

Angular velocity and centripetal acceleration relationship

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During the last few years, the growing boom of smartphones has given rise to a considerable number of applications exploiting the functionality of the sensors incorporated in these devices. A sector that has unexpectedly taken advantage of the power of these tools is physics teaching, as reflected in several recent papers.¹⁻¹⁰ In effect, the use of *smartphones* has been proposed in several physics experiments spanning mechanics, electromagnetism, optics, oscillations, and waves, among other subjects. Although mechanical experiments have received considerable attention, most of them are based on the use of the accelerometer.¹⁻⁸ An aspect that has received less attention is the use of *rotation sensors* or *gyroscopes*.⁹⁻¹⁰ An additional advance in the use of these devices is given by the possibility of obtaining data using the accelerometer and the gyroscope simultaneously. The aim of this paper is to consider the relation between the centripetal acceleration and the angular velocity. Instead of using a formal laboratory setup, in this experiment a smartphone is attached to the floor of a merry-go-round, found in many playgrounds. Several experiments were performed with the roundabout rotating in both directions and with the smartphone at different distances from the center. The coherence of the measurements is shown.

Experimental setup

The experimental setup, shown in Fig. 1, consists of a smartphone placed in a box made with polyurethane foam and fixed to the floor of the merry-go-round using two strong neodymium magnets. An LG Optimus P990 2X (Sensors: three-axis accelerometer KXTF9 Kionix, accuracy 0.001 m/s^2 , three-axis gyroscope MPU3050 Invensense, accuracy 0.0001 rad/s) similar to the one used in Ref. 10 was used. It was oriented with the display pointing upward and the short end parallel to the radial direction, as shown in the figure. Measurements that are relevant in this experiment are those reported by the rotation sensor according to the z -axis and the radial acceleration corresponding to the x -axis.

Rotatory motion

The merry-go-round was propelled in a counterclockwise direction and allowed to come to a stop by the effect of friction,

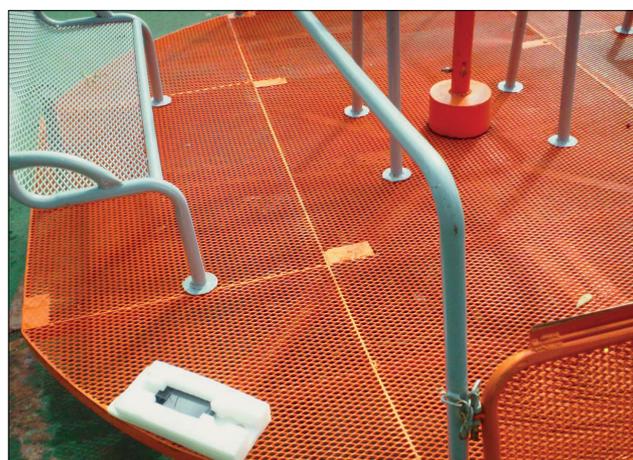


Fig. 1. Smartphone mounted on a merry-go-round.

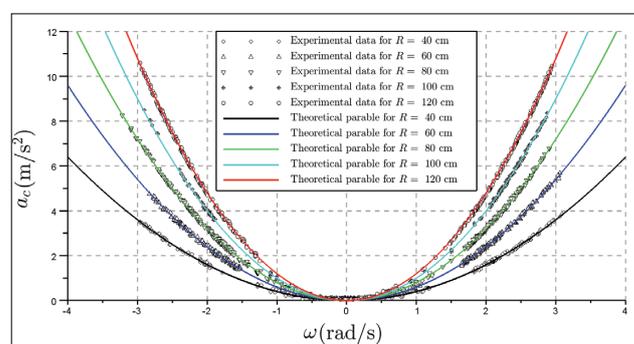


Fig. 2. Centripetal acceleration as a function of the angular velocity for different distances indicated in the legend box.

