

The fragility of resonances

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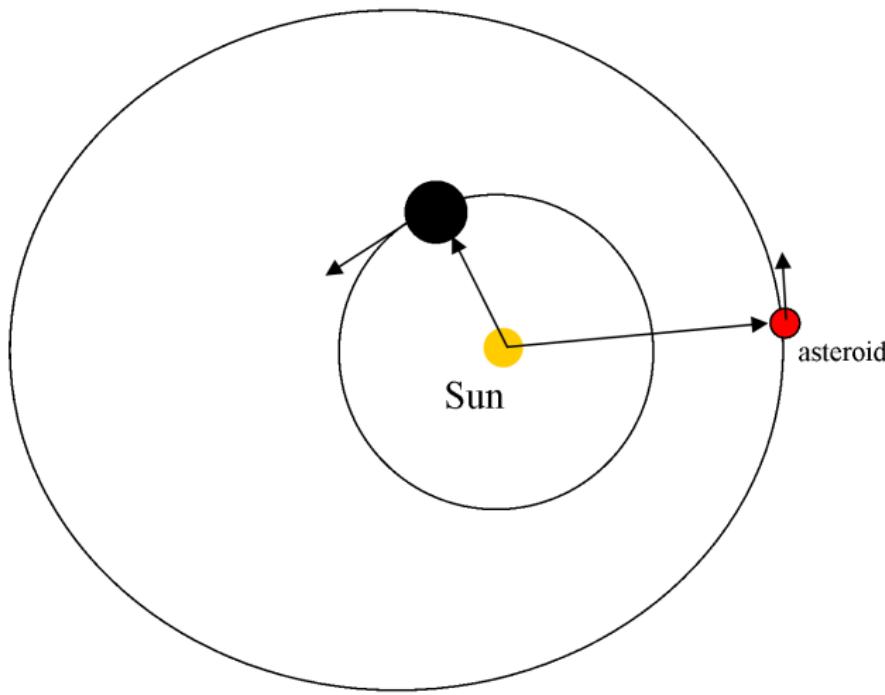


X Taller de Ciencias Planetarias, Maldonado, Marzo 2020

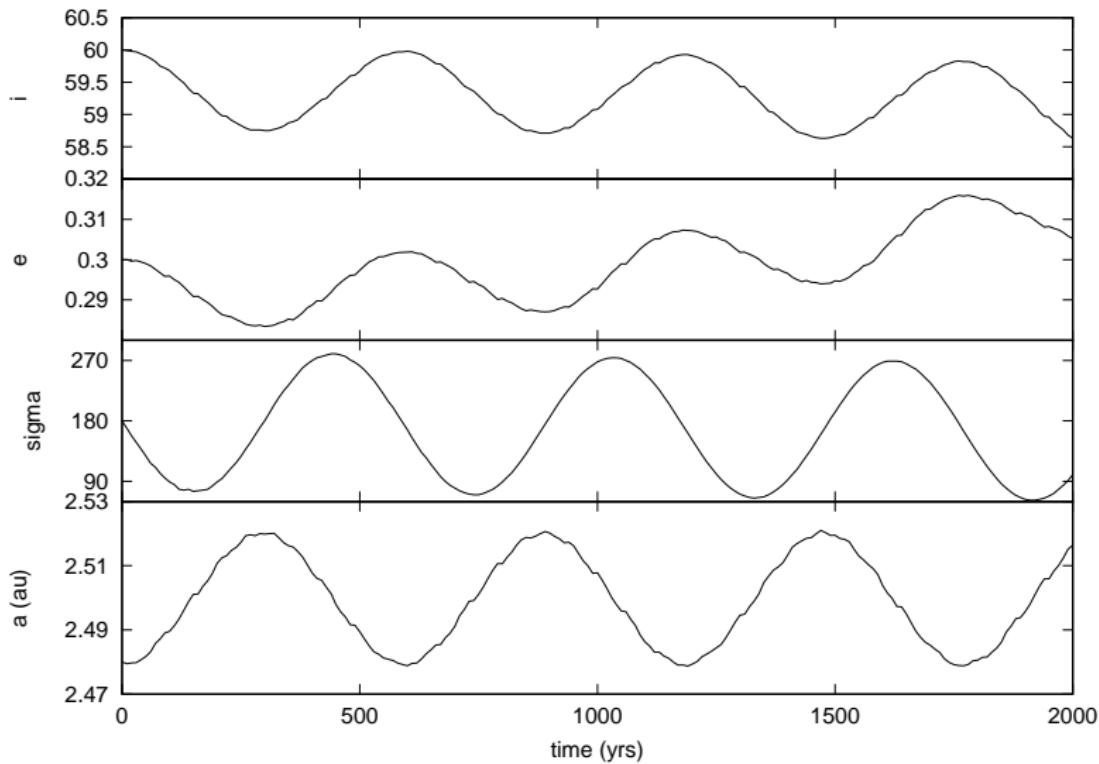


Orbital resonance $k_p:k$

$$kn - k_p n_p \simeq 0 \quad \Rightarrow \quad \sigma = k\lambda - k_p \lambda_p + cte$$

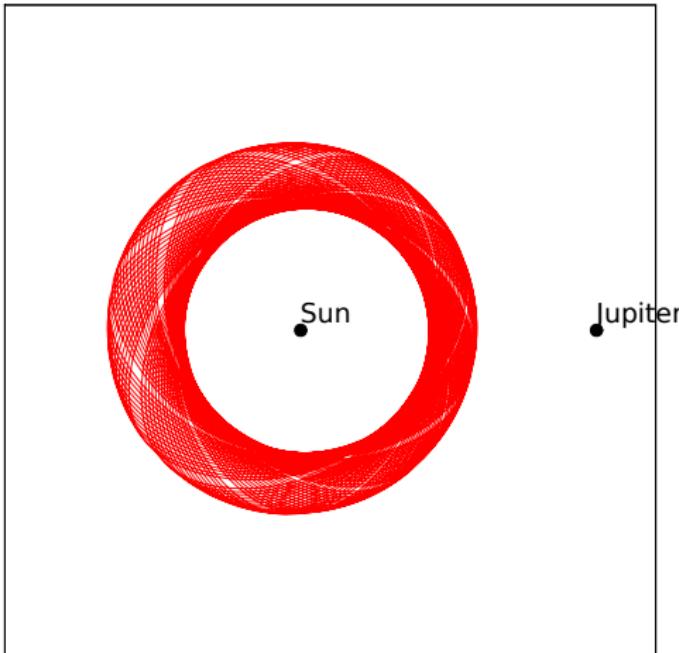


Librations



NON resonant asteroid: symmetry

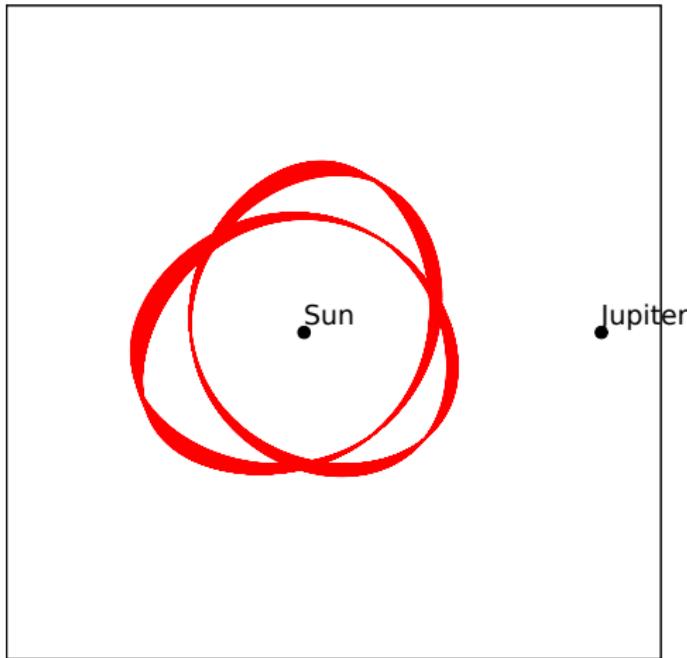
after several orbital revolutions:



