Effect of the Nonlinear Gain in the Visibility of a Semiconductor Laser with Incoherent Feedback in the Coherence Collapsed Regime

Cristina Masoller, Cecilia Cabeza, and Aníbal Sicardi Schifino

Abstract—We study the influence of the gain saturation parameter $c$ on the visibility of a semiconductor laser subject to optical feedback in the coherence collapsed state. The experimental measurements are compared with the results obtained integrating the Lang and Kobayashi equations and a good agreement is found varying the amount of feedback and using different values of $c$. The analysis reveals that there is a best value of $c$ that fits the experimental measurements and that, for this value of $c$, the values of $\gamma$ found are in the expected relation with the attenuation used experimentally.

I. INTRODUCTION

As it is well known, the semiconductor laser dynamics is strongly modified by optical feedback from an external reflector [1]–[7]. The effect of the feedback depends on various factors, such as the intensity of the feedback rate, the delay time on the external cavity, the injected current, etc. While weak feedback levels result in linewidth narrowing and improved frequency stability, moderate to strong feedback levels might induce the laser to switch to a state of significant spectral broadening and dynamical complexity, that has been called coherence collapsed state. Using the Lang and Kobayashi equations and a linear model of optical gain, we recently showed that the laser dynamics in this state is chaotic, calculating the Lyapunov spectrum and fractal dimension of the underlying attractor [8]. However, the laser dynamics depends on the form of optical gain considered and a more realistic model should include the effect of gain saturation. Thus, we studied the influence of the gain saturation parameter $c$ in the dynamics of the laser [9] and showed that an increase of $c$ tends to suppress the occurrence of instabilities and to shift to higher values of the feedback the onset of coherence collapse.

In the present paper, we report the effect of gain saturation in the visibility, i.e., the absolute value of the field-autocorrelation function, of a semiconductor laser in the coherence collapsed regime. An experimental and theoretical study on the spectral properties of a semiconductor laser has been recently reported by several authors. Assuming an important simplifying hypothesis over the feedback, Cohen and Lenstra [10] derived an analytical expression for the visibility of a single-mode semiconductor laser operating well above threshold in the coherence collapsed state. They have considered the feedback as a noise and assumed that the phase and amplitude fluctuations of the feedback electric field satisfy a bivariate Gaussian distribution. With this assumption the field autocorrelation function can be determined self-consistently from the rate equations of Lang and Kobayashi without the spontaneous emission noise. Cohen et al. [11] measured the spectrum and the visibility of a semiconductor laser with incoherent optical feedback from a distant reflector, i.e., the external round trip time $\tau$ exceeds the coherence time of the laser (they used an external cavity of length $L = 3.55$ m). The values of the intensity of the feedback $\gamma$ and the damping rate of the relaxation oscillation $\lambda_R$ were determined from a fit of the self-sustained noise theory to the observed visibilities. They found that the measured visibilities could be well reproduced using their analytical theory, but at the price that the damping rate of the relaxation oscillation $\lambda_R$ had to be assumed to depend almost linearly on the feedback rate $\gamma$. As already noted by the authors, this is an unphysical assumption since the damping rate of the relaxation oscillation is a property of the solitary laser and thus should be independent on the feedback.

In a posteriori study, Hammel et al. [12] measured the visibility of a semiconductor laser subject to incoherent feedback, for various levels of feedback between zero and the maximum feedback for which the laser still operated in a single longitudinal mode (they used an external cavity of length $L = 5$ m). The measured visibilities were compared with the results obtained integrating the Lang and Kobayashi equations including a spontaneous emission term, and a good agreement was found over the whole range of feedback studied. However, this agreement required an unusually high value of the linewidth enhancement factor ($\alpha = 10 \pm 1$). Although the authors give a possible explanation for this surprisingly high value, at present this interpretation remains unclear.

In the present paper, the measured visibilities of a semiconductor laser with incoherent feedback from an external cavity of length $L = 1.5$ m, in the coherence collapsed regime, are well reproduced integrating the full Lang and Kobayashi equations (also including a spontaneous emission
tern), but using a lower value of the linewidth enhancement factor ($\alpha = 4.4$) and varying the intensity of the gain saturation parameter $\epsilon$. Although a measured visibility can be well reproduced using different values of $(\gamma, \epsilon)$, we find that an optimal fit (in where the values found of the feedback rate $\gamma$ are in the expected relation with the attenuation used in the measurements) can only be done for a single “optimal value” of $\epsilon$. This “optimal value” is of the order of the experimentally determined value for this type of semiconductor laser ($\epsilon \approx 2 - 4 \times 10^{-18}$ cm$^3$) and of the order of the value used by Hammel et al. [12].

