Resonancias Planetarias

Tabaré Gallardo

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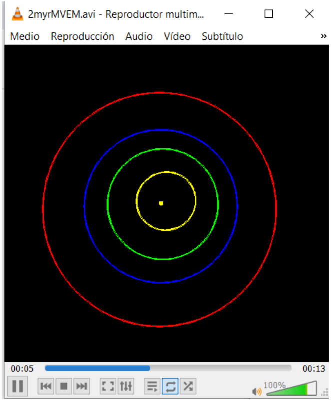
Seminarios de Fisica FIng-FCien, Abril 2021

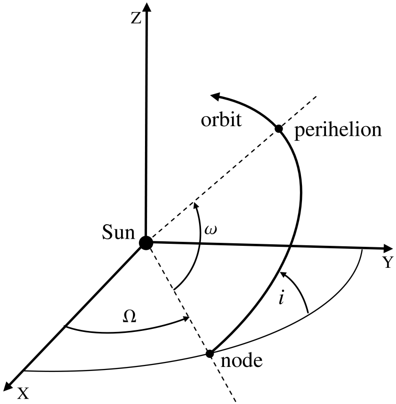
Inner Solar System



2 Myr

Orbital Resonances

Commensurability between **frequencies** associated with orbital motion: nodes (Ω), pericenters (ϖ) and mean motions (n)

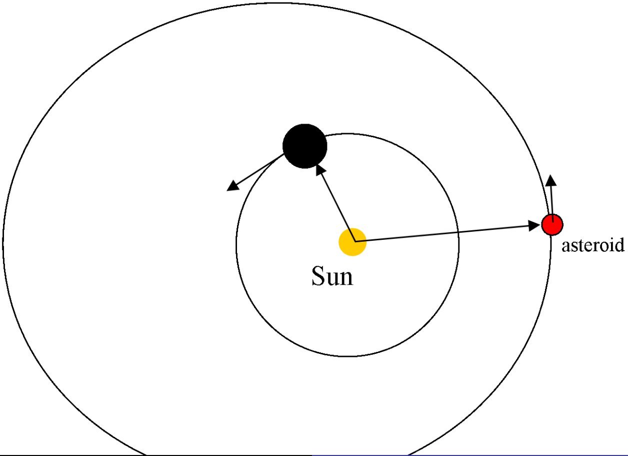


- secular resonances (10^6 yr)
- mean-motion res. (10^2 yr)

2-body resonance k_p/k

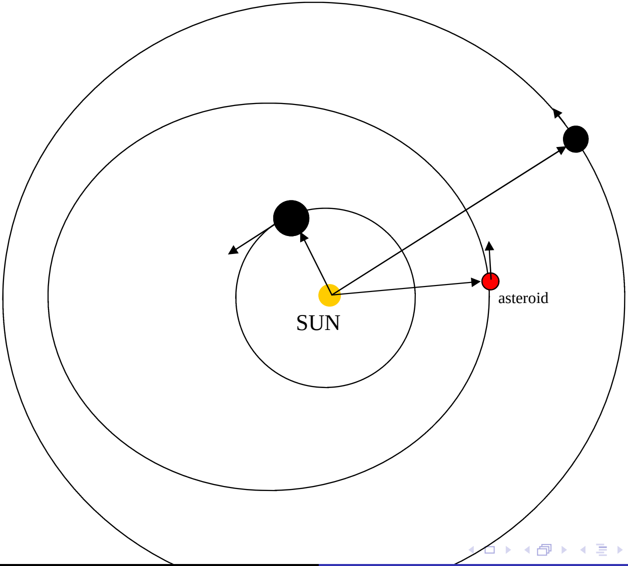
$$kn - k_p n_p \simeq 0$$

example: $3n - 2n_p \simeq 0$
resonance 2/3

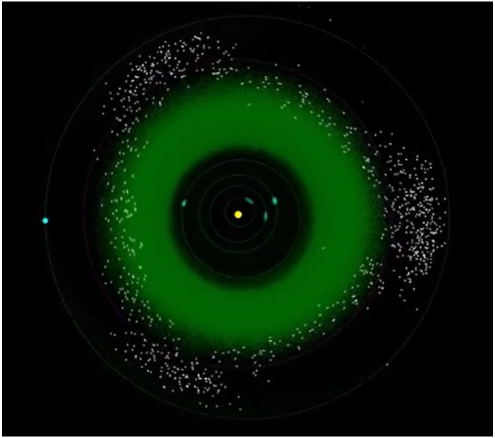


3-body resonances (3BRs)

$$k_0 n_0 + k_1 n_1 + k_2 n_2 \simeq 0$$

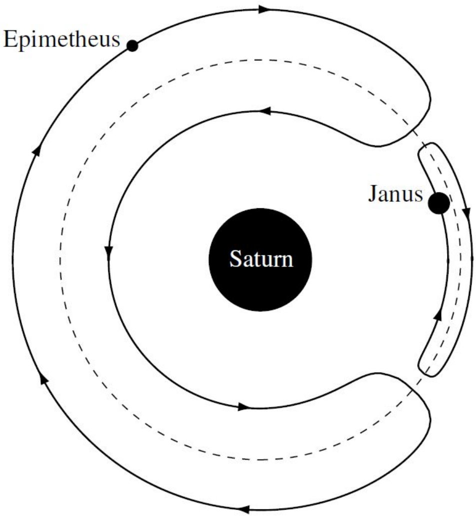


Hildas and Trojans



Hildas (3/2) and Trojans (1/1)

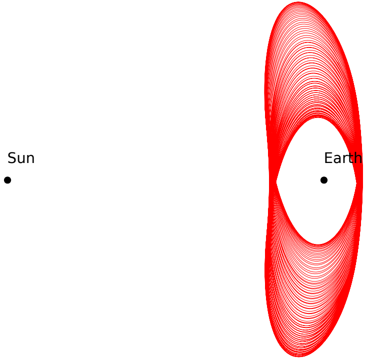
Janus - Epimetheus 1/1 (*planetary* case)



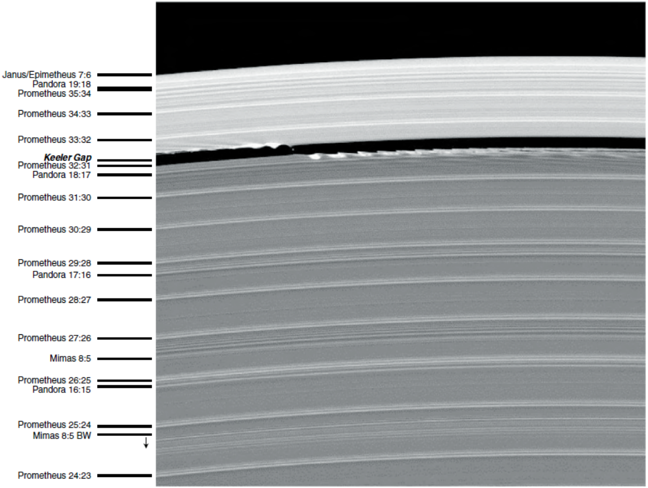
Murray and Dermott, 1999.