(almost) no dynamical effects

Resonant asteroid: asymmetry

after several orbital revolutions:

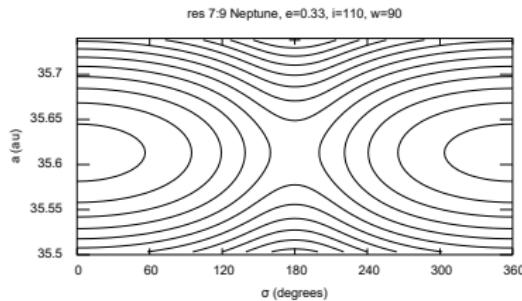


Yes, there are dynamical effects

Resonant Hamiltonian

$$\mathcal{H}(a, e, i, \omega, \sigma) = \boxed{-\frac{\mu}{2a} - n_p \frac{k_p}{k} \sqrt{\mu a}} - \mathcal{R}(a, e, i, \omega, \sigma)$$

- resonant motion: $\mathcal{H} = cte$
- analytical R : series expansions
- numerical $R(\sigma)$: fixed $(a, e, i, \omega) \rightarrow \mathcal{H}(a, \sigma) = F(a) + R(\sigma)$
- $\mathcal{H}(a, \sigma)$: simple libration theory



analytic versus numeric...

Handwritten notes from a lecture on oscillations and resonance, comparing analytic and numeric methods.

Top Left: Analytic solution for a mass-spring system. It shows the displacement x over time t for different initial conditions. The equations involve parameters ω_0 , β , and ϕ .

Bottom Left: A series of plots showing the displacement x versus time t for various initial conditions. The plots illustrate how the system's behavior changes from damped oscillations to non-oscillatory motion as the damping ratio β varies.

Top Right: Numerical simulation results. It shows a plot of displacement x versus time t with a smooth curve representing the numerical solution. Below it, a plot of velocity v versus time t shows a corresponding smooth curve.

Bottom Right: Numerical simulation results. It shows two plots: one for displacement x versus time t and another for velocity v versus time t , both showing smooth curves.

VS

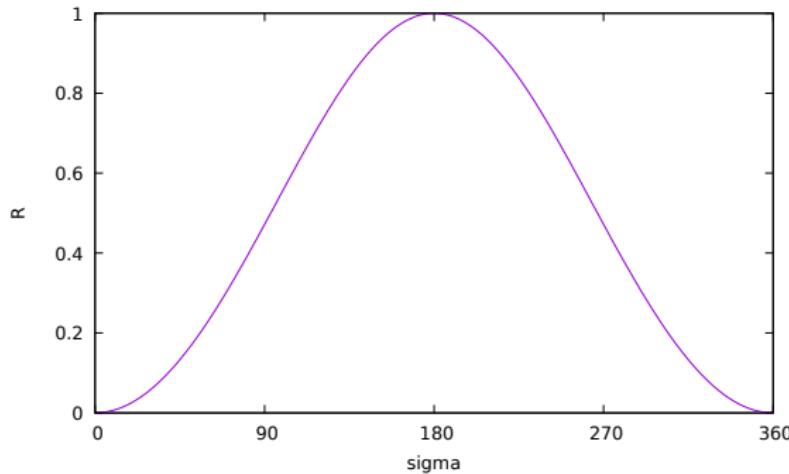


Disturbing function $R(\sigma)$

For **small** (e, i) :

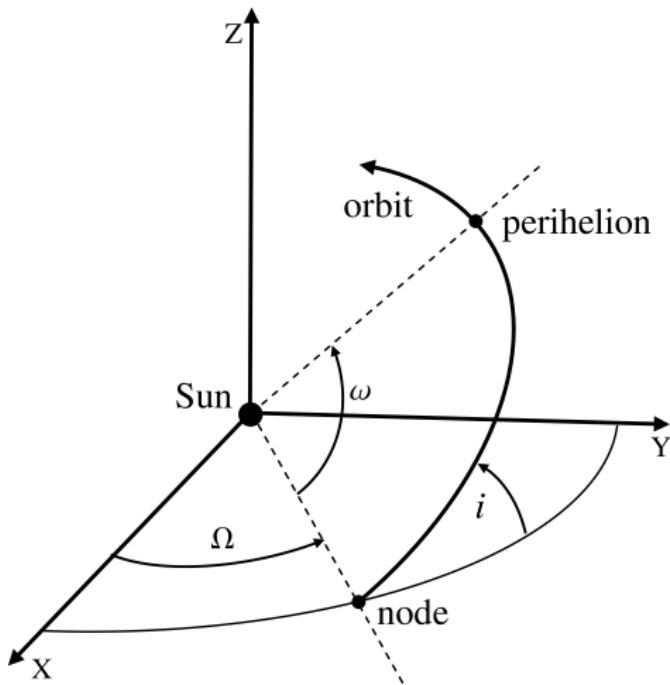
$$R(\sigma) \simeq A(e, i) \cos(\sigma)$$

Strength = $A(e, i)$



Minima of $R(\sigma)$ define the stable equilibrium points.

What happens at high inclinations?

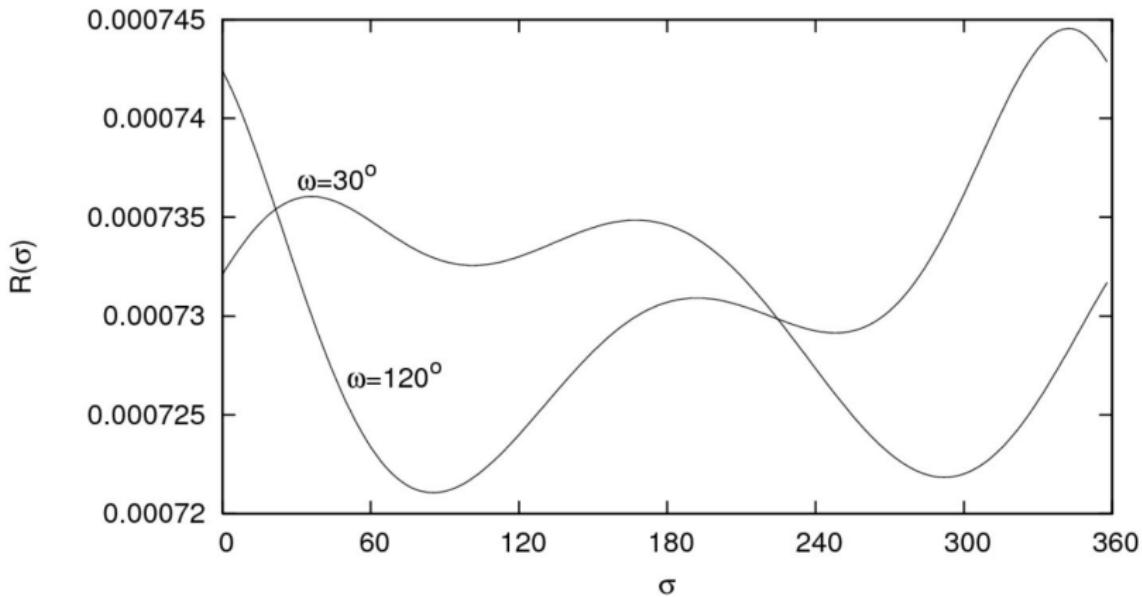


Do they make sense resonances in **retrograde** orbits?

