# The fragility of resonances

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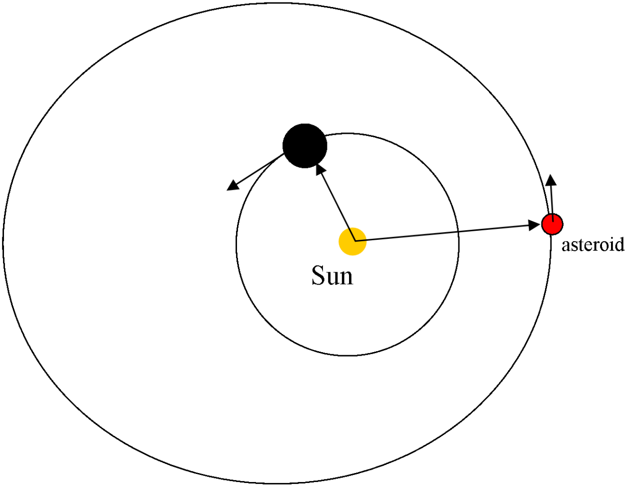
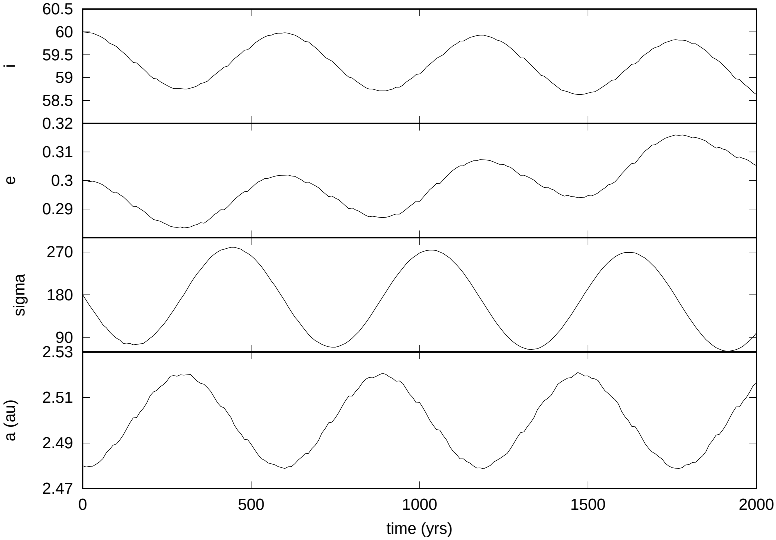
X Taller de Ciencias Planetarias, Maldonado, Marzo 2020

# Orbital resonance $k_p$:$k$

$$kn - k_p n_p \simeq 0 \qquad \Rightarrow \qquad \sigma = k\lambda - k_p\lambda_p + cte$$



# Librations

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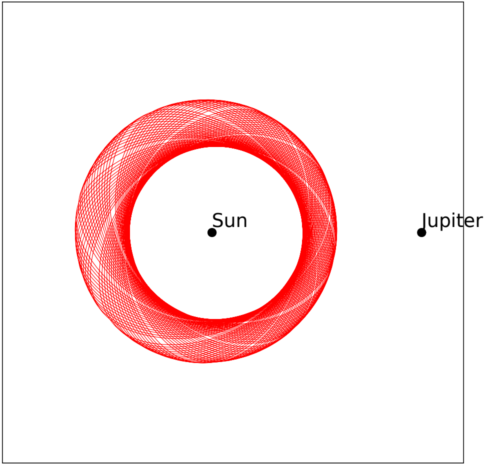
# NON resonant asteroid: symmetry

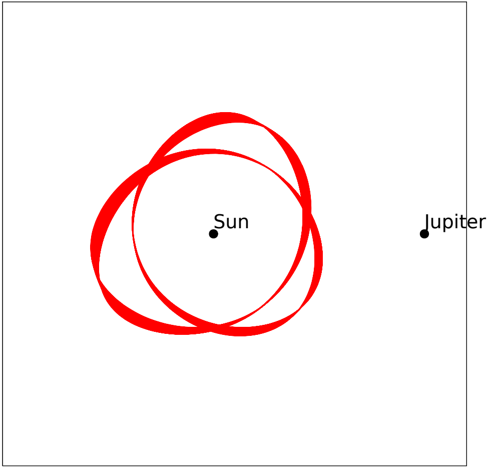
after several orbital revolutions:




(almost) no dynamical effects

# Resonant asteroid: asymmetry

after several orbital revolutions:

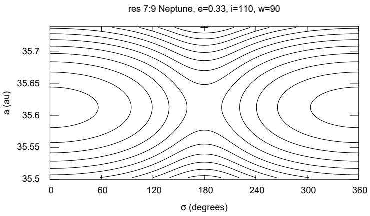



Yes, there are dynamical effects

# Resonant Hamiltonian

$$\mathcal{H}(a,e,i,\omega,\sigma) = \boxed{-\frac{\mu}{2a} - n_p \frac{k_p}{k}\sqrt{\mu a}} - \mathcal{R}(a,e,i,\omega,\sigma)$$

- resonant motion: $\mathcal{H} = cte$
- analytical $R$: series expansions
- numerical $R(\sigma)$: fixed $(a, e, i, \omega) \longrightarrow \mathcal{H}(a,\sigma) = F(a) + R(\sigma)$
- $\mathcal{H}(a,\sigma)$: simple libration theory



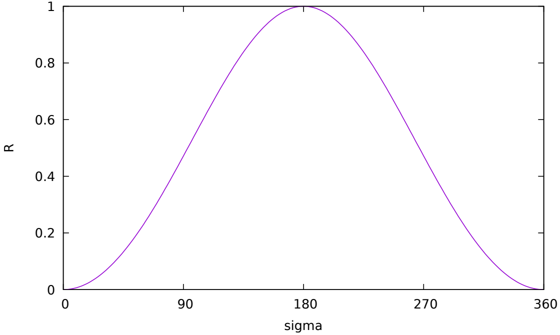
# analytic versus numeric...

vs

For **small** $(e, i)$:

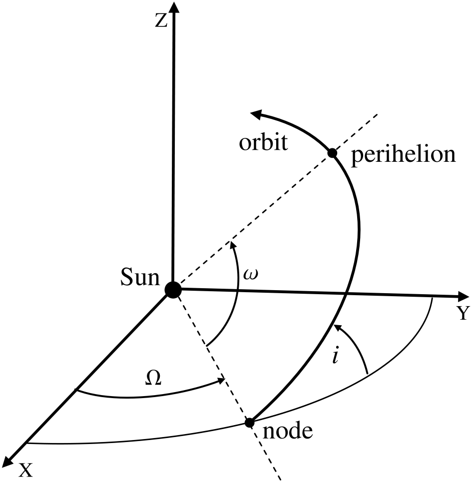
$$R(\sigma) \simeq A(e, i)\cos(\sigma)$$

Strength $= A(e, i)$




Minima of $R(\sigma)$ define the stable equilibrium points.

# What happens at high inclinations?




Do they make sense resonances in **retrograde** orbits?

# 3D Resonance Theory history

- Jefferys and Standish (1966): periodic orbits with $0^\circ < i < 180^\circ$
- $\sim$ 1985, num. integ.: **retrograde resonances in comets**
- Roig et al. (1998): $R$ expansion including **inclination**
- Ellis and Murray (2000): Expansion around $(e = 0, i = 0^\circ)$
- Gallardo (2006): numerical $R(\sigma)$ for arbitrary $(e, i)$. **Strength depends on** $(e, i, \omega)$

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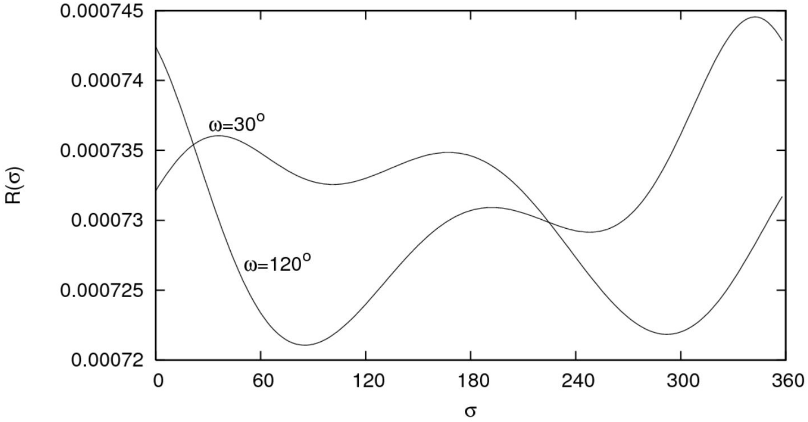
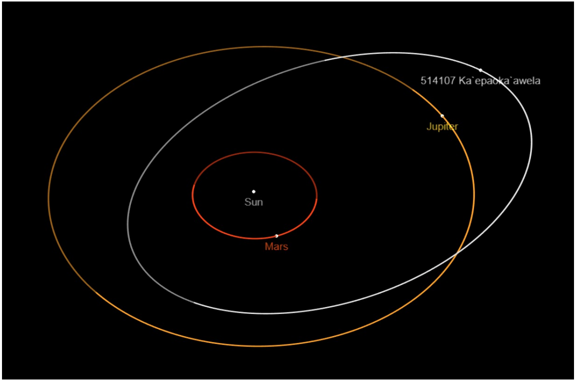
# $R(\sigma)$ dependence on $\omega$




