Strength, stability and three dimensional structure of mean motion resonances in the Solar System

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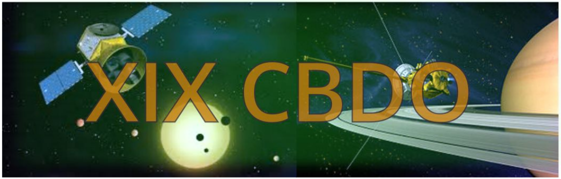
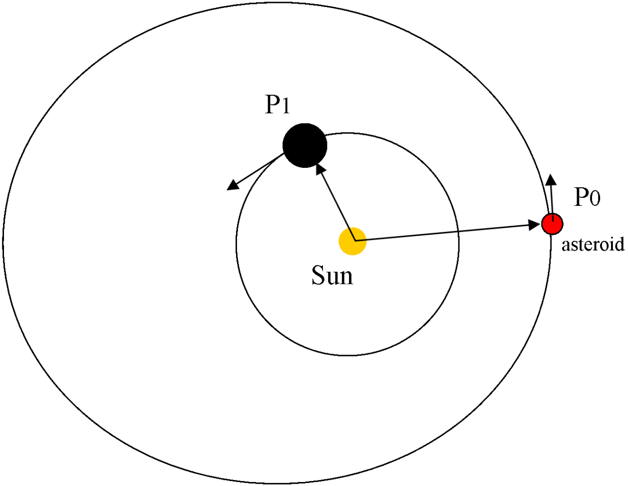


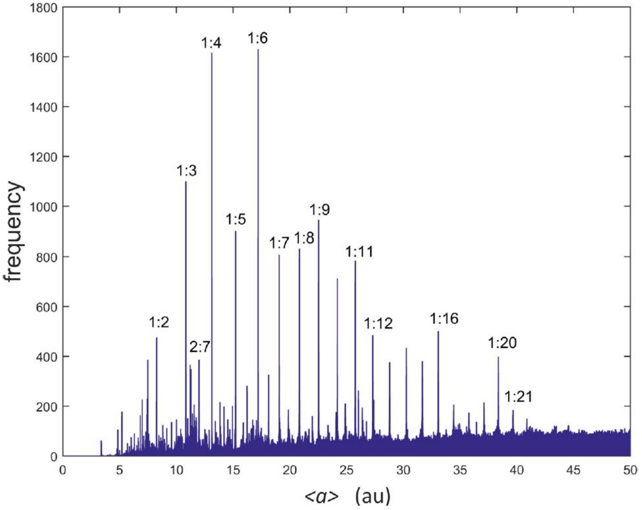
INPE, December 2018

Orbital resonance

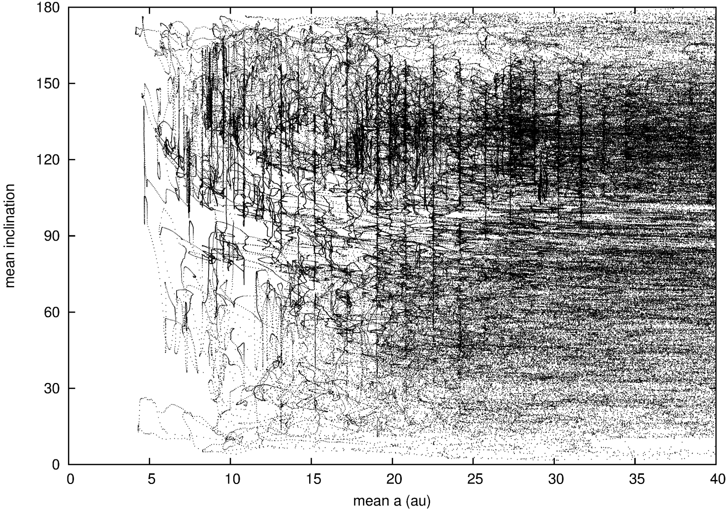
$$k_0 n_0 - k_1 n_1 \simeq 0 \qquad \Rightarrow \qquad \sigma = k_0\lambda_0 - k_1\lambda_1 + \gamma$$



orbital states of long period comets:



Direct and RETROGRADE 1:N resonances



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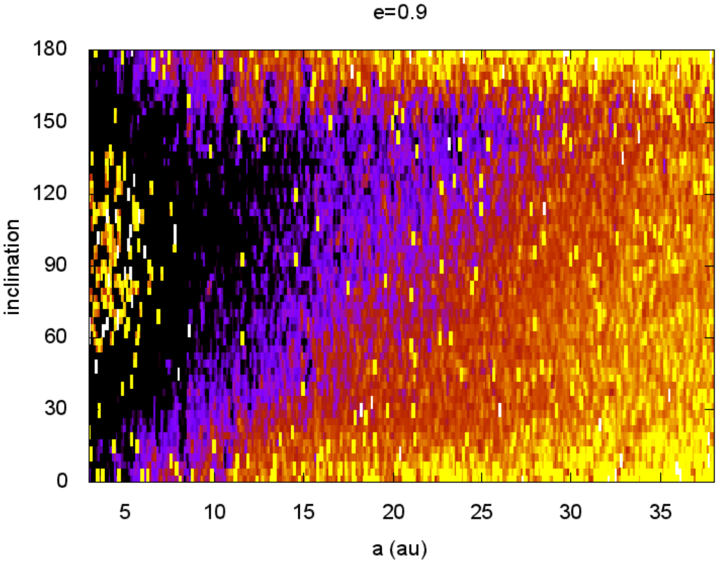
Dynamical map: planetary perturbations



Retrograde resonances

- resonances 1:N are especially popular
- even for retrograde orbits (perturbations destroy direct ones)

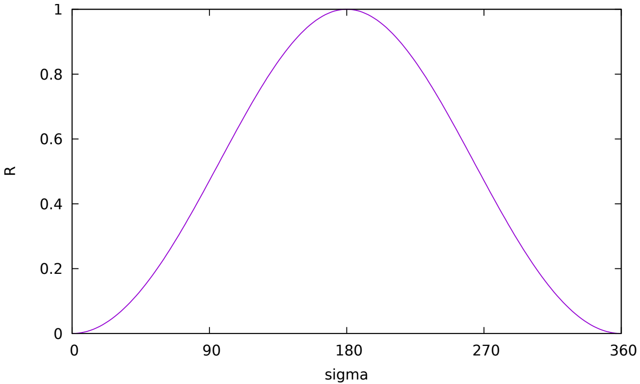
- why 1:N?
- stronger?
- how to calculate the resonance' strength for high inclination orbits?

Disturbing function $R(\sigma)$

Classic theory: **for low** e, i

$$R(\sigma) \simeq A\cos(\sigma)$$

$R(\sigma)$: mean disturbing function
Minima of $R(\sigma)$ define the stable equilibrium centers.



For low e, i

Resonant disturbing function:

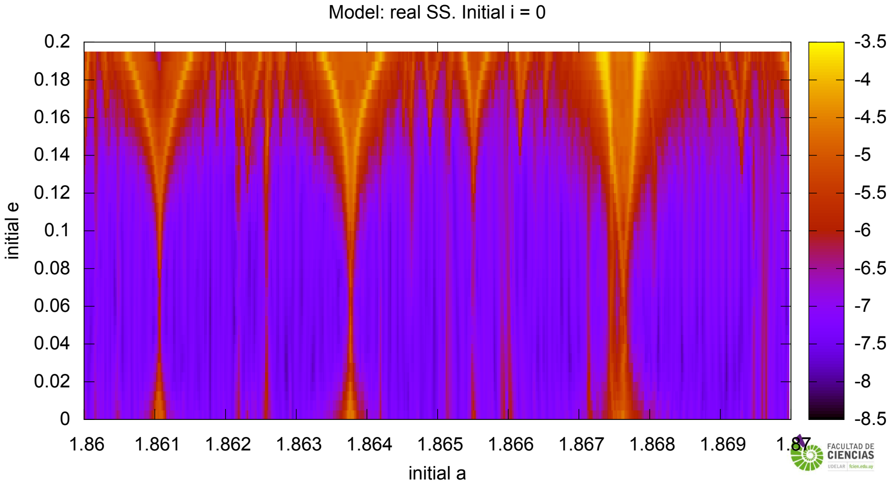
$$R(\sigma) \simeq A\cos(\sigma)$$

- *strength* $\equiv A$
- $A \propto e^q \qquad (q = |k_0 - k_1| = \text{order})$
- *width* (in au) $\propto A^{1/2}$

Dynamical map: Δa

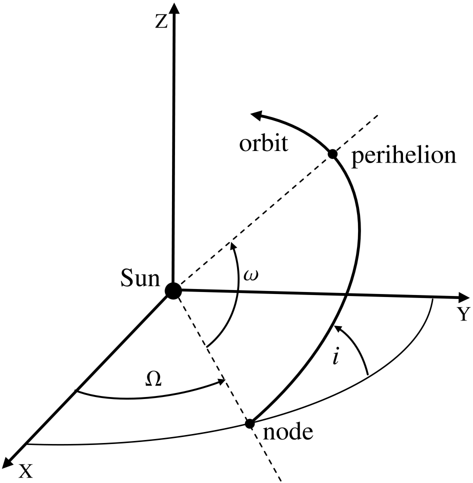
Numerical integrations of test particles in the real Solar System.

larger $e \longrightarrow$ stronger $\longrightarrow$ wider



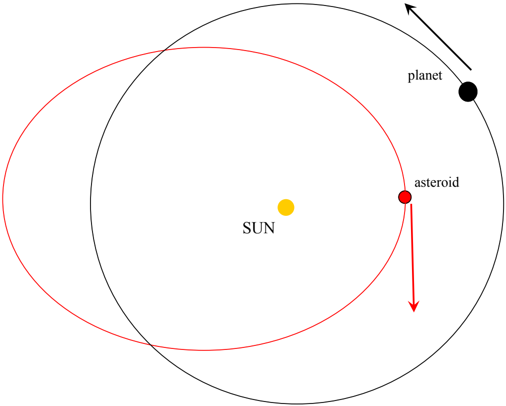
What happens for large inclination orbits?

width?
strength?



COPLANAR retrograde ($i = 180°$)

Morais, Giuppone, Namouni



They found strength $\propto e^{k_0+k_1}$ instead of $e^{|k_0-k_1|}$

The problem of large (e, i)

For large e, i

$$R(\sigma) \simeq A\cos(\sigma) + B\cos(\sigma_1) + \dots$$

$$\sigma_1 = \sigma + \gamma$$

$$\dots$$

- $R(\sigma)$ is the overlap of several sinusoidal terms
- Strength_1 = A, Strength_2 = B ...
- there is not a unique representative critical angle

$$\Downarrow$$

NUMERICAL calculation of $R(\sigma)$

Numerical calculation of $R(\sigma)$

Idea due to Schwarzschild (1903), Schubart (1964), ..., Gallardo (2006, 2019):