3D Resonance Theory history

- Jefferys and Standish (1966): periodic orbits with $0^\circ < i < 180^\circ$
- ~ 1985 , num. integ.: **retrograde resonances in comets**
- Roig et al. (1998): R expansion including **inclination**
- Ellis and Murray (2000): Expansion around $(e = 0, i = 0^\circ)$
- Gallardo (2006): numerical $R(\sigma)$ for arbitrary (e, i) . **Strength depends on (e, i, ω)**

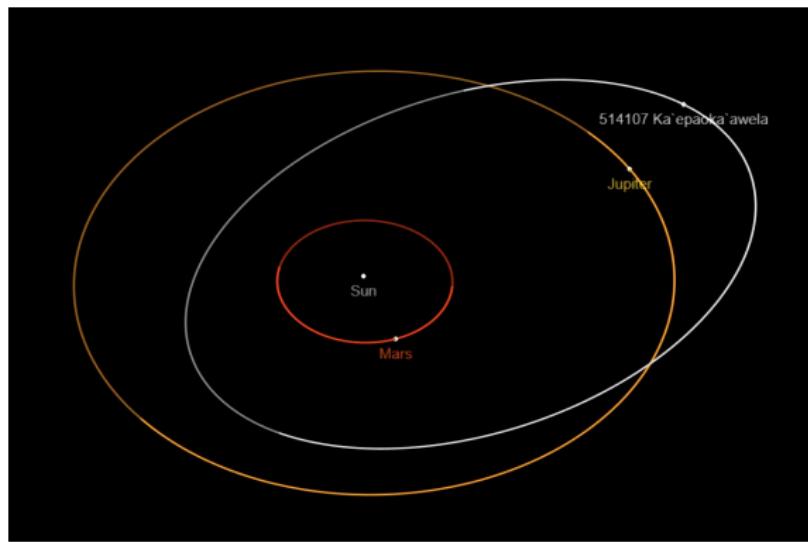
$R(\sigma)$ dependence on ω



$$\Delta R(\sigma) \simeq A(e, i, \omega)??$$

3D Resonance Theory history

- Morais and Giuppone (2012): stability of planar retrograde co-orbital, confirmed with BZ509



- Saillenfest et al. (2016): secular evolution of resonant motion

3D Resonance Theory history

- Namouni and Morais (2018): disturbing function for arbitrary inclinations ($e < 0,6$). Strength depends on (e, i) .
- Gallardo (2019): numerical $R(\sigma)$ again, several examples showing that $\omega \rightarrow$ Strength.
- Lei (2019): **rearrangement** of the NM18 disturbing function, ω appears as a coefficient affecting the strength. Analytical theory of librations.

$$\mathcal{R}_{p,q} = \mathcal{C}_{p,q} \cos \sigma + \mathcal{S}_{p,q} \sin \sigma + \mathcal{C}_{2p,2q} \cos 2\sigma + \mathcal{S}_{2p,2q} \sin 2\sigma +$$

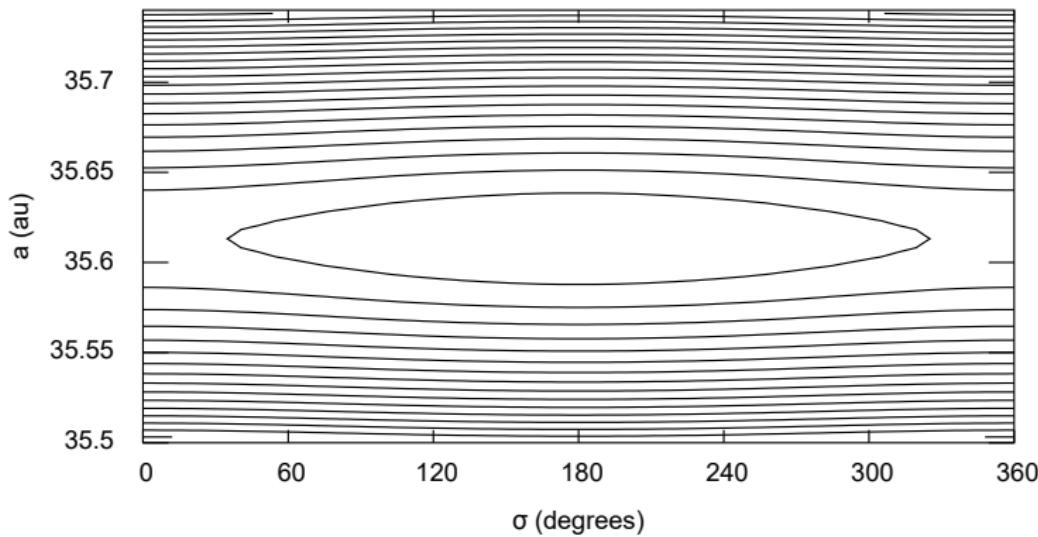
- Gallardo (2020): theory of librations using numerical $R(\sigma)$.
FRAGILITY: $\Delta a(\omega)$.
- Namouni and Morais (2020): librations strongly affected by ω