This paper is organized as follows. Section II describes the experimental setup; the numerical solution of the rate equations is discussed in Section III. In Section IV we present the fits of the numerically calculated visibilities to the experimental results and finally, the conclusions are presented in Section V.

II. EXPERIMENT

The experimental setup is similar to the one used in [11], [12] and thus we will give just a brief description. The experiment (Fig. 1) was carried out with a 780 nm AlGaAs semiconductor laser (Sharp...). The length of the external cavity was $L = 1.5$ m, and from the observed visibility curves (see, for example Fig. 2) we conclude that during our feedback experiments the coherence length of the output light never exceeded the length of the external cavity, so that the feedback was incoherent. The laser was operated at 19°C and well above threshold; for all the experiments reported in this paper the pumpfactor is 2, i.e., the injection current was 100% above threshold ($I_{th} = 40$ mA). The temperature and injection current of the laser chip were stabilized to better than 0.1°C and 100 µA, respectively.

The light emitted by the laser was collimated with the lenses $L_1$ and $L_2$ and the amount of feedback was controlled by using the variable attenuator VA shown in Fig. 1. A Mach–Zender interferometer was used to measure the visibility $V(t)$. The interferometer was constructed with corner-cube reflectors, to avoid unintentional optical feedback. The length of one arm of the Mach–Zender interferometer was modulated by a piezoelectric element, so that the output power varied between constructive ($P_{\text{max}}$) and destructive ($P_{\text{min}}$) interference. We measured the visibility, defined as

$$V(\tau) = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}}.$$  (1)

The visibility can be shown to be equal to the modulus of the normalized field-autocorrelation function $G_F(\tau)$ [13]

$$V(\tau) = |G_F(\tau)| = \langle E(0 + \tau)E(0) \rangle \langle \langle E(t) \rangle \rangle^2.$$  (2)

This allows us to compare the visibility calculated integrating the rate equations, with the measured visibility.

In Fig. 2, we present experimentally observed visibility curves for an increasing feedback level and identical experimental conditions (the measurements are given by the data points, while the solid lines represent numerical fits). Notice the collapse of the coherence at $\approx 0.1$ ns and its restoration at multiples of $2\pi/\Omega$, where $\Omega$ is the angular frequency of the relaxation oscillations. We have measured visibility curves for feedback levels large enough to be in the region of coherence collapse, but lower than the levels for which the output intensity of the laser presents large noise burst, indicating that the laser becomes multimode (this is the same region covered by the experiments reported in [11] and corresponds roughly to the right part of Fig. 3 of [12]).

III. NUMERICAL SIMULATION

The rate equations governing the behavior of a single-mode laser diode with weak to moderate feedback are the Lang and
Kobayashi equations [1] for the amplitude $E_o(t)$ and the phase $\phi(t)$ of the electric field and the average carrier density $N(t)$ in the active region

$$\frac{dE_o(t)}{dt} = \frac{1}{2}\left[G(N) - 1/\tau_p\right]E_o(t) + \gamma E_o(t - \tau) \cos[\Delta(t)] + \frac{R}{2VE_o(t)^2},$$

$$\frac{d\phi(t)}{dt} = \frac{1}{2}\alpha[G(N) - 1/\tau_p] - \gamma \frac{E_o(t - \tau)}{E_o(t)} \sin[\Delta(t)],$$

$$\frac{dN(t)}{dt} = J - \frac{N(t)}{\tau_e} - G(N)E_o(t)^2.$$

In these equations, the field amplitude $E_o(t)$ is normalized such that $VE_o(t)^2$ is the total photon number in the laser wave-guide (where $V$ is the volume of the active region), $\tau$ is the delay time ($\tau = 2L/c$ where $L$ is the length of the optical path and $c$ is the velocity of light), $\tau_e$ is the carrier lifetime or population inversion lifetime, and $\tau_p$ is the photon lifetime. $\Delta(t) = \omega_o \tau + \phi(t) - \phi(t - \tau)$ is the phase delay (where $\omega_o$ is the laser frequency without feedback at the threshold of the laser operation), $\alpha$ is the linewidth enhancement factor, $R$ is the rate of spontaneous emission into the lasing mode, and $J$ is the current density in carriers per unit volume and unit time ($J = I/eV$, $I$ being the injected current and $e$ the electron charge). The optical gain per unit time is $G(N) = G_o(N - N_t)(1 - \epsilon E_o(t)^2)$ with $G_o$ being the modal gain coefficient, $N_t$ the carrier density at transparency and $\epsilon$ is the gain saturation parameter. $\gamma$ is the amplitude feedback rate, i.e., $\gamma^2$ is proportional to the feedback intensity $I_{fb}$, which is inversely proportional to the experimental attenuation.