$$\Delta R(\sigma) \simeq A(e,i,\omega)??$$

# 3D Resonance Theory history

- Morais and Giuppone (2012): stability of planar retrograde co-orbital, confirmed with BZ509




- Saillenfest et al. (2016): secular evolution of resonant motion

# 3D Resonance Theory history

- Namouni and Morais (2018): disturbing function for arbitrary inclinations ($e < 0{,}6$). Strength depends on $(e, i)$.
- Gallardo (2019): numerical $R(\sigma)$ again, several examples showing that $\omega \longrightarrow$ Strength.
- Lei (2019): **rearrangement** of the NM18 disturbing function, $\omega$ appears as a coefficient affecting the strength. Analytical theory of librations.

$$\mathcal{R}_{p,q} = \mathcal{C}_{p,q} \cos\sigma + \mathcal{S}_{p,q} \sin\sigma + \mathcal{C}_{2p,2q} \cos 2\sigma + \mathcal{S}_{2p,2q} \sin 2\sigma +$$

- Gallardo (2020): theory of librations using numerical $R(\sigma)$. **FRAGILITY**: $\Delta a(\omega)$.
- Namouni and Morais (2020): librations strongly affected by $\omega$

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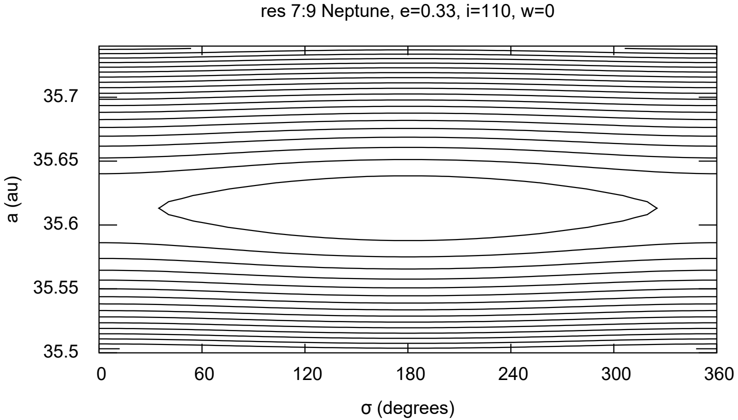
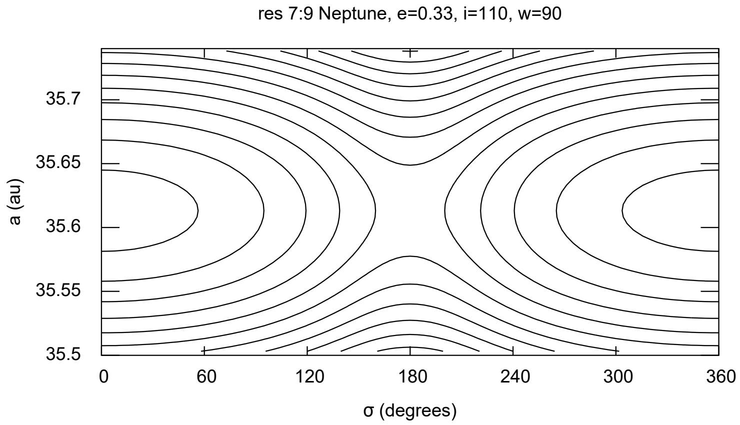
# Origin of fragility

