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The occurrence of high-order resonances and Kozai mechanism in the scattered disk

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Abstract

By means of numerical methods we explore the relevance of the high-order exterior mean motion resonances (MMR) with Neptune that a scattered disk object (SDO) can experience in its diffusion to the Oort cloud. Using a numerical method for estimate the strength of these resonances we show that high-eccentricity or high-inclination resonant orbits should have evident dynamical effects. We investigate the properties of the Kozai mechanism (KM) for non-resonant SDO's and the conditions that generate the KM inside a MMR associated with substantial changes in eccentricity and inclination. We found that the KM inside a MMR is typical for SDO's with Pluto-like or greater inclinations and is generated by the oscillation of ω inside the mixed (e, i) resonant terms of the disturbing function. A SDO diffusing to the Oort cloud should experience temporary captures in MMR, preferably of the type 1:N, and when evolving inside a MMR and experiencing the KM it can reach regions where the strength of the resonance drops and consequently there is a possibility of being decoupled from the resonance generating by this way a long-lived high-perihelion scattered disk object (HPSDO).

Keywords: Celestial mechanics; Resonances; Trans-neptunian objects

1. Introduction

It is not well established whether the trans-neptunian population drops abruptly at around 50 AU or continues far away in a not known function of the heliocentric distance (Morbidelli et al., 2003). But since the work of Duncan and Levison (1997) the numerical simulations show a diffusion from the Edgeworth–Kuiper belt through more extended regions of the Solar System up to the Oort cloud (Fernandez et al., 2004). Then we expect some transient population with very high semi-major axis evolving in the called scattered disk (SD), a region that can be defined by $q > 30$ AU and $a > 50$ AU. This is sustained by the discovery of several objects with $a > 100$ AU.

It has been found in recent numerical experiments the capture of hypothetical scattered disk objects (SDOs) in high-order exterior mean motion resonances (MMR) with Neptune and following very stable evolutions for timescales of gigayears (Gomes et al., 2005). It has also been found (Duncan and Levison, 1997) that objects captured in MMR also experiment the Kozai mechanism (KM) (Kozai, 1962) which is a long-term coupled evolution of (e, i, ω) maintaining an approximately constant value of $H = \sqrt{1-e^2}\cos i$. Remarkably large changes in e and i are associated with oscillations of ω and this particular type of Kozai mechanism is usually known as Kozai resonance (KR). The MMR provides a the constancy of a and the KR can inject for long timescales the objects in high-perihelion high-inclined orbits or HPSDOs following Gomes et al. (2005).

There is an extended literature about asteroids in MMR with the planets, mainly with Jupiter, and about trans-neptunian objects in MMR with Neptune. But, almost all works are referred to low-order resonances inside $a < 60$ AU. There are some exceptions that we should point out. For example, a temporary capture into the exterior 3:13 resonance with Neptune was already reported by Duncan and Levison (1997) for a very eccentric orbit. They also noted the KR is responsible for the increase of the perihelion distances. In a very different context, Chambers (1997) studied the stability of comets in high-order

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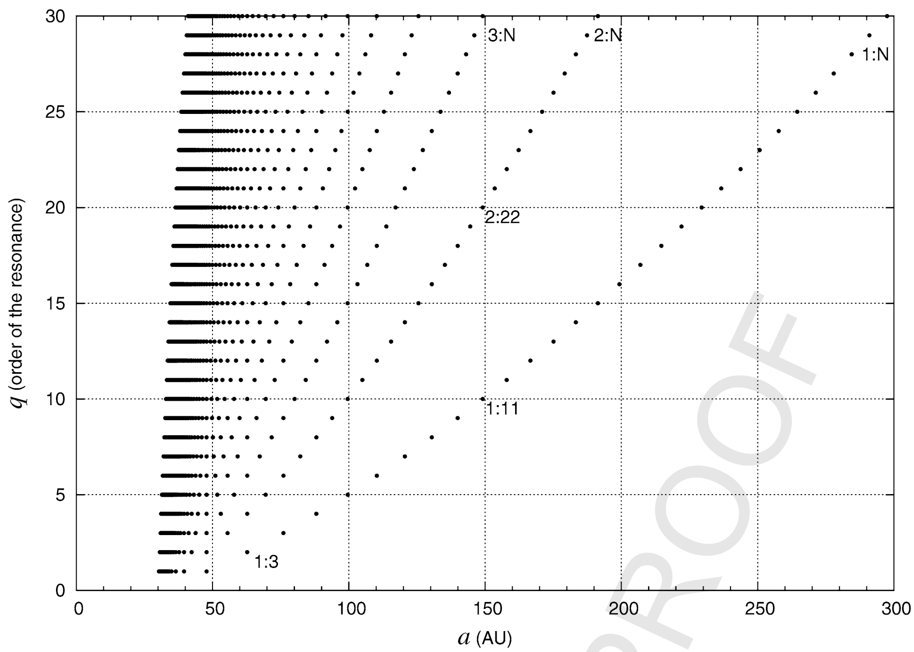

Fig. 1. Approximate localization of the exterior mean motion resonances with Neptune. High values of the order q should correspond to less important resonances from a dynamical point of view. Resonances of the type $1{:}N$ are the lowest-order resonances that a SDO encounter when diffusing outwards.

exterior resonances with Jupiter. By means of a frequency map analysis Robutel and Laskar (2001) identify several high-order exterior MMR with Neptune in the region $a < 90$ AU. They found that for high-inclination resonant orbits the chaotic diffusion is greater than for low-inclination ones; we will come back to this point later. Gladman et al. (2002) showed a particle temporarily captured in the 1:5 resonance with an evident KR associated. They also reported that clones of 2000 CR_{105} were temporarily captured in high-order MMR. Kuchner et al. (2002) studied the evolution of high-inclination KBOs and they found some MMR acting together with the KM in the region $a < 47$ AU. Fernandez et al. (2004) reported captures in resonances up to the resonance 1:13. They found some cases were a MMR is associated with the KR producing high variations in perihelion distances. Gomes et al. (2005) reported the capture at resonances of very high-order like 1:24. They found that an association between high-order MMR with Neptune and the KR is the responsible for large excursions in eccentricity and inclination. They also found that this association MMR + KR seems to be the rule for high-inclination orbits.

We know by basic celestial mechanics that the strength of a resonance is approximately proportional to the eccentricity of the resonant orbit elevated to the order of the resonance and consequently we generally do not consider high-order resonances because their strengths should be vanishingly small. Then, why in the SD so high-order resonances can have so strong effect to become evident? Why for Pluto-like or greater inclination orbits in MMR the KR with big Δe and Δi seems to be the rule? Why it does not appear independently of a MMR? For example, at several figures from Gomes et al. (2005) it seems that the KR appears almost immediately that a MMR is reached; why a so strong link between KR and MMR?

In this work we will try to answer that questions exploring the dynamics of the lowest order resonances in the SD. Even the lowest-order resonances in this region correspond to high-order resonances (Fig. 1). In Section 2 after briefly presented the sketch of the disturbing function for a SDO we will propose a method for measure the strength of a resonance and we will apply it to several high-order MMR. In order to exemplify, in the next sections we will focus on the resonance 1:11 and we will explore in which circumstances the KR appears inside and outside the MMR. At Section 3 we analyze the solutions generated by the secular terms exclusively. At Section 4 we analyze the solutions generated by the secular plus resonant terms looking for the circumstances that generates the KR with strong variations Δe, Δi. At Section 5 we discuss the origin of the KR in the SD and the conclusions are presented.