2004 GU9: Earth quasi satellite, resonance 1/1

- NOT due to the Earth's attraction (like a satellite)
- but to Earth's mean perturbation on the asteroid's heliocentric orbit



Saturn rings



Lissauer and de Pater, Fundamental Planetary Science

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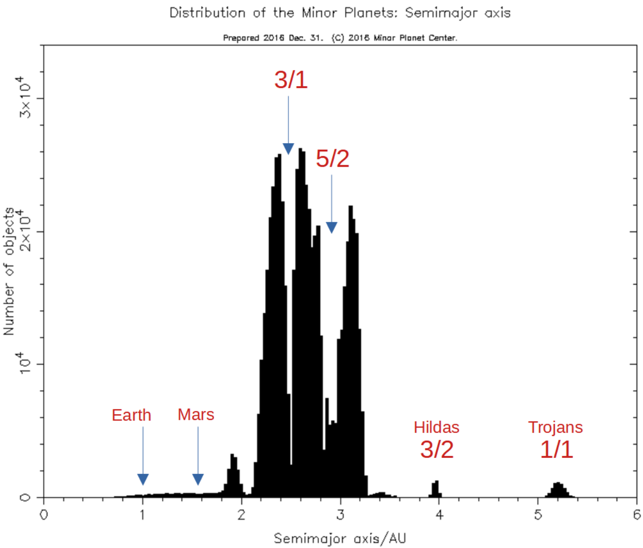
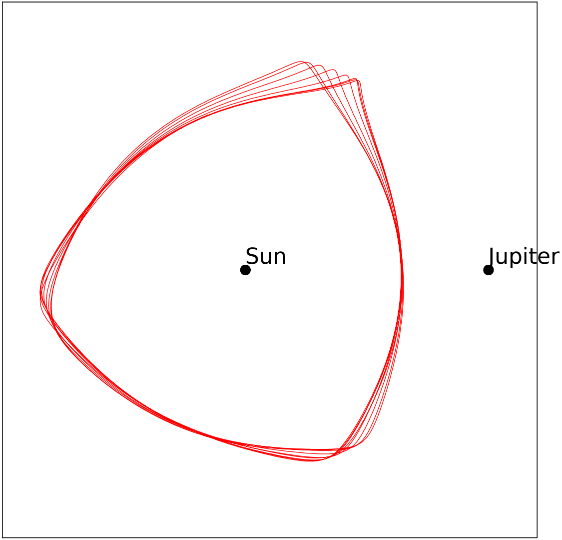
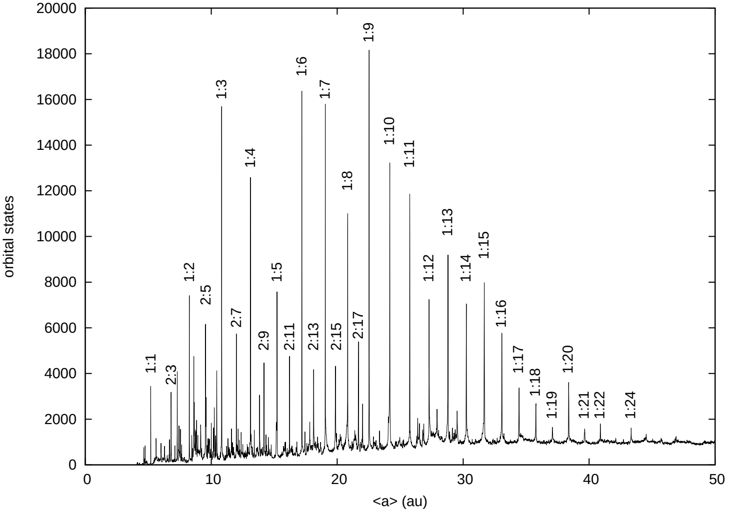
1866: Kirkwood gaps



Hildas 3/2



HISTOGRAM of orbital states (from numerical integrations)

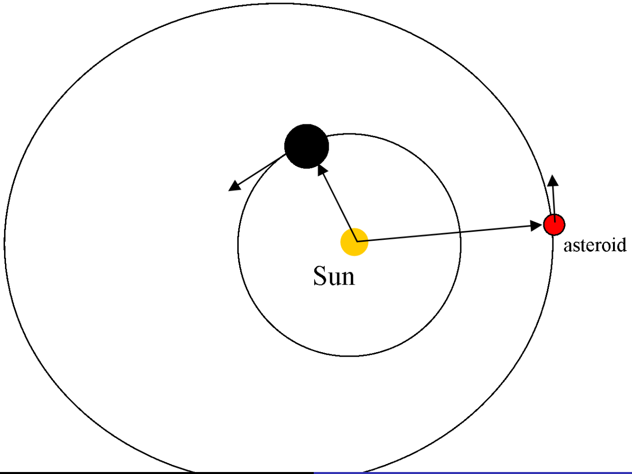


Fernandez et al., 2016

Orbital resonance k_p/k

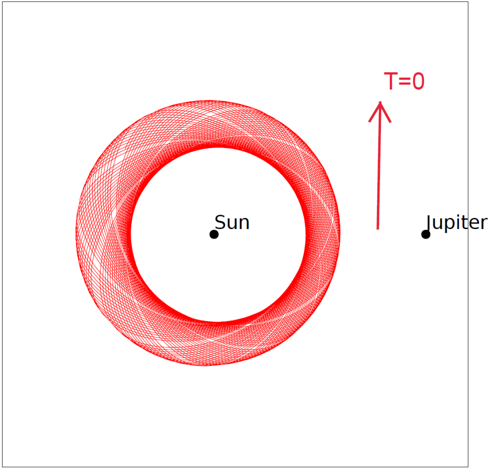
$$kn - k_p n_p \simeq 0 \qquad \Rightarrow \qquad \sigma = k\lambda - k_p\lambda_p + cte$$