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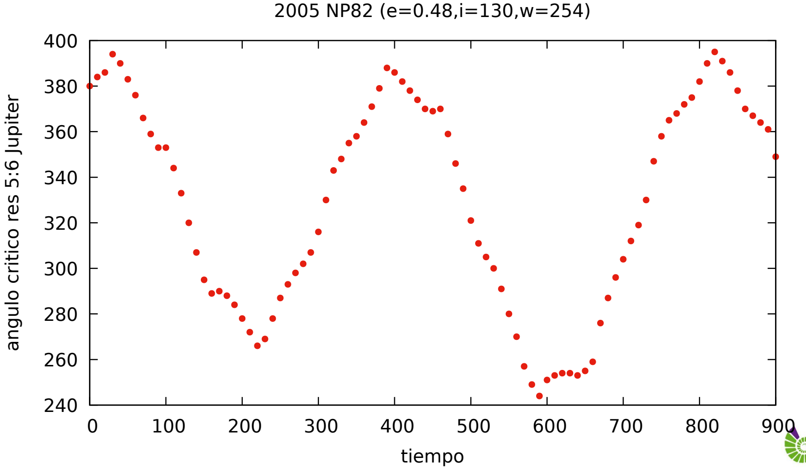
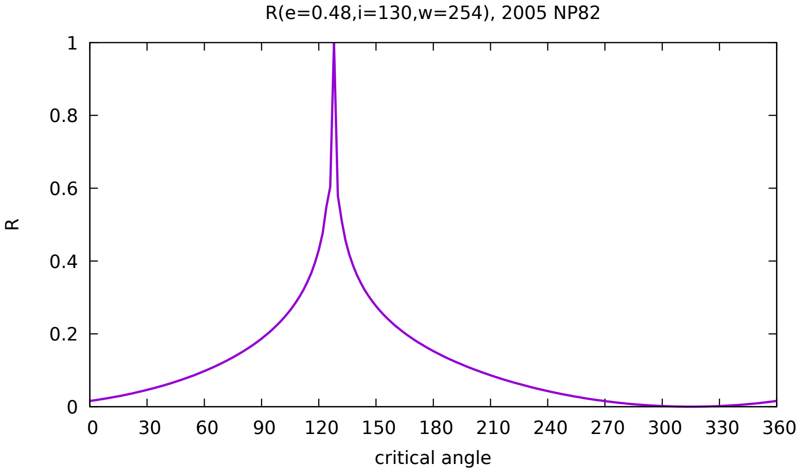
Model: fixed orbits, circular planet, arbitrary asteroid.

Captured in resonance 5:6 with Jupiter, with $i = 130°$.
$\Rightarrow$ equilibrium at $\sigma \sim 320°$

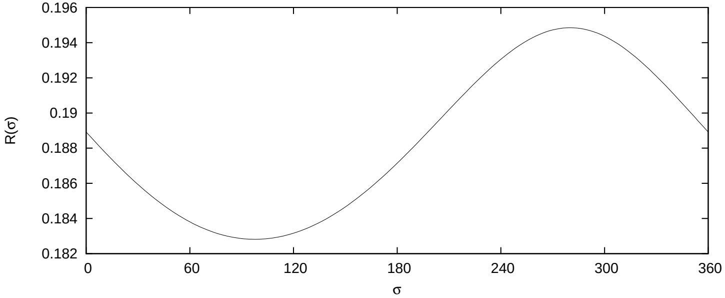


Numerical disturbing function $R(\sigma)$



equilibrium center at $\sigma \sim 320^\circ$ ✓

Definition of Strength SR

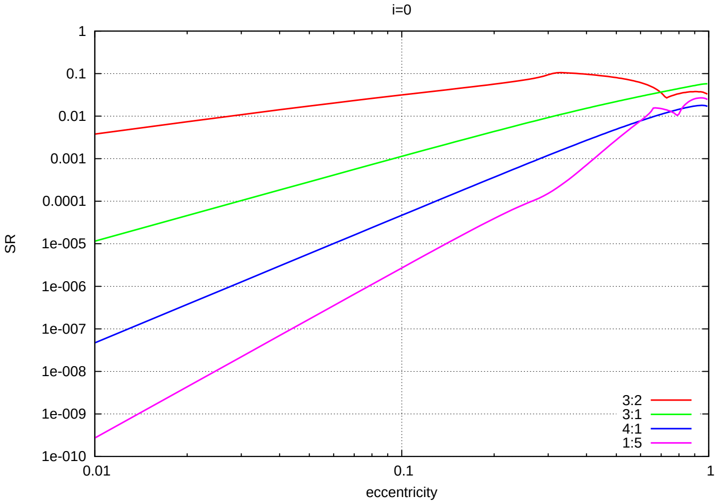


Strength ($\sim$ semiamplitude):