2. The disturbing function for SDOs and the strength of a resonance

The time evolution of a SDO can be analyzed via the Lagrange's planetary equations which depend on the disturbing function R. In order to construct an analytical theory for the dynamics of a SDO we need an expression for R. We will follow the expansion of Ellis and Murray (2000) (EM) which, as the authors explain, allows the calculation, to any order, of the terms associated with any individual argument without the need for expanding the entire disturbing function. Considering a system composed by the Sun, Neptune and a SDO with orbital elements $(a, e, i, \varpi, \Omega)$ the usual expression of the expansion for R is a series of terms of the form:

$$R = \sum C \cos(\varphi) \tag{1}$$

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by:Jolanta p. 3

being C a function of the form

$$C = A(\alpha) e_{\mathrm{N}}^{k_3} e^{k_4} s_{\mathrm{N}}^{k_5} s^{k_6} \tag{2}$$

with $s = \sin(i/2)$, $A(\alpha)$ being a function of $\alpha = a_{\mathrm{N}}/a$ and

$$\varphi = j_1\lambda_{\mathrm{N}} + j_2\lambda + j_3\varpi_{\mathrm{N}} + j_4\varpi + j_5\Omega_{\mathrm{N}} + j_6\Omega, \tag{3}$$

where subscript N denotes Neptune. The j_i are integers verifying $\sum j_i = 0$ with $j_5 + j_6$ being always even (D'Alembert rules) and $k_i \geqslant |j_i|$. If φ is a quick varying angle the effect of the corresponding term will vanish in the long-term evolution even if the coefficient C is not vanishingly small. Also if φ is a slow varying angle but C is vanishingly small the term again will not have a dynamical effect in the motion of the particle. Then if we are interested in a correct description of the long-term dynamical evolution of the particle we must take into account all the slow varying terms with non-negligible coefficients C; these are the resonant and secular terms. As e_{N}, e, s_{N}, s are less than 1 we say that the corresponding term is of order $(k_3 + k_4 + k_5 + k_6)$. It is possible to simplify the analysis taking a circular orbit for Neptune with zero inclination, that means $e_{\mathrm{N}} = s_{\mathrm{N}} = 0$. With this reasonable approximation the number of terms involved drop considerably. It is also possible to take into account in R the terms due to all major planets.

The secular terms are those not depending on λ nor λ_{N} and, in general, a q-order resonance $|p+q|{:}|p|$ occurs when the general critical angle

$$\sigma = -(p+q)\lambda_{\mathrm{N}} + p\lambda + j_3\varpi_{\mathrm{N}} + j_4\varpi + j_5\Omega_{\mathrm{N}} + j_6\Omega \tag{4}$$

librates or have a slow time evolution. Due to the small time variation of the angles $(\varpi_{\mathrm{N}}, \varpi, \Omega_{\mathrm{N}}, \Omega)$ this occurs approximately for

$$-(p+q)\lambda_{\mathrm{N}} + p\lambda = \theta \approx \text{constant}, \tag{5}$$

which is equivalent to

$$\frac{n}{n_{\mathrm{N}}} \simeq \frac{p+q}{p}, \tag{6}$$

where the n's are the mean motions. Then

$$\frac{1}{\alpha} = \frac{a}{a_{\mathrm{N}}} \simeq \left(\frac{p}{p+q}\right)^{2/3}. \tag{7}$$

The integer p is known as the degree of the resonance and we follow the notation that takes $p < 0$ for exterior resonances and $p > 0$ for interior resonances. At Fig. 1 we show the approximate localization of the exterior resonances with Neptune deduced from the formula (7). For example, assuming that Neptune is the only perturbing planet and being in a circular orbit with zero inclination it is possible to show that the disturbing function has only 6 terms with coefficients of order 10 corresponding to the resonance 1:11 (where $p = -11$ and $q = 10$). The 6 associated critical angles are defined from

$$\sigma_j = \big(\lambda_{\mathrm{N}} - 11\lambda + (10 - 2j)\varpi + 2j\Omega\big) \tag{8}$$

with $j = 0, \ldots, 5$. For low-inclination orbits we generally consider only σ_0 but for high-inclination orbits the other σ's become relevant, this will be evident along this work. In this case the resonance is called mixed (e, i) MMR because the coefficients of the resonant terms are combinations of e and i (see Section 4). The σ_j are not independent because $\sigma_j = \sigma_0 - 2j\omega$, then for the study of the long-term evolution it is enough to follow the angles σ_0 and ω, for example, where the first one is related to the MMR and the second one to the KR.

In analogy with Schubart (1968), in order to explore numerically the function R for exterior resonant orbits and by means of a numerical integration of the exact disturbing function, we computed the value of the mean disturbing function

$$R(\theta) = \frac{1}{2\pi|p|} \int_0^{2\pi|p|} R\big(\lambda_{\mathrm{N}}, \lambda(\lambda_{\mathrm{N}}, \theta)\big)\, \mathrm{d}\lambda_{\mathrm{N}} \tag{9}$$

for a given set of values of $(\alpha, e, i, \varpi, \Omega, \theta)$, where we have assumed

$$-(p+q)\lambda_{\mathrm{N}} + p\lambda = \theta = \text{constant} \tag{10}$$

and where $R(\lambda_{\mathrm{N}}, \lambda)$ is evaluated numerically, without the use of a series expansion as in (1). This mean disturbing function is valid for a particle that strictly satisfies the condition above but we will consider it representative also for a SDO with a slow time evolution of the critical angle σ, at least during the period of time in which the integral (9) is calculated. We repeat for a series of values of θ between $(0, 2\pi)$ obtaining a numerical representation of $R(\theta)$. By its way this function give us a numerical representation of the resonant disturbing function. We call DR to the maximum relative variation of $R(\theta)$:

$$\mathrm{DR} = \frac{R_{\max}(\theta) - R_{\min}(\theta)}{\langle R \rangle} \tag{11}$$

being $\langle R \rangle$ the mean value of R with respect to θ. The function DR depends on (e, i, ω) and is a kind of *sensibility* to the critical angle because it measures the effects of θ on the disturbing function. If DR ~ 0 we have R almost independent of the critical angle and the coefficients C from (2) will be vanishingly small. Consequently it can be taken as an indicator of the strength and the dynamical effects on the particles evolving near that resonance. For low-eccentricity and low-inclination orbits DR should follow the function e^q.

As an example we show at Fig. 2 the results for the 10th-order resonance 1:11 for two inclinations and for the case $\omega = 0°$. For comparison is showed the trend of the function e^{10} which represents the leader term in the expansion of the resonant disturbing function for low-inclination orbits. The trend of DR follows very well the function e^{10} for low-inclination orbits but big discrepancies show up for high-inclination ones. This is due to the relative weight of the resonant terms depending on i that show up at non-zero inclinations. It is evident that in spite of the order of the resonance for high eccentricities the strength grows considerably. It is also clear the strength in general is greater for high-inclination orbits, except at very high eccentricities.

This procedure is applied to the resonances of the type 1:N which are the lowest-order resonances that a SDO find when diffusing outwards. The results are shown at Fig. 3 for low-inclination orbits and at Fig. 4 for high-inclination orbits. We

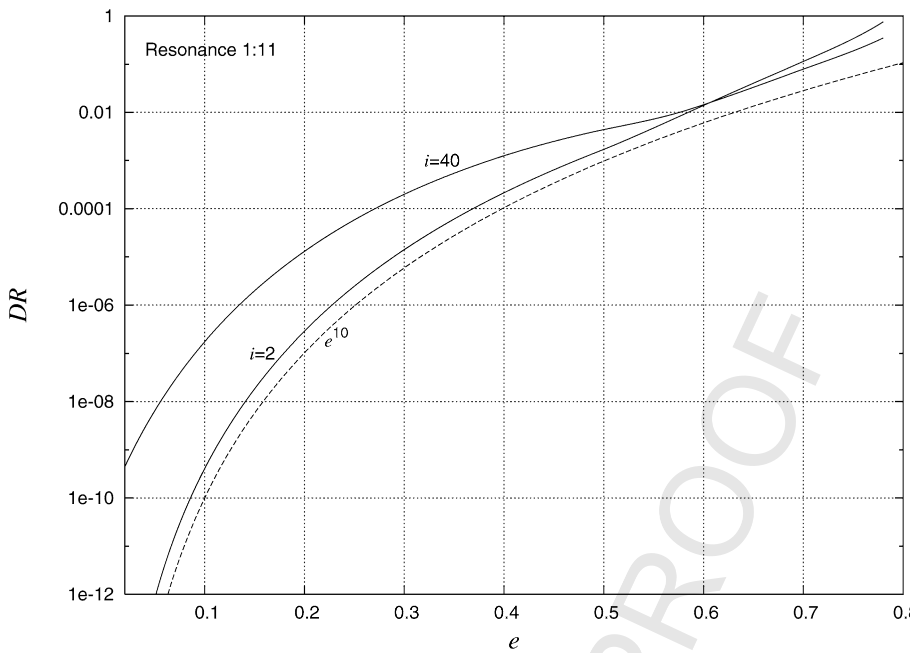

Fig. 2. The strength DR(e, i, ω) of the resonance 1:11 for low- and high-inclination orbits computed numerically for $\omega = 0°$. For comparison it is shown the trend of the function e^{10}, the leader term in the expansion of the resonant disturbing function for the case $i \sim 0°$, which approximately follows the trend for low-inclination orbits.