$$a \simeq \left(\frac{k}{k_p}\right)^{2/3} a_p \text{ (Kepler)}$$



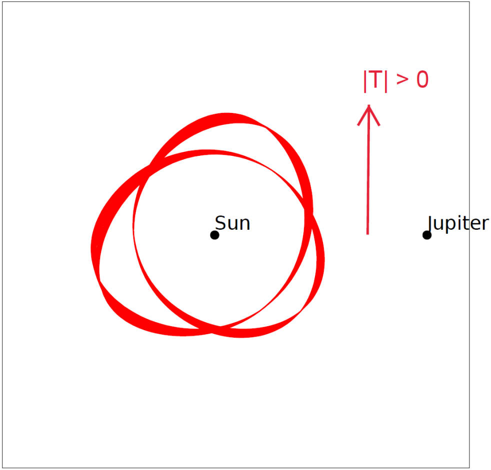
Non resonant asteroid: relative positions

Mean perturbation is radial: Sun-Jupiter

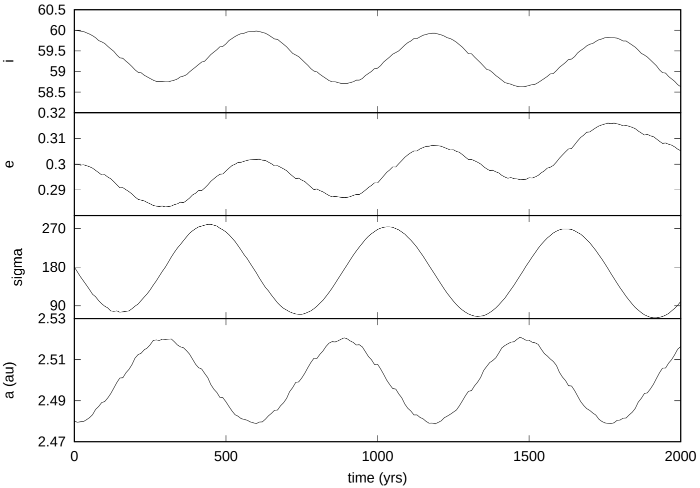


Resonant asteroid

Mean perturbation has a **transverse** component.

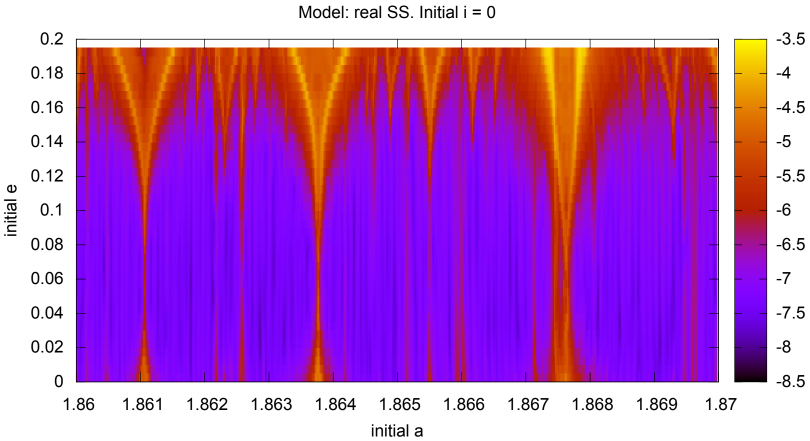


Librations = vibration

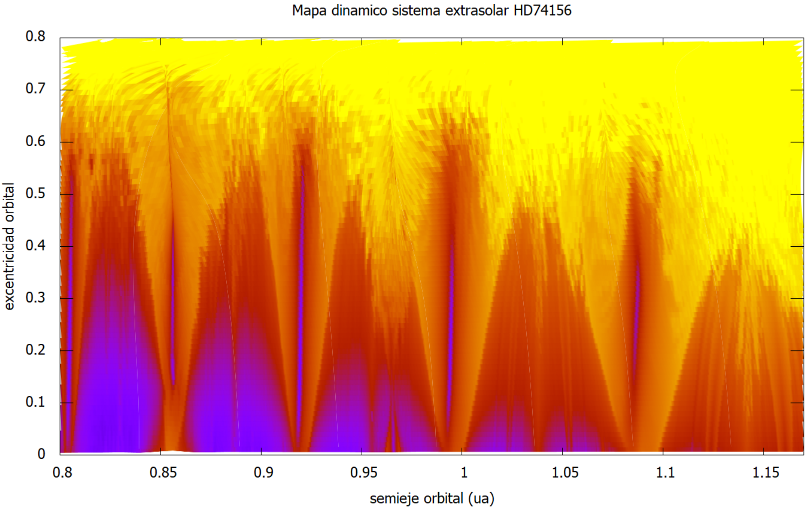


Gallardo 2019

Resonance forest in the Solar system



Extrasolar system HD 74156



Resonant dynamics

- **it is not** an instantaneous effect
- **cumulative effect** of planetary perturbation
- makes the orbit to vibrate: **librations**
- libration provides **stability** against perturbations
- **unusual** long term evolution (large orbital changes?)