$$\mathcal{H}(a, \sigma, \omega = 0^\circ)$$

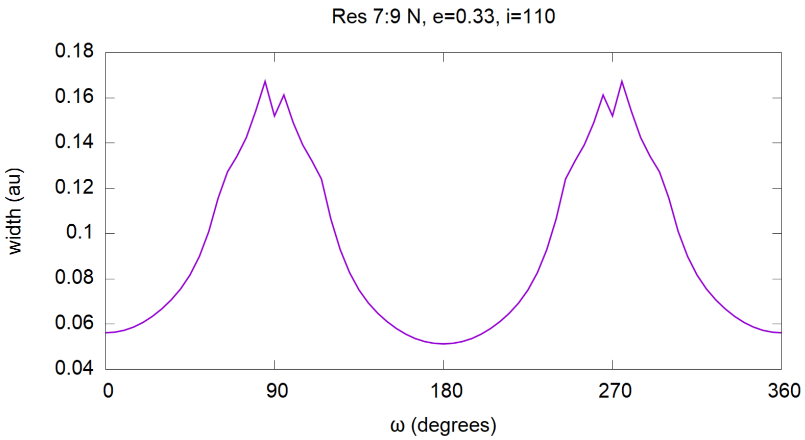


# Origin of fragility

$$\mathcal{H}(a, \sigma, \omega = 90^\circ)$$



# $\Delta a(\omega)$

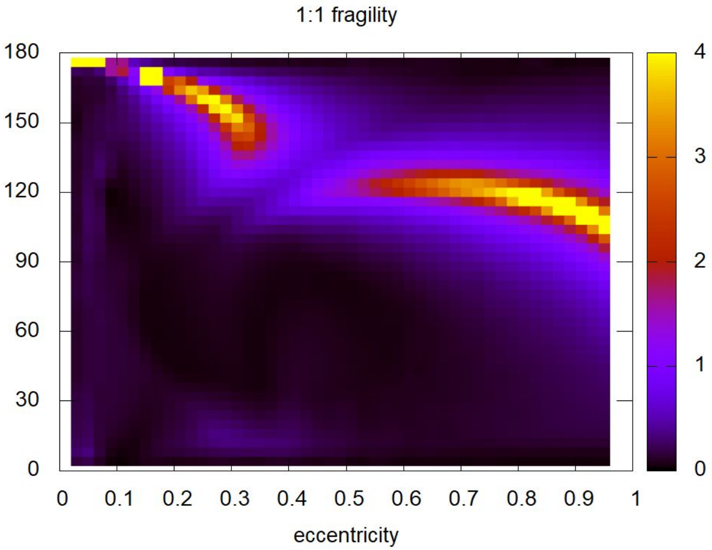



Fragility:

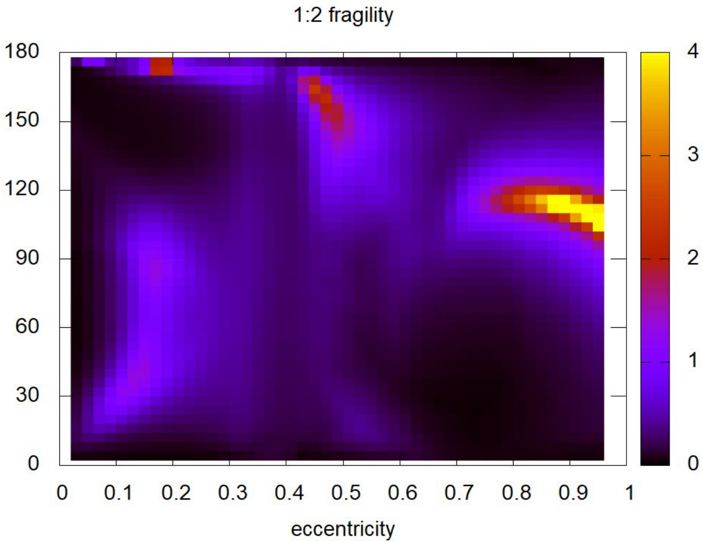
$$f(e,i) = \frac{\Delta a_{max} - \Delta a_{min}}{\Delta a_{min}}$$