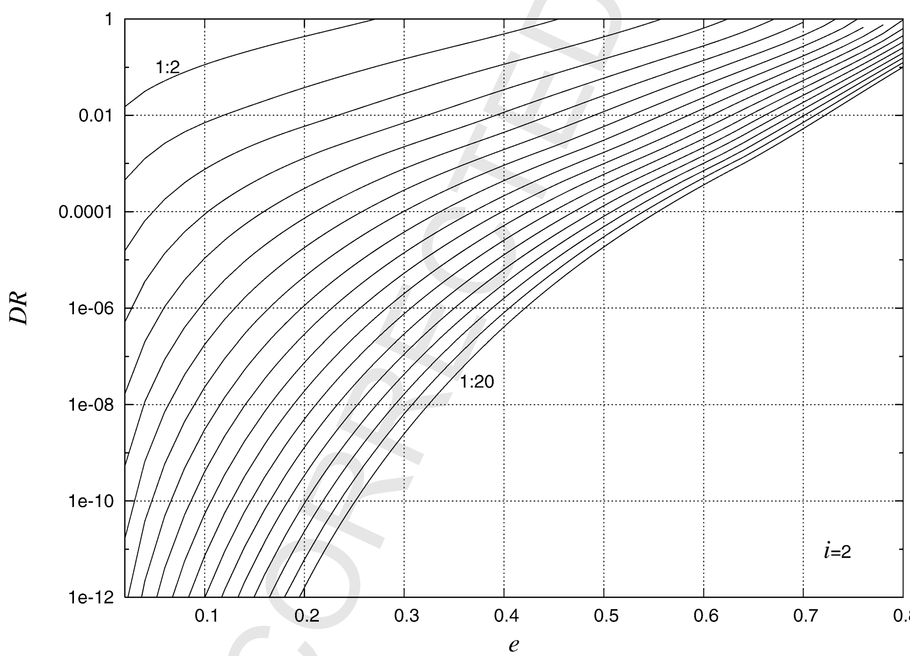

Fig. 3. The strength DR as a function of the eccentricity of the SDO's for low-inclination ($i = 2°$) resonant orbits of the type 1:N. DR follows very approximately the trend of the functions e^q being q the order of the resonance. Case $\omega = 0°$.

excluded orbits with perihelion distances less than 31 AU in order to avoid the strong perturbations by Neptune. To have an idea of the necessary strength for a resonance to subsist in the outer Solar System we have calculated DR for the known candidates to Twotinos (Chiang et al., 2003) obtaining DR > 0.5 for all cases. Values of DR less than 10^{-4}, for example, certainly cannot produce large dynamical effects, that means, locking in resonance. On the other hand, high values for DR only produce significant dynamical effects when the critical angle librates.

We note that the object 2000 CR_{105} has a barycentric semi-major axis, $a \simeq 221$ AU, near the 1:20 resonance ($a_{1:20} \simeq 222$ AU). If this object were located in that resonance, taking into account its eccentricity ($e \sim 0.8$), inclination ($i \sim 22°$), and argument of perihelion ($\omega \sim 317°$), the corresponding value for the strength of the resonance would be DR $\simeq 0.076$ which we consider is high enough for a resonance to show up in the SD. There are other resonances near the semi-major axis of this object although of very high order like, for example,

S0019-1035(05)00468-9/FLA AID:7841 Vol.●●●(●●●) yicar7841 [+model] P.5 (1-13)
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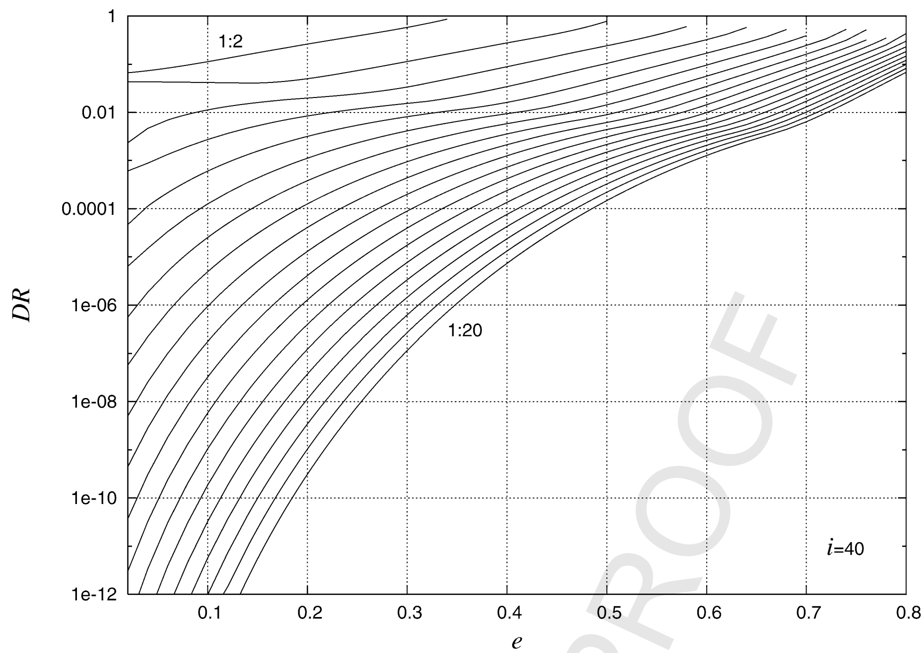

Fig. 4. The strength DR as a function of the eccentricity of the SDOs for high-inclination ($i = 40°$) resonant orbits of the type $1{:}N$. The functions DR are greater than the low-inclination corresponding ones and they depart from the trend of e^q. Case $\omega = 0°$.

$a_{2:39} \simeq 218$ AU. In spite of being of very high order this resonance is also strong (DR $\simeq 0.057$) but with a nominal value of $a_{2:39}$ more separated from 221 than $a_{1:20}$. In other words, taking into account the strength of the resonances near the semi-major axis of 2000 CR_{105} and the difficulties in the orbital determination of this kind of objects it would not be surprising that this object were captured in the resonance 1:20.

Looking at Figs. 3 and 4 we find, as a general rule, that for higher eccentricities we expect stronger dynamical effects. This is a confirmation of that we could deduce looking at the expansion of the disturbing function where the leader resonant terms are proportional to e^q. Then it is possible that high-order resonances show up in the SD when the e grows enough. Another interesting result, as we have already pointed out, is that higher-inclination resonant orbits should produce stronger resonant effects, that means high-inclination orbits could be sustained in resonance with smaller eccentricities than low-inclination orbits. It is interesting to note that Chambers (1997) in a different framework concluded that librations will be more prevalent in comets with highly inclined orbits. We do not analyze here the stability of this kind of orbits but we find the resonant effects should be more evident for high-inclination orbits. This is in agreement with Robutel and Laskar (2001) where it is showed that high-inclination resonant orbits in the region $a < 90$ AU have a diffusive process more evident than low-inclination resonant orbits.

All this indicate that mixed MMR with Neptune involving (e, i) should have an important role in the scattered disk. But to invoke a high-inclination high-perihelion population captured in high-order MMR with Neptune it is necessary to have an idea of the stability of these orbits. In order to show the importance of the eccentricity and inclination in the strength of a high-order MMR and to have an idea of the stability we have numerically integrated the system composed by the Sun, Neptune and some particles at the 1:11 resonance and looked for the evolution of the critical angles σ_j. Then we repeated the experiments including Uranus. We found that orbits with $e < 0.2$ never resonate due, probably, to the vanishingly small resonant potential compared with the short-period perturbations produced by Neptune. Orbits with $e > 0.3$ can evolve in resonance and for higher eccentricities and inclinations we obtained more critical angles σ_j librating, but when including Uranus only some high-inclination orbits remains locked in the resonance (see Figs. 5 and 6). Then, for a given eccentricity and due to short-period planetary perturbations, high-inclination resonant orbits have more chances of surviving with respect to the low-inclination ones.

In the next sections we will investigate which terms of the expansion of the disturbing function for a resonant SDO are relevant for the dynamical evolution, how that evolution could be and why the KR appears. We will focus on the specific 1:11 high-order MMR but the reasoning can be generalized to other high-order resonances.