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1846: discovery of Neptune

quasi resonance Uranus - Neptune:

$$n_{Uranus} \sim 2n_{Neptune}$$

quasi resonance Saturn - Uranus:

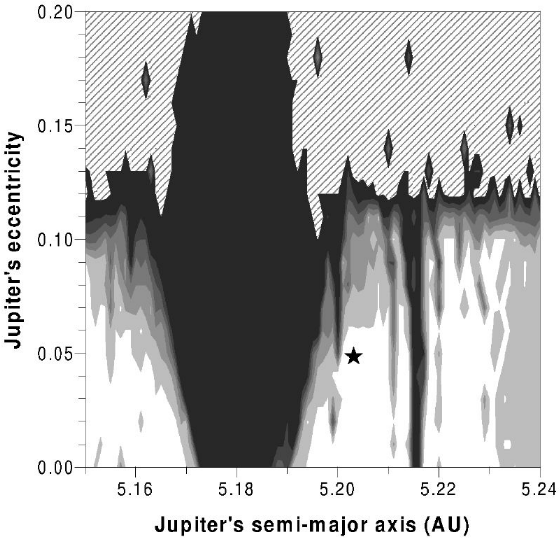
$$n_{Saturn} \sim 3n_{Uranus}$$

quasi resonance: Jupiter - Saturn

$$2n_{Jupiter} \sim 5n_{Saturn}$$

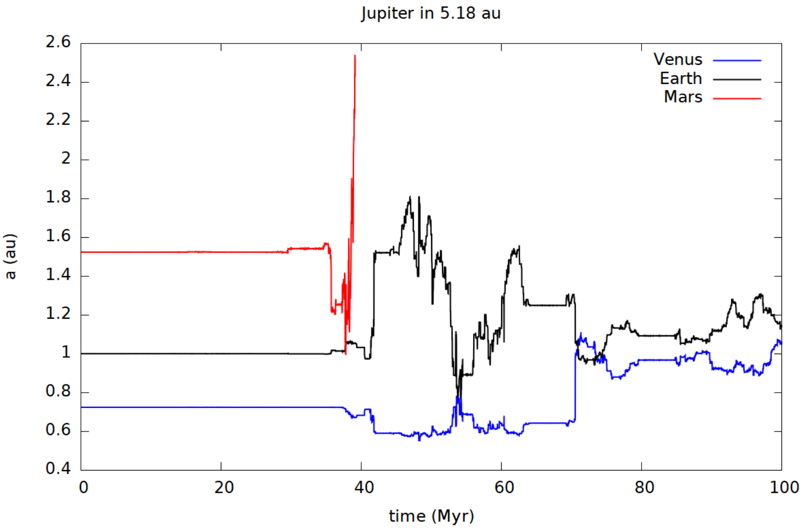
Why the planets are so close to resonances?
$\rightarrow$ **planetary migration**

2:5 resonance near Jupiter



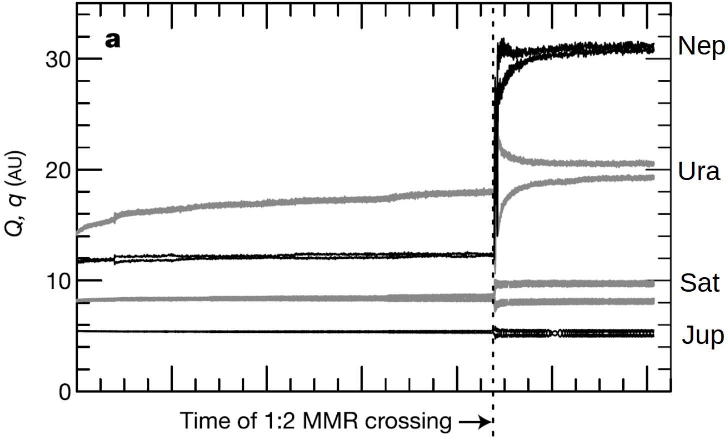
Michtchenko and Ferraz-Mello, 2001.

Experiment: Jupiter shifted 0.02 au (res 2:5)



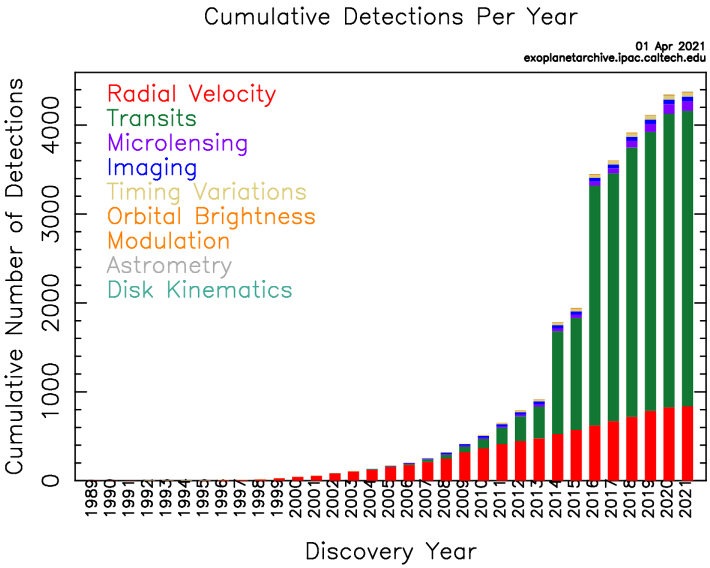
don’t do this at home ...

Fernández-Ip migration:



Gomes et al. 2005

Exoplanets


Cumulative Detections Per Year
01 Apr 2021
exoplanetarchive.ipac.caltech.edu
Radial Velocity
Transits
Microlensing
Imaging
Timing Variations
Orbital Brightness
Modulation
Astrometry
Disk Kinematics
Cumulative Number of Detections
0
1000
2000
3000
4000
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
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2008
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2014
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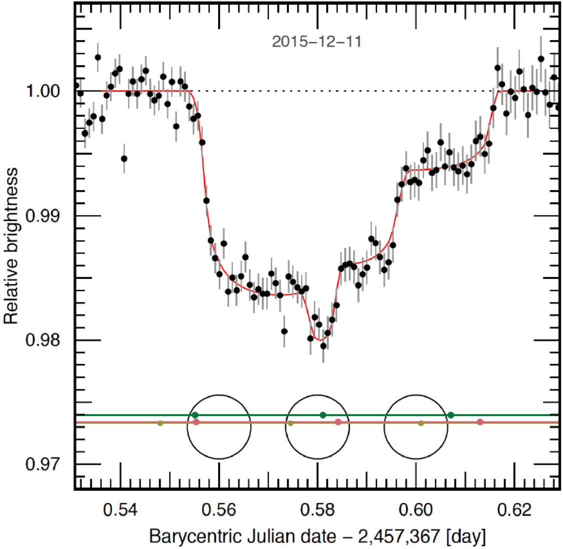
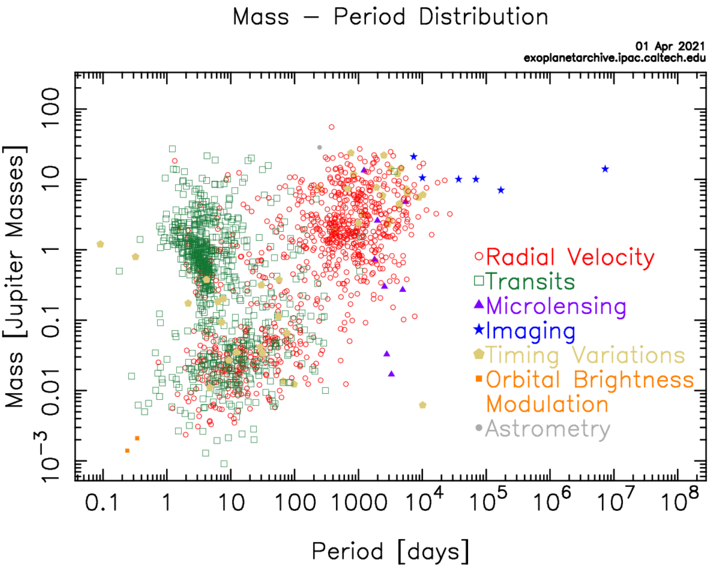
2016
2017
2018
2019
2020
2021
Discovery Year



