
Morais and Namouni 2013

2015 BZ509: discovered in January 2015

$a = 5.12$ au, $e = 0.38$, $i = 163°$

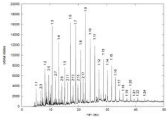


Resonances of Long Period Comets



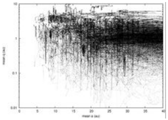
Fernandez et al., in preparation

Resonances of Long Period Comets



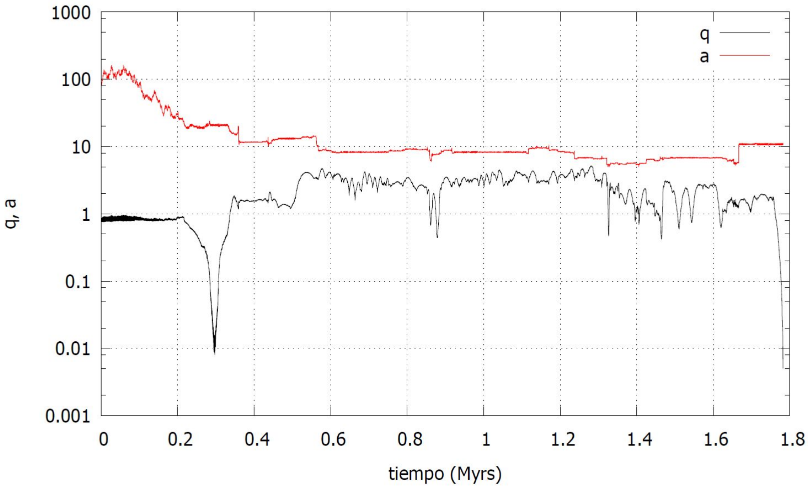
Fernandez et al., in preparation

Resonances of Long Period Comets

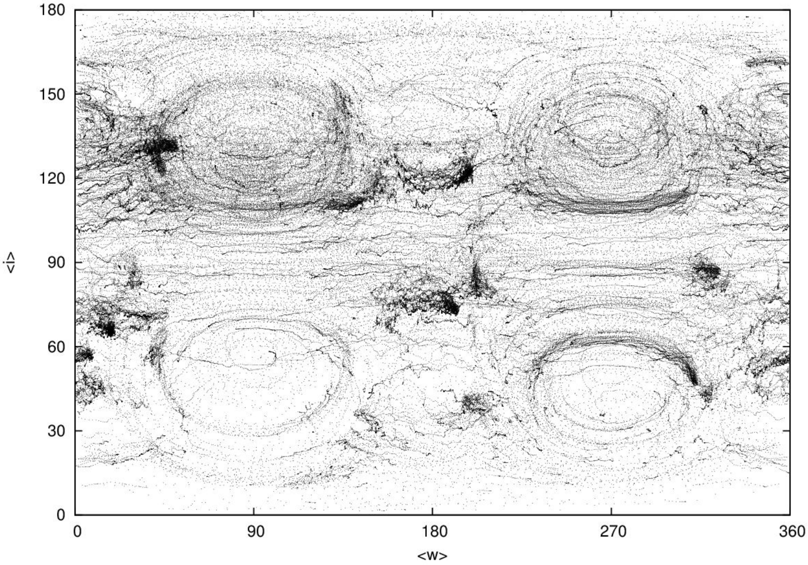


Fernandez et al., in preparation

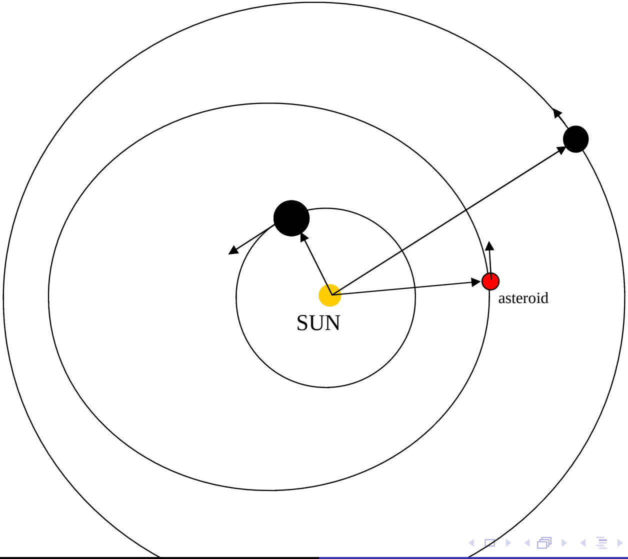
LP comet ending as sungrazer



Orbital states in space (ω, i): KL mechanism



Orbital commensurability: $k_0 n_0 + k_1 n_1 + k_2 n_2 \simeq 0$



TBRs are weak and numerous

$$k_0 n_0 + k_1 n_1 + k_2 n_2 \simeq 0$$