# Trans Neptunian Region

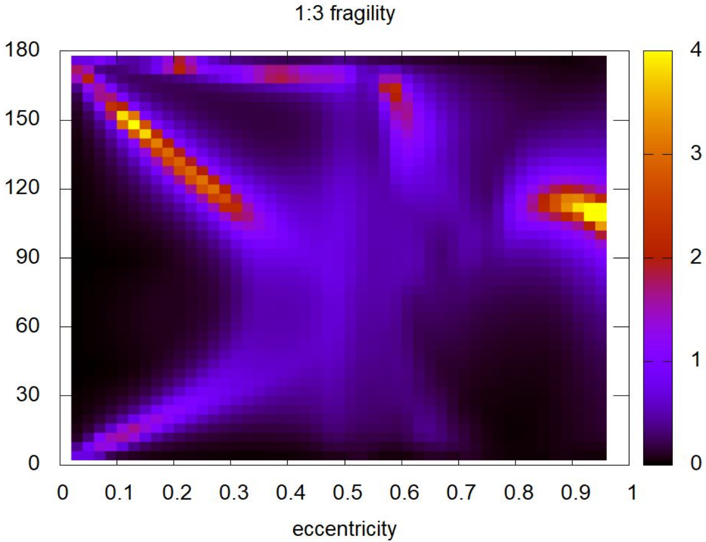
# 1:1 Neptune


1:1 fragility
180
150
120
90
60
30
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
eccentricity
4
3
2
1
0

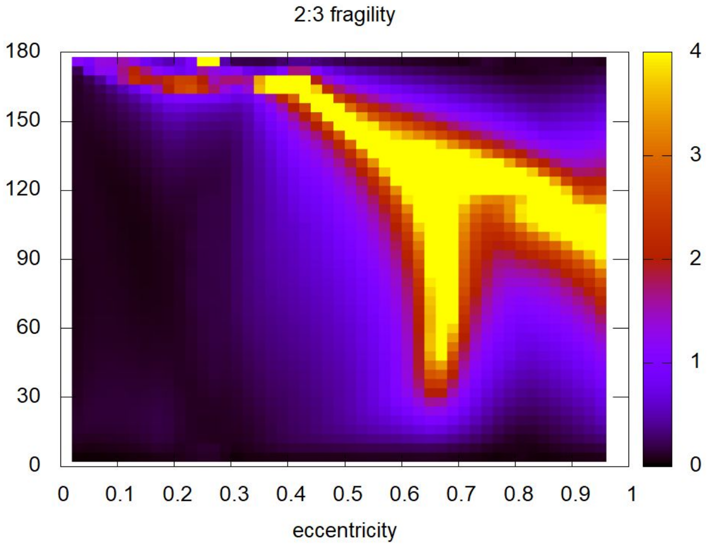
# 1:2 Neptune


1:2 fragility
180
150
120
90
60
30
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
eccentricity
4
3
2
1
0

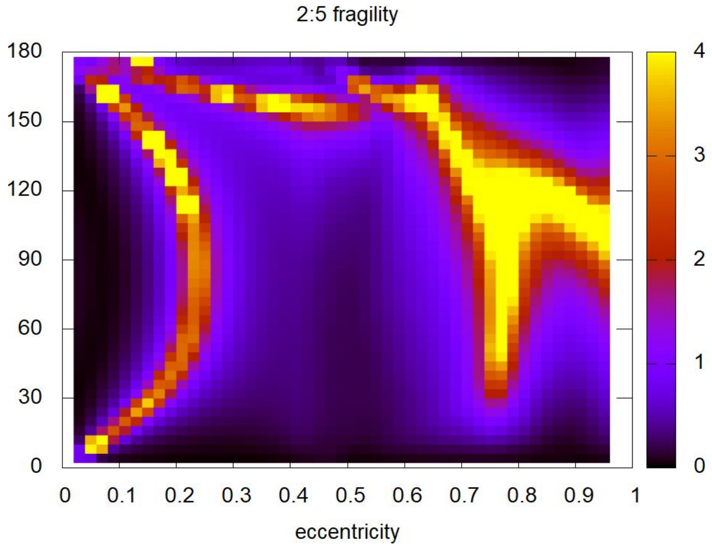
# 1:3 Neptune


1:3 fragility
180
150
120
90
60
30
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
eccentricity
4
3
2
1
0