Let us now discuss what are the reasonable values to be adopted for these parameters in the numerical simulation. We estimated the effective dimensions (based on the divergence angles specified by Sharp) and found a value of $V = 1.2 \times 10^{-16}$ m$^3$ for the volume of the active region. $\omega_o = 2\pi c / \lambda_o$, $\lambda_o = 780$ nm is the lasing pulsation at solitory laser threshold in the absence of reflections. The length of the external cavity used is $L = 1.5$ m and thus the delay time is $\tau = 10$ ns. The values $\tau_p$ and $\tau_e$ generally accepted are in the range 1–2 ps and 1–2 ns respectively, and we have used the values $\tau_p = 1.4$ ps $\tau_e = 1$ ns. For the linewidth enhancement factor of our laser we used the value $\alpha = 4.4$ [11] and for the rate of spontaneous emission $R = 1.1 \times 10^{12}$ s$^{-1}$.

The threshold current of our laser is $I_{th} = 40$ mA, thus $J_{th} = 2.083 \times 10^{33}$ m$^{-3}$s$^{-1}$. Since the pump factor is 2, $J = 4.166 \times 10^{33}$ m$^{-3}$s$^{-1}$. The threshold carrier density was calculated from $N_{th} = J_{th} \tau_e$ and the carrier density at transparency $N_o$ was calculated from $G(N_{th}) = G_o(N_{th} - N_o) = 1/\tau_p$. Finally, the modal gain coefficient $G_o$ was calculated from the relaxation oscillation frequency of the solitary laser $\omega_o$, using the relation $\omega_o^2 = G_o (J - J_{th})$.

In order to study the influence of the gain saturation parameter $\epsilon$ on the visibility of the laser, in the numerical simulation we have used different $\epsilon$ values. We have increased the value for $\epsilon$ from $\epsilon = 0$ m$^3$ to realistic values between $0.2 \times 10^{-23}$ m$^3$ and $0.5 \times 10^{-23}$ m$^3$. Since the relaxation oscillation frequency of the laser subject to optical feedback $\Omega$ is related with the relaxation oscillation frequency of the solitary laser $\omega_o$, and the gain saturation parameter $\epsilon$ in the form [2]

$$\Omega^2 - \omega_o^2 = \Omega \cot \left( \frac{\Omega \tau}{2} \right) \left[ \frac{1}{\tau_o} + \tau_o \omega_o^2 + \epsilon (J - J_{th}) \right].$$

(6)

for each value of $\epsilon$ used, we numerically adjusted the value of $\omega_o$ in order that the obtained value of $\Omega$ fitted the experimental one (from (6), it can be derived that there exists a linear relation between the values of $\epsilon$ and $\omega_o^2$ that leave $\Omega$ fixed). Notice that for different values of $\epsilon$ we used different values of $\omega_o$ and thus, different values of $G_o$ and $N_0$. The values found for $G_o$ and $N_0$ that fit the experimental measures, although depend on the value of $\epsilon$ used on the fit, are in the range of the typical values used for this type of laser ($G_o \approx 8 - 12 \times 10^{-13}$ m$^3$s$^{-1}$ $y N_0 \approx 1.1 - 1.4 \times 10^{24}$ m$^{-3}$). It is important to notice that since $\Omega$ depends of the external cavity parameters only on $\tau$ but not on $\gamma$ (see (6)), the measured visibilities can be adjusted varying only $\gamma$, i.e., once we fix $\epsilon$, the value of $\omega_o$ and thus, the values of $G_o$ and $N_o$ remain fixed. Table I contains the values of the parameters used on the fits that are independent of $\epsilon$, and Table II contains for each fit, the values of the parameters that depend on the value of $\epsilon$ used.