3. Secular dynamics near the resonance 1:11

Looking carefully at expression (1) of the original expansion we will find several terms depending on λ. Taking into account we are studying a far object with an orbital period 11 times the orbital period of Neptune we can doubt if all the arguments φ depending on λ are quick varying variables. By semi-analytical methods or direct numerical integration it is possible to obtain the period of the small amplitude librations of the associated critical angles σ_j for this resonance. The libration period results to be at least an order of magnitude greater (around 10^4–10^5 yr) than the circulation period of λ (1820 yr), then we can discard

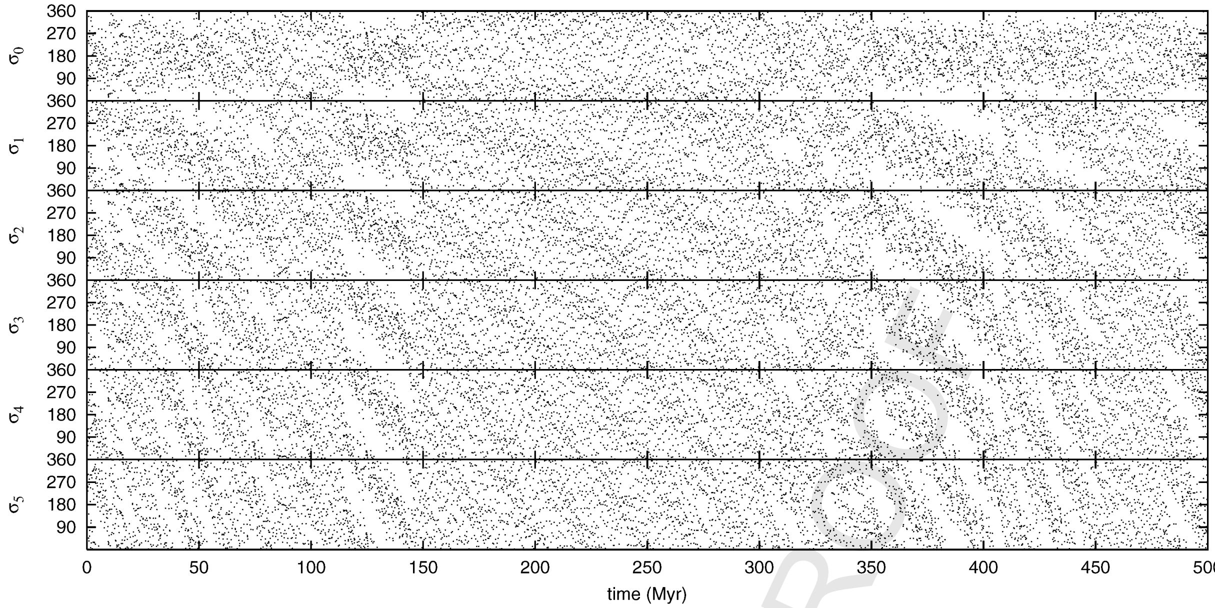

Fig. 5. Librations of a low-inclination ($i \simeq 2°$) resonant orbit with $e \simeq 0.5$ strongly perturbed by Uranus. The critical angles σ_j are defined following expression (8). The strength of this resonant orbit corresponds to $\mathrm{DR} \simeq 0.0017$.

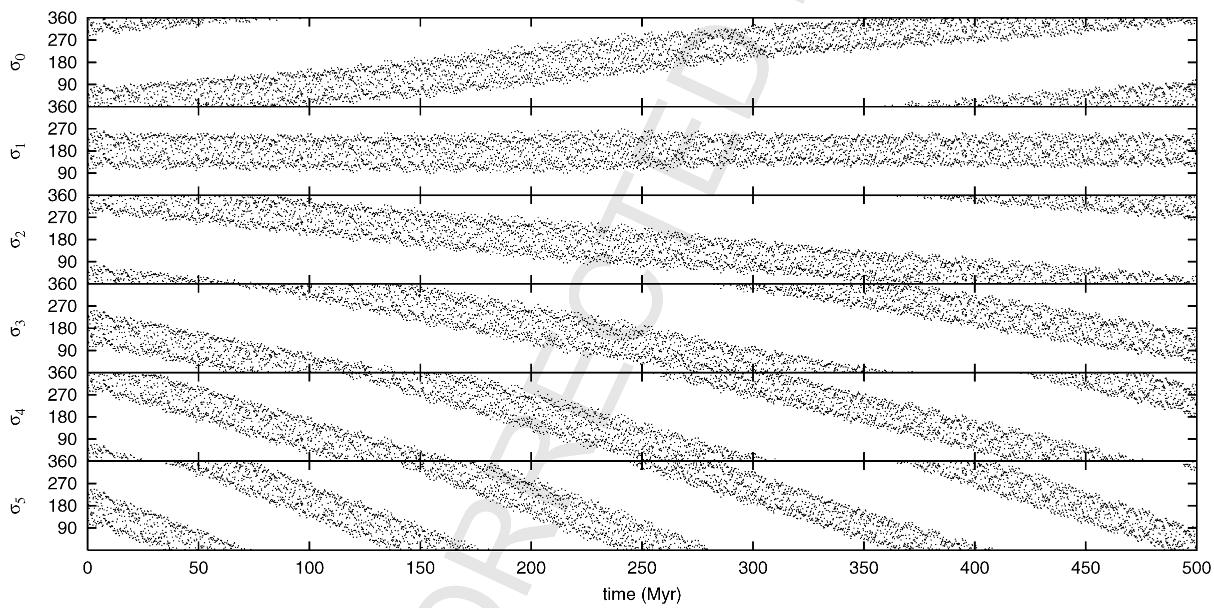

Fig. 6. Librations of a high-inclination ($i \simeq 40°$) resonant orbit with the same eccentricity ($e \simeq 0.5$) as Fig. 5 surviving to the perturbations by Uranus. The critical angles σ_j are defined following expression (8) and in this case $\mathrm{DR} \simeq 0.0044$. This stability is attributed to greater values of DR than in low-inclination orbits.

the terms depending on λ and consider only the resonant and secular terms.

As we are analyzing a 10th-order resonance we will consider the secular terms of the EM expansion of the disturbing function for a SDO due to Neptune up to order 10 which due to D'Alembert rules results:

$$R_{\mathrm{sec}} = \sum_{j,k=0}^{10} B_{jk}(\alpha) e^j s^k + \cos(2\varpi - 2\Omega) \sum_{j,k=2}^{10} C_{jk}(\alpha) e^j s^k + \cos(4\varpi - 4\Omega) \sum_{j,k=4}^{10} D_{jk}(\alpha) e^j s^k, \quad (12)$$

where B_{jk}, C_{jk}, and D_{jk} are functions of α. We can consider the secular terms due to all four giant planets but this does not introduce notable qualitative changes in the dynamical evolution. Secular resonances do not occur at this far region of the Solar System because the rates of change of the node and perihelion of the particle are very low in comparison with the corresponding values for the planets (or more properly: in comparison with the fundamental frequencies of the Solar System). This justify the elimination of the secular terms involving ϖ_{N} and Ω_{N}.

It is also possible to evaluate numerically R_{sec} by means of

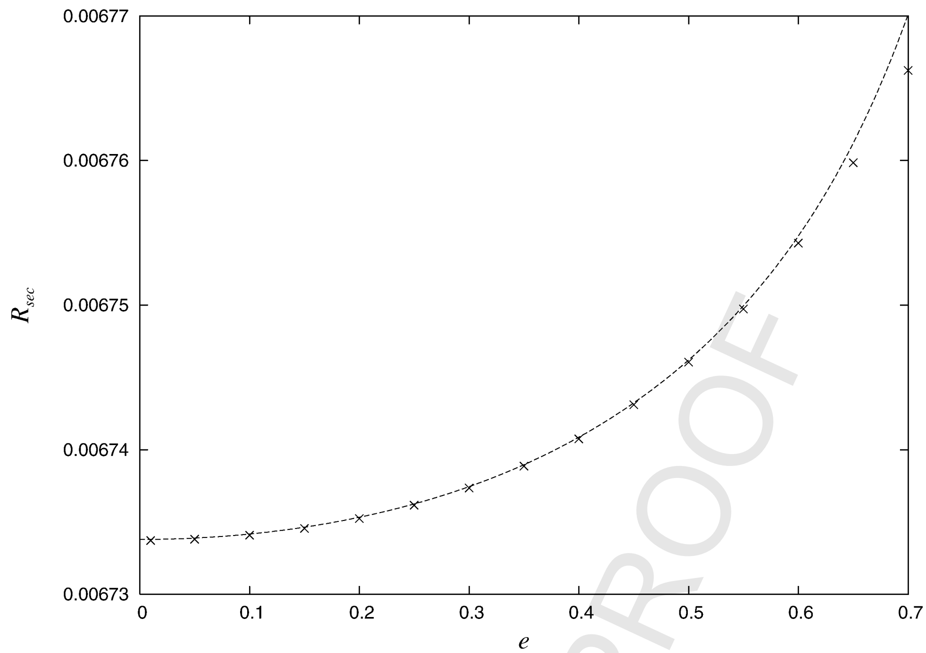

Fig. 7. Comparison between the EM analytical expansion and numerical computation of the exact secular disturbing function (crosses) using expression (13) evaluated at $i = 39.8°$, $\omega = 57.3°$ and taking $G = m_N = 1$.