- Given two planets, an **infinite** family of TBRs is defined:

$$n_0 \simeq \frac{-k_1 n_1 - k_2 n_2}{k_0}$$

- **how strong** are they?
- They are **weak**: $\propto m_1 m_2$.
- superposition generates **chaotic diffusion**.

Three Body Resonances

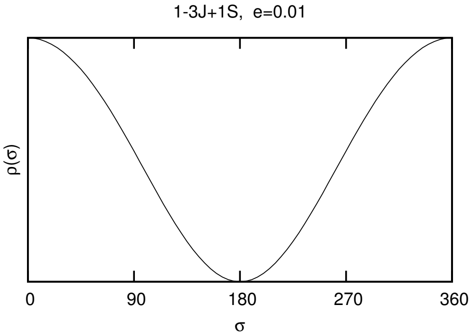
- A VERY complicated problem
- An expansion for the resonant disturbing function can be obtained as a summation of terms of the type

$$Ce_0^{|k_3|}e_1^{|k_4|}e_2^{|k_5|}\sin(i_0)^{|k_6|}\sin(i_1)^{|k_7|}\sin(i_2)^{|k_8|}\times$$

$$\times\cos(k_0\lambda_0+k_1\lambda_1+k_2\lambda_2+k_3\varpi_0+k_4\varpi_1+k_5\varpi_2+k_6\Omega_0+k_7\Omega_1+k_8\Omega_2)$$

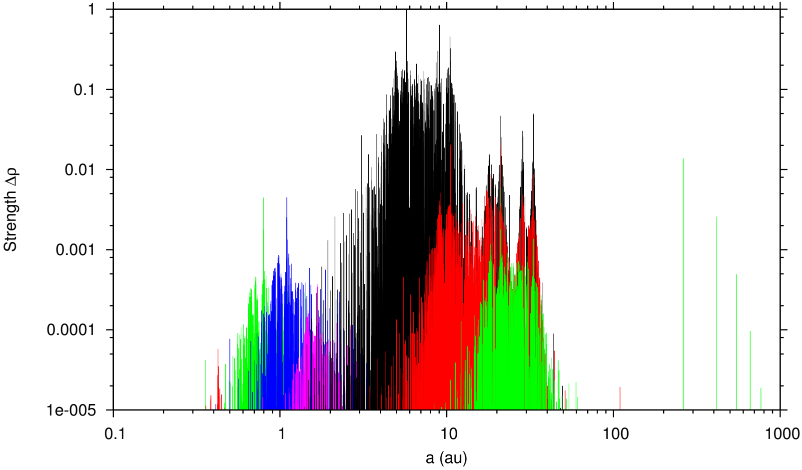
- being C also a VERY complicated expression