Equations (3)-(5) were integrated using a standard sixth-order Runge-Kutta integration routine with a time increment $\Delta t = 10$ ps. The delay term of (3)-(5) required that the values of the electric field and phase were stored in memory and reused in the next round-trip interval. We analyzed the trajectory (after a certain number of round trips in order to eliminate transient effects) using a standard FFT algorithm (the transforms were done with 65 536 data points with a time
TABLE I
NUMERICAL VALUES USED IN THE CALCULATIONS LEADING TO FIGS. 2–7 THAT DO NOT DEPEND ON $\epsilon$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of the active region</td>
<td>$V = 1.2 \times 10^{-16}$ m$^3$</td>
</tr>
<tr>
<td>Vacuum wavelength $\lambda_0$</td>
<td>780 nm</td>
</tr>
<tr>
<td>Length of the optical path $L$</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Delay time $\tau$</td>
<td>10 ns</td>
</tr>
<tr>
<td>Photon lifetime $\tau_p$</td>
<td>1.4 ps</td>
</tr>
<tr>
<td>Carrier lifetime $\tau_s$</td>
<td>1 ns</td>
</tr>
<tr>
<td>Linewidth enhancement factor $\alpha$</td>
<td>4.4</td>
</tr>
<tr>
<td>Spontaneous emission rate $R$</td>
<td>$1.1 \times 10^{12}$ s$^{-1}$</td>
</tr>
<tr>
<td>Threshold current $J_{th}$</td>
<td>$2.083 \times 10^{33}$ m$^{-3}$ s$^{-1}$</td>
</tr>
<tr>
<td>Injected current $J$</td>
<td>$4.166 \times 10^{33}$ m$^{-3}$ s$^{-1}$</td>
</tr>
<tr>
<td>Threshold carrier density $N_{th}$</td>
<td>$2.083 \times 10^{24}$ m$^{-3}$</td>
</tr>
</tbody>
</table>

TABLE II
NUMERICAL VALUES USED IN THE CALCULATIONS THAT DEPEND ON $\epsilon$

<table>
<thead>
<tr>
<th>Fig</th>
<th>$\epsilon$ (x10$^{-23}$ m$^3$)</th>
<th>$\omega_0$ (GHz)</th>
<th>$G_0$ (x10$^{-12}$ m$^3$ s$^{-1}$)</th>
<th>$N_0$ (x10$^{24}$ m$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>41.0</td>
<td>8.068</td>
<td>1.198</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>41.5</td>
<td>8.266</td>
<td>1.219</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>41.8</td>
<td>8.367</td>
<td>1.231</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>42.2</td>
<td>8.540</td>
<td>1.248</td>
</tr>
<tr>
<td>6</td>
<td>0.4</td>
<td>42.7</td>
<td>8.751</td>
<td>1.267</td>
</tr>
<tr>
<td>7</td>
<td>0.5</td>
<td>43.0</td>
<td>8.852</td>
<td>1.278</td>
</tr>
</tbody>
</table>

IV. RESULTS

Figs. 2–7 show the fit between the measured and the numerically calculated visibility, for different values of the saturation parameter $\epsilon$ and the feedback intensity $\gamma$ in the coherence collapse regime. We have increased the value for $\epsilon$ from $\epsilon = 0$ m$^3$ (Fig. 2) to more realistic values ($\epsilon = 0.5 \times 10^{-23}$ m$^3$, Fig. 7). In each fit, the only free parameter is the feedback intensity $\gamma$, the other parameters are fixed and their values are given in Tables I and II. Although an adequate fit of the curves is possible for all the values of $\epsilon$ used, notice that for low values of $\epsilon$ (Figs. 2–4) the experimental curve is below the numerical fit one, while for higher values of $\epsilon$ (Figs. 6 and 7) occurs the contrary. We conclude that for values of $\epsilon$ in a neighborhood of $0.3 \times 10^{-23}$ m$^3$ the fit is more accurate. This optimal value of $\epsilon$ found is also in agreement with the value of $\epsilon$ experimentally determined for this type of lasers ($\epsilon \approx 2 \times 10^{-18}$ cm$^3$ [6, 11]).

Fig. 8 shows the dependence of the feedback intensity $\gamma$ versus the experimental attenuation, for each fit shown in Figs. 2–7, in a linear-logarithmic scale. As $\gamma^2$ is proportional to feedback intensity $I_{fb}$, and $I_{fb}$ is inversely proportional to the used attenuation, a quadratic dependence of $\gamma$ with the attenuation is expected. Therefore, in the linear-logarithmic graphic, a slope of $-0.5$ is theoretically predicted.