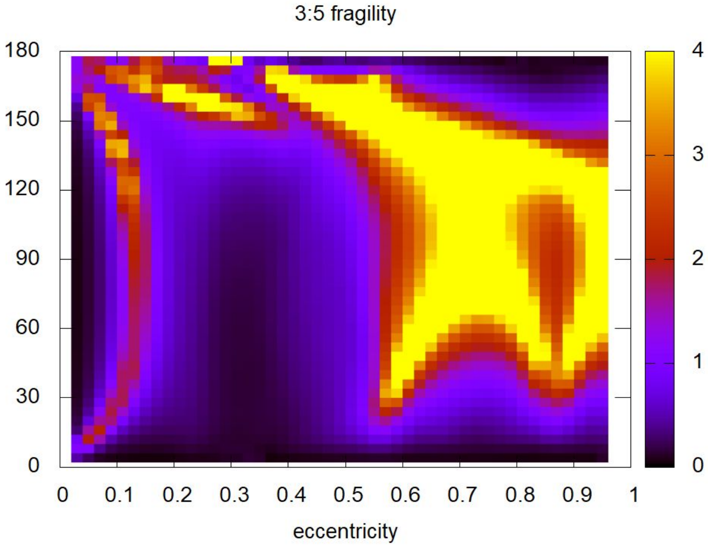
# 2:3 Neptune


2:3 fragility
180
150
120
90
60
30
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
eccentricity
4
3
2
1
0

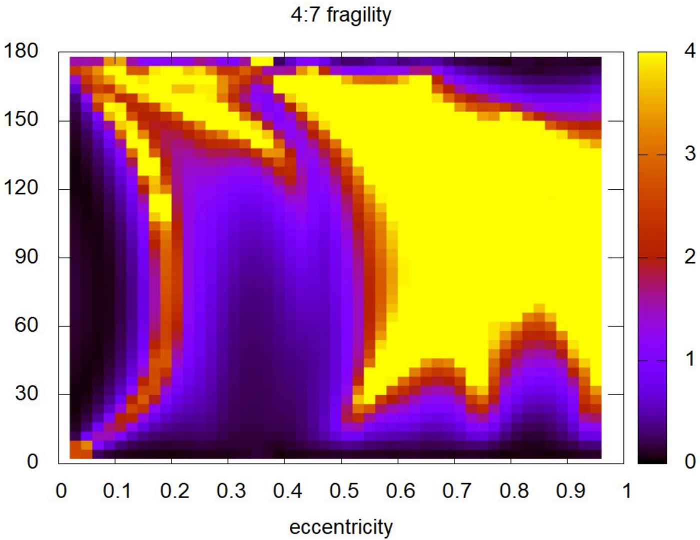
# 2:5 Neptune


2:5 fragility
180
150
120
90
60
30
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
eccentricity
4
3
2
1
0

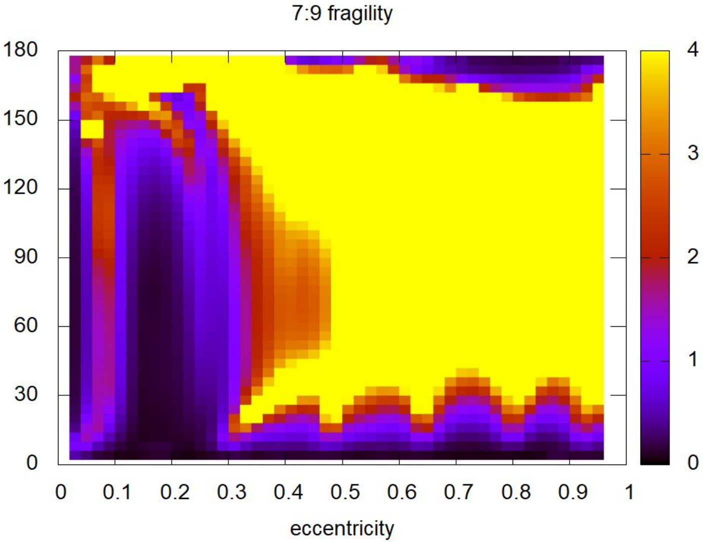
# 3:5 Neptune


3:5 fragility
180
150
120
90
60
30
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
eccentricity
4
3
2
1
0

# 4:7 Neptune


4:7 fragility
180
150
120
90
60
30
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
eccentricity
4
3
2
1
0