Origin of fragility

$$\mathcal{H}(a, \sigma, \omega = 0^\circ)$$

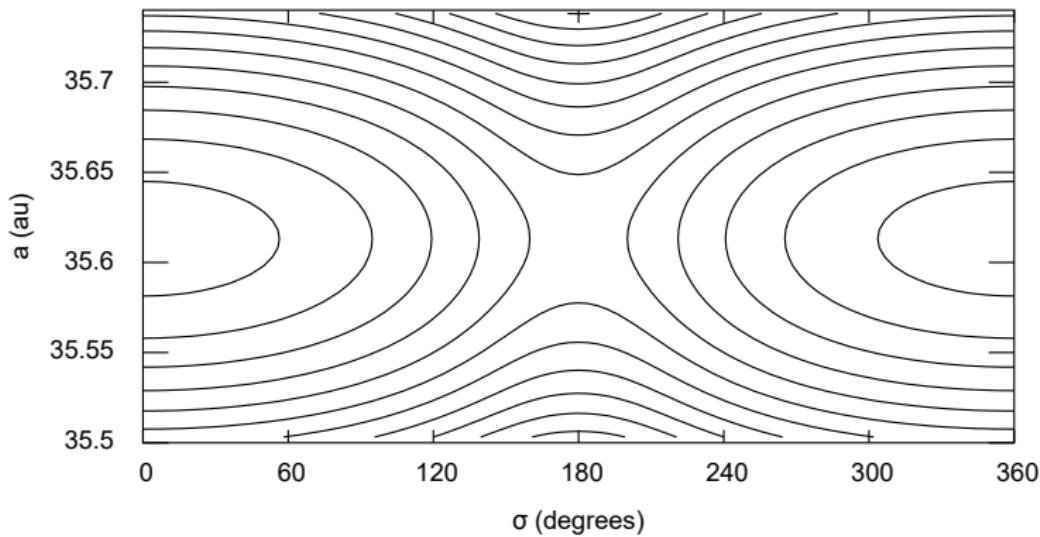
res 7:9 Neptune, e=0.33, i=110, w=0



Origin of fragility

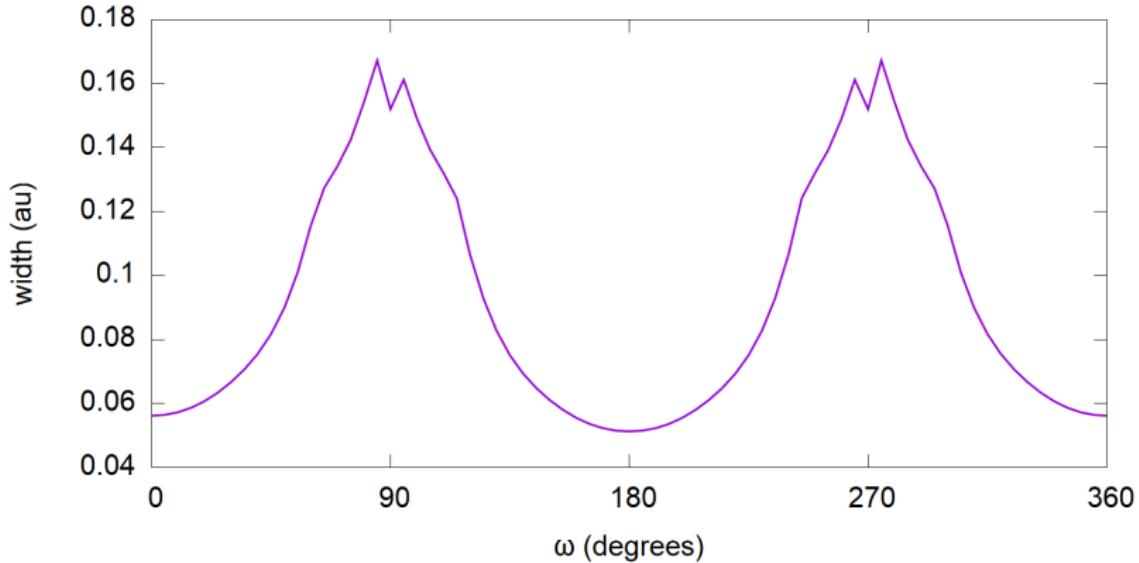
$$\mathcal{H}(a, \sigma, \omega = 90^\circ)$$

res 7:9 Neptune, e=0.33, i=110, w=90



$$\Delta a(\omega)$$

Res 7:9 N, e=0.33, i=110



Fragility:

$$f(e, i) = \frac{\Delta a_{max} - \Delta a_{min}}{\Delta a_{min}}$$

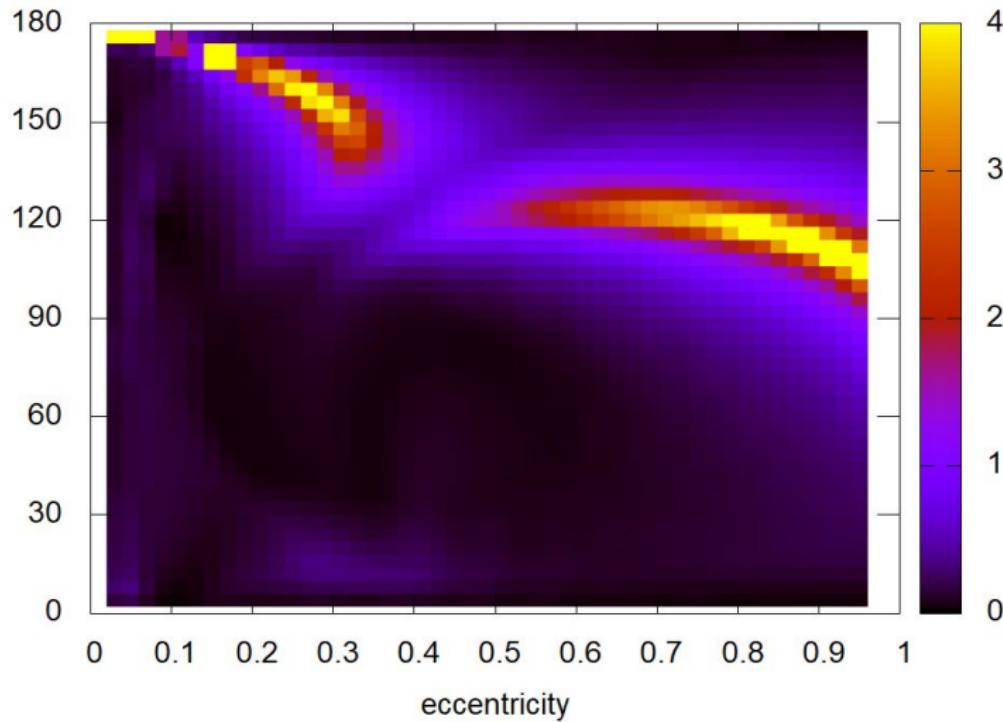
Trans Neptunian Region



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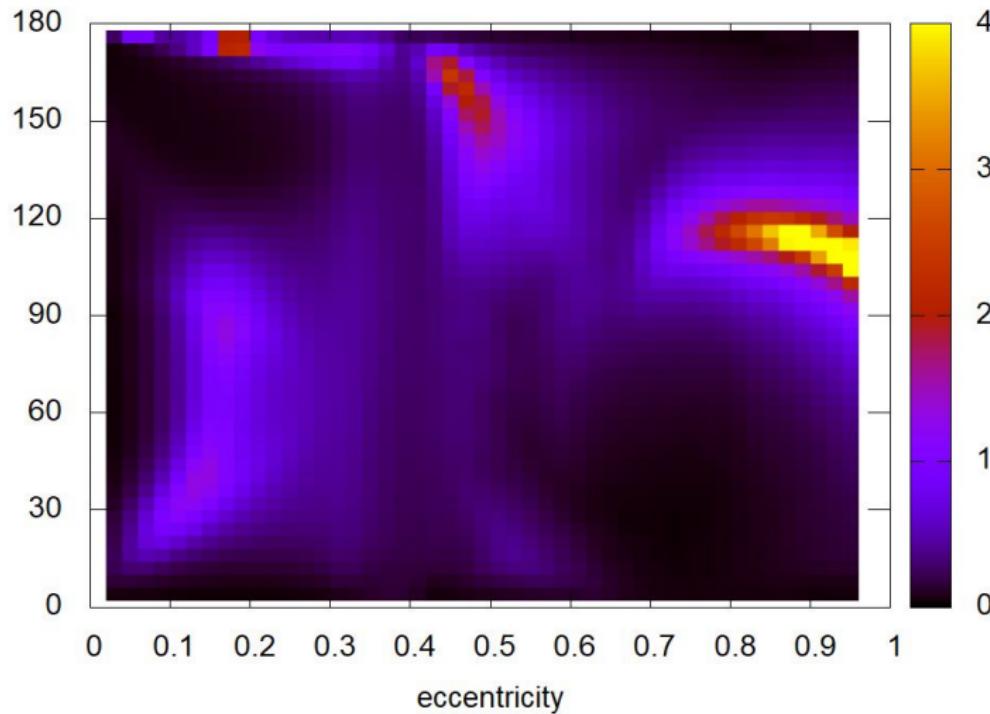
1:1 Neptune