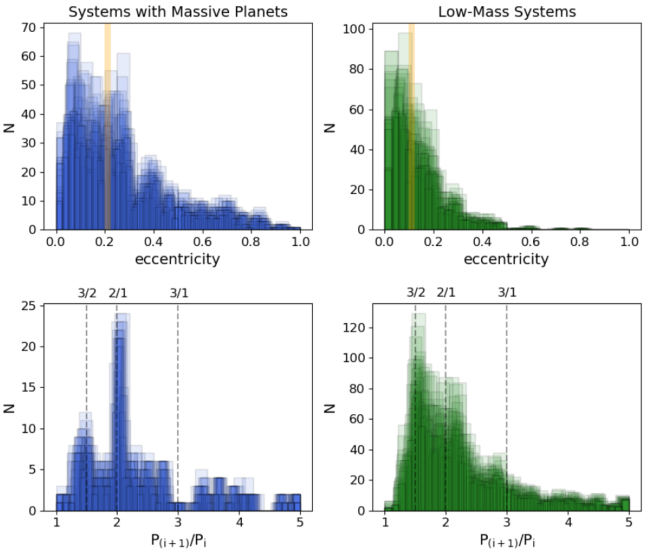
Guillon et al. 2017

Exoplanets



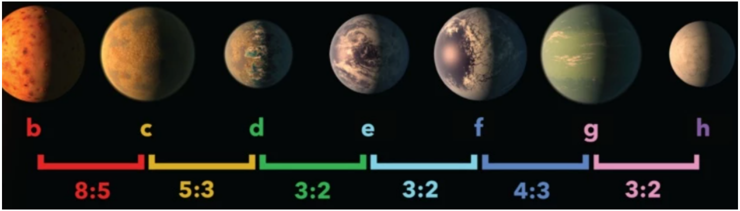
Mass – Period Distribution
01 Apr 2021
exoplanetarchive.ipac.caltech.edu
Mass [Jupiter Masses]
100
10
1
0.1
0.01
10^{-3}
0.1
1
10
100
1000
10^4
10^5
10^6
10^7
10^8
Period [days]
Radial Velocity
Transits
Microlensing
Imaging
Timing Variations
Orbital Brightness Modulation
Astrometry

Extrasolar systems: resonances



Beaugé and Ferraz-Mello, 2021

TRAPPIST-1 system



it is not an effect of chance

1784: Laplacian resonance

$$3n_{Europa} - n_{Io} - 2n_{Ganymede} \simeq 0$$

They are also in commensurability by pairs:

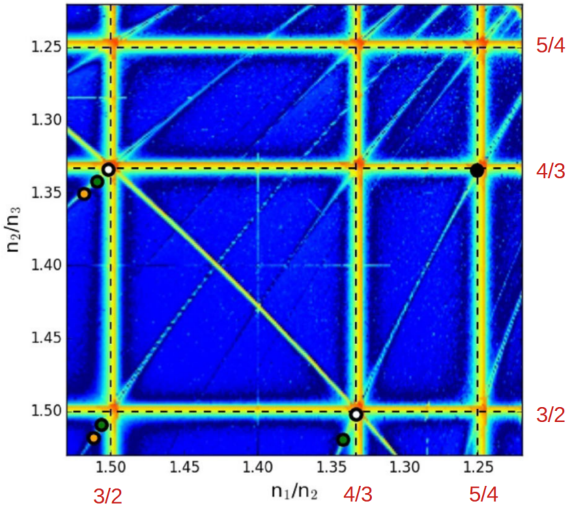
$$2n_{Europa} - n_{Io} \simeq 0$$

$$2n_{Ganymede} - n_{Europa} \simeq 0$$

$$\Downarrow$$

It must be the consequence of some physical mechanism.