$\rho(\sigma)$: numerical estimation of $R(\sigma)$



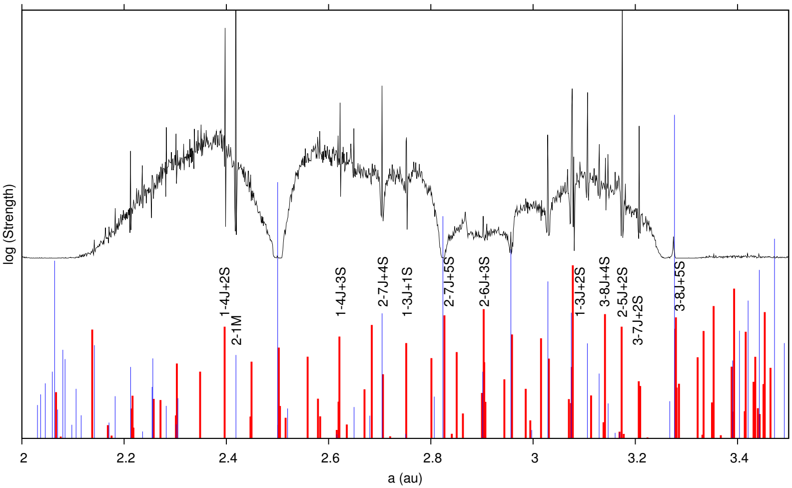
- **large variations** of ρ with σ is indicative of a **strong** resonance
- **small variations** of ρ with σ is indicative of a **weak** resonance
- an **extreme** of $\rho(\sigma)$ at some σ means there is an **equilibrium point**

Atlas of TBRs: global view (for $e = 0.15$)



Gallardo 2014

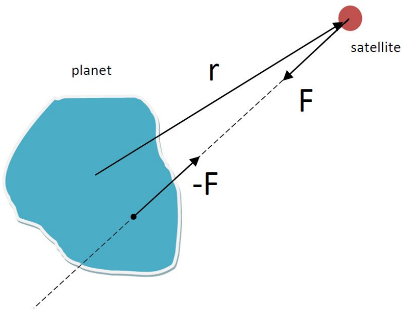
Effects on the distribution of asteroids



Gallardo 2014

Spin-Orbit Resonances

Irregular bodies: Angular Momentum exchange



A not central $V(\vec{r})$ generates a force on the satellite:

$$\vec{F} = -m_{sat}\nabla V$$

- reaction on the planet:

$$\vec{M} = \vec{r} \wedge (-\vec{F}) = \vec{r} \wedge m_{sat}\nabla V$$

- $\vec{L}_{pla}$ variation:

$$\frac{d\vec{L}_{pla}}{dt} = \vec{M}$$

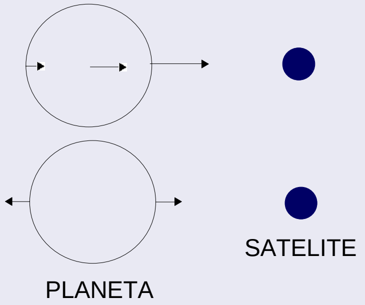
- $\vec{L}$ conservation:

$$\vec{L}_{pla} + \vec{L}_{orb} = \text{constant}$$

$\Rightarrow$ angular momentum exchange $\Delta\vec{L}_{orb} = -\Delta\vec{L}_{pla}$

Tides

Tides: a common cause of angular momentum exchange.



acceleration due to the satellite:

$$\alpha = G\frac{m_{sat}}{r^2}$$

tides on the planet:

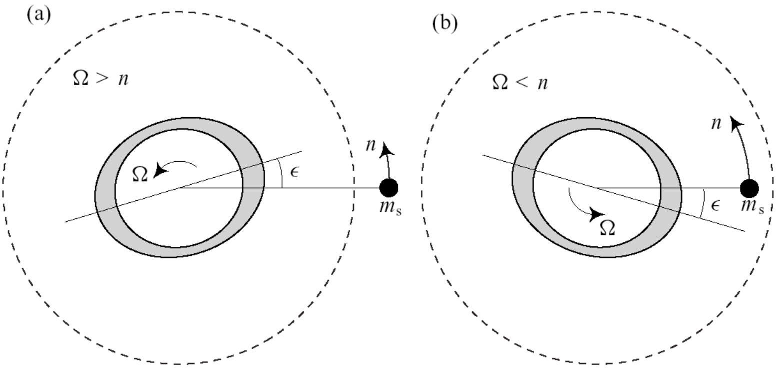
$$\Delta\alpha = 2G\frac{m_{sat}}{r^3}\Delta r$$

where $\Delta r = R_{pla}$

tides $\Rightarrow$ deformation

Tides and continuous Angular Momentum exchange

Response to tides are not instantaneous:

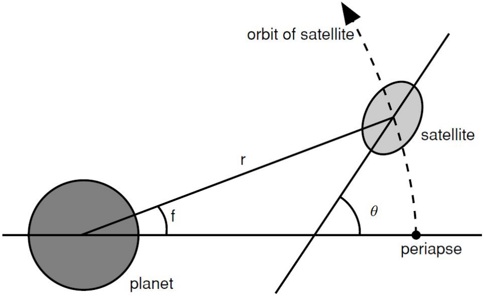


Murray and Dermot, 1999

The bulge is systematically ahead (the planet rotation slows down) or back (the planet rotation accelerates).

Spin-orbit resonance

Now, look at the satellite



Encyclopedia of the Solar System, chap. 42

from Euler's equations

$$\dot{\vec{L}}_{sat} = \vec{M}$$

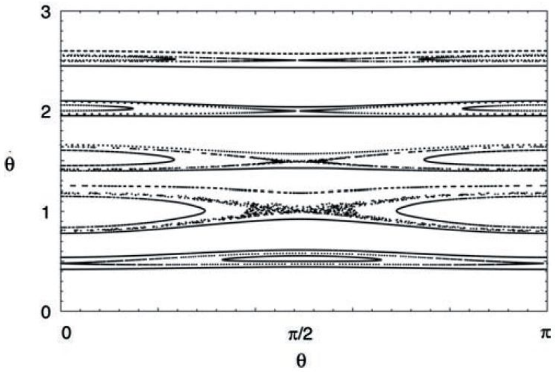
$$\Rightarrow \ddot{\theta} = \frac{\omega_0^2}{2r^3} \sin 2(f - \theta)$$

with

$$r = \frac{a(1 - e^2)}{1 + e\cos f}$$

$$\omega_0^2 = \frac{3(B - A)}{C} G m_p$$

Case $e = 0.1$ and $\omega_0 = 0.2$:

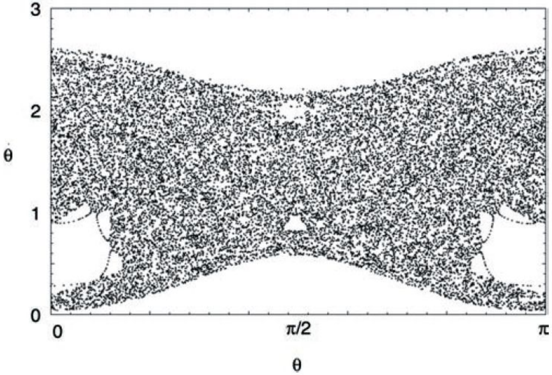


Encyclopedia of the Solar System, chap. 42

Resonances 1:2, 1:1, 3:2, 2:1, 5:2 are showed.

Spin-orbit resonance: chaos

Case of Hyperion $e = 0.1$ and $\omega_0 = 0.89$:



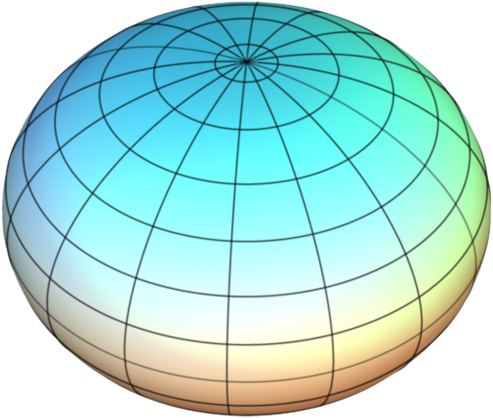
Encyclopedia of the Solar System, chap. 42