In Fig. 8(a) and (b) each curve was labeled with the corresponding value of $\epsilon$ (in units of $10^{-23}$ m$^3$) and a reference line with a slope of $-0.5$ was draw between the lines corresponding to the best values of $\epsilon$. 
Notice the bend of the curves at low attenuation. A possible explanation for this bend is that for low attenuation, the laser is not yet in the fully developed coherence collapsed state, and in this regime, the spontaneous emission noise might become more relevant in the dynamics. Principally, a modification of the phase equation is expected. Thus, we expect that the predictions of the model won’t be as good in this regime as are in the fully developed coherence collapse.

In Fig. 8(a), from a linear regression analysis we conclude that the best value of $\epsilon$ is in the range $0.3 \times 10^{-23}$ m$^3$, $0.4 \times 10^{-23}$ m$^3$, and the Correlation Coefficient is closest to $-1$, and the Regression Coefficient, i.e., the slope, is closest to $-0.5$. However, if we disregard the values of $\gamma$ for the lowest attenuation, and we fit using the other three values, we obtain slightly different slopes (Fig. 8(b)) and the best value of $\epsilon$ is in the range $(0.1 \times 10^{-23}$ m$^3$, $0.2 \times 10^{-23}$ m$^3$).

Finally, Fig. 9 shows the relation between $\omega_0^2$ and $\epsilon$ used in Figs. 2–7. The almost linear relation can be explained using (6). Nevertheless this equation was derived in the context of stable single external cavity mode operation and is valid in the approximation $\omega_0^2 \gg \gamma^2$ and $\omega_0^2 \gg \gamma \left[ 1/\tau_a + \tau_p \omega_0^2 + \epsilon (J - J_{th}) \right]$, it is relevant in the coherence collapse regime. Since (6) depends on the external cavity parameters only on $\tau$ and not on the intensity of the feedback $\gamma$, it is reasonable to expect that, 

\begin{align*}
\text{(a)} & \quad \text{Visibility of the semiconductor laser with increasing amount of feedback, } \epsilon = 0.4 \times 10^{-23} \text{ m}^3, \text{ and (a) } \gamma = 4.35 \text{ GHz; (b) } \gamma = 4.65 \text{ GHz; (c) } \gamma = 5.05 \text{ GHz; (d) } \gamma = 5.65 \text{ GHz.} \\
\text{(c)} & \quad \text{Fig. 6. Visibility of the semiconductor laser with increasing amount of feedback, } \epsilon = 0.5 \times 10^{-23} \text{ m}^3, \text{ and (a) } \gamma = 4.90 \text{ GHz; (b) } \gamma = 5.15 \text{ GHz; (c) } \gamma = 5.80 \text{ GHz; (d) } \gamma = 6.10 \text{ GHz.} \\
\text{(c)} & \quad \text{Fig. 7. Visibility of the semiconductor laser with increasing amount of feedback, } \epsilon = 0.5 \times 10^{-23} \text{ m}^3, \text{ and (a) } \gamma = 4.90 \text{ GHz; (b) } \gamma = 5.15 \text{ GHz; (c) } \gamma = 5.80 \text{ GHz; (d) } \gamma = 6.10 \text{ GHz.} \\
\text{(c)} & \quad \text{Fig. 8. Logarithmic of feedback rate } \gamma \text{ versus the attenuation of intensity. The values of } \gamma \text{ are the values obtained from the numerical fits, and the curves are labeled with the corresponding value of } \epsilon \text{ (in units of } 10^{-23} \text{ m}^3). \text{ A reference line with a slope of } -0.5 \text{ is draw between the lines corresponding to the best values of } \epsilon. \text{ The attenuation was measured (in arbitrary units) by the variable attenuator VA in Fig. 1. (a) All values of } \gamma \text{ are used and the expected quadratic dependence occurs for values of } \epsilon \text{ between } \epsilon = 0.3 \times 10^{-23} \text{ m}^3 \text{ and } \epsilon = 0.4 \times 10^{-23} \text{ m}^3. \text{ (b) If we disregard the values of } \gamma \text{ for attenuation } \geq 0.7, \text{ the best value of } \epsilon \text{ is in the range } (0.1 \times 10^{-23} \text{ m}^3, 0.2 \times 10^{-23} \text{ m}^3).}
\end{align*}
as the feedback increases, the original frequency \( f_0 = \Omega / 2\pi \)
of the periodic regime that appears when the ECM becomes unstable is still present (i.e., has the same value, independent of \( \gamma \)) and additional frequencies appear (for example, in the quasiperiodic regime, the two frequencies involved are \( f_0 \) and \( f_1 = 1/\tau \)). Confirming this hypothesis, experimentally we observe that the first maximum and minimum of the visibility curve \( V(\tau) \) occur for values of \( \tau \) that do not depend on the attenuation used. Also, to adjust the value of \( \omega_r \) in order that the obtained value of \( \Omega \) fitted the experimental one is a better approach that the one commonly used it the literature, where the abscissa of the first maximum \( (V_{\text{max}}) \) and the first minimum \( (V_{\text{min}}) \) of the visibility curve are assumed to occur at \( V_{\text{max}} \approx V(2\pi/\omega_r) \) and \( V_{\text{min}} \approx V(\pi/\omega_r) \), respectively.