$$R_{sec} = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} R(\lambda_N, \lambda)\, d\lambda_N\, d\lambda, \tag{13}$$

where λ and λ_N are now independent so the condition (10) does not apply. Fig. 7 compares $R_{sec}(e, i \simeq 40°, \omega \simeq 57°)$ from (12) with a numerical evaluation from (13) which do not use any series expansion. It is known that this kind of expansion diverges for $e > 0.66$ (Ferraz-Mello, 1994) and near that limit the convergence is slowly reached, that means, it is necessary to consider more and more terms.

We checked the EM expansion secular part (12) with the numerical integration (13) and it represents well the exact secular disturbing function at least in the range ($e < 0.6$, $i < 70°$). Then we consider it appropriate for a secular non-resonant model in this region.

3.1. *Kozai mechanism for non-resonant orbits*

We are interested in the determination of the circumstances that allow for the KM with significative changes in eccentricity and inclination. The Lagrange's secular equations for de/dt and di/dt using (12) are expressions of the form

$$\frac{d(e, i)}{dt} = (f_2, g_2)\sin(2\omega) + (f_4, g_4)\sin(4\omega), \tag{14}$$

where the coefficients f_j and g_j are functions depending on α, e, i. A particle evolving this way will show the KM characterized by coupled evolution in e, i, ω maintaining the parameter $H = \sqrt{1 - e^2}\cos i$ as a constant of motion. This can be shown writing dH/dt as function of de/dt and di/dt and then substituting with the corresponding planetary equations for e and i. Taking into account that $\partial R_{sec}/\partial\varpi + \partial R_{sec}/\partial\Omega = 0$ we obtain $dH/dt = 0$.

The quick circulation of ω is a guarantee that the sign of the temporal derivatives (14) switches time to time avoiding an uncontrolled growing of eccentricities and inclinations. That guarantee can be missed if $\dot\omega \sim 0$ and if the f's and g's are not vanishingly small (KR). In order to identify initial conditions conducing to evolutions with high variations in e and i a good starting point will be to investigate when $\dot\omega \sim 0$. It is known that beyond Neptune this situation only occurs for very high-inclination orbits (Thomas and Morbidelli, 1996).

We made an analysis of the rates of changes of ϖ, Ω, and ω for a SDO's with $a = a_{1:11}$ but considering only the secular terms due to the perturbations of the four major planets in the disturbing function. We solved numerically the system of Lagrange's planetary equations including the equations given by (14) obtaining the time evolution of e, i, ϖ, Ω with initial conditions in the region ($e < 0.6$, $i < 70°$).

We found $\dot\varpi > 0$ for $i < 45°$ and $\dot\varpi < 0$ for higher inclinations. On the other hand, we have $\dot\Omega < 0$ in all range of inclinations and eccentricities studied. At Fig. 8 we show $|\dot\omega|$ in grey scale calculated from the Lagrange's planetary equations. Almost null values are showed as black regions. This figure shows very slight variations according to the initial values adopted for ω. The Kozai resonance appears around the region where $\dot\omega \sim 0$ and that situation only occurs for high-inclination orbits ($i \sim 63°$, also known as *critical inclination*) being $\dot\omega > 0$ for $i < 63°$ and $\dot\omega < 0$ otherwise. Kuchner et al. (2002) found the same result but for high-inclination classical KBOs.

At Fig. 9 we show an example of an hypothetical SDO with $a = a_{1:11}$ that was obtained integrating numerically the Lagrange's planetary equations considering only the secular terms of the disturbing function for all the major planets. At this high inclination the coefficients f's and g's are greater than at low

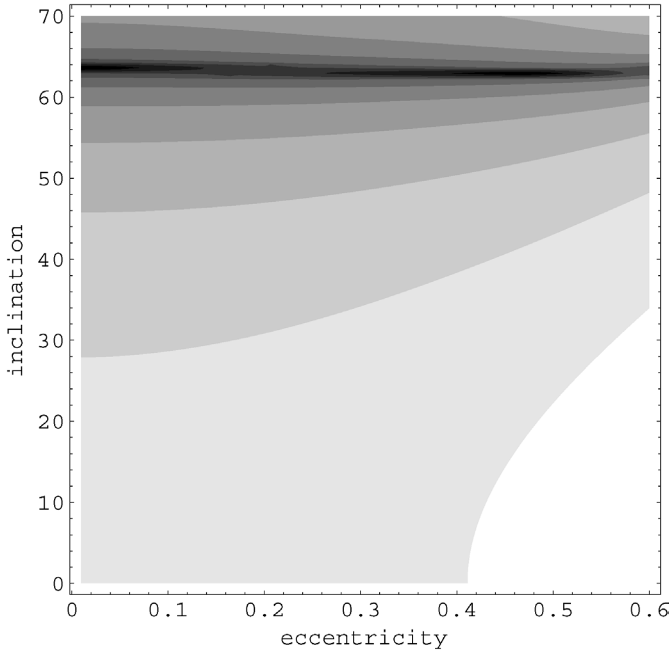

Fig. 8. Values for $|\dot{\omega}|$ in gray scale from the Lagrange's planetary equations considering only the secular terms given by R_{sec} from Eq. (12) for an hypothetical SDO with $a = a_{1:11}$. Black regions correspond to $\dot{\omega} \sim 0$. The region below $i \sim 63°$ corresponds to $\dot{\omega} > 0$ and for higher inclinations $\dot{\omega} < 0$.

inclination contributing even more to $\mathrm{d}e/\mathrm{d}t$ and $\mathrm{d}i/\mathrm{d}t$. Note the constancy of H which reflects not other thing than the precision of the numerical solution.

By this way we calculated the orbital evolution for several SDOs with different initial e, i and we confirm that at very high inclinations the KR dominates the evolution, but for low inclinations the smallness of f's and g's and the relatively quick variation of ω produces no more than very small amplitude oscillations of e, i. As we explain in the next section we also investigated the secular evolution in the region of higher eccentricities by means of the numerical integration of the full equations of motion finding a similar behavior than the described here.

Then from this analysis we conclude that for typical inclinations in the scattered disk and considering only the secular perturbations we cannot expect nothing but a constant time evolution of ϖ, Ω, and ω, providing by that way very low amplitude oscillations in e and i. In the next section we will investigate the more interesting dynamical effects generated by the resonant terms.