Lindblad resonances

Lindblad resonances

Consider an axially symmetric planet

$$V(r,\phi) = -\frac{GM}{r}\left[1 - J_2 P_2(\sin\phi)\left(\frac{R}{r}\right)^2 + \dots\right]$$



Lindblad resonances

mean motion $n^2 = \frac{GM}{r^3}\left[1 + \frac{3}{2}J_2\left(\frac{R}{r}\right)^2 + \ldots\right]$

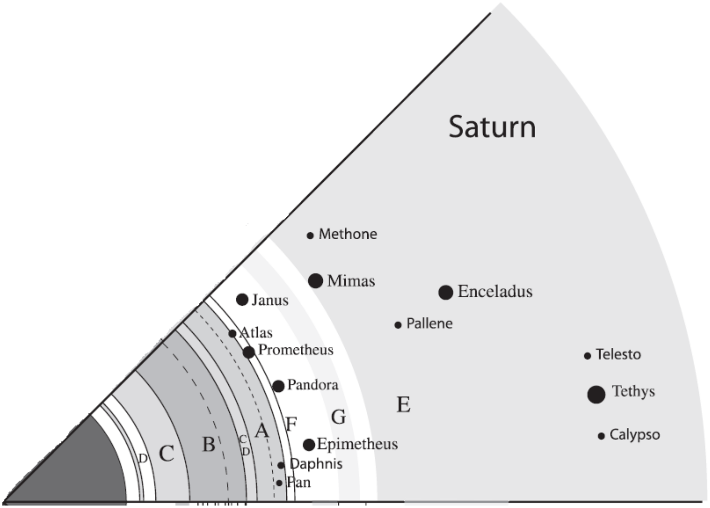
radial frequency $k^2 = \frac{GM}{r^3}\left[1 - \frac{3}{2}J_2\left(\frac{R}{r}\right)^2 + \ldots\right]$

vertical frequency $\mu^2 = \frac{GM}{r^3}\left[1 + \frac{9}{2}J_2\left(\frac{R}{r}\right)^2 + \ldots\right]$

a spherical planet verifies $n = k = \mu$

Lindblad resonances

Saturn rings and satellites



Encyclopedia of the Solar System, chap. 27

Lindblad resonances

⇒ resonances occur at r when:

$$\frac{n(r)}{n_s} = \frac{j+k+l}{j-1}$$

with strength

$$\propto e^{|l|}(\sin i)^{|k|}$$

⇒ strongest **horizontal** ($i = 0$) resonances occur for $l = k = 0$:

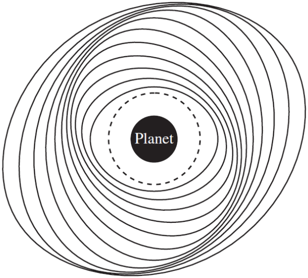
$$\frac{n(r)}{n_s} = \frac{j}{j-1} \qquad (29:28)$$

⇒ strongest **vertical** ($i \neq 0$) resonances occur for $k = 1, l = 0$:

$$\frac{n(r)}{n_s} = \frac{j+1}{j-1} \qquad (4:2)$$

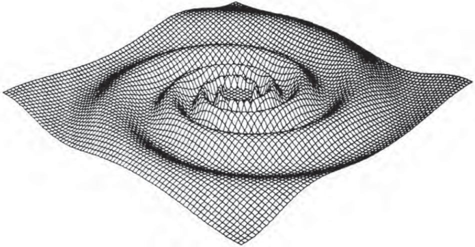
⇒ formation of spiral **density** (e, horizontal) waves and spiral **bending** (i, vertical) waves.