$$SR = < R > - R_{min}$$

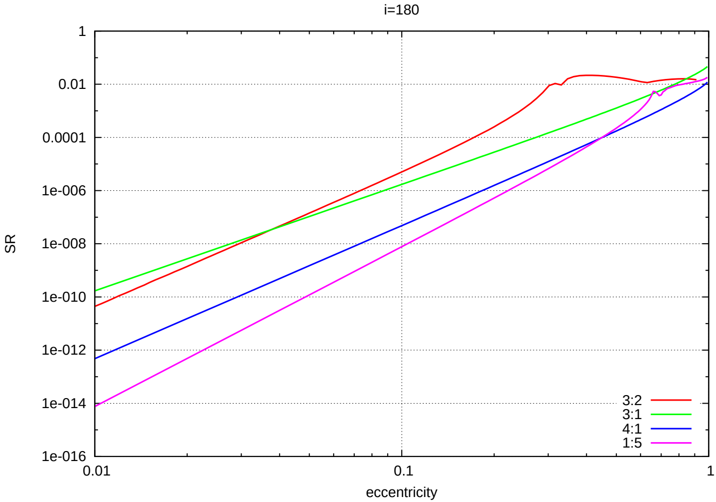
$SR(e, i, \omega)$ independent of $\sigma, \sigma_1, \ldots$

Numerical Strength SR for $i = 0^\circ$



$$SR \propto e^{|k_0 - k_1|} \checkmark$$

Numerical Strength SR for $i = 180°$



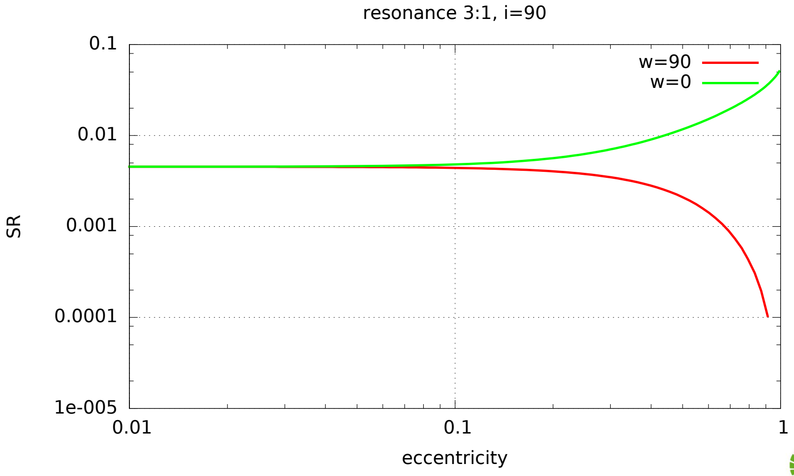
$$SR \propto e^{|k_0+k_1|} \checkmark$$

SR for $i = 90°$:

The ω factor

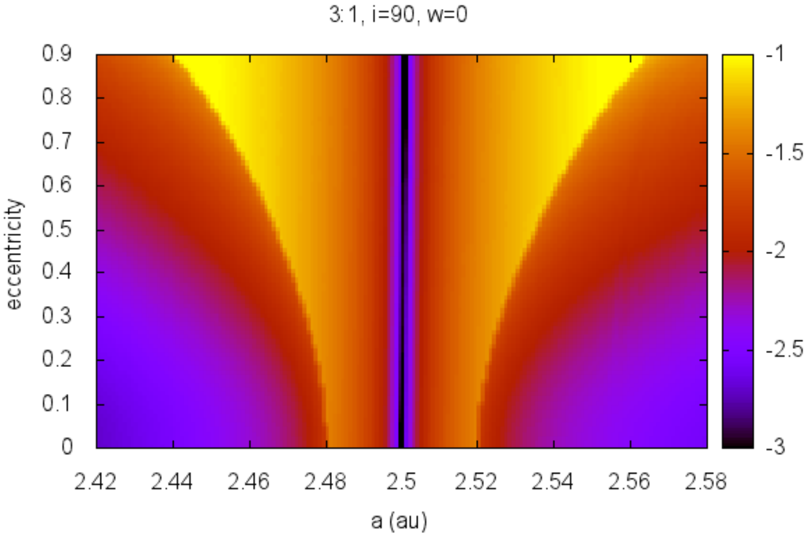
Weird behavior for $i = 90°$

strong dependence with ω

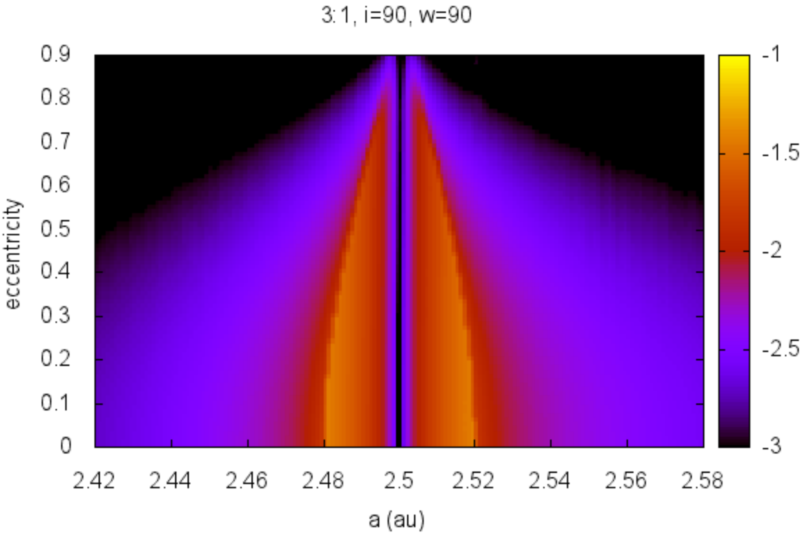


WIDTH by dynamical map for $\omega = 0^\circ$

larger $e \longrightarrow$ stronger $\longrightarrow$ wider



larger $e \longrightarrow$ weaker $\longrightarrow$ it shrinks !!!