4. Long-term evolution inside the resonance 1:11

From the EM expansion it is possible to show that assuming circular- and zero-inclination orbit for Neptune the principal terms of the resonance 1:11 are:

$$R_{\mathrm{res}} = \sum_{j=0}^{5} A_{2j}(\alpha) e^{10-2j} s^{2j} \cos(\sigma_j), \tag{15}$$

where σ_j is given by (8) and the A_{2j} are functions depending only on α being the first three terms the most important ones. A_2 is of an order of magnitude greater than A_0 and A_4 but considering they are multiplied by $e^{10-2j} s^{2j}$ it results that the two first terms ($j = 0, 1$) are the most important being of the same order for non-negligible inclinations. These 6 resonant terms are all 10th-order on the small parameters e, s and are the most important terms corresponding to the resonance because the next ones following in the expansion of the disturbing function are of order 14th. One must take care of working inside the convergence domain of the resonant part of the expansion, which is the case using the EM expansion provided $e < 0.66$ as we have mentioned earlier. Also, if we are planing to use the 10th-order expression (15) we should work well under that limit because of the slowness of the convergence of the series. This series truncated at 10th order, for example, fails in reproducing the asymmetric librations characteristics of the resonances of the type 1:N (Beaugé, 1994) that for the resonance 1:11 we

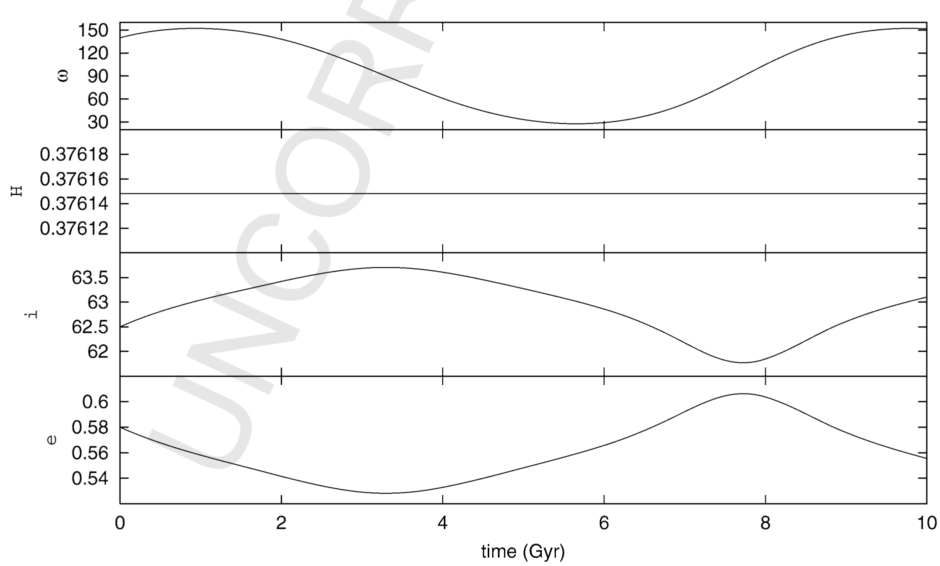

Fig. 9. The KR at very high inclinations. A very slow evolution of a SDO with $a = a_{1:11}$ inside the KR obtained integrating numerically the Lagrange's planetary equations considering only the secular terms of the disturbing function due to the four major planets. Note the constancy of H.

found they appear for $e > 0.5$, then we cannot use it to resolve the corresponding Lagrange's planetary equations for such eccentricities. It would be necessary to take higher harmonics in order to obtain the correct asymmetric librations.

For low eccentricities the expression (15) is valid and the equations for $\mathrm{d}e/\mathrm{d}t$ and $\mathrm{d}i/\mathrm{d}t$ considering only resonant terms in the disturbing function are expressions of the form:

$$\frac{\mathrm{d}(e,i)}{\mathrm{d}t} = \sum_{j=0}^{5} (f'_j, g'_j) \sin(\sigma_j), \tag{16}$$

where the f'_j and g'_j are functions depending on (α, e, i). The Lagrange's planetary equations for the resonant case are substantially more complicated than the secular case (see, for example, Murray and Dermott (1999, p. 252)). The system is completed with the equations for the other elements and we can solve it numerically. We can obtain different solutions from different initial conditions and we can also generate different families of solutions from the different terms of (15). This procedure would allow us to have an idea of the relevance and the effect of each term in the resonant motion. From the several numeric experiments we performed we can advance that the resonant terms depending on σ_0 and σ_1 are the main responsible of significative changes in e, i.

The corresponding equations for the complete (secular plus resonant) evolution of $\mathrm{d}e/\mathrm{d}t$ and $\mathrm{d}i/\mathrm{d}t$ are combinations of expressions similar to (14) and (16):

$$\frac{\mathrm{d}e}{\mathrm{d}t} = f_2 \sin(2\omega) + f_4 \sin(4\omega) + \cdots + f'_0 \sin(\sigma_0) + f'_1 \sin(\sigma_1) + \cdots, \tag{17}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = g_2 \sin(2\omega) + g_4 \sin(4\omega) + \cdots + g'_0 \sin(\sigma_0) + g'_1 \sin(\sigma_1) + \cdots. \tag{18}$$

As we are interested in SDO's showing some evolution in a this imply $q \lesssim 36$ and $e > 0.76$ for the resonance 1:11. Then we cannot use these expressions to resolve the Lagrange's planetary equations so, to explore the high-eccentricity orbits, we will move to numerical integrations of the full equations of motion. Anyway, in our qualitative analysis of the obtained numerical solutions we will use the expressions (17) and (18) as an approximate description of the terms affecting $\mathrm{d}e/\mathrm{d}t$ and $\mathrm{d}i/\mathrm{d}t$.

The existence of the resonant terms breaks the constancy of H but maintain

$$\sqrt{a}\left(H - \frac{(p+q)}{p}\right) = \text{constant} \tag{19}$$

(Gomes, 1997). Then, the librations imposed to a by the resonant terms will produce small amplitude oscillations in H.

4.1. KM for high-eccentricity resonant orbits

We performed a series of numerical integrations of SDOs all them with initial $a = 149.08$ AU, $e = 0.7$, $i = 20°$, $\omega = 90°$ and with initial conditions corresponding for orbits from outside the resonance to the deep resonance. The only perturbing planet considered was Neptune and it was assumed in circular- and zero-inclination orbit with initial $\lambda_N = 304.3°$.

4.1.1. Outside the resonance: circulation of σ_0

Fig. 10 corresponds to a SDO which evolves outside the resonance with circulation of all the critical angles. The argument of the perihelion circulates with $\dot{\omega} > 0$ as expected from our secular study extrapolating to $e > 0.6$. It is a typical secular evolution without significative changes in e, i. The quick circulation of all the σ_j produces no net effect on Eqs. (17) and (18) and only the more slow evolution of ω introduces the long-period small amplitude coupled oscillations of e, i trough the

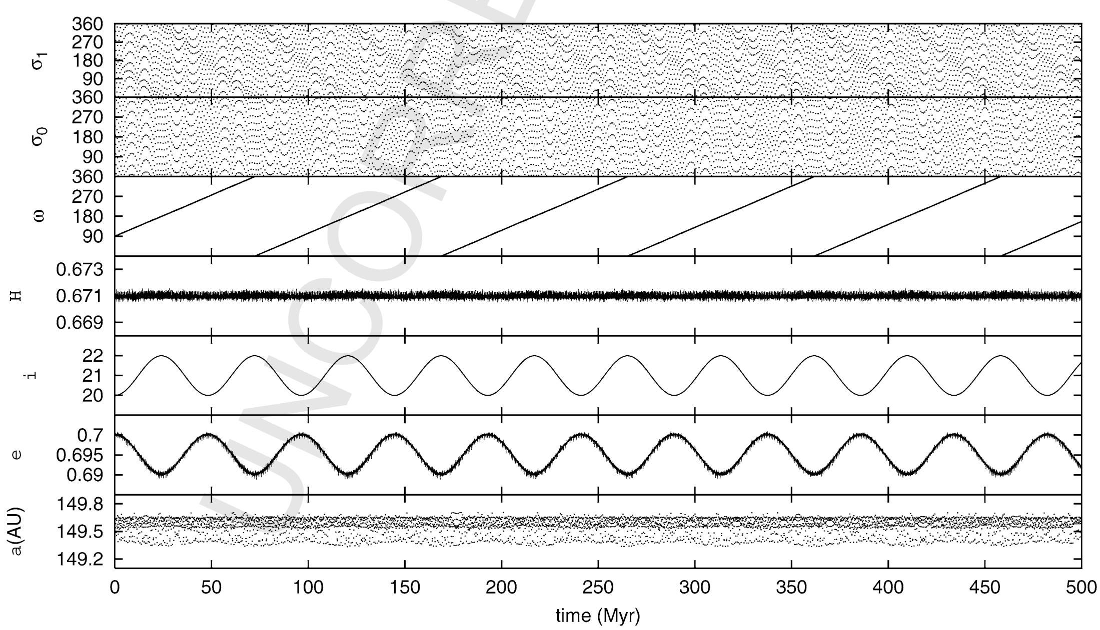

Fig. 10. A numerical integration of the full equations of motion for a SDO evolving outside but near the resonance 1:11. For this inclination the KM is present producing small amplitude librations of e, i. Neptune was taken with $e_N = i_N = 0$.

