Planetary resonances in the Outer Solar System

Tabaré Gallardo

Facultad de Ciencias, UdelaR, Uruguay



XXI CBDO, São José dos Campos, December 2022



XVII Latin American Regional IAU Meeting



27 de noviembre al 1 de diciembre de 2023

Centro de Conferencias Intendencia Municipal de Montevideo





In planetary resonances we do not distinguish between interior and exterior resonances. (Hildas are interior and Plutinos are exterior)

э

イロト 不得 トイヨト イヨト

Quasi - resonant Outer Solar System

critical angles $\sigma = k_1 \lambda_1 - k_2 \lambda_2 + \dots$



• all $d\sigma/dt < 0 \Rightarrow$

A B > A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A

• exterior planet is always to the left of the resonance

< ⊒

FACULTAD DE

- since XVII: intriguing systematic variations in a_J, a_S
- Laplace (~ 1784): grande inégalité $2\lambda_J 5\lambda_S$
- Varadi et al. (1999): *Jupiter, Saturn, and the Edge of Chaos* ⇒ instability generated by 2:5.
- Michtchenko and Ferraz-Mello (2001, Icarus, AJ): *Resonant Structure of the Outer Solar System in the Neighborhood of the Planets*

↓ mutual planetary resonances between JSUN are chaotic



Michtchenko and Ferraz-Mello 2001



- frequency analysis
- 1.5 Myr
- space (a, e)



(日)

< ⊒



A simple experiment shifting Jupiter to the resonance

eccentricities



イロト イポト イヨト イヨ

Characterization of the resonances





< ∃→

A D > A A > A

A&A 646, A148 (2021)

Semianalytical model for planetary resonances Application to planets around single and binary stars

🔟 Tabaré Gallardo¹, 🝺 Cristián Beaugé² and Cristian A. Giuppone²

It works very well for arbitrary (e, i) (at least for now...)



イロト イポト イヨト イヨト

Libration amplitudes and period



Resonance
$$k_1\lambda_1 - k_2\lambda_2$$
 $(k_1 \le k_2)$

- SAME libration period
- different libration amplitude

$$\Delta a_{less-massive} > \Delta a_{more-massive}$$

$$\frac{\Delta a_1}{\Delta a_2} \sim \frac{m_2}{m_1} (\frac{k_1}{k_2})^{4/3}$$

• • • • • • • • • • •

< ⊒

Model for Jupiter – Saturn 2:5 resonance

Hamiltonian 2:5 resonance





э

イロト イロト イヨト イヨト

Model: resonance widths near Saturn



・ロト ・ 理 ト ・ モ ト ・ モ ト



э

Dynamical map near Saturn: $log(\Delta a)$



Tabare Gallardo Planetary resonances in the Outer Solar System

FACULTAD DE CIENCIAS

Numerical integration of an example of 2J:5S





Planetary resonances in the Outer Solar System

FACULTAD DE

Model: resonance widths near Uranus



・ロト ・ 理 ト ・ モ ト ・ モ ト



э

Dynamical map near Uranus



Tabare Gallardo Planetary resonances in the Outer Solar System

FACULTAD DE CIENCIAS In the planetary case

- dynamical maps are wider than theoretical widths
- because in each computation the other planet is shifted from the nominal position
- good for studying structure



イロト 不得 トイヨト イヨト

libration period: 13200 yr



Hamiltonian 1:3 resonance

FACULTAD DE

Model: resonance widths near Neptune





FACULTAD DE CIENCIAS

э

Dynamical map near Neptune





ヘロト ヘロト ヘビト ヘビト

Model for Uranus – Neptune 1:2 resonance

libration period: 27700 yr



Hamiltonian 1:2 resonance

Using current orbital elements (just shifting *a* to the EXACT resonance), the libration amplitudes and periods would be:

- res 2:5 \rightarrow Jupiter (0.004 au) Saturn (0.04 au), 3260 yr
- res 1:3 \rightarrow Saturn (0.004 au) Uranus (0.1 au), 13200 yr
- res 1:2 \rightarrow Uranus (0.08 au) Neptune (0.2 au), 27700 yr

Can we set up a resonant chain in the OSS?



イロト イポト イヨト イヨ

Artificial EXACT resonant chain





Uranus



Tabare Gallardo Planetary resonances in the Outer Solar System

- the model GBG21 predicts quite well the properties of mutual two-body resonances
- in the present OSS the quasi resonance Jupiter Saturn is by far the most important

- an OSS in resonant chain would generate strong chaos in Uranus due to superposition of (strong) resonances with Saturn and Neptune.
- can a resonant chain exist involving strong mutual two-body resonances (like Uranus)?
- or maybe weak (low *e*) two-body resonances is a requirement?



イロト イポト イヨト イヨト

Obrigado!



https://sites.google.com/view/mmresonances/



э

ヘロト ヘロト ヘヨト ヘヨト