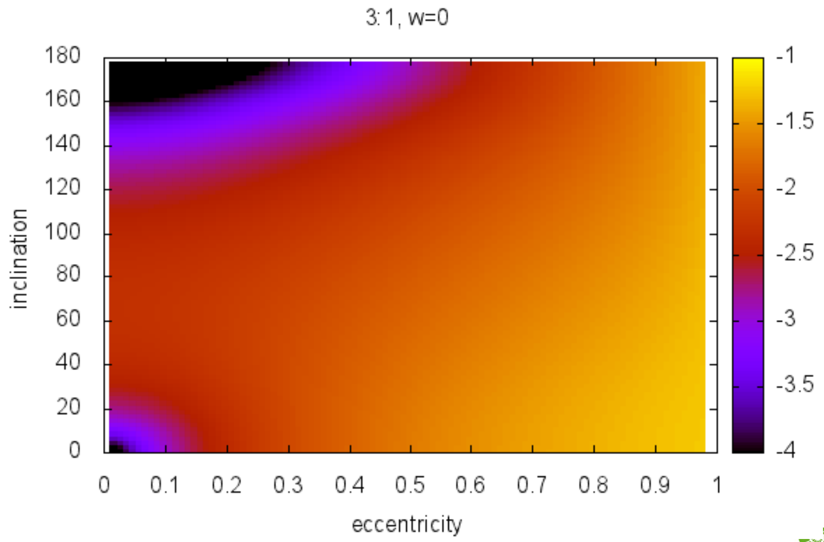


$SR(e, i)$ for $\omega = 0^\circ$

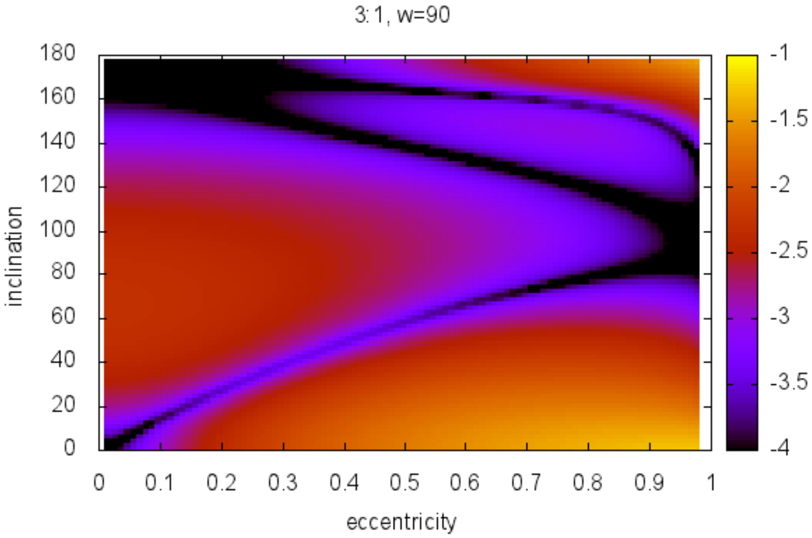
versus

for $\omega = 90^\circ$

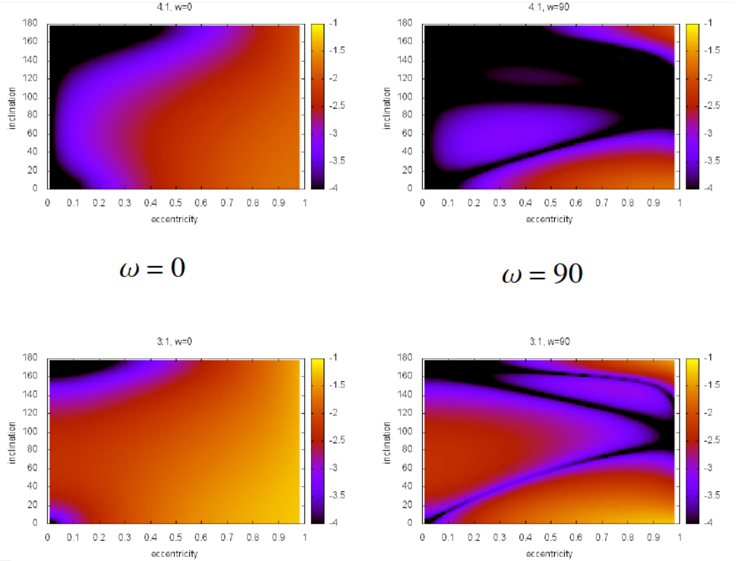
$SR(e,i)$ for resonance 3:1, $\omega = 0°$



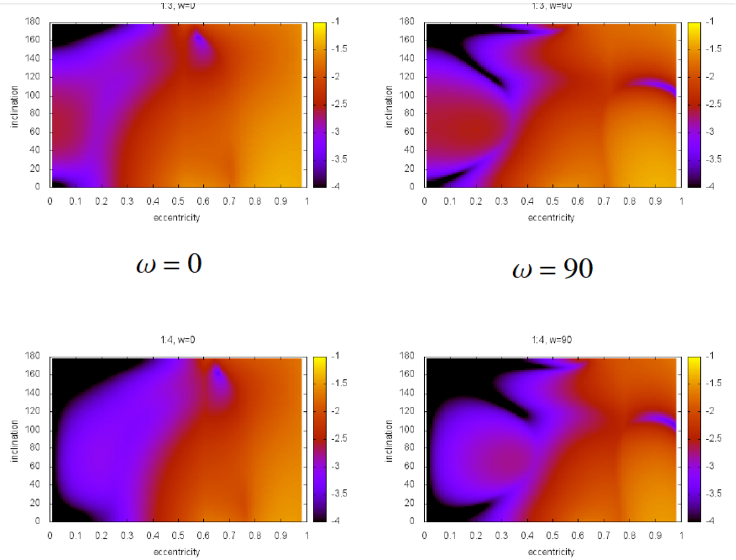
$SR(e, i)$ for resonance 3:1, $\omega = 90^\circ$



