Kozai Dynamics in the Trans-Neptunian Region

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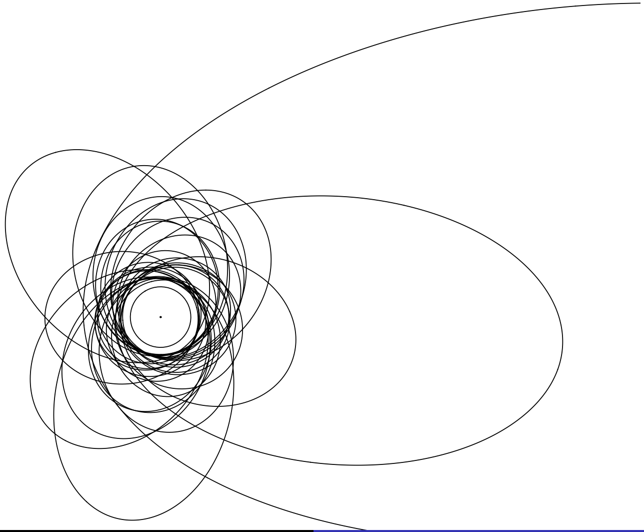
Motivation: orbital scattering in the TNR



Why scattered orbits decoupled from Neptune?

- non discovered planets?
- stellar companion?
- passing stars?
- **what about Kozai dynamics?**

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Icarus

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Survey of Kozai dynamics beyond Neptune

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Kozai-Lidov secular dynamics

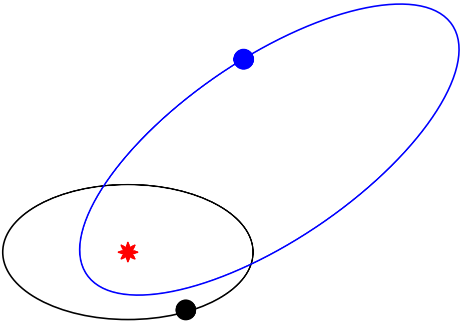
- Kozai model: coplanar circular non-perturbed orbits for the planets.
- $K(-, g, -, L, G, H) \longrightarrow L, H$ constants
- a and $H = \sqrt{1 - e^2} \cos i$ constants

$$\boxed{K(\omega, q, a, H) = -\frac{\mu}{2a} - R_{sec}}$$

History: looking for R_{sec}

- Lidov 1961-1962: satellite perturbed by Moon, analytical R_{sec}.
- Kozai 1962: asteroids perturbed by Jupiter, analytical R_{sec}.
- Bailey et al. 1992: comet perturbed by Jupiter, **numerical** R_{sec}.
- Thomas & Morbidelli 1996: TNOs perturbed by JSUN, numerical R_{sec}.
- Kozai 2004: extension to the TN case.

Numerical method by Bailey et al. (1992) and Thomas & Morbidelli (1996):

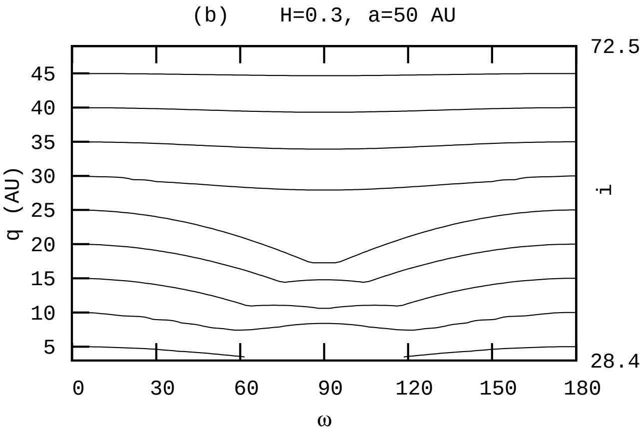


$$R_{sec} = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} R(\lambda_{pla}, \lambda) \, d\lambda_{pla} \, d\lambda$$