Dynamical map



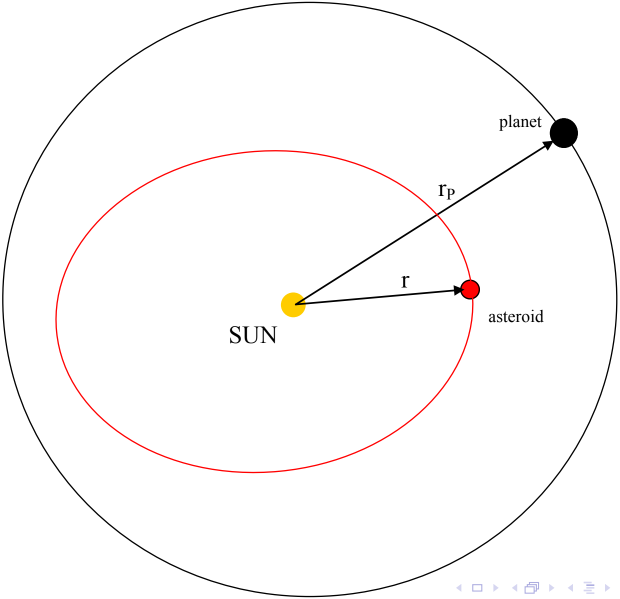
Charalambous et al. 2018

Resonance model

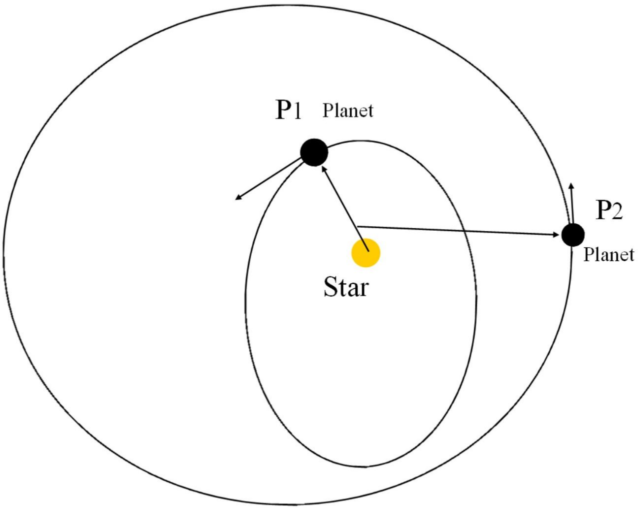
- Restricted case (asteroid): Gallardo (2020)
- Planetary case (massive bodies): Gallardo, Beaugé, Giuppone (2021)

$\rightarrow$ General semi-analytical models

Restricted (asteroidal) resonance



Planetary resonance



Hamiltonian for star + 2 planets

$$H = -k^4 \frac{m_0^3 m_1^3}{2(m_0 + m_1)L_1^2} - k^4 \frac{(m_0 + m_1)^3 m_2^3}{2(m_0 + m_1 + m_2)L_2^2} - R,$$

where

$$R = k^2 m_0 m_2 \left(\frac{1}{\Delta_{02}} - \frac{1}{r_2} \right) + k^2 m_1 m_2 \left(\frac{1}{\Delta_{12}} - \frac{1}{r_2} \right),$$

$$H(\lambda_1, \lambda_2, L_1, L_2)$$

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Hamiltonian: star + 2 planets

Canonical transformation:

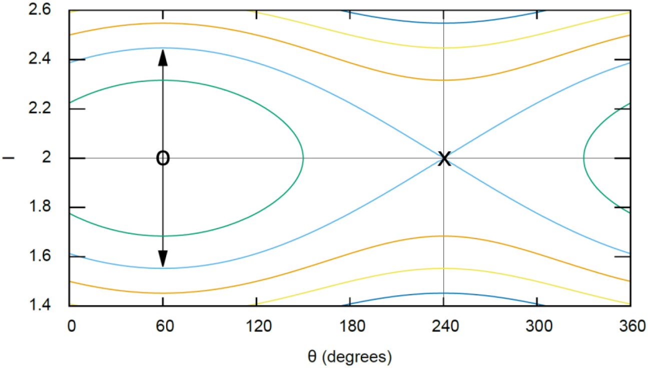
$$H(\lambda_1, \lambda_2, L_1, L_2) \rightarrow H(\theta, \lambda_2, I_1, I_2)$$

$$\boxed{\begin{array}{ll} \theta = k_1\lambda_1 - k_2\lambda_2, & I_1 = L_1/k_1 \\ \lambda_2, & I_2 = L_2 + k_2 L_1/k_1 \end{array}}$$

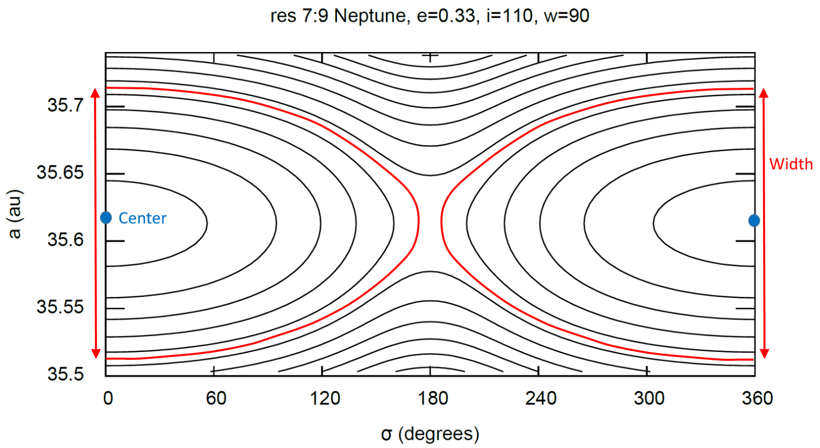
Numerical averaging in fast variable λ_2:

$$H(\theta, \lambda_2, I_1, I_2) \rightarrow H(\theta, -, I_1, I_2)$$

Level curves of $H(\theta, I_1)$

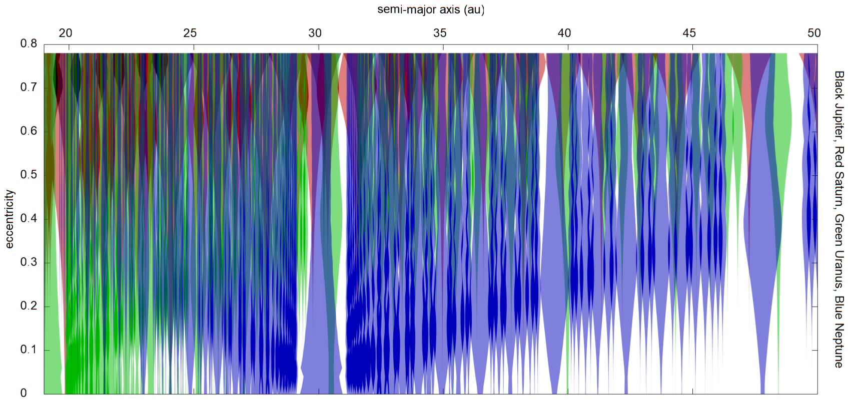


Level curves of $H(\sigma, a_1)$



Width, centers, periods

Atlas of resonances: stable widths



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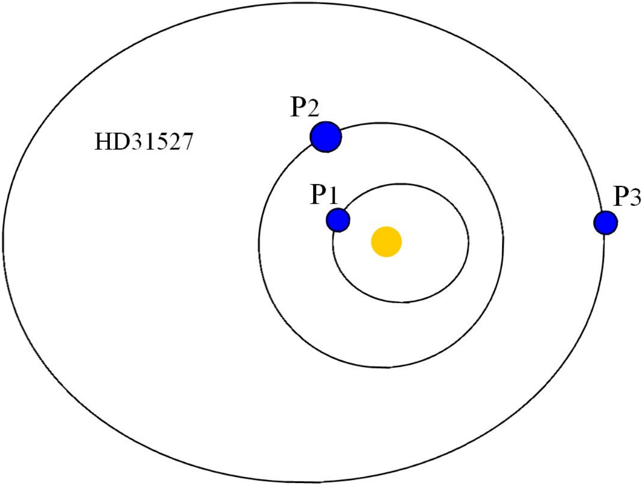
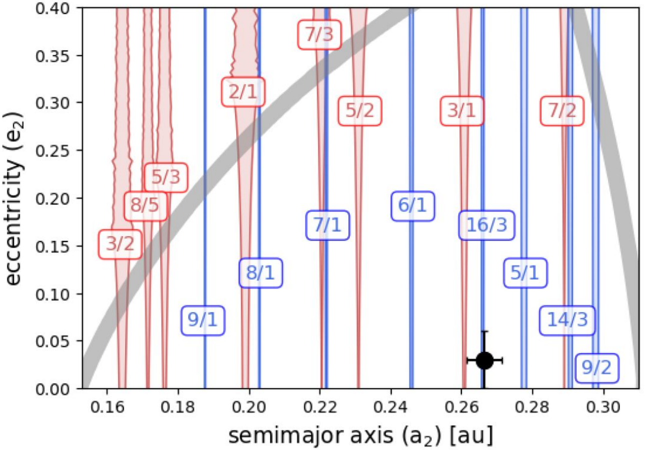
Extrasolar HD31527

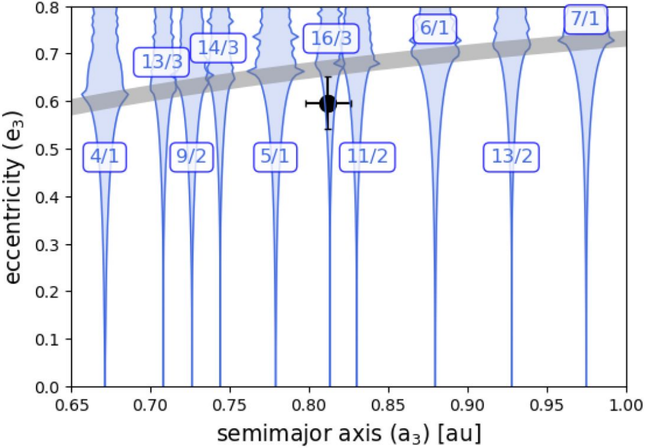


HD31527: atlas for middle planet P2



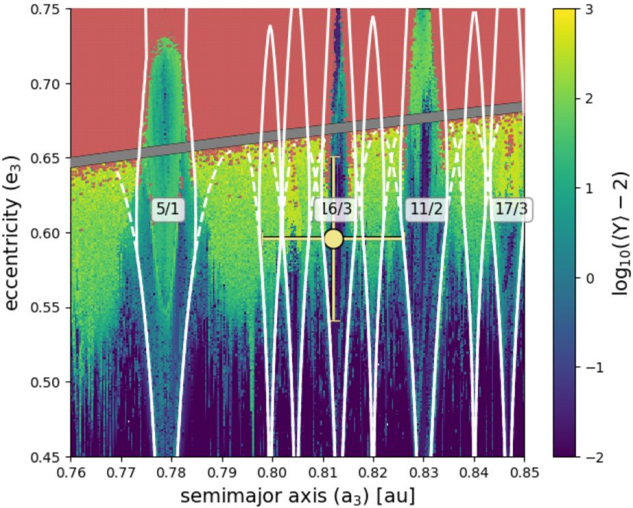
Gallardo, Beaugé, Giuppone, 2021

HD31527: atlas for exterior planet P3



Gallardo, Beaugé, Giuppone, 2021

Dynamical map: exterior planet P3



Gallardo, Beaugé, Giuppone, 2021

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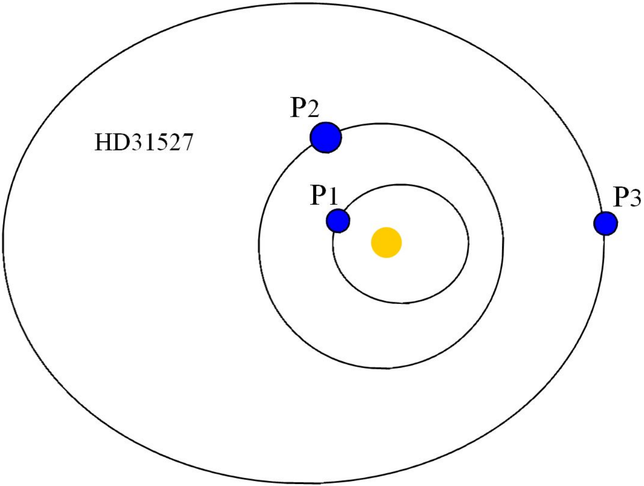
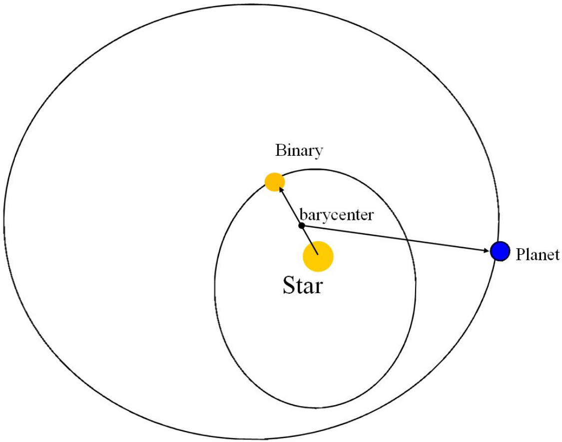
Planets P2 and P3 in resonance 16/3