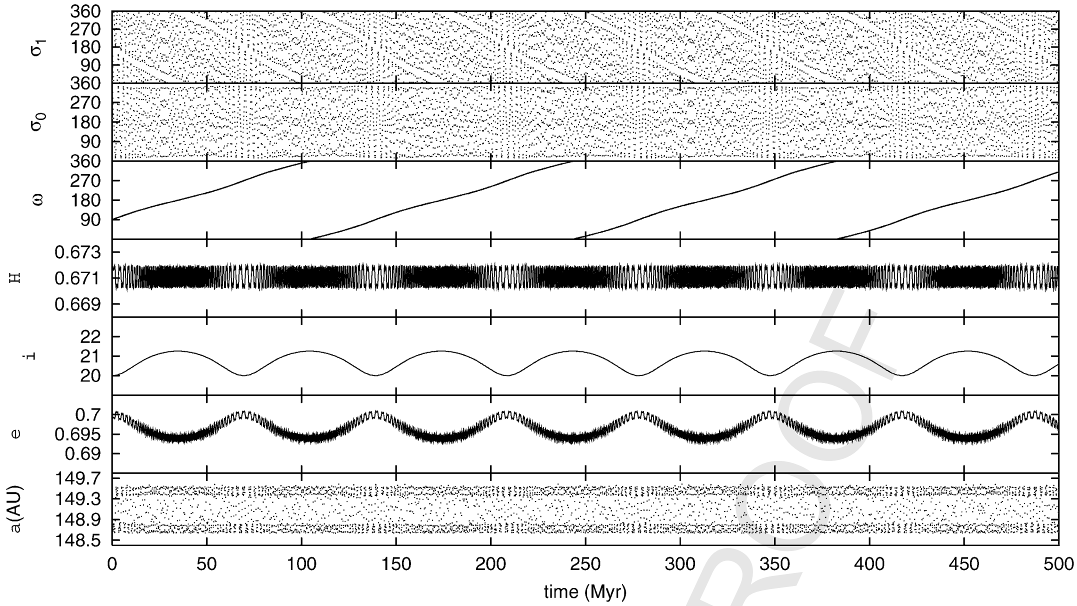

Fig. 11. A numerical integration of the full equations of motion for a SDO evolving in the shallow resonance with external high amplitude librations of σ_0. These librations are around a center located at $\sigma_0 = 180°$ and wrap the asymmetric librations around centers located at $\sigma_0 \sim 90°$ and $\sigma_0 \sim 270°$. The evolution is similar to Fig. 10 plus a short-period oscillation of e and H due to the resonant terms.

small amplitude secular terms like f_2, f_4, g_2, g_4. The KM is present producing the coupled evolution of (e, i, ω) maintaining constant the parameter H.

4.1.2. *Shallow resonance: external high amplitude libration of σ_0*

Fig. 11 corresponds to a SDO evolving with very high amplitude librations of σ_0. These librations are exterior to the region covered by the smaller amplitude asymmetric librations in a similar way as a horseshoe trajectory wraps the tadpoles in the case of trojans. Also, as in the previous case, ω slowly circulates. The critical angles σ_j with $j > 0$ show the same behavior than σ_0 but with an associated drift that increases as j increases. The drift is clearly due to the evolution of ω, that is easily explained because $\sigma_j = \sigma_0 - 2j\omega$. Again, no drastic changes in e, i are obtained which we attribute to the quasi-circulation of all critical angles. The resonance produces very small amplitude oscillations in e and a and the associated oscillations in H given by Eq. (19).

4.1.3. *Deep resonance: internal asymmetric libration of σ_0*

Fig. 12 corresponds to a SDO evolving with small amplitude librations of the principal critical angle σ_0 and important associated variations Δe, Δi due of the KR. Time to time the librations switch between both libration centers. The critical angles σ_j also librate with the associated drift in their libration centers due to the oscillations of ω. We also confirmed that the libration period is the same for all the σ's which is due to the fact that $\dot{\sigma}_j = \dot{\sigma}_0 - 2j\dot{\omega}$ and $|\dot{\omega}| \ll |\dot{\sigma}_0|$.

Finally, Fig. 13 corresponds to a SDO evolving with very small amplitude librations of the principal critical angle σ_0 but in this case ω is circulating. Consequently, the other critical angles also librate with the associated drift due to the circulation of ω producing a net slow circulation of the libration centers. In this case the variations in e, i are modest in comparison with Fig. 12. All this suggests that terms like $f_1' \sin(\sigma_1)$ and $g_1' \sin(\sigma_1)$ (i.e., the terms depending on $\sigma_1 = \sigma_0 - 2\omega$) are the responsible for the different behavior between Figs. 12 and 13 as we will explain below.

Note that in Figs. 11–13 the resonance is showed by means of the librations of a and σ_0, however, the oscillations of e and i over a libration period of σ_0 are very small. If we use the EM expansion for estimating the resonant terms that appear in (17) and (18) we find they are 2 orders of magnitude greater than the secular ones but evidently they are not big enough to produce an important oscillation on e, i after a libration period as can be clearly seen in all the figures. We remark: the secular terms are not relevant compared with the resonant ones, and these last ones have no enough importance to produce large librations in e, i. Why then we do see the long term high amplitude variations in e, i?

This apparent contradiction is explained looking for example at the libration center of σ_1 which is not fixed but slowly migrates or switches between different values. Sometimes, for example when σ_1 librates around 0° or 180°, terms like $f_1' \sin(\sigma_1)$ do not contribute to $\mathrm{d}e/\mathrm{d}t$ and the eccentricity will be near a maximum or minimum. On the other hand, when σ_1 librates around 90° or 270° those terms contribute sustainedly making the eccentricity to change substantially. The contribution given by the terms depending on the critical angle σ_0 are not the most important ones for the variations of $\mathrm{d}e/\mathrm{d}t$ and $\mathrm{d}i/\mathrm{d}t$ and this is showed at Fig. 13 were the time evolution of e, i is linked to the libration center of σ_1. As $\sigma_1 = \sigma_0 - 2\omega$ it results that the evolution of the libration center of σ_1 is determined by the time evolution of ω.

When σ_0 and ω librate the time evolution of e, i become the most drastic because all the quasi-constant contribution from all the resonant terms act together. This corresponds with Fig. 12.

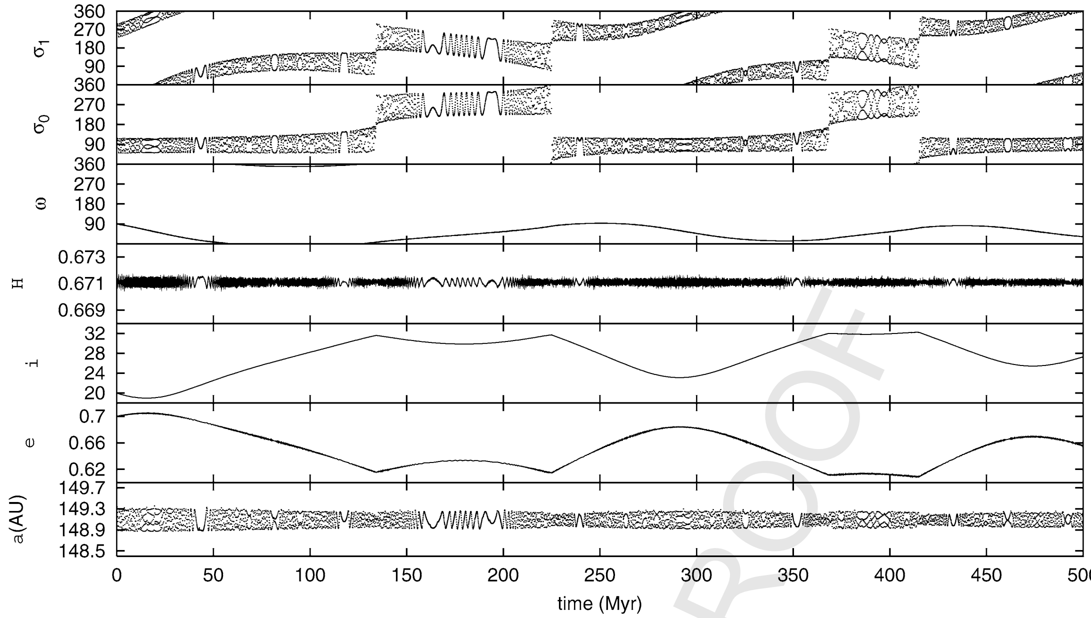

Fig. 12. A numerical integration of the full equations of motion for a SDO evolving in the deep resonance with all critical angles librating. The short-period oscillations due to the resonant terms have very small amplitude but there is a long-term high amplitude trend due to the KR.