Spiral waves



Fundamental Planetary Science

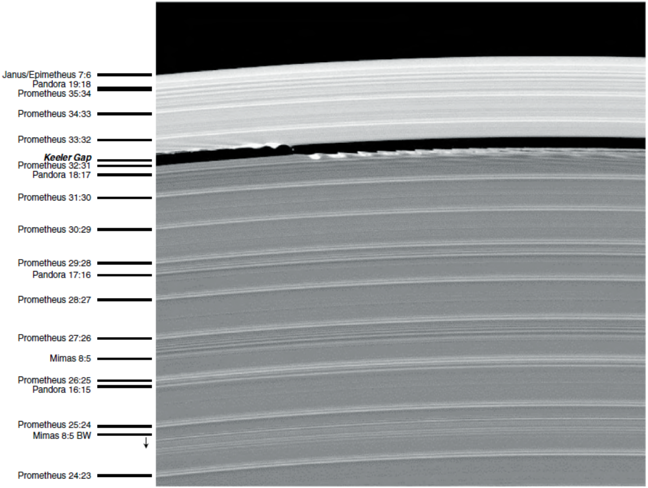
spiral density waves
(due to e)



Fundamental Planetary Science

spiral bending waves
(due to i)

Saturn rings



Lissauer and de Pater, Fundamental Planetary Science

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- Modern Celestial Mechanics, Morbidelli 2011
- Methods of Celestial Mechanics, Beutler 2005
- Encyclopedia of the Solar System, McFadden et al. (eds) 2007
- Fundamental Planetary Science, Lissauer and de Pater 2013

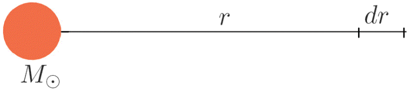
Orbital integrations

- JPL database: ssd.jpl.nasa.gov/sbdb-query.cgi
- MPC: www.minorplanetcenter.net
- AstDyS: hamilton.dm.unipi.it/astdys
- numerical integrators for beginners: Solevorb, Evorb, ORSA,...
- numerical integrators for experts: Mercury, Swift, HNBody,...

Appendix

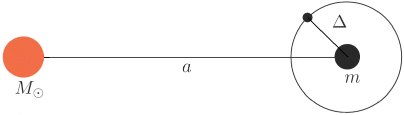
Close encounters?

$$\alpha = GM_{\odot}/r^2$$



A small departure dr generates a tide:

$$d\alpha = 2GM_{\odot}\frac{1}{r^3}dr$$



Solar tide on satellite's orbit:

$$d\alpha = 2GM_{\odot}\frac{1}{a^3}\Delta$$

$$\alpha_{pla} = Gm/\Delta^2$$

$d\alpha = \alpha_{pla}$ occurs for

$$\Delta_{lim} \sim a\left(\frac{m}{2M_{\odot}}\right)^{1/3}$$

$$\Rightarrow \Delta_{Hill} = a\left(\frac{m}{3M_{\odot}}\right)^{1/3}$$

Planetary system: Angular Momentum Deficit

$$\vec{L} = \sum_{j=1}^{N} \vec{L}_j = (C_x, C_y, C) = \text{constant}$$

invariable plane: $\perp \vec{L}$

$$L_z = C \simeq \sum_{j=1}^{N} m_j \sqrt{a_j(1 - e_j^2)} \cos i_j$$

L for circular coplanar orbits in IP is

$$L(0,0) = \sum_{j=1}^{N} m_j \sqrt{a_j}$$

$$AMD = L(0,0) - L_z(\text{actual}) = \text{constant}$$

AMD = departure from coplanar circular orbits

Mystery of meteorites solved

- metallic meteorites have Cosmic Ray Exp. times of $\sim$ 100 Myrs
- carbonaceous meteorites have Cosmic Ray Exp. times of $\sim$ 1 Myrs

Mechanism:

- collision starts exposure to cosmic rays
- Yarkovsky:
  - metallic fragments migrate slowly (large CRE)
  - others fragments migrate quickly (small CRE)
- they reach a resonance at different times
- resonance quick delivery to Earth