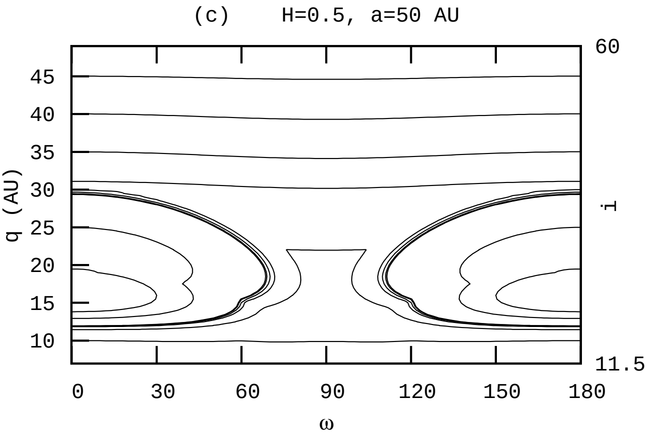
Non Resonant Kozai Dynamics

$$\begin{aligned} K &= constant \\ a &= constant \\ H = \sqrt{1-e^2}\cos i &= constant \end{aligned}$$

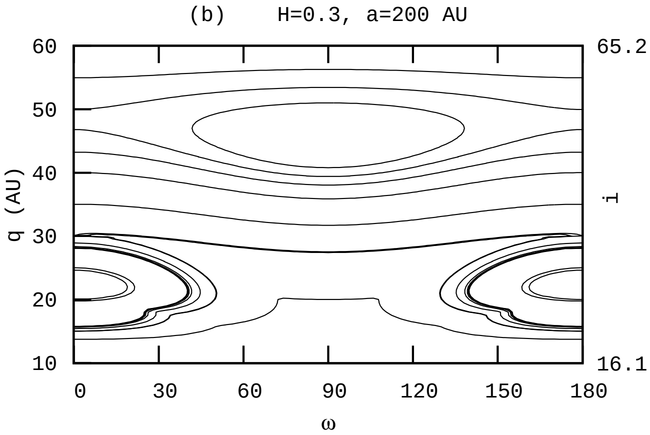
$K = \text{constant}$



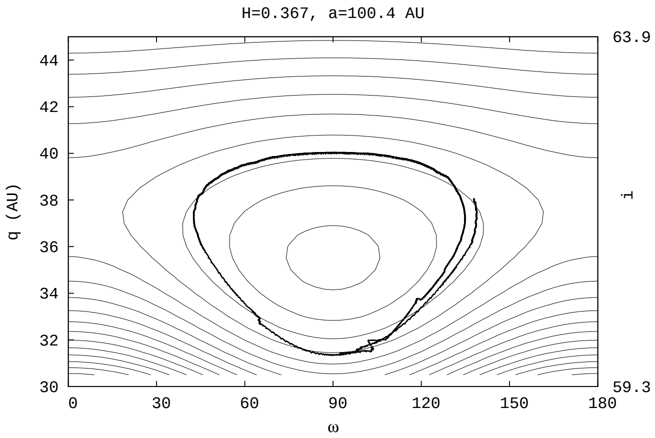
K = constant


(c) H=0.5, a=50 AU
q (AU)
45
40
35
30
25
20
15
10
60
i
11.5
0
30
60
90
120
150
180
ω

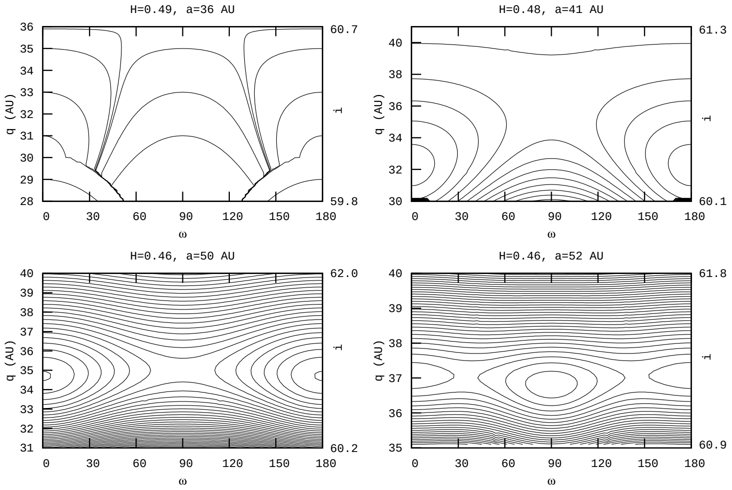
New equilibrium point at $\omega = 90°, i \sim 63°$