# 7:9 Neptune

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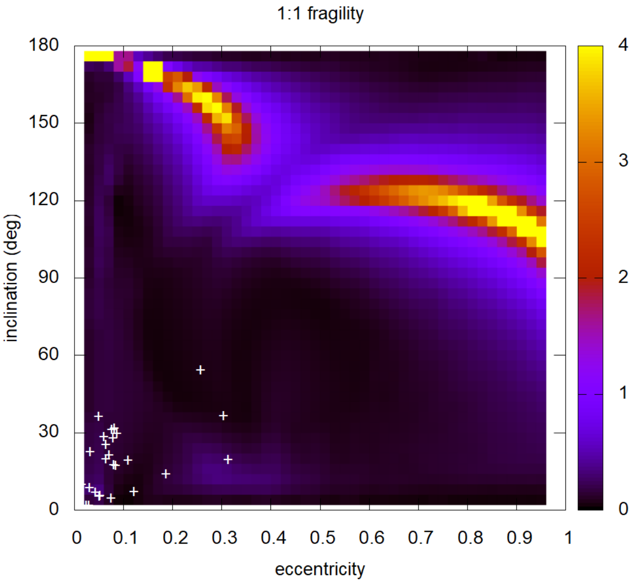
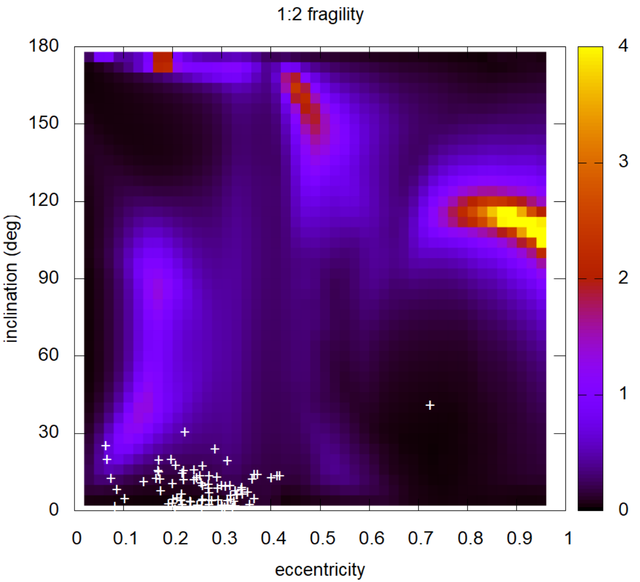
# Populations