1:1 fragility



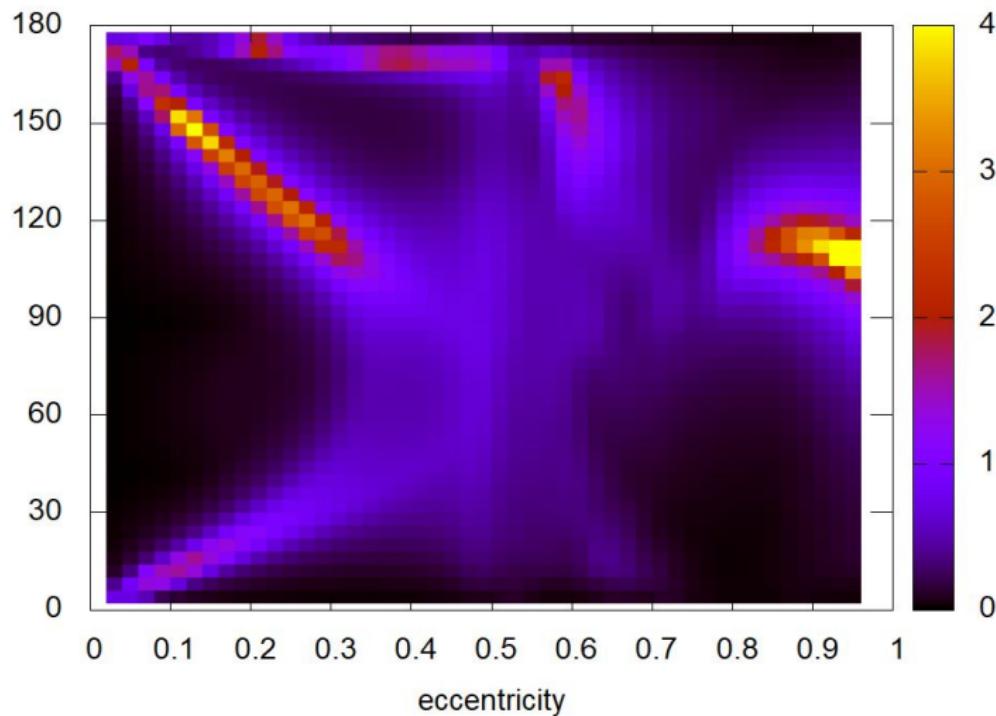
1:2 Neptune

1:2 fragility



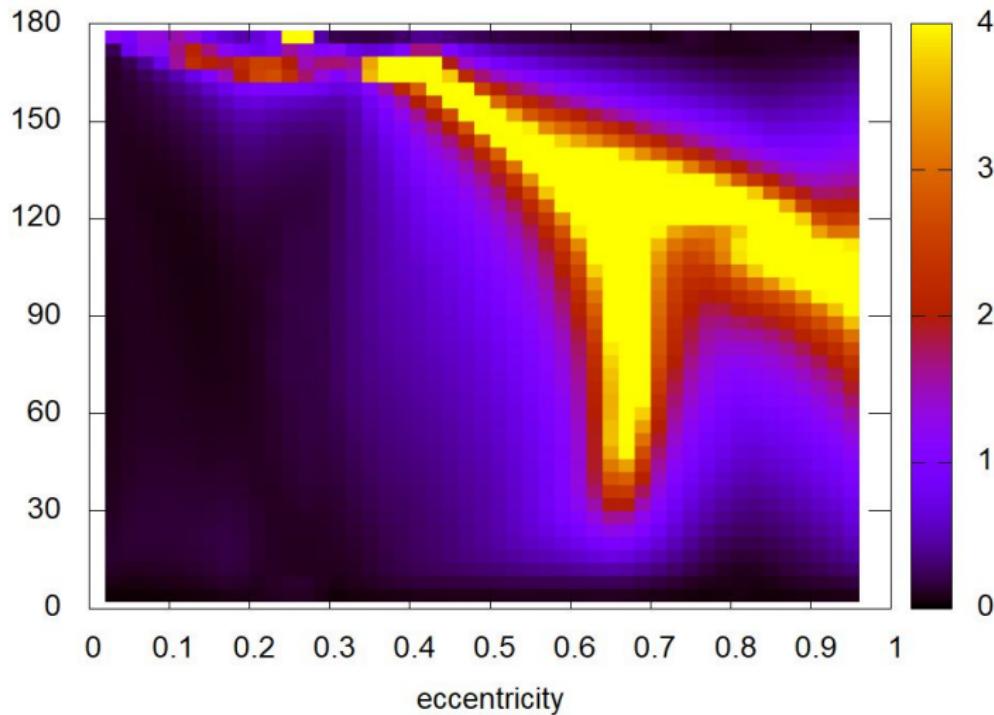
1:3 Neptune

1:3 fragility



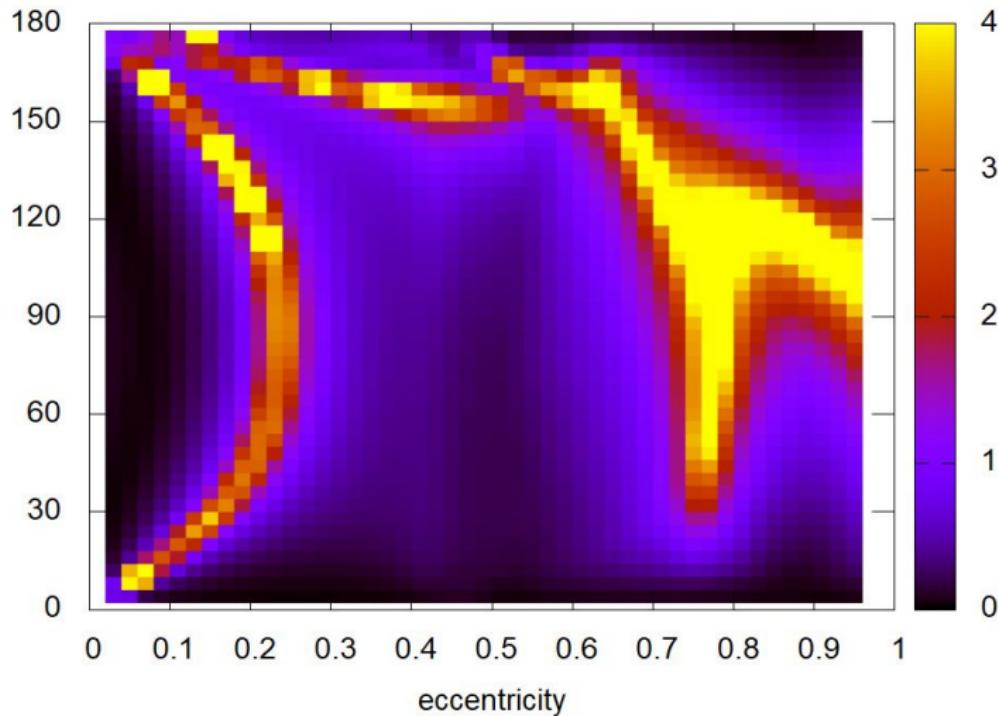
2:3 Neptune

2:3 fragility



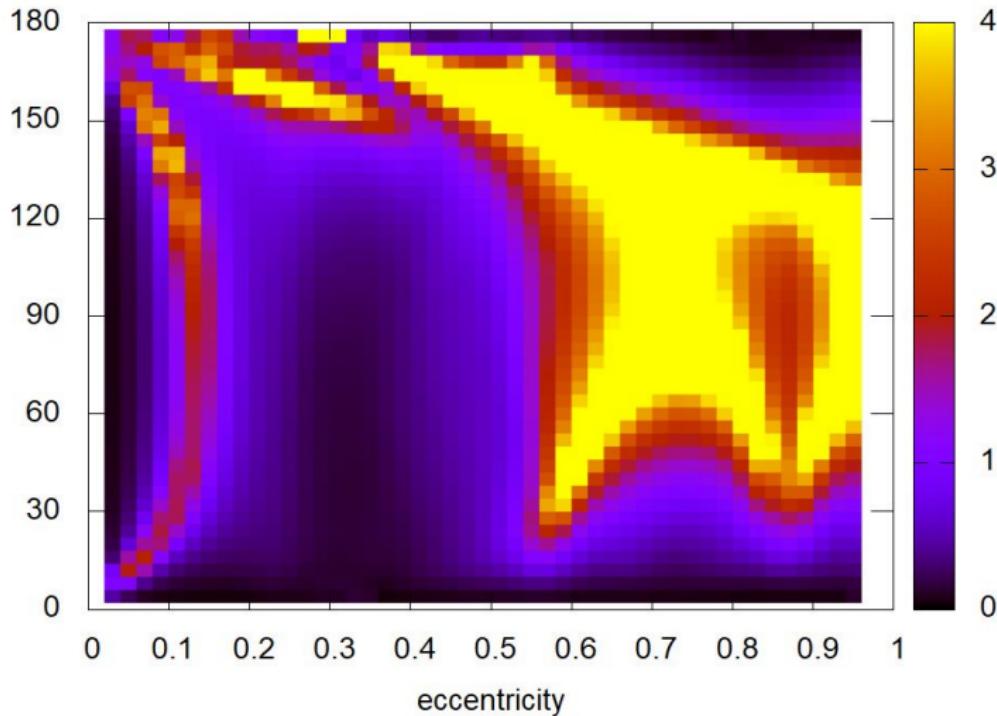
2:5 Neptune

2:5 fragility



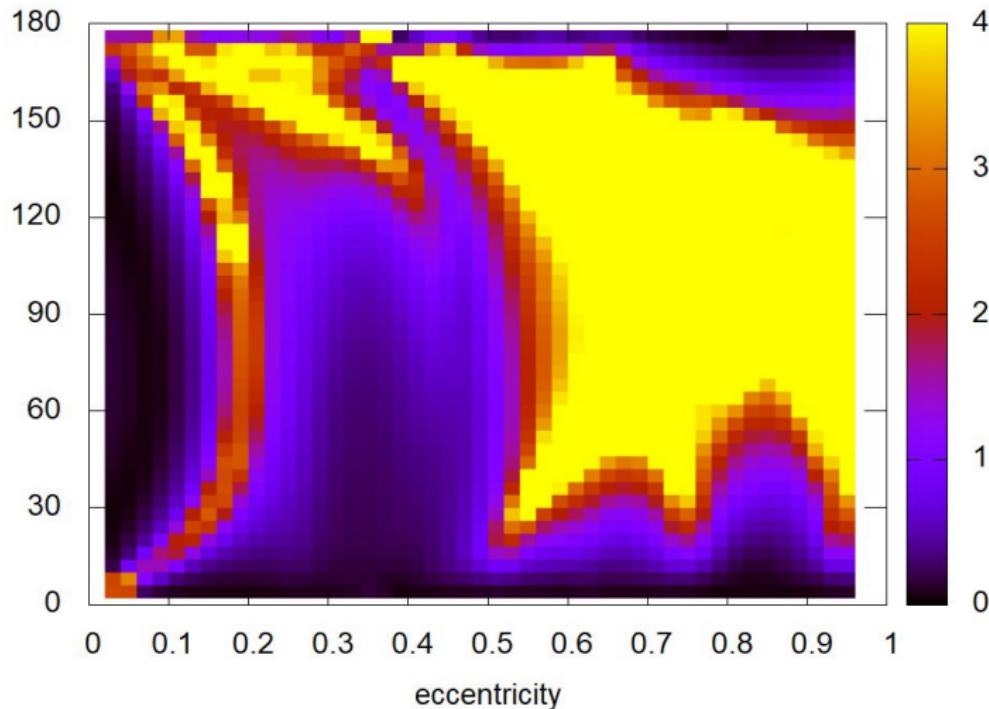
3:5 Neptune