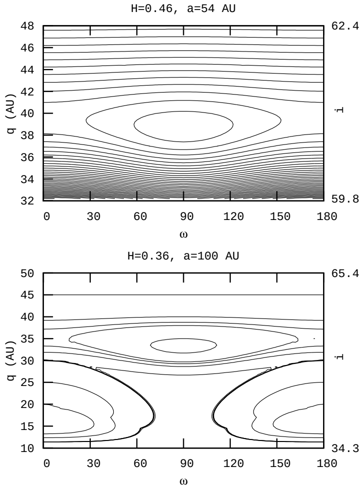
Numerical integration of the exact equations versus secular model:



Metamorphosis from $a = 36$ to $a = 52$ AU



Metamorphosis from $a = 54$ to $a = 500$ AU

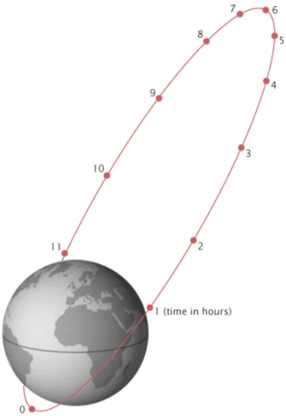


Then:

- There is a family of equilibrium points at $i \sim 60^\circ - 63^\circ$.
- $i \sim 63^\circ$ is very similar to the critical inclination in the Earth 's artificial satellite problem, generated by the Earth 's oblateness

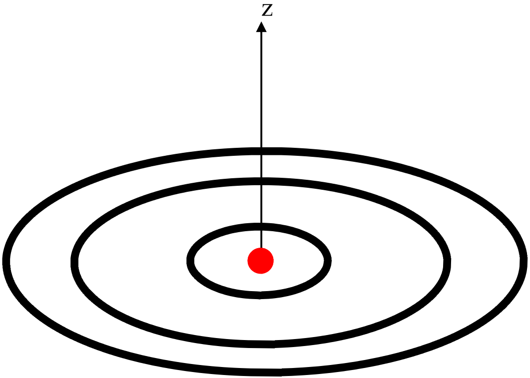
$\Rightarrow$ maybe the secular effect of JSUN is analogue to a solar oblateness

Molniya



(wikipedia)

Our model: spherical Sun + circular rings of matter



Disturbing function

$$R = \frac{C}{4}\frac{\mu}{r^3}(1 - 3\frac{z^2}{r^2}) + \frac{9E}{64}\frac{\mu}{r^5}(1 - 10\frac{z^2}{r^2} + \frac{35}{3}\frac{z^4}{r^4}) + \dots$$

where

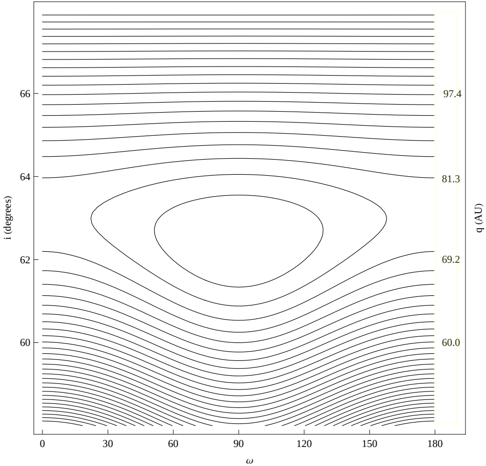
$C = \sum_j^N m_j a_j^2$ and $E = \sum_j^N m_j a_j^4$

Secular disturbing function

We calculate analytically

$$R_{sec}(\omega, e, i, a) = \frac{1}{2\pi} \int_0^{2\pi} R(\lambda)\, d\lambda$$