It is important to notice that the results shown in Figs. 2–7 are independent of the values employed for \( \tau_p \) and \( \tau_e \), i.e., if these values are changed in the range of the usual values employed, the results shown in Figs. 2–7 do not change significantly. The same occurs with the rate of spontaneous emission \( R_s \) that could have been omitted without any significant change in the figures (this is explained the fact that the effects of spontaneous emission noise are negligently in the coherence collapsed state). On the contrary, variations in the value of \( \alpha \) causes important changes in the graphics of Figs. 2–7. As has been reported previously, we find that the effects of \( \alpha \) and \( \epsilon \) are opposite [6], [14]. While increasing \( \alpha \) tends to favor the occurrence of instabilities and chaos, increasing \( \epsilon \) causes a damping of the system. We will discuss this point further in the next section.

V. CONCLUSION

Since the numerical results presented in the previous section rely strongly on the integration of the rate equations (3)–(5), let us discuss here the numerical accuracy and convergence of the integration procedure used.

First, the results obtained depend on the time interval that we use to let transients relax \( t_{\text{relax}} \). After a few test we found that \( t_{\text{relax}} = 500 \text{ ms} \) (that corresponds to 50 round-trips in the external cavity) was enough to eliminate any start-up effect. In addition, it is important that the trajectory analyzed covers the underlying attractor. For low values of the feedback, the attractor is homogeneous and 16384 points (that corresponds to approximately 16 round-trips in the external cavity) are enough to use in the FFT algorithm, but in the coherence collapsed region that is studied in this paper, a longer trajectory is required in order to cover completely the underlying chaotic attractor (we used approximately 65 round-trips in the external cavity).

To conclude, we find a good agreement for the visibility curves from experiment and numerical simulation, the later done with different values of the gain saturation parameter \( \epsilon \) between 0 m\(^2\) and 0.5 \times 10^{-23} \text{ m}^3. \) However, we find that the optimal fit is for \( \epsilon \approx 0.3 \times 10^{-23} \text{ m}^3 \) and that for this value of \( \epsilon \), the values of the feedback intensity \( \gamma \) found for each curve are in the expected relation with the attenuation used in the measurements.

We want to point out that in [12] a much larger value of the linewidth enhancement factor \( \alpha \) had to be used in order to fit the measured visibilities. It remains unclear to us the reason of this unexpected high value, since the rate equations and the parameters they use are similar to ours except of the length of the external cavity (which is three times longer than ours) and of the value of the modal gain coefficient \( G_m \) (which three times lower than ours). The later might be the reason of the unusually high value of \( \alpha \), since decreasing the value of \( G_m \), as well as increasing the value of \( \epsilon \) causes a decrease in the optical gain, i.e., increases the dumping of the system. Thus, a larger value of \( \alpha \) should be used in order to compensate the dumping and favor the occurrence of instabilities and chaos.

In our numerical calculations, we found that we could adjust the measured visibility using different values of \( \alpha \), but since in this work we concentrate on the study of the dependence of the visibility curves on the feedback intensity and on the nonlinear gain suppression factor, we chose to use a typical (and fixed) value of \( \alpha \) and leave the study of the dependence of the laser dynamics on the value of \( \alpha \) for future work. Also, we could have relied on the measured attenuation to get the variation of \( \gamma \), and leave only the absolute feedback level as a "free" parameter, but since we want to make emphasis on the opposite effect that have the variations of \( \epsilon \) and \( \gamma \), we preferred to leave both parameters equally free.

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*Cristina Masoller,* photograph and biography not available at the time of publication.

*Cecilia Cabeza,* photograph and biography not available at the time of publication.

*Anibal Sicardi Schifino,* photograph and biography not available at the time of publication.