3:5 fragility



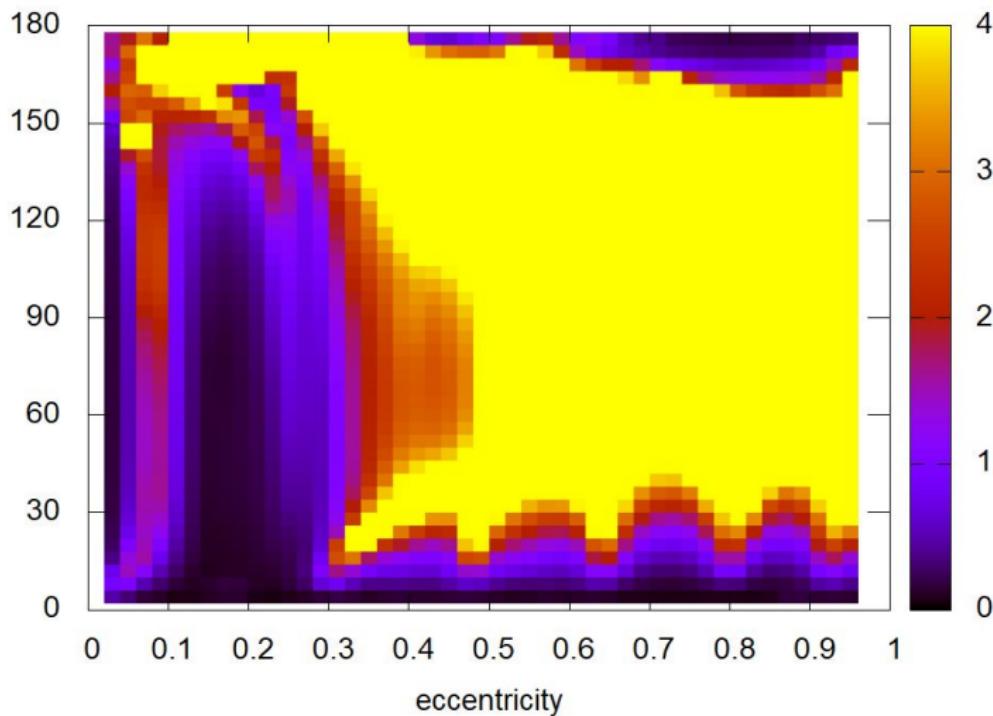
4:7 Neptune

4:7 fragility



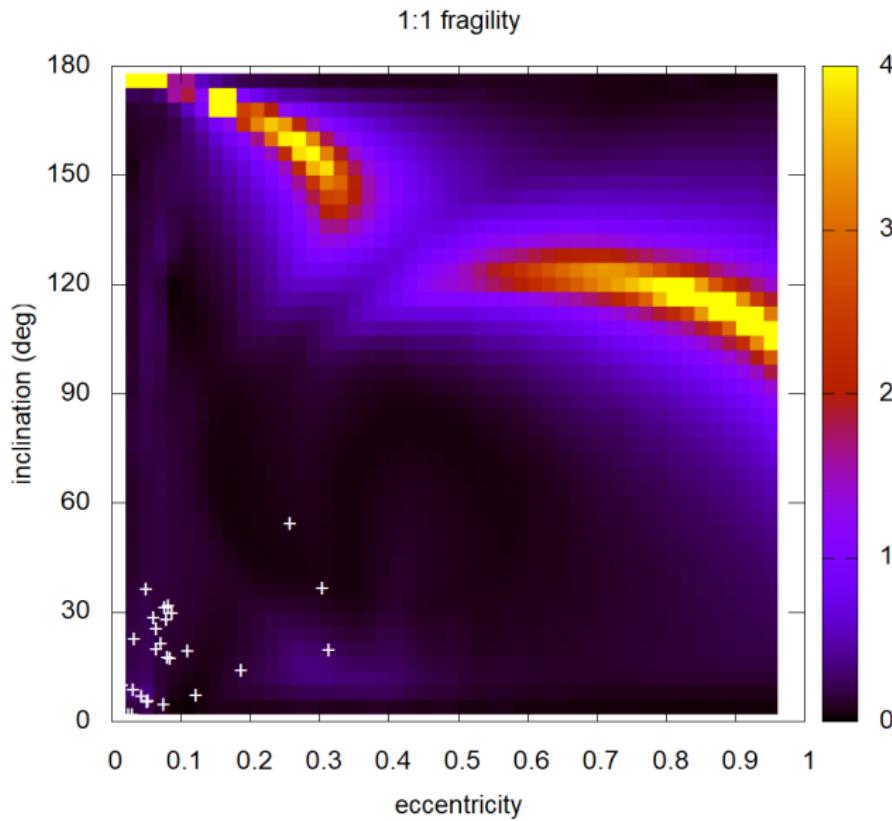
7:9 Neptune

7:9 fragility

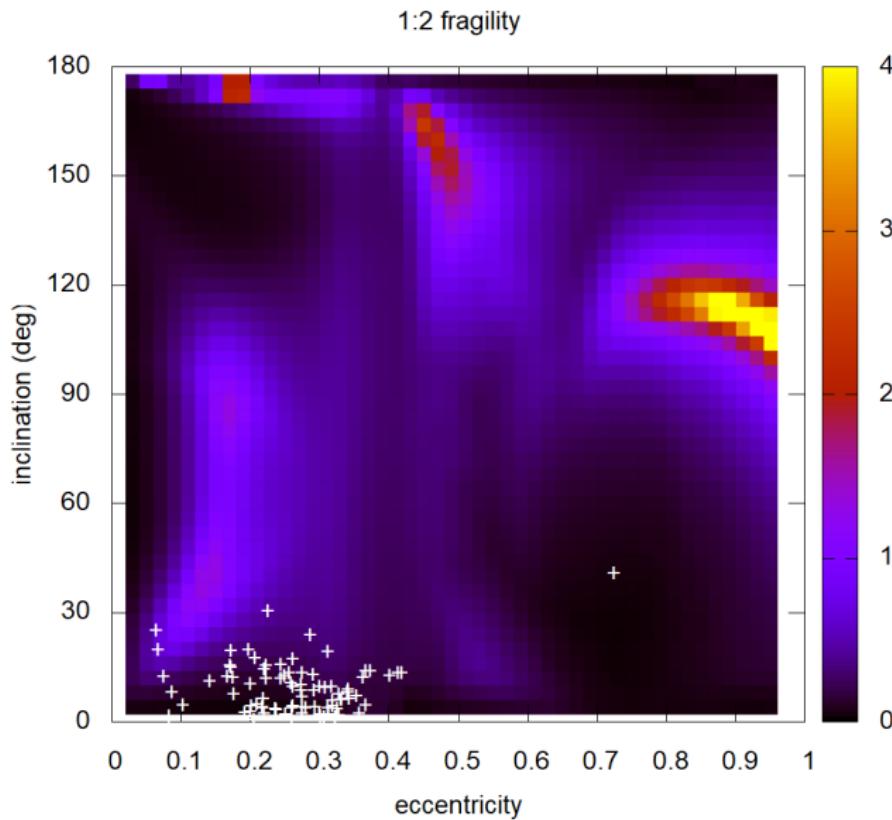


Populations

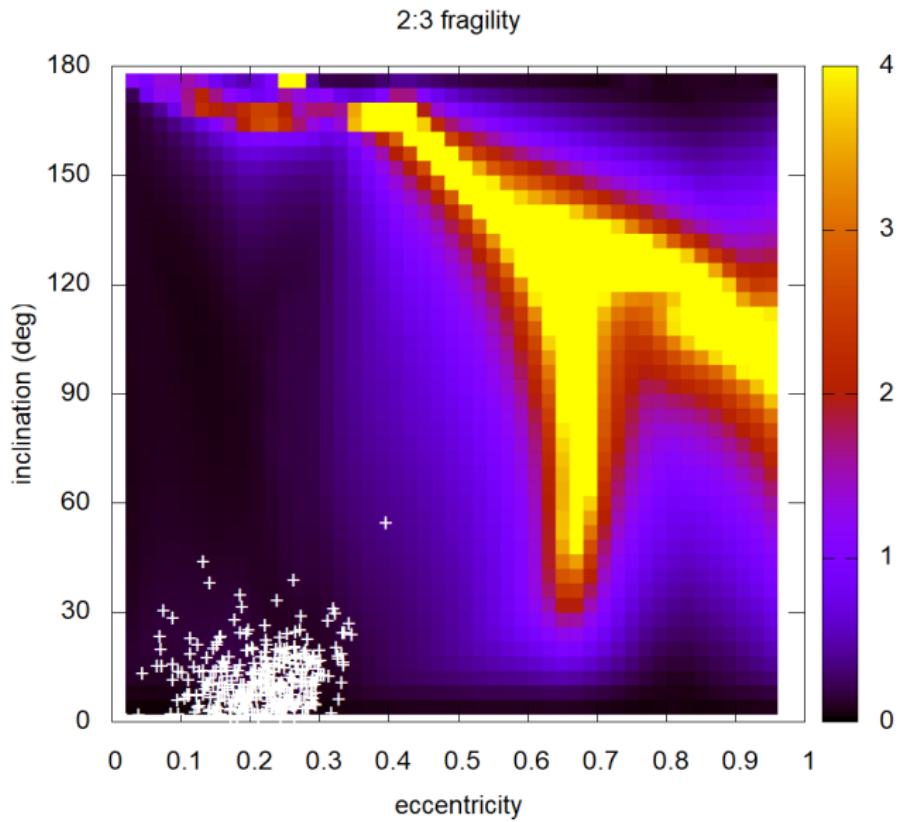
1:1 Neptune fragility + population



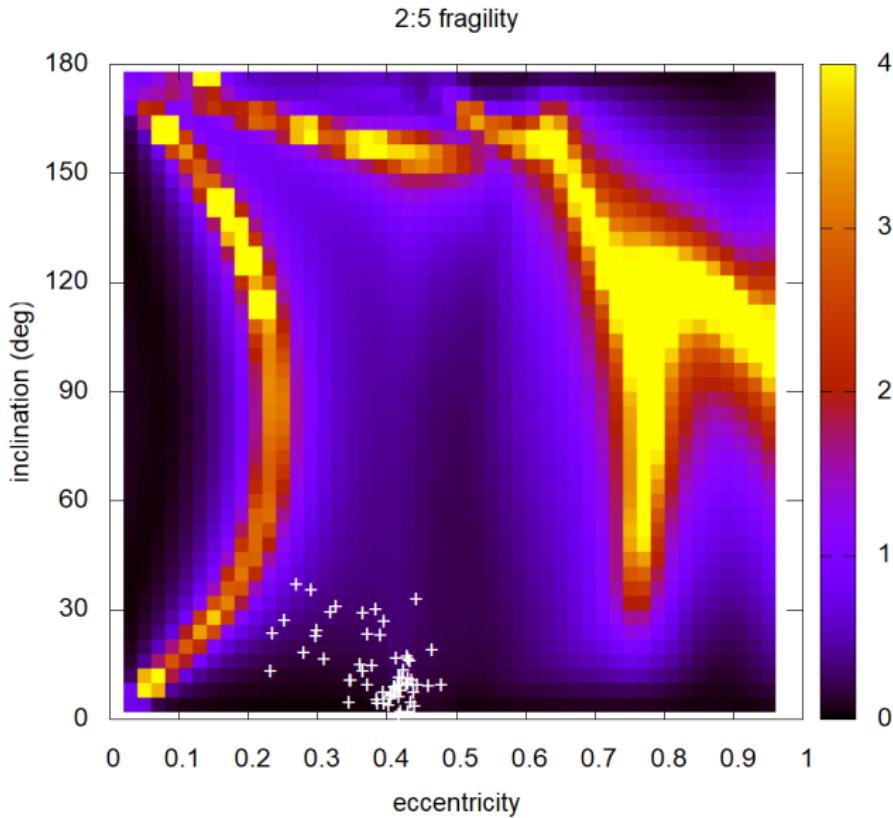
1:2 Neptune fragility + population