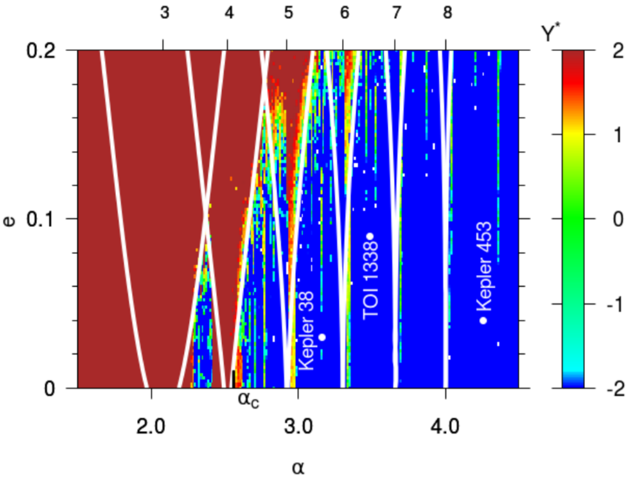


Binary + planet

Chaos close to the binary

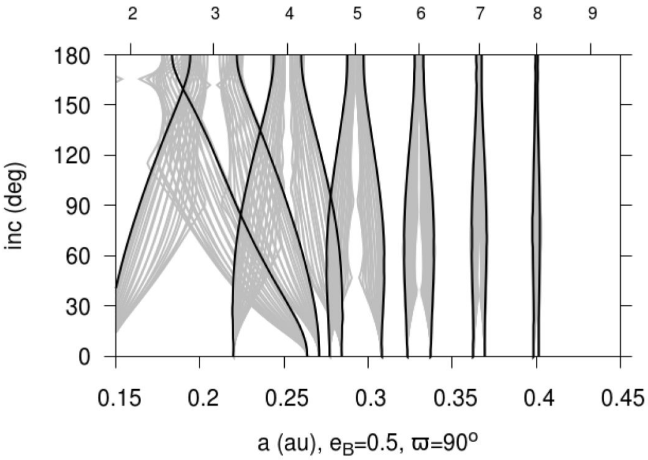


Binary: dynamical map + model



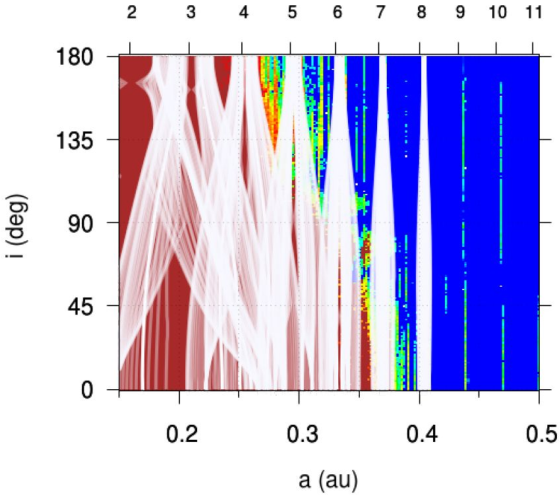
Gallardo, Beaugé, Giuppone, 2021

Inclined binary: atlas of resonances



Gallardo, Beaugé, Giuppone, 2021

Chaos: superposition of resonances



Gallardo, Beaugé, Giuppone, 2021

Summary

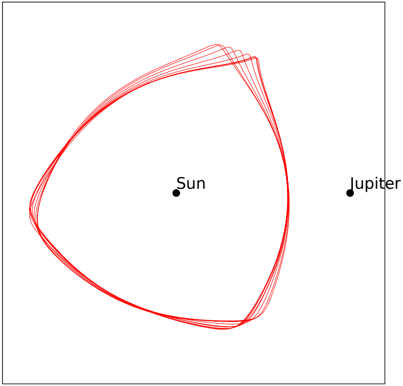
- 2-body and 3-body resonances
- restricted and planetary case
- planet orbiting a binary system
- chaos in binaries: resonance overlap
- resonant chains: just the addition of 2BRs?

Some references

- *Semianalytical model for planetary resonances*, Gallardo, Beaugé, Giuppone 2021
- *Resonances in the asteroid and TNO belts: a brief review*, Gallardo 2018.
- Atlas of MMRs site: www.fisica.edu.uy/∼gallardo/atlas

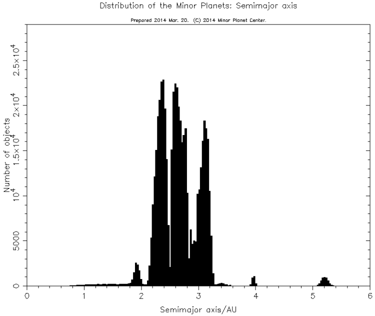
¡Gracias!

1875: resonant asteroids (153) Hilda 3:2

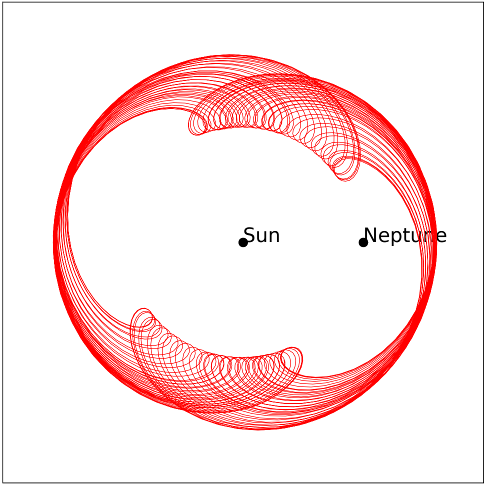


$$2n_{Hilda} = 3n_{Jup}$$

$$a_{Hilda} = (\frac{2}{3})^{2/3} a_{Jup} = 3{,}97\text{ua}$$



(134340) Pluto in exterior resonance 2:3



$$\mathcal{R}(\theta) = \frac{1}{2\pi k_1} \int_0^{2\pi k_1} R(\lambda_2, \lambda_1(\lambda_2, \theta)) d\lambda_2$$

... after some algebra...

$$\Delta a_1 = \sqrt{a_1}\frac{\sqrt{(m_0+m_1)}}{m_0 m_1} k_1 2\sqrt{2}\sqrt{\frac{-\Delta\mathcal{R}}{k^2 H_{II}}}$$

$$\Delta a_2 = \sqrt{a_2}\frac{\sqrt{(m_0+m_1+m_2)}}{(m_0+m_1)m_2} k_2 2\sqrt{2}\sqrt{\frac{-\Delta\mathcal{R}}{k^2 H_{II}}}$$

Gallardo, Beaugé, Giuppone, 2021