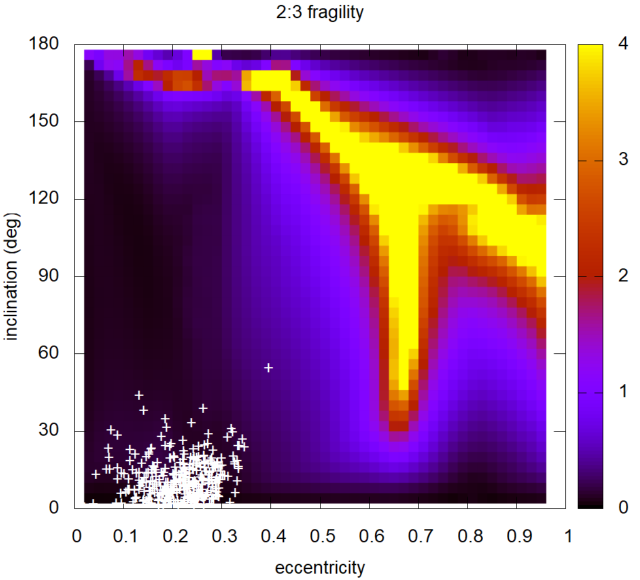
# 1:1 Neptune fragility + population



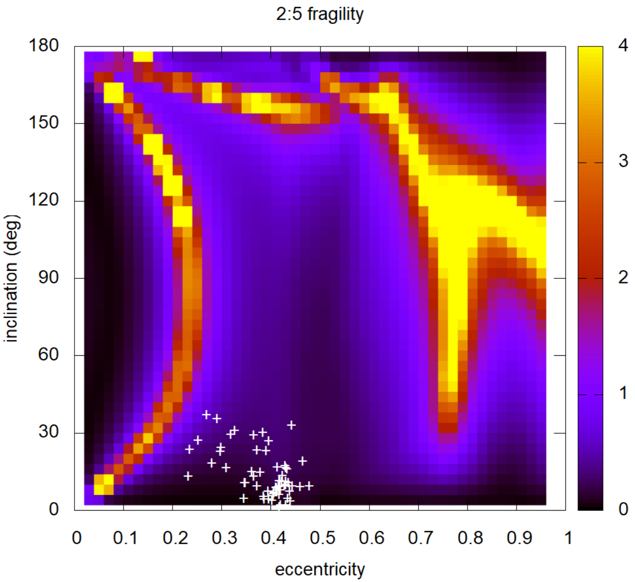
# 1:2 Neptune fragility + population



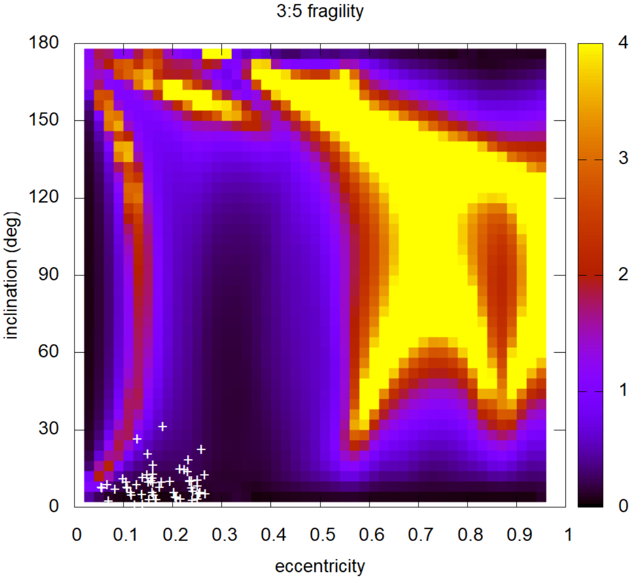
# 2:3 Neptune fragility + population



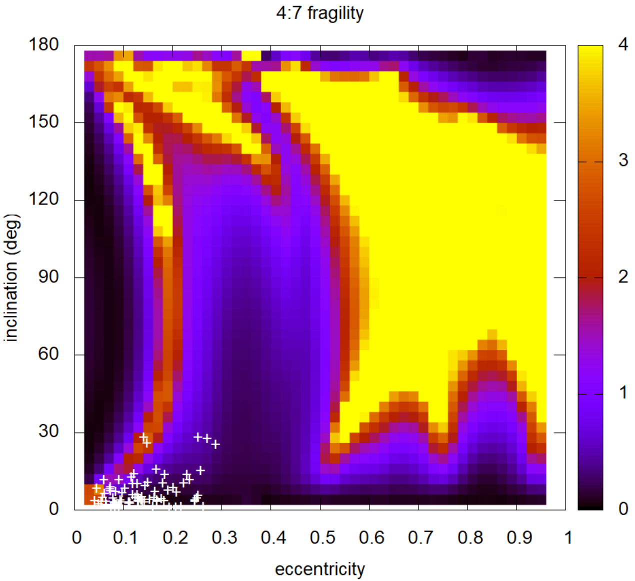
# 2:5 Neptune fragility + population



# 3:5 Neptune fragility + population



# 4:7 Neptune fragility + population

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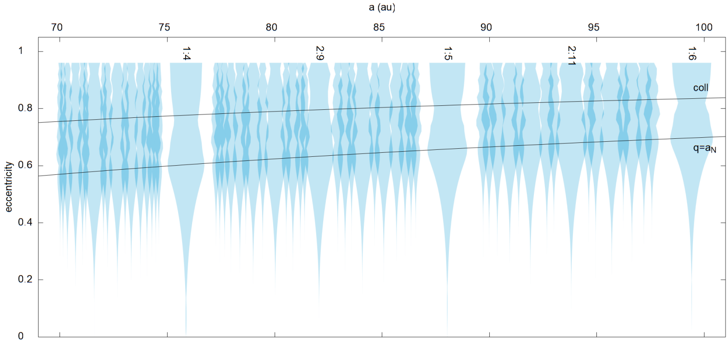
# SUMMARY

- planar case: $e \longrightarrow$ strength, width,...
- NON-planar case: $(e, i, \omega) \longrightarrow$ strength, width,...
- $\omega(t) \longrightarrow$ fragility $f(e, i)$
- $k_p$:$k$, high $k_p \longrightarrow$ high fragility
- **primordial populations** $\longrightarrow$ low fragility regions
- population 4:7 with Neptune **shifted** to high fragility region
- Gallardo (2020, CMDA): semi-analytical model
- www.fisica.edu.uy/~gallardo/atlas

# é isso!


a (au)
70
75
80
85
90
95
100
1
0.8
0.6
0.4
0.2
0
eccentricity
1:4
2:9
1:5
2:11
1:6
coll
q=a_N