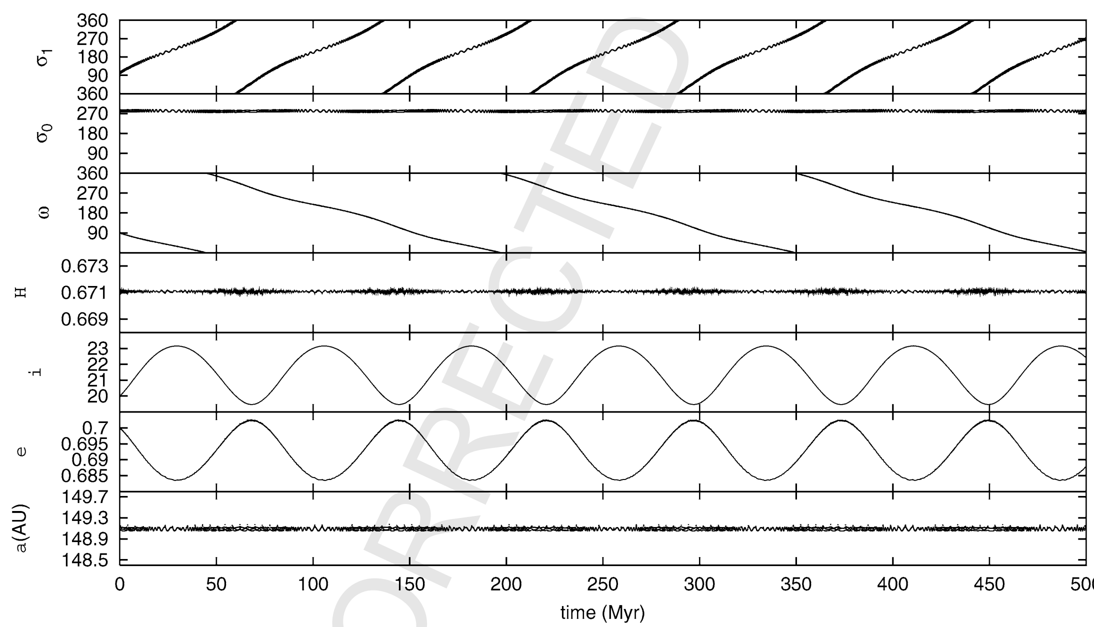

Fig. 13. A numerical integration of the full equations of motion for a SDO evolving in the deep resonance with very small amplitude libration of σ_0 but circulation of the other σ_j. The KM produces modest long-term oscillations of e, i.

See inclusive how the sign of $\mathrm{d}e/\mathrm{d}t$ and $\mathrm{d}i/\mathrm{d}t$ changes when the libration center of σ_j switches between $\sim 90^\circ$ and $\sim 270^\circ$ accordingly with the change of sign of $\sin(\sigma_j)$. The evolution of e, i is modulated by the libration center of σ_1 which is determined by the evolution of ω. This extreme case with all σ_j librating occurs when ω librate (KR) and now this is possible at $i < 63^\circ$ due to the effects of the resonant terms in the time evolution of ω which is defined by an equation analogous to (18). This is consistent with the Figs. 12–14 of Gomes et al. (2005) where the exterior high amplitude librations around 180° are not associated with significative changes in e, i whereas librations around 90° and 270° with associated librations of ω do. When we consider the actual eccentricity and inclination of Neptune's orbit the Δe, Δi obtained are something greater due to the associated forced modes due to Neptune. Another interesting point is that the libration amplitude of the σ_j grow for lower eccentricities allowing switching between libration centers and eventually breaking the resonant motion.

An alternative way to see the KR inside a mean motion resonance is located at different inclination and has a stronger strength is the following. Write σ_1 as $\sigma_0 - 2\omega$ in (17) and (18) and expand $\sin(\sigma_1)$ as: $\sin(\sigma_0)\cos(2\omega) - \cos(\sigma_0)\sin(2\omega)$. Then average (17) and (18) over a libration period of σ_0. It is evident that $\oint \cos(\sigma_0)\,d\sigma_0 \neq 0$, where $\oint$ represents the integral over a libration cycle, and that $\oint \sin(\sigma_0)\,d\sigma_0$ is also non-zero in case of asymmetric librations. Thus the averaged equations (17) and (18) contain terms proportional to f_0', f_1', g_0', g_1' unlike the secular equations (14). As said before, the primed coefficients are larger than the non-primed coefficients f and g. Thus the equations describing the KR in a mean motion resonance are significantly different from those applicable to the purely secular case. This explains why the KR appears at a different inclination, and its strength is in general stronger than outside a mean motion resonance.

5. Discussion and conclusions

The secular equations excluding resonant terms provide the possibility of the Kozai resonance with relevant changes Δe, Δi for a SDO but only for very high-inclination orbits ($i \sim 63°$, the *critical inclination*). The KR for lower inclination orbits is obtained only when considering the full equations of motion for a resonant case and we have provided arguments that indicate that the resonant mixed e, i terms are responsible for such behavior.

The strong changes in e, i that we observe inside the resonance are not due to the effects of the secular terms directly but to the resonant ones and more specifically due to the oscillations of the libration center of σ_1 generated by the oscillations of ω (KR). That is why the KR is linked to MMR. That is why Gomes et al. (2005) found captures in the KR almost immediately that an object is captured in asymmetric librations for those orbits having Pluto-like or greater inclinations, which is a condition for the mixed e, i resonant term to show up and for the KR to be installed.

With respect to the mechanism of trapping into the MMR, looking at examples presented by Gomes et al. (2005) we can conclude that the SDO first is captured in exterior librations by stochastic evolution in a when evolving to the Oort cloud. Then, temporary captures in high amplitude asymmetric librations follows producing the most evident orbital changes associated with KR. We did not investigated the details of the transitions between exterior librations and the asymmetric ones but it is possible that the variations Δe, Δi due to the KM or even the forced modes due to Neptune's eccentricity that we did not considered here could have some role in the process.

We can summarize some conclusions of this work:

1. High-order exterior MMR (and specially those of the type $1{:}N$) with Neptune have significant dynamical effect in the SD specially on orbits of high e and/or high i.
2. For a given eccentricity, resonances with libration centers located at high inclination are stronger than resonances with libration centers located at low inclination.
3. Whereas outside a MMR significant Δe and Δi due to KR only appears at very high inclinations, inside a MMR this occurs at Pluto-like or greater inclinations due to the oscillation of ω inside the mixed (e, i) resonant terms. This behavior should be the rule for SDOs with Pluto-like o greater inclinations captured in MMR.

Taking all this into account a possible evolutive path for a SDO can be summarized as follows:

1. According to Fernandez et al. (2004), the SDO experiments a stochastic evolution in a maintaining the perihelion distance $31 \lesssim q \lesssim 36$ AU and with stochastic evolution for i.
2. As a grows, necessarily e grows and also the strength DR of the resonances near a. If the associated critical angles circulate those resonant terms does not show up.
3. When a reaches some a_{res} (and specially when $a \simeq a_{1:N}$) the critical angles start to librate and the SDO can be locked in MMR. We calculated the parameter DR for the captures showed by Gomes et al. (2005) and in general they correspond to DR > 0.01, it seems that for a lesser strength of the resonance the possibility of being locked in resonance in the real outer Solar System drops significantly. All resonances of the kind $1{:}N$ we have studied here for orbits with $31 \lesssim q \lesssim 36$ AU satisfy this conditions, then the capture in resonances of the type $1{:}N$ should not be surprising.
4. If the i is Pluto-like or greater, mixed (e, i) resonant terms dominate the evolution and the KR appears allowing big correlated Δe, Δi and the SDO is injected at least temporarily in timescales of 10^8–10^9 yr in a high-perihelion high-inclination high-order MMR. By this way the object contributes to maintain a long-lived population of HPSDOs (Gomes et al., 2005). Trajectories in the space (e, i, ω) due to the KM do not follow a constant value for DR being smaller for the lower values of e. That means it would be possible that due to the planetary perturbations, perturbations by agents external to the Solar System or other mechanisms like planetary migration the resonance broke at high-perihelion high i and freezing the SDO in a region with a very long lifetime.

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