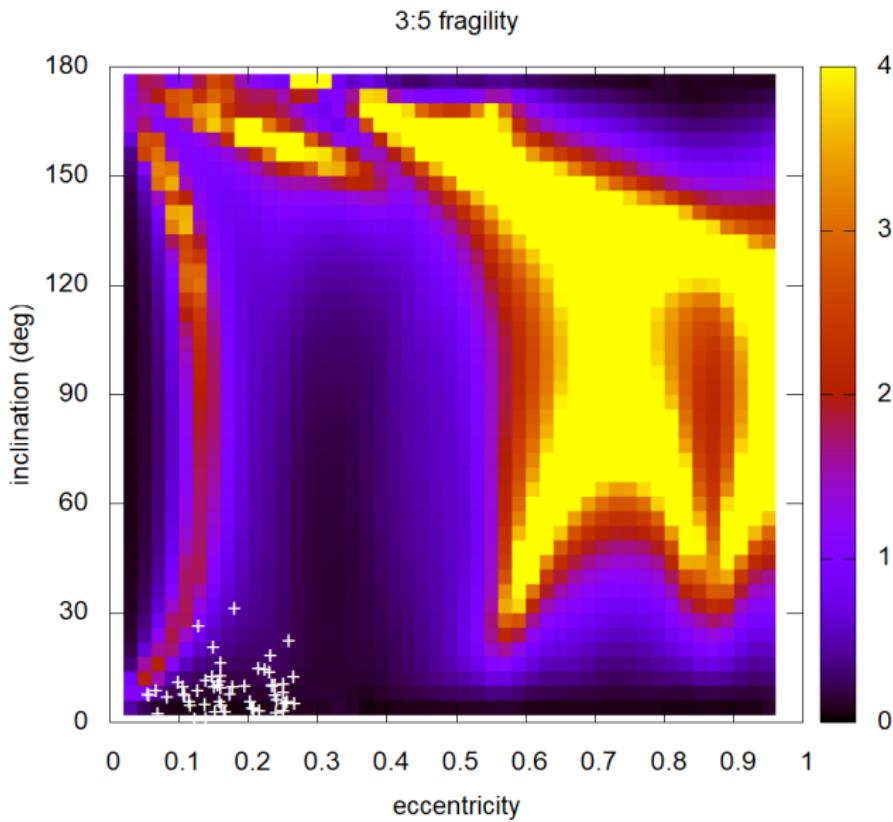
2:3 Neptune fragility + population



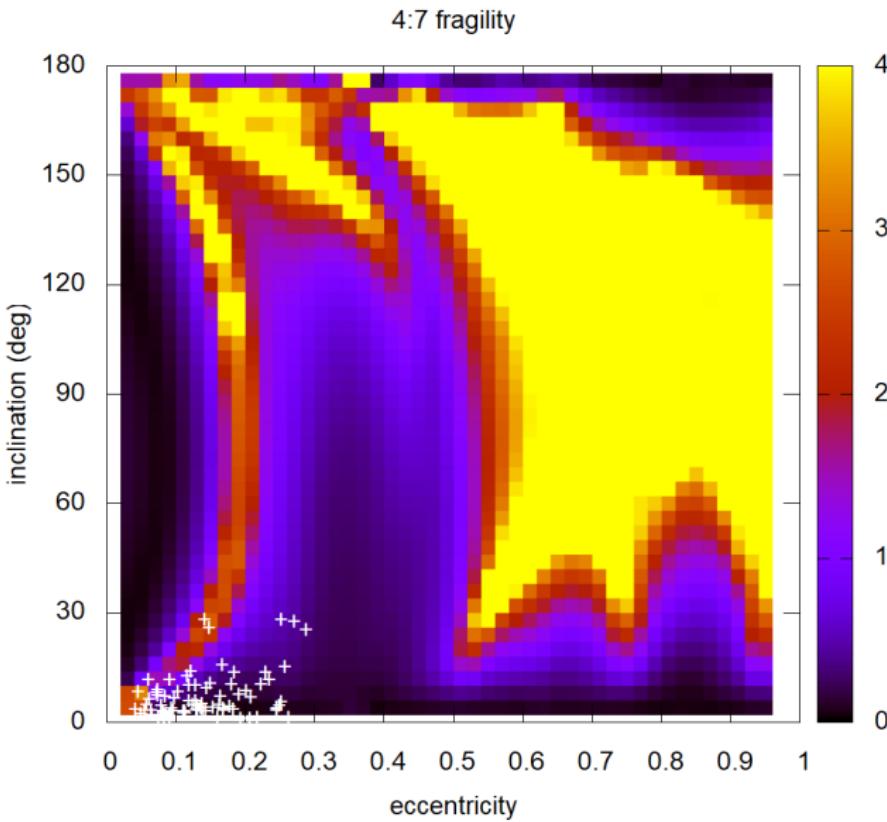
2:5 Neptune fragility + population



3:5 Neptune fragility + population



4:7 Neptune fragility + population



SUMMARY

- planar case: $e \rightarrow$ strength, width,...
- NON-planar case: $(e, i, \omega) \rightarrow$ strength, width,...
- $\omega(t) \rightarrow$ fragility $f(e, i)$
- $k_p:k, \quad \text{high } k_p \rightarrow \text{high fragility}$
- **primordial populations** \rightarrow low fragility regions
- population 4:7 with Neptune **shifted** to high fragility region
- Gallardo (2020, CMDA): semi-analytical model
- www.fisica.edu.uy/~gallardo/atlas

é isso!