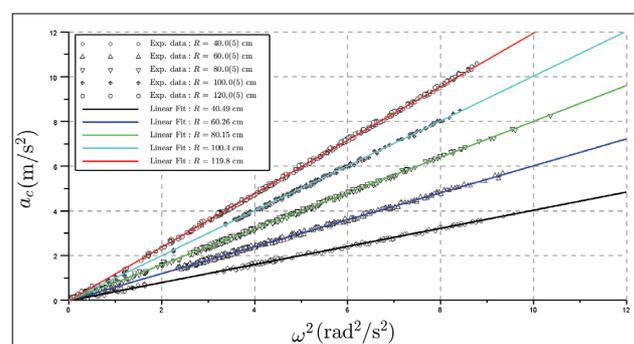


Fig. 3. Centripetal acceleration as a function of the angular velocity squared for different distances.

and then propelled again but in the clockwise direction. The experiment was repeated for different distances of the smartphone to the rotation center: 40, 60, 80, 100, and 120 cm. The angular velocity is measured with the z -component of the gyroscope, while the centripetal acceleration is measured with the x -axis of the accelerometer.

The measurements obtained are summarized in Figs. 2 and 3, where the centripetal acceleration a_c is plotted as a function of the angular velocity ω and the angular velocity squared ω^2 , respectively. The linear and parabolic fits

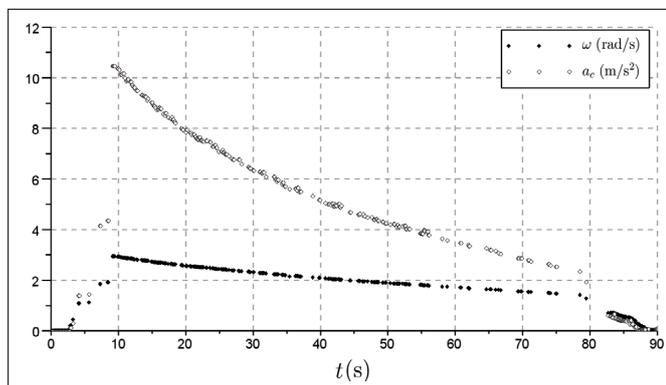


Fig. 4. Angular velocity and centripetal acceleration as a function of time, corresponding to one of the realizations shown in the previous figures. The distance is $R = 120$ cm and the merry-go-round is spinning counterclockwise.

included in these figures reveal that both magnitudes are related by the well-known relationship

$$a_c = \omega^2 R, \quad (1)$$

where R is the distance from the smartphone to the axis of the merry-go-round. The coefficients given by the fit correspond to the distances with very good agreement.

To complete the analysis in Fig. 4, the considered magnitudes are plotted as functions of time for one of the realizations. We observe in this figure the different stages of the motion. During the first seconds, the merry-go-round is pushed fiercely. Next, between approximately 10 and 80 s, it is slowing down gradually. Finally, in the last seconds, the merry-go-round is abruptly stopped. It is worth noting that, due to limitations of the smartphone, the sampling rate is not uniform. In addition, comparing with Fig. 2, we note that the wide gap about $\omega^2 \sim 2 \text{ rad}^2/\text{s}^2$ is a consequence of the violent stopping process. The analysis and comparison of the different figures can be the origin of a stimulating classroom discussion.

Conclusion

A basic kinematic relationship between angular velocity and centripetal acceleration was verified using smartphone sensors. The coherence of the measurements taken with the

different sensors was shown. This experiment illustrates the simplicity of using a smartphone in physics experiments. It is worth mentioning that the experiment proposed here is not easy to implement in a traditional laboratory. Indeed, angular velocity measurements require rotation sensors that are not easily coupled to rotating devices such as a merry-go-round. In addition, traditional sensors available in most laboratories are not only considerably more expensive than smartphones, but also need wired connections. A similar experimental setup, without using smartphones, is far more complex than that proposed in this paper.

The experiment could also be conducted in a classroom, e.g., with a rotating disk, if no adequate merry-go-round is available.

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