Kozai Maps using R_{sec}

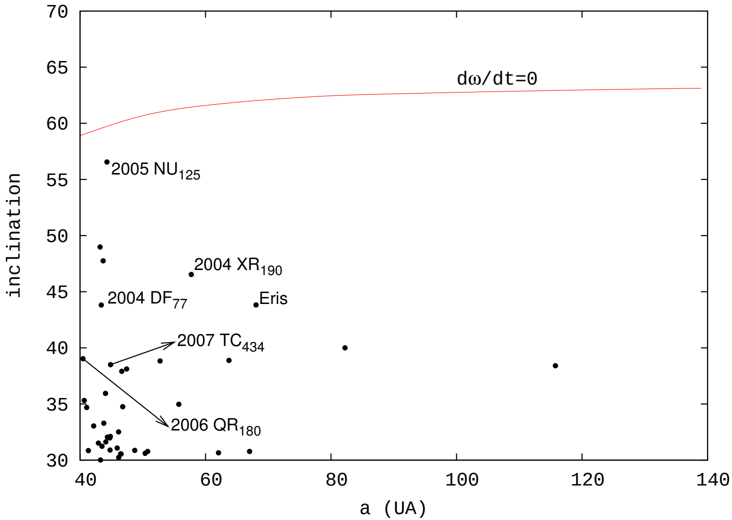


Equation for ω

$$\frac{d\omega}{dt} = \frac{3Ck}{16a^{7/2}(1-e^2)^2}(3 + 5\cos 2i) + \dots$$

$$\Longrightarrow i \sim 63{,}4^\circ \longrightarrow \frac{d\omega}{dt} = 0$$

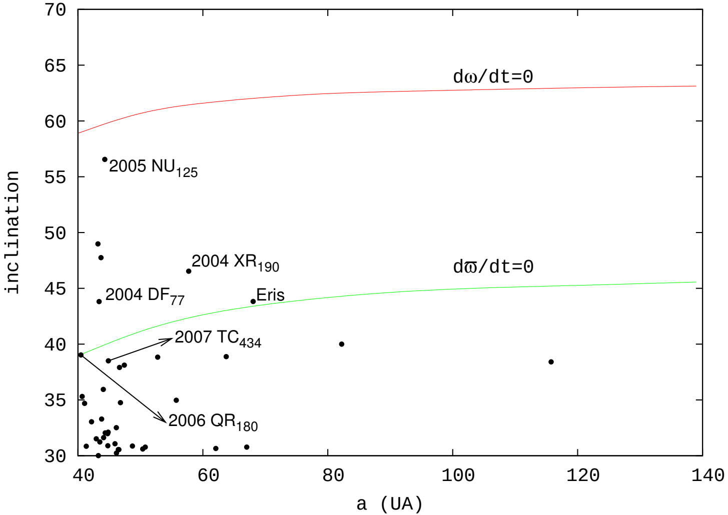
like Molniya satellites

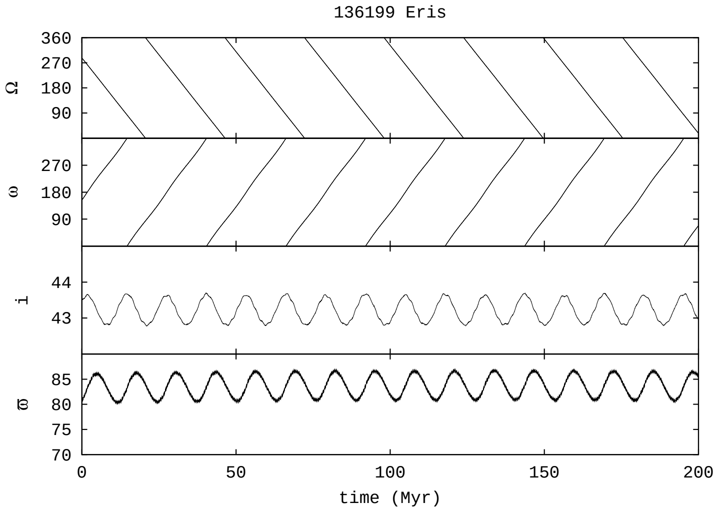

dω/dt=0
2005 NU125
2004 XR190
2004 DF77
Eris
2007 TC434
2006 QR180
inclination
a (UA)
70
65
60
55
50
45
40
35
30
40
60
80
100
120
140

Equation for $\varpi = \Omega + \omega$

$$\frac{d\varpi}{dt} = \frac{3Ck}{16a^{7/2}(1-e^2)^2}(3 - 4\cos i + 5\cos 2i) + \ldots$$

$$\frac{d\varpi}{dt} = 0 \longrightarrow i \sim 46^\circ$$


dω/dt=0
dϖ/dt=0
2005 NU$_{125}$
2004 XR$_{190}$
2004 DF$_{77}$
Eris
2007 TC$_{434}$
2006 QR$_{180}$
inclination
a (UA)
70
65
60
55
50
45
40
35
30
40
60
80
100
120
140