INTERIOR resonances: $\omega = 0°$ and $\omega = 90°$

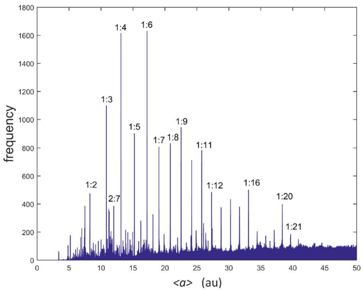


EXTERIOR resonances 1:N, $\omega = 0°$ and $\omega = 90°$



Conclusions

- resonances 1:N are **strong for all** (i, ω)
- all others are weak for some (i, ω), especially **near polar orbits**
- as ω changes over time, resonances 1:N survive
- there are **stable resonant regions** between the giant planets, in particular for retrograde orbits in resonances 1:N, where comets can survive



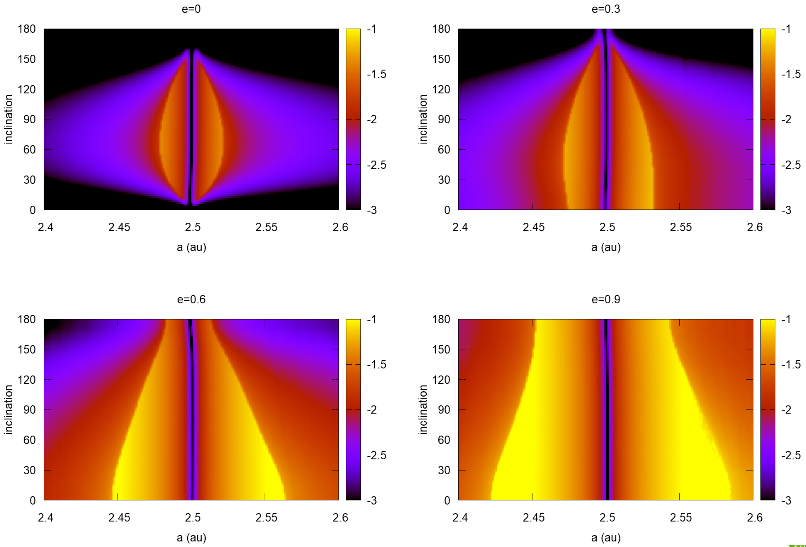
Muito obrigado

see codes in

www.fisica.edu.uy/∼gallardo/atlas

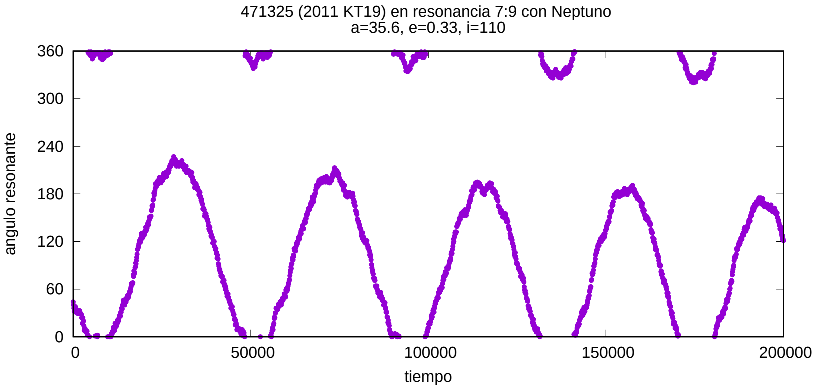
Extras

WIDTHS in (a, i), 3:1



Resonant with Neptune

$$i = 110^\circ$$



Coorbital with Jupiter

$$i = 163^\circ$$