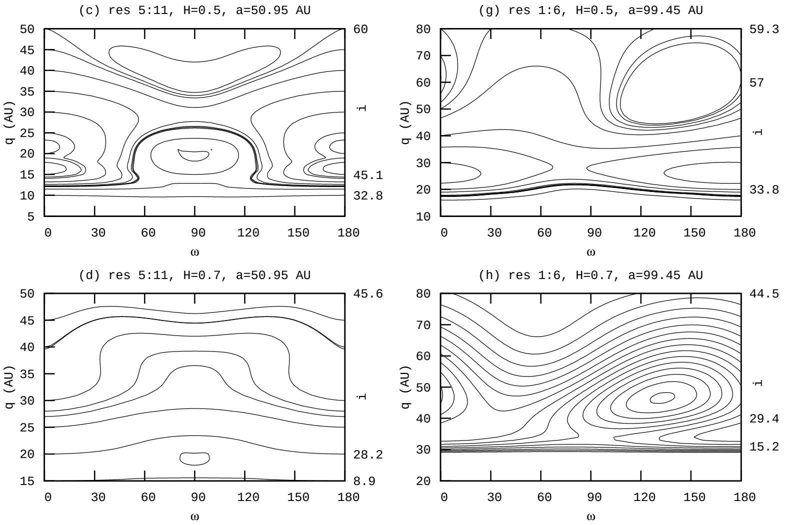

136199 Eris
Ω
360
270
180
90
ω
270
180
90
i
44
43
ϖ
85
80
75
70
0
50
100
150
200
time (Myr)

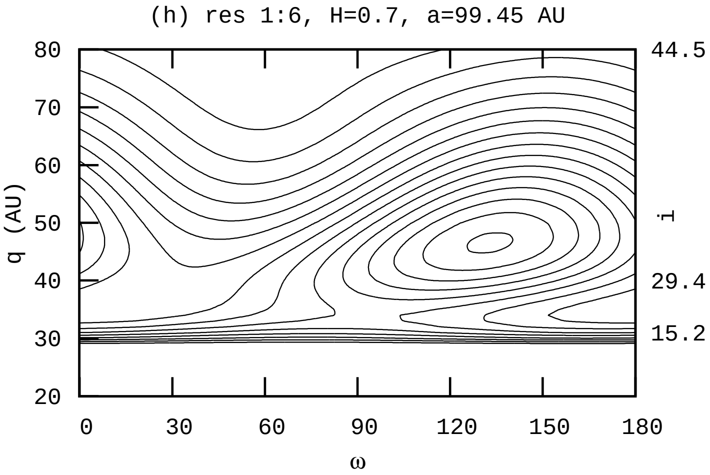
Some conclusions

- There is a barrier at $q = 30$ AU. No connections between $q < 30$ and $q > 30$.
- The Solar System is dynamically equivalent to an oblate Sun.
- There is a family of equilibrium points (at $i \sim 63^\circ$) around which large oscillations in q can occur.
- $\dot{\varpi} = 0$ at $i \sim 46^\circ$, resonance between ω and Ω.
- Eris is the only known TNO with $\dot{\varpi} \sim 0$

Resonant Kozai Dynamics

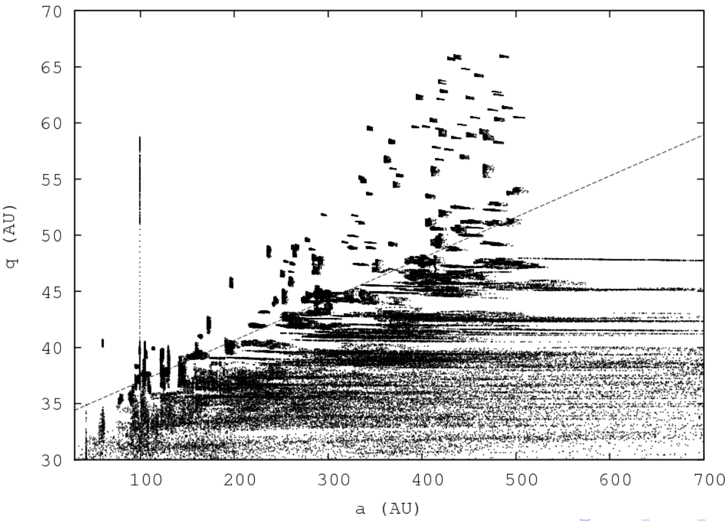
$$\sigma = (\mathrm{p} + \mathrm{q})\lambda_N - \mathrm{p}\lambda - \mathrm{q}\varpi$$


(c) res 5:11, H=0.5, a=50.95 AU
q (AU)
i
ω
50
45
40
35
30
25
20
15
10
5
60
45.1
32.8
0
30
60
90
120
150
180
(g) res 1:6, H=0.5, a=99.45 AU
q (AU)
i
ω
80
70
60
50
40
30
20
10
59.3
57
33.8
0
30
60
90
120
150
180
(d) res 5:11, H=0.7, a=50.95 AU
q (AU)
i
ω
50
45
40
35
30
25
20
15
45.6
28.2
8.9
0
30
60
90
120
150
180
(h) res 1:6, H=0.7, a=99.45 AU
q (AU)
i
ω
80
70
60
50
40
30
20
44.5
29.4
15.2
0
30
60
90
120
150
180

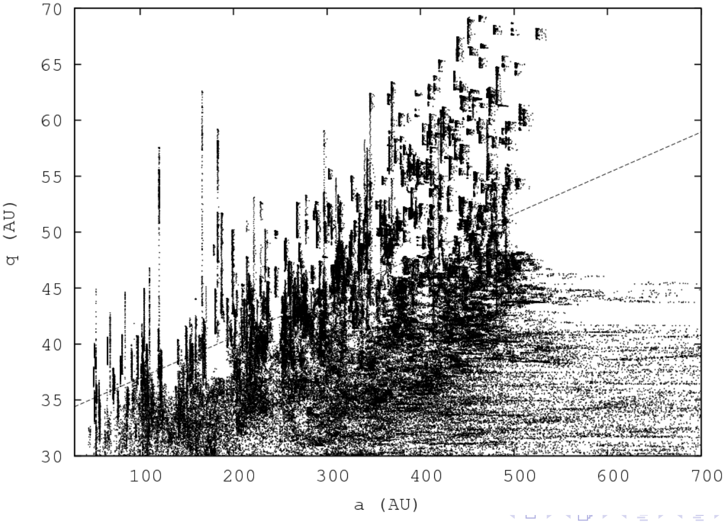

(h) res 1:6, H=0.7, a=99.45 AU
q (AU)
80
70
60
50
40
30
20
44.5
i
29.4
15.2
0
30
60
90
120
150
180
ω

Diffusion in (a, q)

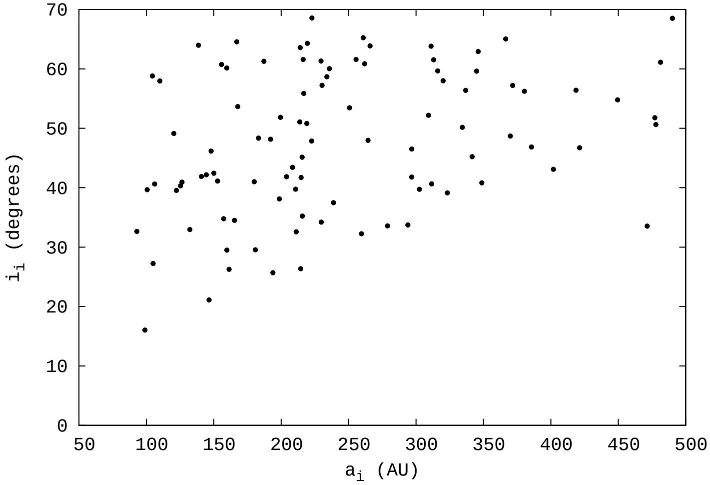
Diffusion al low inclinations ($i < 20°$)



Diffusion al high inclinations ($i > 20°$)



Orbits with $\Delta q > 5$ AU



More conclusions (resonant orbits)

- No barrier at $q = 30$ AU. Large Δq allowed.
- Resonances 1:N have *asymmetric* Kozai maps and Δq are larger.
- Large Δq implies a minimum $i_m(a) \propto a$

Acknowledgments

- Comite Organizador do CBDO
- CSIC
- PEDECIBA
- **e VOCES!**

Eris by 200 Myr:

http://youtu.be/CA1XPj_DklY

References

- *Survey of Kozai Dynamics Beyond Neptune*, Icarus 220, 392-403.